The Janus Cosmological Model

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At a time when the ΛCDM model is experiencing increasing difficulty in accounting for observations, we expand the physical and mathematical foundations of its challenger, the Janus Cosmological Model. We discuss recent observations suggesting that dark energy, if it may have been more significant in the past, makes today the interpretation derived from the Standard Model problematic, while these observational data confirm one of the predictions of the Janus model. We discuss the possible impact of taking negative energy states into account in quantum mechanics.

Keywords: dark matter, dark energy, biometric models, runaway phenomenon, very large structure, spiral structure, negative mass, negative energy states, dipole repeller, negative gravitational lensing, twin universe model, primeval antimatter, tow-folds cover of a manifold, enantiomorphy, ΛCDM model, Janus Model, symplectic groups, momentum, T-symmetry, C-symmetry, PT-symmetry, CPT-symmetry

I. AN APPROACH WITH HEURISTIC ORIGIN

New observational data from the James Webb Telescope have created a serious crisis in the field of cosmology, casting doubt on the validity of the standard ΛCDM model. At the same time, all efforts to capture and identify the components of dark matter have been complete failures for decades. It is therefore perfectly justified to consider that this dark matter, with positive mass, simply does not exist and that the phenomena it is supposed to account for can be attributed to an entirely different cause. It should be noted that theorists have also been unable to give a credible identity to this other invisible component, dark energy. For ten years, the Janus Cosmological Model has proposed an alternative that boils down to the introduction of negative mass and energy components into the universe. Such an approach was initially carried out heuristically, by adopting laws of interaction between positive and negative masses according to:

- Masses of the same sign attract each other

- Masses of opposite signs repel each other.

This scheme immediately produced interesting results. Let us call ρ and $\overline{\rho}$ the positive and negative mass densities. Assuming that $|\overline{\rho}| \gg \rho$, numerical simulations produced a rapid separation between the two types of masses from the early 1990s. Negative masses then have a shorter accretion time (Jeans time).

$$\bar{t}_J = \frac{1}{\sqrt{4\pi G\overline{\rho}}} \ll t_J = \frac{1}{\sqrt{4\pi G\rho}} \tag{1}$$

As a result, they immediately give rise to a regular distribution of spheroidal conglomerates, the positive mass then gathering in the interstitial space, thus adopting a structure reminiscent of contiguous bubbles [1].

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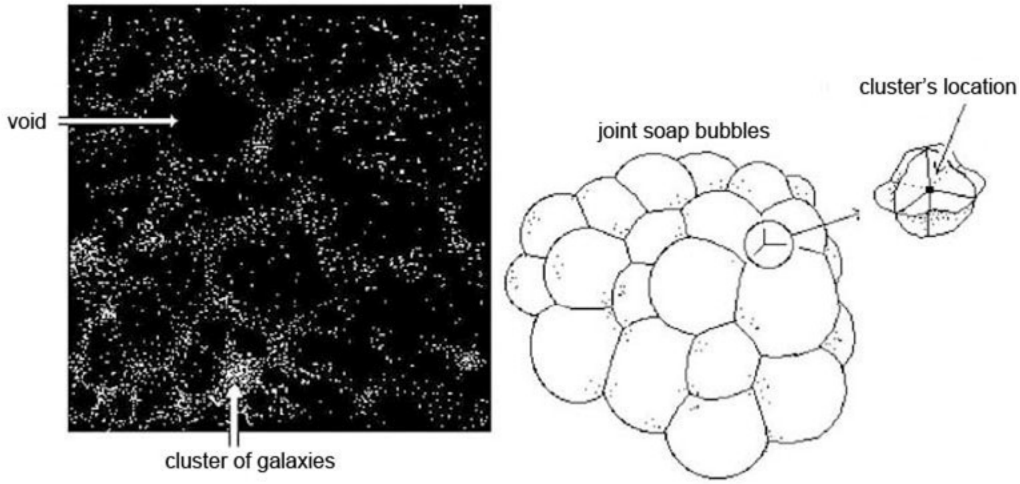

FIG. 1. Very large-scale structure in contiguous bubbles

Considering a three-dimensional structure that is that of a foam made up of bubbles of equal volume, this corresponds to a Weaire-Phelan structure [2]. Under these conditions, gravitational instability will cause galaxies to gather along segments, representing the junction of three bubbles, while at the meeting points of these segments will be located clusters of galaxies. The negative mass content will therefore play the same role as the dark matter - dark energy ensemble, with a similar distribution.

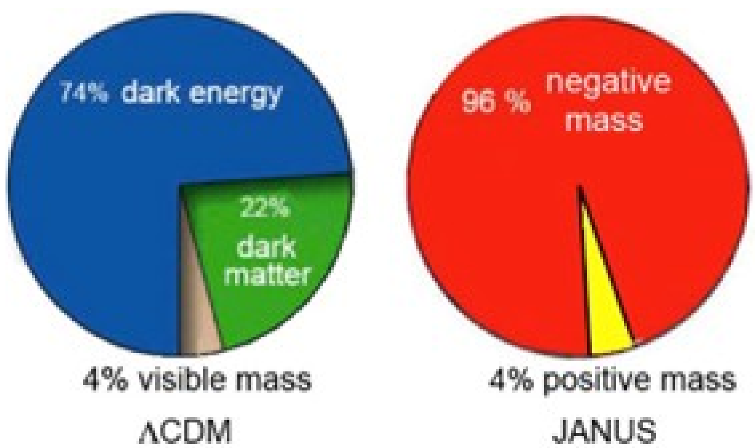

FIG. 2. Comparative distribution of components for the λCDM model and the Janus Model

In 2017, the repeller dipole was discovered [3], which appears as a gap around which the surrounding galaxies experience a repulsive force [4].

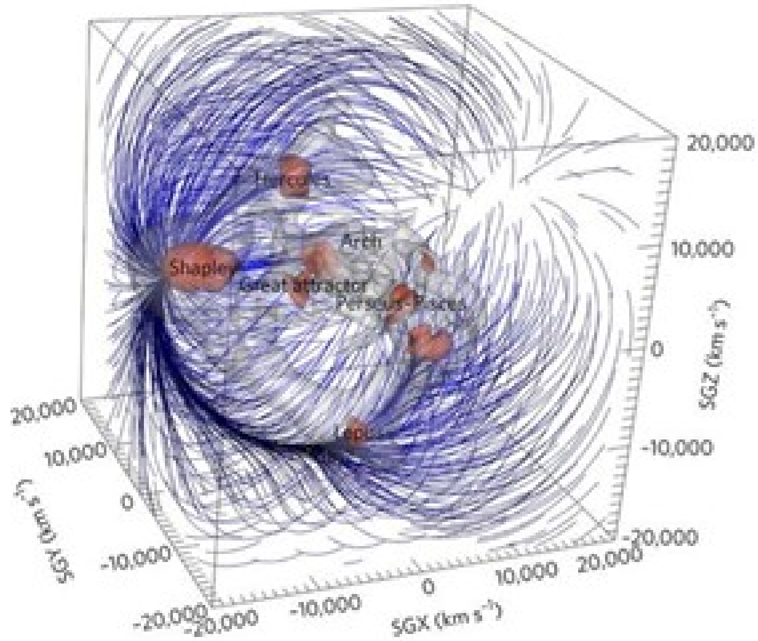

FIG. 3. The dipole Repeller [3]

From this time on, it was concluded that, during this constitution of the large-scale structure of the universe, the fact that the positive mass is suddenly compressed by two adjacent clusters of negative mass must, by causing its rapid heating and its no less rapid cooling by radiative losses, lead to a pattern of galaxy formation much faster than the classical pattern. Indeed, flat plate structures are optimal for ensuring rapid dissipation of thermal energy.

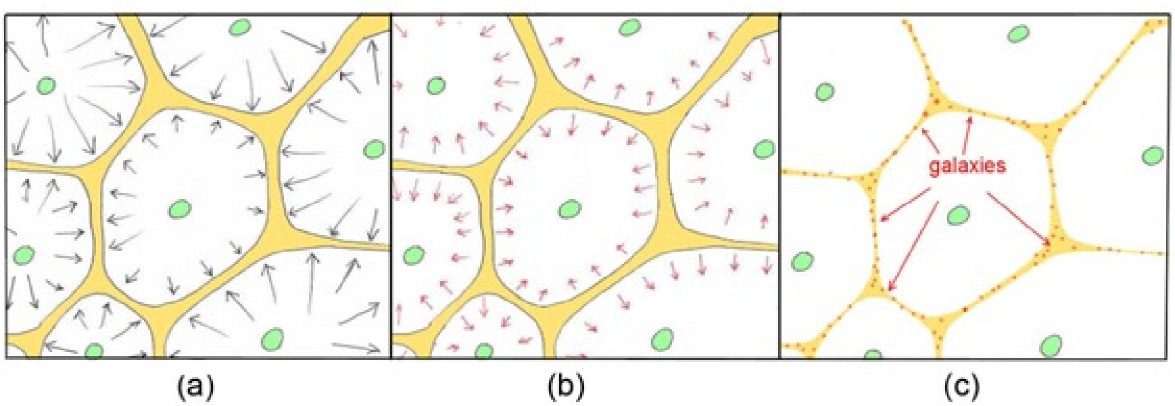

FIG. 4. 2D didactic image evoking the formation diagram of galaxies in the Janus model [5].

Negative mass clusters are shown in green. In fig. 4-a, they exert a counterpressure on the plates constituting the contiguous bubble structure of positive mass, which heats up rapidly, at the very moment of the creation of this large-scale structure. In fig. 4-b, the rapid radiative loss causes the temperature in these plates to drop, making them gravitationally unstable. In fig. 4-c, all galaxies form at the same time, in the first hundred million years. At the same time as galaxies form, negative mass infiltrates between the galaxies and exerts a retro-compression on them, ensuring their confinement.

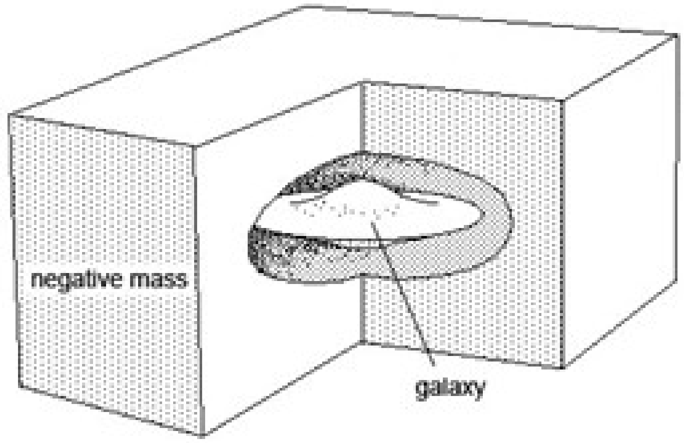

FIG. 5. Galaxy confined in a negative mass environment.

Their rotation curves are profoundly affected, exhibiting a plateau at the periphery. By adding a vast halo of positive-mass dark matter to the visible mass, rotation rate measurements could be accounted for. The negative mass environment plays a similar role [5]. The order of magnitude of the density in the vicinity of the galaxy is then comparable in absolute value to the density in the halo, i.e., low, which ensures its stability given that the associated Jeans time is then of the order of the age of the universe. The gravitational force exerted on the components of the galaxy, created by this gap in the negative mass distribution, is then equivalent to the attractive field of its negative image [4].

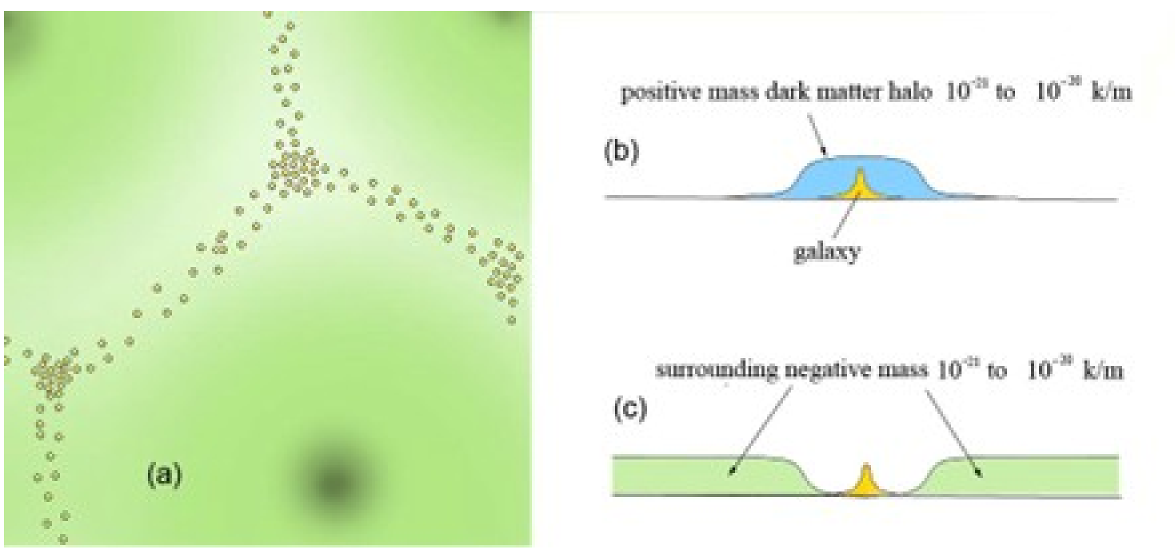

FIG. 6. Comparative actions of a dark matter halo and the equivalent gap in a negative mass distribution [4].

In fig.6-a, a schematic representation of the large-scale structure. In yellow, the galaxies, in green, the dark matter. At the center of each of the cells in Figure 6-a, a negative-mass spheroidal cluster. In 6-b, the galaxy, in yellow with its vast dark-matter halo, in blue. In fig.6-c, the same galaxy, housed in a negative-mass gap, a negative image of the halo, ensuring the same confinement and the same rotation curve profile. What is true for confinement is also true for gravitational lensing effects. A gap in a uniform negative-mass distribution creates positive lensing identical to that created by the equivalent halo.

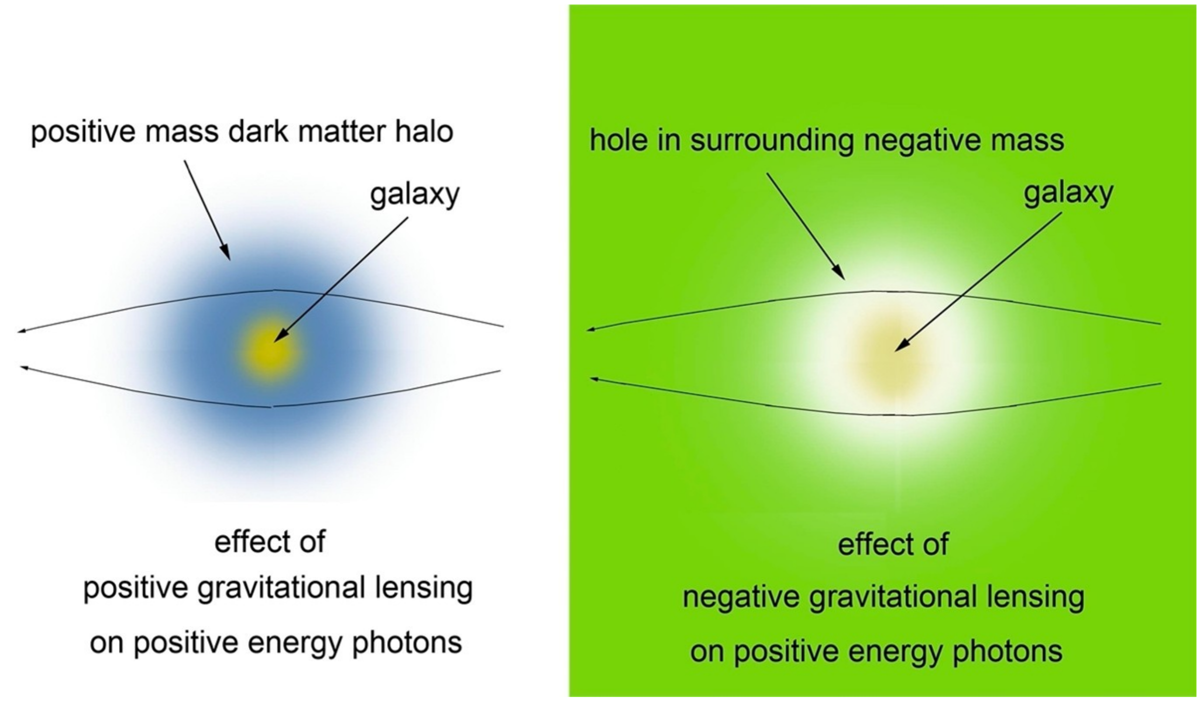

FIG. 7. Gravitational lensing effects in the vicinity of galaxies.

What is true for galaxies also applies to galaxy clusters. Starting from such a configuration, by giving the galaxy a rotational movement, numerical simulations show the immediate formation, in a few turns, of a barred spiral structure, lasting for more than thirty turns.

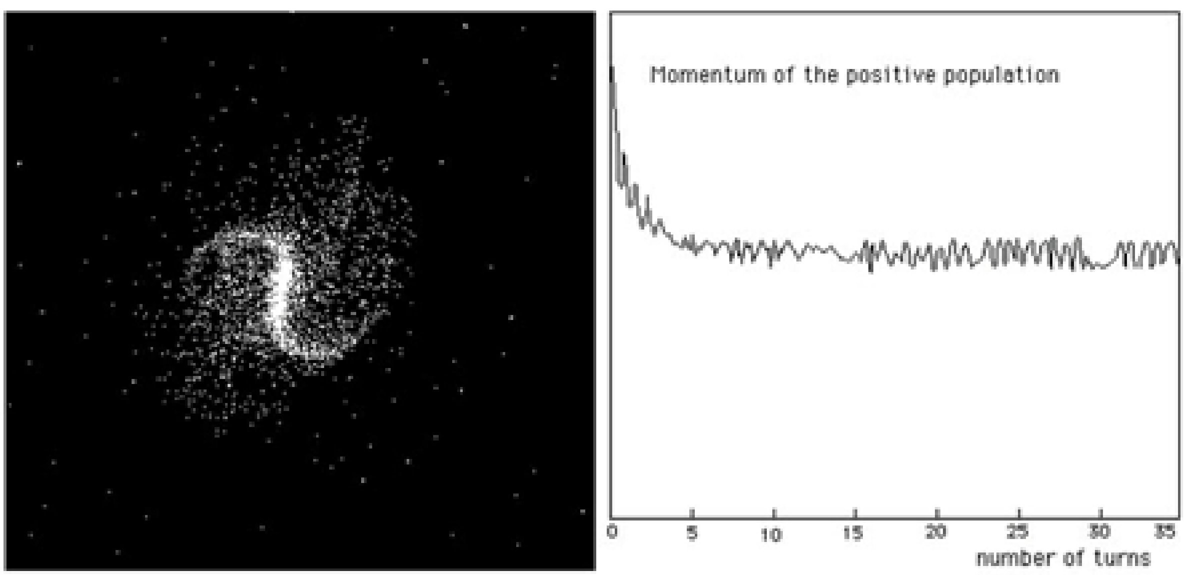

FIG. 8. Results of numerical simulations [5].

Since the Janus model presents itself as the challenger to the standard ΛCDM model, it is important to note the points where it provides a clear and coherent answer, where the dominant model does not. Indeed, for decades, when this spiral structure is introduced as initial conditions, it dissipates in a very small number of turns. The Janus model shows that these structures reflect the way in which non-collisional systems, such as galaxies, transfer momentum to their environment, a straight line curve, not from near to far, by viscous constraint, but through an action at a distance linked to the density waves that originate there, which have their counterpart in the negative mass environment. Thus, those who try to model this spiral structure by looking for what gives rise to it and what makes it last are like physicists trying to understand the waves of the sea, forgetting what creates it: the wind. These spiral waves therefore reflect a dissipative phenomenon. They will therefore persist without limitation over time. Finally, let us point out that through these first simulations of spiral structure we had played on the parameter (m in the figure) representing the ratio between the absolute value of the density in the environment, compared to the order of magnitude of the density in the galaxy. It was then possible to show that high values caused the spiral arms to fold into an annular formation.

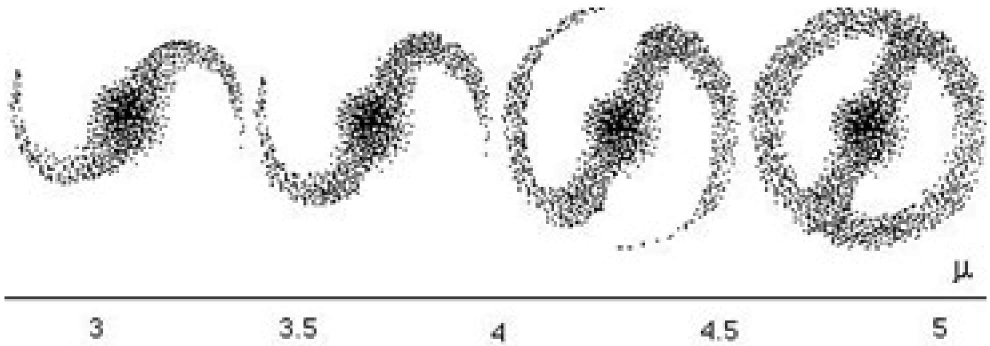

FIG. 9. Evolution of the shape of galaxies as a function of density contrast [5].

If work has not yet been undertaken in this area, with resources finally up to the task, it is undoubtedly because the simulation tools used are not designed to include negative masses in their data. Furthermore, the objections raised by H. Bondi [6] in 1957, which will be discussed extensively in the rest of the article, are sufficient to maintain strong scepticism about the possible introduction of negative masses in astrophysics as in cosmology. Let us recall the result of introducing such masses into Einstein's equation, in the model of general relativity:

- Positive masses attract their own kind as well as negative masses

- Negative masses repel their own kind as well as negative masses.

In the framework of general relativity, this leads to the appearance of a phenomenon that immediately violates the physical principle of action-reaction: the runaway phenomenon, in which pairs of particles of opposite masses undergo uniform acceleration, without their overall energy being modified.

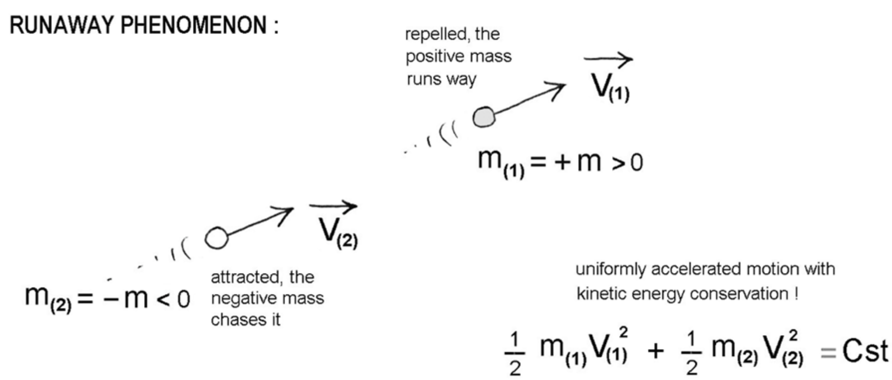

FIG. 10. Runaway phenomenon [5].

At this point, there is only one conclusion: if we want to go further, we will have to move beyond the framework of general relativity or, more precisely, include it in a broader geometric context. Before making this leap, we will discuss the twin model suggested by Andrei Sakharov in 1967 (ref.[7–9]).

II. 1967 : FIRST EXTENSION OF THE COSMOLOGICAL MODEL BY ANDREI SAKHAROV

Since the discovery of the CMB lent credence to the Big Bang model, it has emerged that these photons were the result of the fantastic matter-antimatter annihilation that occurred in the distant past, leaving only one matter particle in a billion. But then, why can't we observe the primordial antimatter particles that should also have survived? To try to answer this important question, Sakharov implemented what could be called the polarity principle. Our universe is spatially oriented and temporally oriented. Sakharov therefore assumes the existence of a second layer of the universe where the arrow of time is reversed and where everything that is "right" becomes "left," and vice versa, in short, a universe PT-symmetrical to ours. Noting the violation of CP symmetry, he suggests that this second universe, which he calls twin, and which he imagines linked to ours by the Big Bang singularity, is CPT-symmetric to ours. This is an idea that is becoming fashionable again at the moment [10], noting that its supporters seem to ignore the one who introduced it half a century earlier. If baryons are formed by the union of three quarks and antibaryons by that of three antiquarks, Sakharov suggests that the first reaction would have been slightly faster than the second. Then, when the energy level would have fallen, there would have remained in our universe sheet, in addition to the remainder of matter, an equivalent remainder in the form of antiquarks, in a ratio of 3/1. The opposite situation in the twin universe. In the figure 11, a 2D didactic image of Sakharov's model.

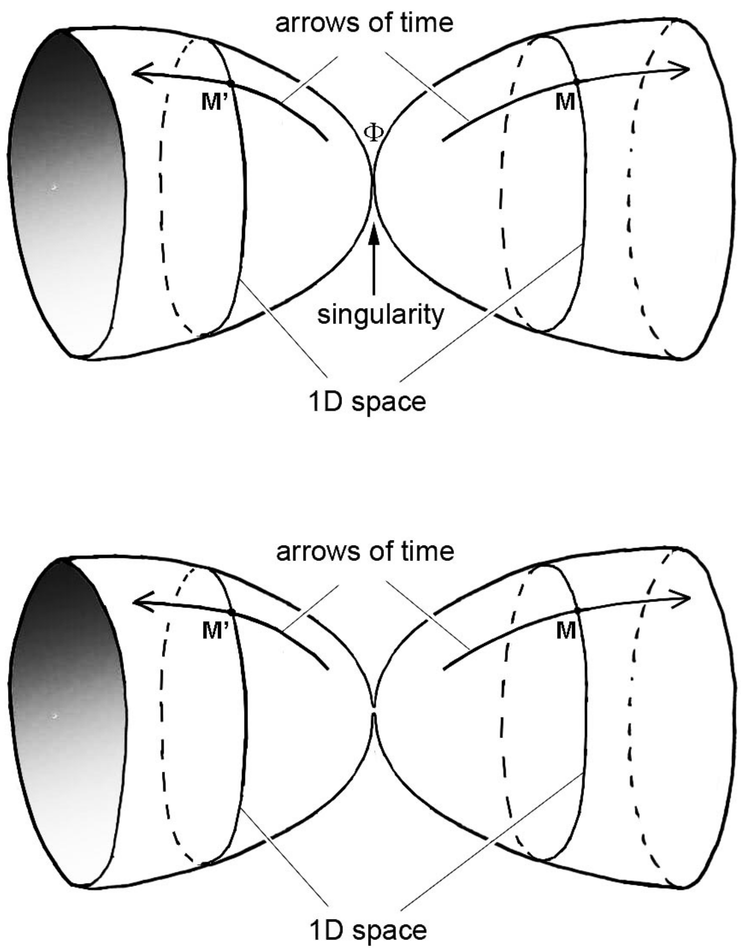

FIG. 11. 2D didactic image of Andrei Sakharov's model, with and without initial singularity.

Let us point out that this idea remains the only one still in the running today. But, you might ask, what does this have to do with negative masses?

III. GEOMETRIC MEANING OF ENERGY AND MASS INVERSION

It is to be found in the work of the mathematician J.M. Souriau [11, 12] which translates an application of symplectic geometry to physics. It is also the expression of the main idea of the mathematician Emmy Noether, according to which every new symmetry is associated with a new invariant. Souriau groups all these invariants in the same space, the momentum space, the construction of which results from the action of a dynamical group on the dual of its Lie algebra. With respect to relativistic physics, this dynamical, or symplectic, group is the isometry group of the Minkowski space, the Poincaré group. Starting from a Lorentzian metric, the metric defining this space is:

$$ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \tag{2}$$

Its Gram matrix:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3}$$

We construct the Lorentz group axiomatically:

$$^tLGL = G \tag{4}$$

By adding the four-vector of space-time translations:

$$G = \begin{pmatrix} \Delta x^0 \\ \Delta x^1 \\ \Delta x^2 \\ \Delta x^3 \end{pmatrix} \tag{5}$$

We obtain the isometry group of the ten-dimensional Minkowski space, the complete Poincaré group, whose matrix representation is:

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \tag{6}$$

The Lorentz group has four connected components, each including certain symmetries. We have the sets:

- L_n , neutral component (including the neutral element)
- L_s , elements reversing space, but not time.
- L_t , elements reversing time, but not space.
- L_{st} , elements reversing both time and space.

Physics, until now, has limited the use of this complete group to the restricted, orthochronous Poincaré subgroup, a subset containing components that do not reverse time:

$$\{L_o\} = \{L_n\} \cup \{L_s\} \tag{7}$$

The action on the dual of the Lie algebra then reveals energy, momentum and spin as components of the momentum space. This subgroup includes P-symmetry, but not T-symmetry. We can then consider opting for the complete Poincaré group, including the subset of its antichronous components:

$$\{L_a\} = \{L_t\} \cup \{L_{st}\} \tag{8}$$

But the action of the components then causes the inversion of energy and mass(see ref.[11] p190 eq.14.67) . We can therefore see that group theory suggests considering the inclusion of negative mass and energy elements in physics, that is to say, including PT-symmetry in the model. Initiated in [13], all this has been largely developed and extended to CPT symmetry, through the Janus group, in reference [14]. In this perspective, Sakharov's twin universe would then be made up of particles of negative energy, and masses when they were endowed with them. But at no time does he consider making these two entities interact.

IV. THE GEOMETRIC AND TOPOLOGICAL CONTEXT

To bring these two entities into interaction, the approach then involves a major paradigmatic leap, of geometric and more precisely topological essence, by using the concept of a two-sheet covering of a manifold.

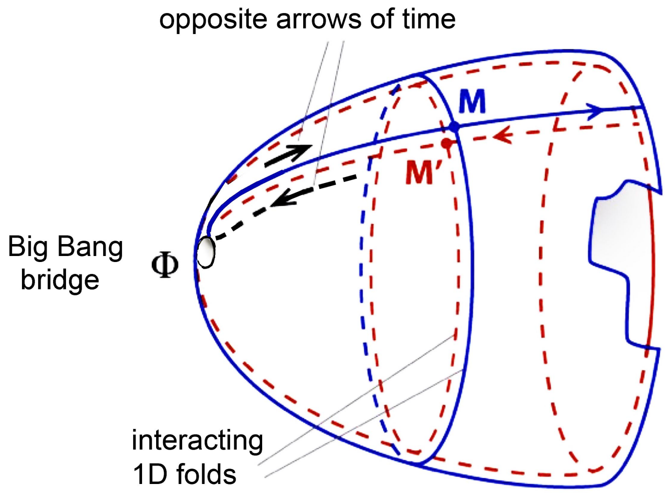

FIG. 12. 2D didactic image of the adaptation of the Sakharov model.

The approach was outlined in 1994 in a now-defunct journal [15]. It consists of considering that the universe could be configured according to the two-sheet covering of an even-dimensional projective, which is then unorientable. For example, an S2 sphere can be configured according to the two-sheet covering of a P2 projective, whose image is the surface described by W. Boy in 1902 (ref [16–18]). We will mesh this sphere by imagining that its "north" and "south" poles represent the "Big Bang" and the "Big Crunch", its equator representing its state of maximum extension. Its meridians become the "timelines" of this closed 2D space-time.

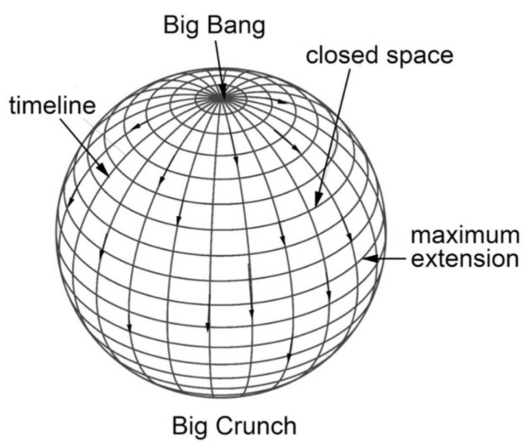

FIG. 13. 2D didactic image of a closed space-time.

We will bring the antipodal points of this sphere into coincidence. We will note in passing that this operation brings its two poles into coincidence, therefore "the Big Bang" and the "Big Crunch". In figure 14-a, the correspondence between the points and their antipodes.

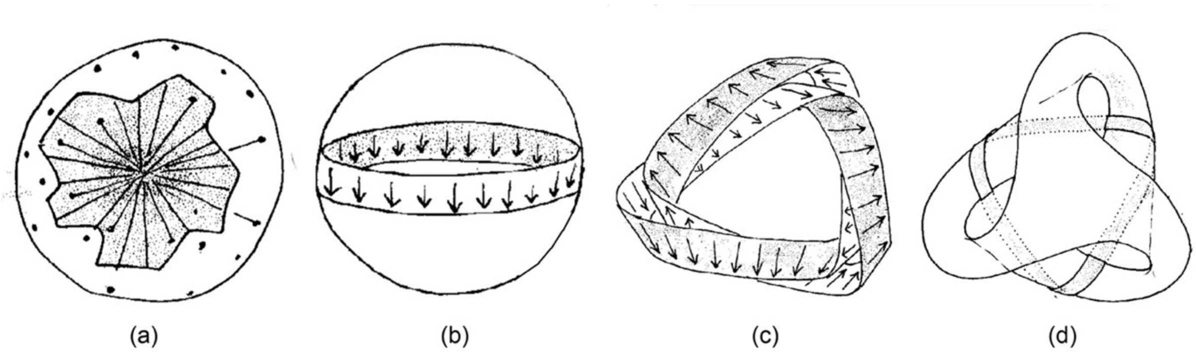

FIG. 14. Configuration of the equatorial neighborhood of the sphere S2 as a two-sheet covering of the equator of the projective P2.

In Figure figure 14-b the time arrows of this equatorial neighborhood. Figure 14-c shows how this strip is configured according to the two-sheet covering of a unilateral, three-half-turn Möbius strip. The Boy surface represents the immersion of the projective P2 in R^3 and the way in which this covering is located on it. We see that the operation has brought into coincidence portions of space-time with opposing time arrows (induced T-symmetry). In the following figure, different views of the Boy surface. The reader will find in reference [18] images arranged in a "flip book" where we can see this covering operation taking place, through a succession of animated images.

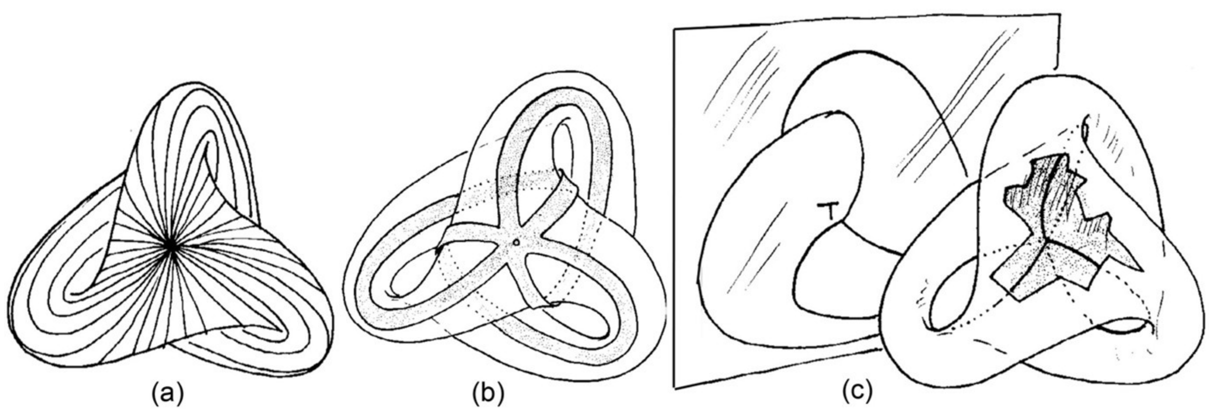

FIG. 15. The Boy surface.

In figure 15-a the meridians of the surface, converging towards its unique pole. In figure 15-b the neighborhoods of three meridians: Möbius strips with a half-turn. In figure 15-c we see, in its mirror image, its triple point, which the cut allows us to see in the object located in the foreground. What must be understood is that this triple point and the trifoliate self-intersection set are only accidents linked to the impossibility of plunging the projective space P2 into R^3. We can only represent it in the form of an immersion, where it then intersects itself. The same is true for the Klein bottle K2 which is then equipped with an intersection curve in the form of a simple circle. Figure 15 illustrates the fact that the two-sheet covering configuration generates a T-symmetric linking the two portions of space-time opposite each other. It also generates an enantiomorphy, an induced P-symmetry. See figure 16.

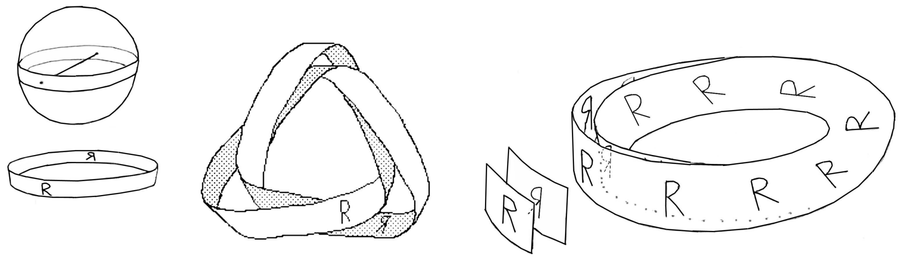

FIG. 16. Induced enantiomorphism relationship between adjacent portions of spacetime. On the left, covering of the equatorial neighborhood. On the right, of a meridian.

Thus this operation results in a PT-symmetry connecting the adjacent portions of space-time. By configuring a four-dimensional space-time as a two-sheet covering of a P4 projective, we also see this PT symmetry appear. In chapter 5 of reference [19] we will find, with the first evocation of the geometric translation of C-symmetry by inversion of an additional dimension, a presentation of relativity in 5 dimensions. We could then consider the configuration of an S5 sphere as a two-sheet covering of a P5 projective. But in this case the desired induced CPT symmetry does not manifest itself because odd-dimensional projectives are orientable. In [13] and [14] the idea is raised that quantum charges can appear as invariants associated with translations in additional dimensions. This can then only concern even-dimensional projectives: P6, P8, P10. The universe is then identified with the covering of an even-dimensional hypersphere, with an Euler-Poincaré characteristic equal to 2, therefore endowed with two poles, according to the two-sheet covering of the projective of the same dimension, which has a unit Euler-Poincaré characteristic, and is therefore each time endowed with a unique pole. Note that this configuration makes it possible to envisage the elimination of the “Big Bang-Big Crunch” singularity. Using a 2D image:

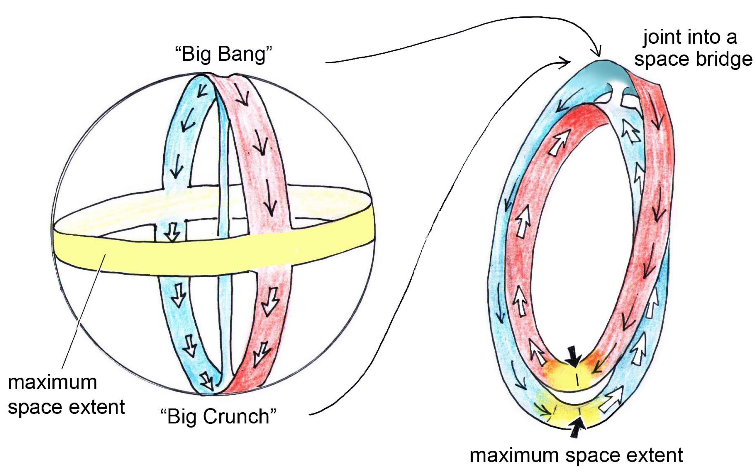

FIG. 17. Elimination of the singularity.

A tubular passage will be provided between the two poles, which the operation has brought into coincidence. On the right, this operation is illustrated on the neighborhood of a world line, a meridian curve of the projective P2. Note that, in this modification of the "initial singularity", an inversion of the arrow of time, a T-symmetry, occurs. The topology then corresponds to the configuration of a torus T2, configured as a two-layer covering of a Klein bottle K2. This space-time thus configured becomes non-contractile. The Klein bottle K2 having a zero Euler-Poincaré characteristic can be provided with a regular tiling, see figure 18-a. Figure 18-b shows the two-layer covering of the two-dimensional Klein bottle.

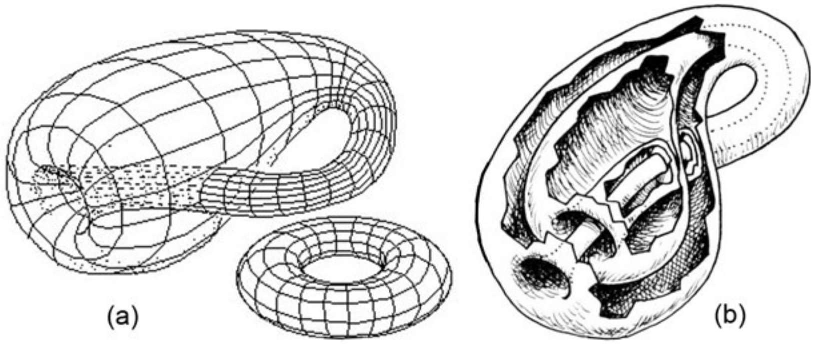

FIG. 18. Klein K2 bottle (a) and its two-layer coating (b).

Adding pairs of dimensions results in the two-sheet covering of hypertorus T4, T6, T8. In 4 dimensions we will have a PT-symmetry, the function operating through a throat sphere. In 2D the passage of a throat circle can be assimilated to the sliding of a triangle crossing "a hole". After passing over the "back" of this 2-surface, the orientations of the black and gray triangles are inverse.

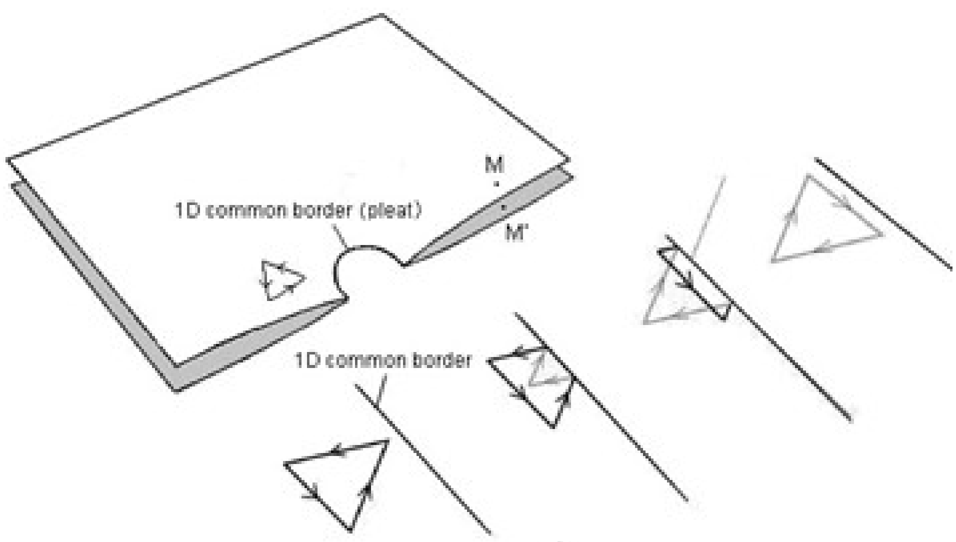

FIG. 19. Inversion of a triangle after crossing the throat circle.

The orientation of a 3D space can be achieved by providing the triangular faces of a tetrahedron with a direction of travel, orienting their normal vectors. Although this is more difficult to imagine, a throat sphere can connect two enantiomorphic three-dimensional spaces. For the triangle, in 2D we operated the crossing of the throat circle of its three vertices. The same thing in 3D with successive crossing of the four points constituting the four vertices of a tetrahedron. We note that the operation reverses the normal vectors of the four faces.

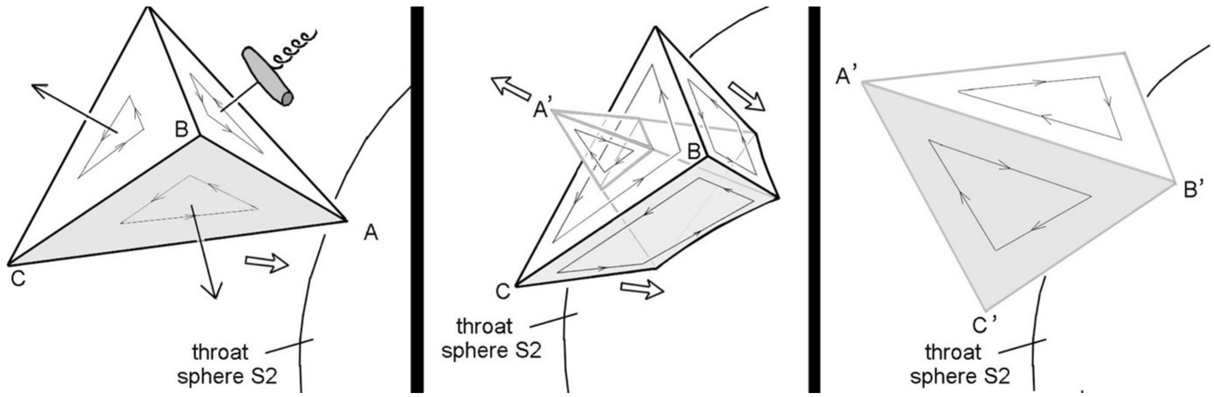

FIG. 20. P-symmetry when crossing the throat sphere.

By limiting ourselves to the four-dimensional, we see here the deployment of the geometric approach that served as a starting point for the development of the Janus cosmological model, which models the interaction between two sheets of space-time, as a covering of an M4 manifold. What about this inversion of the time coordinate? The answer then emerges, as mentioned above, from the theory of dynamical groups, which shows that this inversion of the time coordinate, this T-symmetry, is synonymous with inversion of energy and mass. At the stage we are at, using the tools of topology and group theory, we have a geometric model in the form of a two-sheet covering of a Lorentzian manifold by symmetric CPT sheets. It remains to make these two entities interact.

V. CONSTRUCTION OF BIMETRIC SYSTEMS

The points of the manifold of which our space-time hypersurface is the two-sheet covering are located using coordinates $\xi^0, \xi^1, \xi^2, \xi^3$. We will equip the two adjacent sheets with Lorentzian metrics $g_{\mu\nu}(\xi^i)$ and $\overline{g}_{\mu\nu}(\xi^i)$. The masses m and $\bar{(}m)$ of the two sheets will follow their non-zero length geodesics while the photons, with positive energy in one of the sheets and negative energy in the second, will follow the zero-length geodesics. Using these two metrics we construct the Ricci tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ as well as the corresponding Ricci scalars R and $\overline{R}$. In the model of general relativity the Einstein-Hilbert action is written:

$$J = \int (R - \chi L)\sqrt{-g}\, d^4x \tag{9}$$

Where L is the Lagrangian of matter, χ the Einstein constant and $\sqrt{-g}\, d^4x$ the four-dimensional hypervolume. By performing a variation $\delta g^{\mu\nu}$ in the functional space of Lorentzian metrics, we express that $\delta J = 0$. Taking into account the fact that:

$$\delta \int R\sqrt{-g}\, d^4x \;=\; \int \left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \right) \sqrt{-g}\, d^4x \tag{10}$$

And by posing:

$$\delta \int L\sqrt{-g}\, d^4x \;=\; \int \frac{\delta(L\sqrt{-g})}{\delta g^{\mu\nu}}\, \delta g^{\mu\nu}\, d^4x = \int T_{\mu\nu}\sqrt{-g}\, \delta g^{\mu\nu}\, d^4x \tag{11}$$

We obtain Einstein's equation, without the cosmological constant Λ:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu} \tag{12}$$

By analogy, the action of a bimetric system is written:

$$J = \int \left[(R - \chi L)\sqrt{-g} + (\overline{R} - \overline{\chi}\overline{L})\sqrt{-\overline{g}} + I \right] d^4x \tag{13}$$

Where I is an interaction term. With :

$$\frac{\delta I}{\delta g^{\mu\nu}} = -\sqrt{-g}\, K_{\mu\nu} \qquad\qquad \frac{\delta I}{\delta \overline{g}^{\mu\nu}} = -\sqrt{-\overline{g}}\, \overline{K}_{\mu\nu} \tag{14}$$

We then obtain the system of two coupled field equations:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, [T_{\mu\nu} + K_{\mu\nu}] \tag{15}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R}\, \overline{g}_{\mu\nu} = \overline{\chi}\, [\overline{T}_{\mu\nu} + \overline{K}_{\mu\nu}] \tag{16}$$

In reference [20] the authors designate the two metrics by $g^L_{\mu\nu}$ (left) and $g^R_{\mu\nu}$ (right). By introducing (we adapt their notations to ours) an equivalent elementary hypervolume $(g\overline{g})^{\frac{1}{4}}\, d^4x$, they define their interaction term according to:

$$I = -\mu^4\, (g\overline{g})^{\frac{1}{4}}\, V(g, \overline{g}) \tag{17}$$

By further choosing $\chi = \overline{\chi} = 1$. Their system is then written:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu} + K_{\mu\nu} \tag{18}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R}\, \overline{g}_{\mu\nu} = \overline{T}_{\mu\nu} + \overline{K}_{\mu\nu} \tag{19}$$

$T_{\mu\nu}$ and $\overline{T}_{\mu\nu}$ represent the source terms, representing the two contributions to the matter fields m and $\overline{m}$. The terms $K_{\mu\nu}$ and $\overline{K}_{\mu\nu}$ represent two interaction tensors. ∇_v and $\overline{\nabla}_v$ being the two covariant derivations, constructed from the metrics $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$, we know that we have:

$$\nabla^\nu \left(R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} \right) = \overline{\nabla}^\nu \left(\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R}\, \overline{g}_{\mu\nu} \right) = 0 \tag{20}$$

If the relationships:

$$\nabla^\nu T_{\mu\nu} = \overline{\nabla}^\nu \overline{T}_{\mu\nu} \tag{21}$$

translate the conservation of the two species of matter, this implies a constraint which determines the form of the interaction tensors $K_{\mu\nu}$ and $\overline{K}_{\mu\nu}$:

$$\nabla^\nu K_{\mu\nu} = \overline{\nabla}^\nu \overline{K}_{\mu\nu} \tag{22}$$

The geometric context of reference [20] is not clearly defined and is assumed to be located in the fuzzy world of branes. The second matter remains undefined, and they consider making the two species interact, in a context of massive multigravity, by implementing gravitons with a mass spectrum. Based on the alleged existence of "a gap between light gravitons and heavy gravitons," they decide to consider only the former and then attempt to shape their interaction tensors in different contexts, brane models, Kaluza-Klein models, Connes' non-commutative geometry, without arriving at a result likely to lead to a confrontation with observational data. At no point does the desire appear that the second matter could be assimilated to a population of negative masses.

In reference [21] a symmetry between gravitating and anti-gravitating particles is examined. In a later article reference [22] the author proposes an action and a system of two field equations where in the interaction terms appear the factor $\sqrt{\frac{\overline{g}}{g}}$ and its inverse. But here again the implicit hypothesis appears besides $\chi = \overline{\chi} = 1$ and no tangible result is obtained.

VI. TOWARDS THE SYSTEM OF TWO FIELD EQUATIONS OF THE JANUS MODEL

Adjacent points are identified by a common set of coordinates, which are simple, dimensionless numbers.

$$\{\xi^0, \xi^1, \xi^2, \xi^3\} \tag{23}$$

$$\xi = \sqrt{(\xi^1)^2 + (\xi^2)^2 + (\xi^3)^2} \tag{24}$$

Let us introduce time and space gauges according to:

$$t = \tau \xi^0 \qquad \overline{t} = \overline{\tau} \xi^0 \tag{25}$$

$$r = a \xi \qquad \overline{r} = \overline{a} \xi \tag{26}$$

We will show that by opting for $\overline{\chi} = -\chi$ we will find the diagram of the Newtonian interaction forces between elements, corresponding to the one which, chosen heuristically, has been used profitably. To simplify and find the direction of the forces we will assume identical values of the constant of gravity ($\overline{G} = G$) and of the speed of light ($\overline{c} = c$) in the two sheets, as well as identical space and time gauges ($|\overline{\tau}| = \tau$; $\overline{a} = a$), which amounts to considering locating the points with the same coordinates:

$$(t, x, y, z) \qquad with \qquad r = \sqrt{(x^2 + y^2 + z^2} \tag{27}$$

We can then consider the conditions of the Newtonian approximation:

- Weak fields
- Low speeds compared to the speed of light This translates, geometrically, to the approximation :

$$g_{\mu\nu} \cong \eta_{\mu\nu} + \epsilon \gamma_{\mu\nu} \qquad \overline{g}_{\mu\nu} \cong \eta_{\mu\nu} + \epsilon \overline{\gamma}_{\mu\nu} \tag{28}$$

ϵ being a small parameter and $\eta_{\mu\nu}$ the Lorentz metric. The following two source tensors will also be of the form:

$$T_{\mu\nu} \cong \begin{pmatrix} \epsilon\rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \overline{T}_{\mu\nu} \cong \begin{pmatrix} \epsilon\overline{\rho} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{29}$$

We denote by ρ and $\overline{\rho}$, the densities of the masses m and $\overline{m}$. Considering that the action of one type of mass on the other is also a disturbance, we can write:

$$K_{\mu\nu} \cong \begin{pmatrix} \epsilon K_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \overline{K}_{\mu\nu} \cong \begin{pmatrix} \epsilon\overline{K}_{00} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{30}$$

We know that we can then develop in series of first members where only the first order terms γ_{00} and $\overline{\gamma}_{00}$ remain, in the form of two second derivatives, according to:

$$\frac{\partial^2 \gamma_{00}}{\partial x^2} + \frac{\partial^2 \gamma_{00}}{\partial y^2} + \frac{\partial^2 \gamma_{00}}{\partial z^2} = -\chi \left(\rho + K_{00}\right) \tag{31}$$

$$\frac{\partial^2 \overline{\gamma}_{00}}{\partial x^2} + \frac{\partial^2 \overline{\gamma}_{00}}{\partial y^2} + \frac{\partial^2 \overline{\gamma}_{00}}{\partial z^2} = -\overline{\chi} \left(\overline{\rho} + \overline{K}_{00}\right) \tag{32}$$

Let us first consider, in a stationary regime, a region of space-time where the masses m would be absent. The first equation is then identified with Einstein's equation, with the equation of general relativity. In this Newtonian approximation the motion of a witness particle corresponds to the equation:

$$\frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \epsilon \frac{\partial \gamma_{00}}{\partial x^i} \tag{33}$$

It is the movement of a witness particle subjected to a force deriving from the Newtonian potential:

$$\psi = \frac{c^2}{2} \epsilon \gamma_{00} \tag{34}$$

Two masses m attract each other according to Newton's law, which emerges from the field equation, and it is by identifying the equation:

$$\frac{\partial^2 \gamma_{00}}{\partial x^2} + \frac{\partial^2 \gamma_{00}}{\partial y^2} + \frac{\partial^2 \gamma_{00}}{\partial z^2} = -\chi \rho \tag{35}$$

To the Poisson equation :

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=4\,\pi\,G\,\rho \tag{36}$$

that we determine the value of the Einstein constant χ for which, in this bimetric model, we will give the value:

$$\chi=-\frac{8\,\pi\,G}{c^2} \tag{37}$$

Before we consider the action of the interaction terms, let us instead consider a region of space-time where only negative mass is present. In the Newtonian approximation the second equation becomes:

$$\frac{\partial^2\overline{\gamma}_{00}}{\partial x^2}+\frac{\partial^2\overline{\gamma}_{00}}{\partial y^2}+\frac{\partial^2\overline{\gamma}_{00}}{\partial z^2}=-\overline{\chi}\,\overline{\rho} \tag{38}$$

The motion of a witness particle (of mass $\overline{m}=-1$) will also be Newtonian, corresponding to the equation:

$$\frac{d^2x^i}{dt^2}=-\frac{c^2}{2}\,\epsilon\,\frac{\partial\overline{\gamma}_{00}}{\partial x^i} \tag{39}$$

Which we will connect to a potential:

$$\overline{\psi}=\frac{c^2}{2}\,\epsilon\,\overline{\gamma}_{00} \tag{40}$$

To fit with our heuristic assumptions, we want these particles of negative masses to attract each other. It is therefore necessary that this $\overline{\gamma}$ created by this negative mass distribution $\overline{\rho}$ is equal to the γ_{00} , created by an equivalent distribution of positive masses $\rho=-\overline{\rho}$. Which amounts to saying that we will have to find the same Poisson equation. This will be possible if the equivalent of the Einstein constant in the second system corresponds to:

$$\overline{\chi}=-\chi \tag{41}$$

So our system of two coupled field equations becomes:

$$R_{\mu\nu}-\frac{1}{2}R\,g_{\mu\nu}=\chi\,[T_{\mu\nu}+K_{\mu\nu}] \tag{42}$$

$$\overline{R}_{\mu\nu}-\frac{1}{2}\overline{R}\,\overline{g}_{\mu\nu}=\chi\,\left[\overline{T}_{\mu\nu}+\overline{K}_{\mu\nu}\right] \tag{43}$$

In ref. [20, 21] and [22], the authors, by immediately assuming in their bimetric systems $\chi=\overline{\chi}=1$, could not influence this latitude of choice of the "Einstein constant" $\overline{\chi}$, and in particular its sign, in the second system. We must now consider how masses of opposite signs interact. We should note that nothing, for the moment, imposes the form of the two interaction tensors, $K_{\mu\nu}$ and $\overline{K}_{\mu\nu}$, except for the mathematical constraint of zero divergence ref.[23], to which we will return later. The only detractor of our theory (ref.[23, 24]), who bases his criticism on a non-satisfaction of these Bianchi conditions, has persisted since 2019 in basing himself on the idea that these interaction tensors are identical to the corresponding source tensors, that is to say that:

$$K_{\mu\nu}=\overline{T}_{\mu\nu}\qquad\qquad \overline{K}_{\mu\nu}=T_{\mu\nu} \tag{44}$$

But this has never been the case. Let's return to these two typical configurations where only a single type of matter exists in regions of space. Still applying the Newtonian approximation, we are led to consider two cases:

- Cases where only positive mass is present, the action on a negative reference mass is then associated with Poisson's equation:

$$\frac{\partial^2\overline{\gamma}_{00}}{\partial x^2}+\frac{\partial^2\overline{\gamma}_{00}}{\partial y^2}+\frac{\partial^2\overline{\gamma}_{00}}{\partial z^2}=-4\pi G\,\overline{K}_{00} \tag{45}$$

- Case where only the negative mass is present, the action on a positive witness mass is then associated with the Poisson equation.

$$\frac{\partial^2\gamma_{00}}{\partial x^2}+\frac{\partial^2\gamma_{00}}{\partial y^2}+\frac{\partial^2\gamma_{00}}{\partial z^2}=4\pi G\,K_{00} \tag{46}$$

For the masses of opposing signs to repel each other it is necessary and sufficient that:

$$K_{00} < 0 \qquad\qquad \overline{K}_{00} > 0 \tag{47}$$

We have deliberately used a letter K to denote the interaction tensors $K_{\mu\nu}$ and $\overline{K}_{\mu\nu}$, to avoid that readers confuse them with the source tensors $T_{\mu\nu}$ and $\overline{T}_{\mu\nu}$. This confusion remains possible in ref.[25] where we have used a capital letter T, in a different font:

$$\mathcal{T}_{\mu\nu} \qquad \overline{\mathcal{T}}_{\mu\nu}$$

FIG. 21. Interaction tensors in the reference [25].

the positive mass side and negative energy in the negative side. In terms of observations, the presence of an invisible negative mass conglomerate will result in a negative gravitational lensing effect.

VII. FIRST OBSERVATIONAL CONFIRMATIONS

We have therefore justified the heuristic hypothesis which had provided us with interesting simulation results. We therefore have, as it stands:

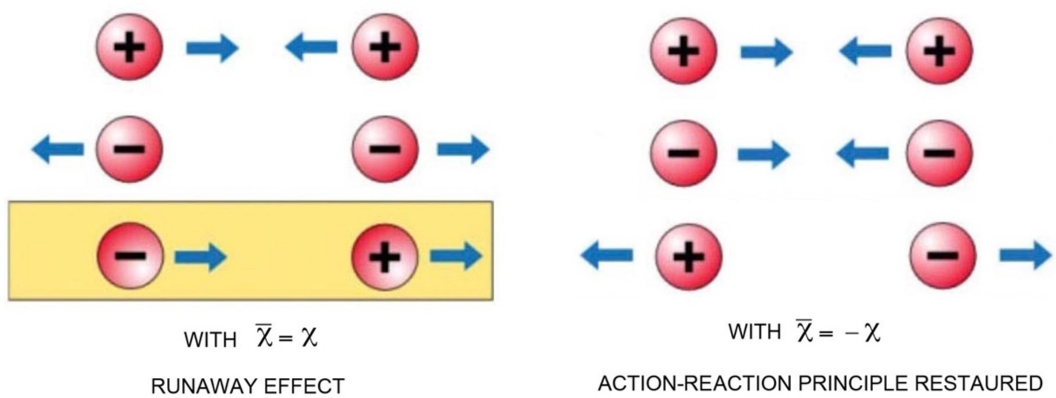

FIG. 22. Comparative bimetric interaction laws.

We can therefore make an initial assessment of the agreements with observations. Two cases arise:
The Janus Model provides alternative interpretations to those of the ΛCDM model, for :

- Large-scale structure.
- Confinement of galaxies.
- Flat rotation curves at the periphery.
- Gravitational lensing effects in the vicinity of galaxies and clusters.

To this we will add another element (ref. [1, 26, 27]), but which is beyond the scope of this article :

- The Janus model provides an alternative interpretation of inflation by proposing a primitive phase in a regime of "variable constants".

In the following, when the ΛCDM model does not provide an explanation, the Janus Cosmological Model :

- Accounts for the runaway motion of galaxies in the vicinity of gaps in the large-scale structure by attributing it to the presence at the center of the structure of a repulsive, negative-mass cluster.
- Explains the non-observation of the invisible components of the universe.

- Gives the identity of these invisible components (negative-mass anti-H and anti-He).
- Explains the early birth of galaxies and stars.
- Justifies the absence of observation of primordial antimatter.
- Explains the formation of perennial spiral structures.
- Predicted the fall of laboratory-created antimatter into the Earth's field.

Regarding the last point, the group approach (Janus group [13, 14, 25], [13], [24]) establishes that the matter-antimatter duality (C-symmetry) also exists in the world of negative masses. There are therefore two antimatters:

- The C-symmetric of our matter, with positive mass.
- The PT-symmetric of our matter, with negative mass.

Hence the prediction, which has been confirmed, that laboratory-created antimatter "falls downward." Furthermore, the "CPT theorem" is based on the assumption that the quantum time-reversal operator T is antisymmetric, and therefore reverses neither energy nor mass. In the Janus model, the symmetric CPT of our positive-mass matter is a copy of this, but assigned a negative mass due to T symmetry. See below. A model must also be falsifiable. The existence of large voids in the very large-scale structure of the universe is accounted for by proposing that at their center lies a spheroidal conglomerate of negative mass. Positive-energy photons, originating from the background, pass through it without difficulty, since they interact with it only in a self-gravitational manner. They undergo a negative gravitational lensing effect, which will have the effect of attenuating the luminosity of distant objects located behind this negative-mass object. A precise luminosity map should then reveal the contours of this invisible object and its diameter.

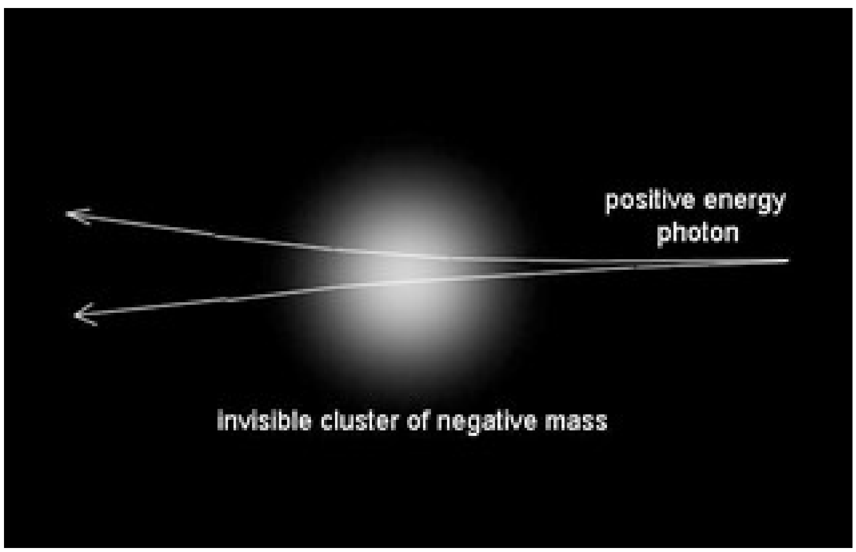

FIG. 23. Effect of dimming the brightness of objects located in the background of a negative mass conglomerate.

Finally, there is an observational datum for which the Standard Model has hitherto provided a very accurate model of CMB fluctuations, based on the fitting of six parameters, including the Hubble constant. But today, due to the so-called "Hubble tension", the value required for the fit to the fluctuation curve turns out to be different from that derived from direct observations.

In any case, the Janus model will also have to account for these fluctuations. This is a work in progress.

VIII. FIRST ATTEMPT AT CONSTRUCTING INTERACTION TENSOR

The Janus model does not consider producing solutions other than those that arise from the symmetry assumptions of general relativity. The first is the construction of a scenario describing the evolution under a hypothesis of homogeneity and isotropy. Under these conditions, metric solutions of the form FLRW were sought. With:

$$d\Omega^2 = d\theta^2 + sin^2\theta \, d\phi^2 \tag{48}$$

It gives :

$$ds^2 = (d\xi^0)^2 - a^2\left(\frac{d\xi^2}{1-k\xi} + \xi^2 d\Omega^2\right) \tag{49}$$

$$d\overline{s}^2 = (d\xi^0)^2 - \overline{a}^2\left(\frac{d\xi^2}{1-\overline{k}\xi} + \xi^2 d\Omega^2\right) \tag{50}$$

An attempt to describe the matter era by making $K_{\mu\nu} = \overline{T}_{\mu\nu}$ and$\overline{K}_{\mu\nu} = T_{\mu\nu}$ leads to the Dirac-Milne model ref.[28], with a linear expansion as a function of time), where the sum $\rho + \overline{\rho}$ is then zero, as are the second members of the two equations, which ensures the satisfaction of the Bianchi conditions in a trivial way. In this model the densities of the two species are equal. If we then try to reconstruct the very large-scale structure of the universe, we no longer obtain a lacunar structure, but a simple percolation:

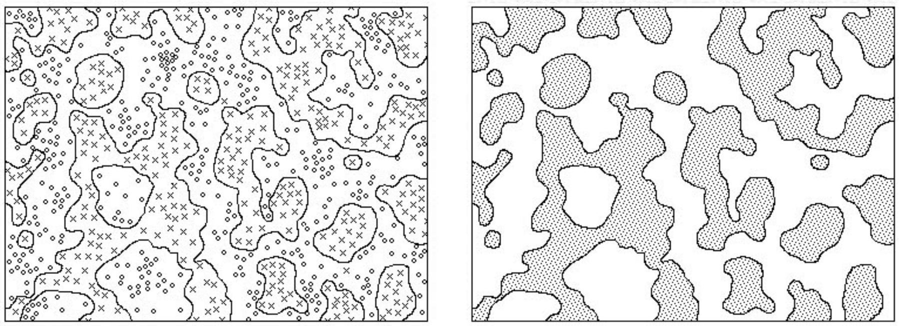

FIG. 24. Large-scale structure for $|\overline{\rho}| = \rho$ [5].

Thus, the particularity of the Janus model is this total asymmetry between the two populations, that is to say: $|\overline{\rho}| \gg \rho$. Is it fundamental? We think not, and that it manifests itself in the primordial era (with "variable constants") from a situation of total symmetry. This is work in progress. Then, one of the two populations is given a secondary role, with respect to the other. Within this population, gravitational instability will only give a set of spheroidal conglomerates, anchoring in some way the lacunar structure of the other population. These conglomerates, comparable to giant protostars, lose energy by emitting negative energy photons, equivalent to the red and infrared radiation emitted by protostars. But their cooling time then exceeds the age of the universe and they cannot evolve, harboring within them fusion reactions that would restart nucleosynthesis, which one can imagine limited to the synthesis of antihelium, before the formation of these clusters. This world then gives birth to neither stars, nor galaxies, nor planets, and life is consequently absent. Since this solution of total symmetry had to be abandoned, still in an unsteady situation with isotropy and homogeneity, we then opted [13] for a modification of the equations according to :

$$R_{\mu\nu} - \frac{1}{2}R\, g_{\mu\nu} = \chi\left[T_{\mu\nu} + \Phi\overline{T}_{\mu\nu}\right] \tag{51}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\,\overline{g}_{\mu\nu} = -\chi\left[\overline{T}_{\mu\nu} + \overline{\Phi}T_{\mu\nu}\right] \tag{52}$$

Where Φ and $\overline{\Phi}$ are functions of the common chronological coordinate. If we limit ourselves to the era dominated by matter, the tensors $T_{\mu\nu}$ and $\overline{T}_{\mu\nu}$ are then limited to the terms:

$$T_{00} = \rho c^2 \qquad\qquad \overline{T}_{00} = \overline{\rho c}^2 \tag{53}$$

By introducing the FLRW solutions into this system we obtain the solution:

$$\Phi = \frac{\overline{a}^3}{a^3} = \overline{\Phi}^{\,-1} \tag{54}$$

And the corresponding compatibility condition, of existence of solution:

$$E \,=\, \rho\, c^2\, a^3 \,+\, \overline{\rho}\, \overline{c}^2\, \overline{a}^3 \,=\, Cst \tag{55}$$

which translates a generalized law of conservation of energy E. What creates the curvature, drives the geometry, and in the case considered the dynamics of the system, is not the mass but the energy. By making the hypothesis that this energy E is predominantly negative, we obtain two exact solutions, with negative curvature index ($\overline{k} = k = -1$)) where the expansion of the positive population is accelerated. This solution also corresponds to that proposed by W.Bonnor [29], in a universe dominated by negative masses.

$$a(\xi^0) = \alpha^2\, ch^2(\xi^0) \tag{56}$$

$$t(\xi^0) = \alpha^2 \left(1 + \frac{sh(\xi^0)}{2} + \xi^0 \right) \tag{57}$$

We then obtain the following curve, which cannot be continued in the radiative era, towards the origin.

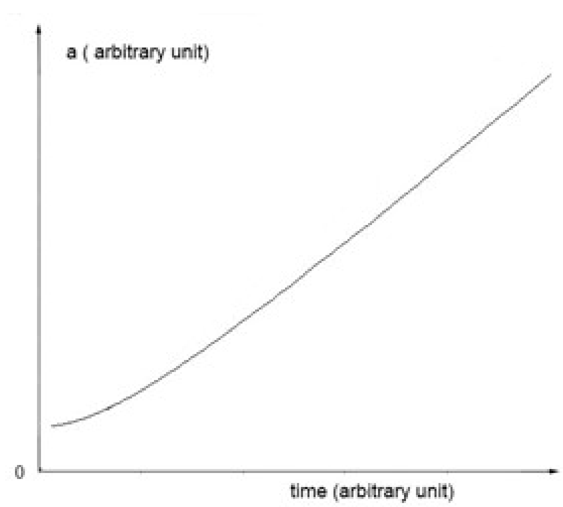

FIG. 25. Expansion of the population of positive masses, during the phase dominated by matter [13], [29].

An expansion law which then turned out to fit with the first observational data.

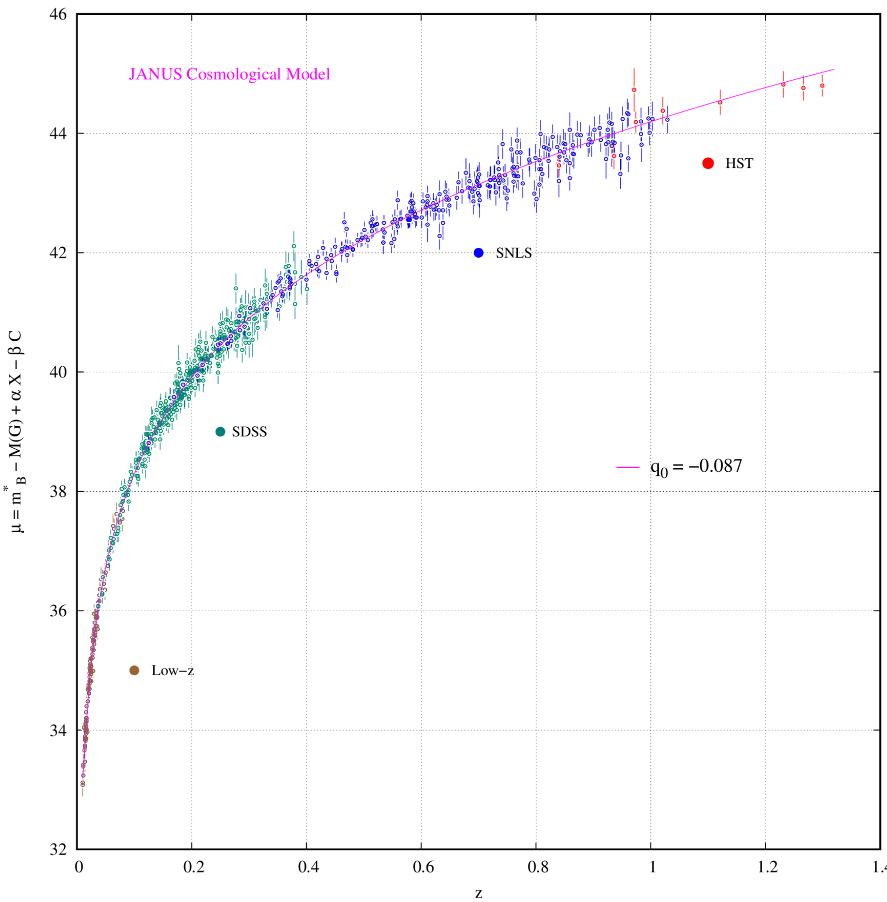

FIG. 26. Comparison with observational data [30]. Magnitude as a function of redshift

Still with a view to falsifiability, the Janus model predicted, ten years ago [13], that the phenomenon of acceleration of cosmic expansion would diminish over time.

IX. NEW OBSERVATIONAL DATA INVALIDATE THE STANDARD MODEL ΛCDM

Recent observational data [31] tend to confirm this prediction, which the ΛCDM model cannot account for, for which the observable universe must imperatively follow an exponential expansion:

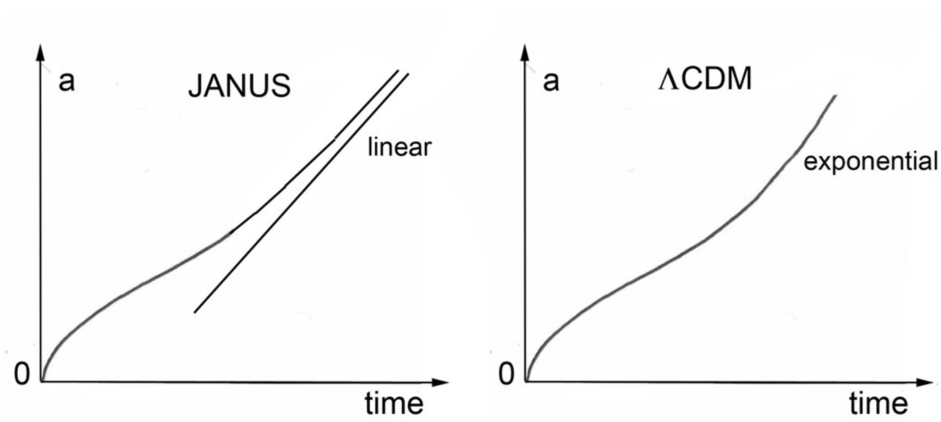

FIG. 27. Comparative schematic evolutions of cosmic expansion

We therefore have a third category of observational comparisons with theoretical predictions. This is the case where:

- The Janus model's predictions are confirmed, for an observational fact that the standard model, the ΛCDM model, is unable to account for.

X. STILL LOOKING FOR THE SHAPE OF INTERACTION TENSORS

We note that relation Eq. (54(55), combined with the FLRW forms of the metrics gives:

$$\frac{\overline{a}^3}{a^3} = \sqrt{\frac{\overline{g}}{g}} \tag{58}$$

Which suggests a reinterpretation of the equations in the form:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi \left[T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}} K_{\mu\nu} \right] \tag{59}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R}\, \overline{g}_{\mu\nu} = -\chi \left[\overline{T}_{\mu\nu} + \sqrt{\frac{g}{\overline{g}}} \overline{K}_{\mu\nu} \right] \tag{60}$$

Still with $K_{00} < 0$ and $\overline{K}_{00} > 0$, to find in Newtonian interaction laws in agreement with the principle of action-reaction. These equations which can then be obtained from another form given to the action [25]. Does this mean that the equations above would be "better"? We are waiting for the physicist-mathematician-geometer who will bring out a totally accomplished, and necessary, form of these interaction tensors. All that can be said at present is that we can give them particular forms which allow them to produce results which are positively confronted with observational data, like this unsteady solution which not only accounts for the acceleration of the expansion, but also predicts that this acceleration must have been more important in the past, which has just been confirmed. After this first result it was shown with tensors, in their mixed form, of the form:

$$K_\mu^\nu = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & \epsilon p & 0 & 0 \\ 0 & 0 & \epsilon p & 0 \\ 0 & 0 & 0 & \epsilon p \end{pmatrix} \qquad \overline{K}_\mu^\nu = \begin{pmatrix} \overline{\rho} c^2 & 0 & 0 & 0 \\ 0 & \epsilon \overline{p} & 0 & 0 \\ 0 & 0 & \epsilon \overline{p} & 0 \\ 0 & 0 & 0 & \epsilon \overline{p} \end{pmatrix} \tag{61}$$

That is to say, derivatives of the tensors and where we assign the pressure terms a small parameter ϵ, by changing their sign, that we obtain, in a stationary regime in spherical symmetry, a satisfaction of the Bianchi conditions to the second order ([32], [25]). We must not forget that these conditions ensuring the mathematical consistency of the equations, in fact cover conservation relations, of energy in section VIII, and of equilibrium between pressure and the force of gravity inside the masses in section X.

XI. GEOMETRIC MODELING OF THE GEOMETRY INSIDE AND OUTSIDE THE DIPOLE REPELLER

In this object the density remains limited. The above approximation can then be suitable. What matters to us then is to construct the induced geometry, the metric $g_{\mu\nu}$, not only outside the object, but also inside it, since positive energy photons can pass through it freely. This is to be able to calculate the attenuation of the brightness of the sources located in the background and to be able to compare this calculation result with the observation data. We will therefore have to construct two metrics: An external metric $g_{\mu\nu}^{ext}$ translating the induced geometry, outside the object. This will have the form:

$$ds^2 = \left(1 - \frac{2\,G\,\overline{M}}{c^2\,r}\right) c^2 dt^2 - \frac{dr^2}{1 - \dfrac{2\,G\,\overline{M}}{c^2\,r}} - r^2\, d\Omega^2 \tag{62}$$

$\overline{M}$ being the (negative) mass of the (invisible) object. For the geometry inside the object, we will assimilate it to a spherical mass filled with a negative mass of constant density $\overline{\rho} < 0$. We will then linearize an interior Schwarzschild metric $g_{\mu\nu}^{int}$, remembering that, technically (in linear and non-linear), everything happens as if a light ray passing through the object, passing through the object, was deflected by a quantity of negative mass equal to $\frac{4}{3}\pi d^3\overline{\rho}$, d being the distance at which the ray would pass from the center of gravity of the object, if it were not deflected.

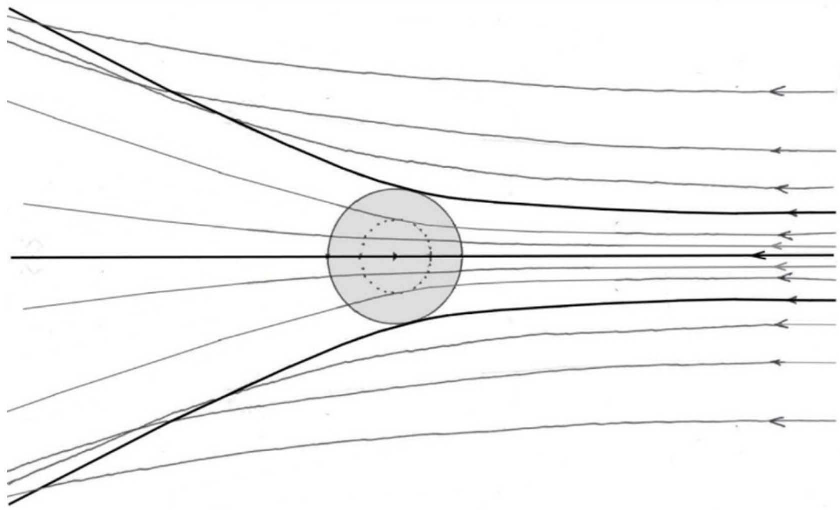

FIG. 28. Schematic deflection of light rays passing through the object or passing nearby

We are therefore perfectly able to construct this calculation data in order to be able to compare it with future observational data. Under these conditions, the rays that will undergo maximum deflection will be those that graze the surface of the object. Such a configuration should make it possible to construct a map where the magnitude attenuation would be maximum in a corona.

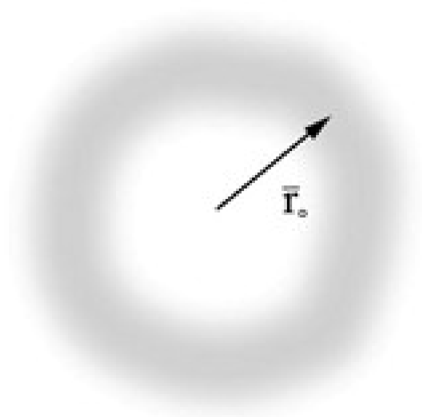

FIG. 29. Annular magnitude attenuation prediction

XII. MODELING THE GEOMETRY INSIDE AND OUTSIDE A NEUTRON STAR

The system of field equations is then:

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} = \chi T_{\mu\nu} \tag{63}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\,\overline{g}_{\mu\nu} = -\chi \overline{K}_{\mu\nu} \tag{64}$$

Karl Schwarzschild provided the pair of exact, non-linear solutions ($g^{int}_{\mu\nu}$, $g^{ext}_{\mu\nu}$) of the first equation, as early as 1916. To complete and construct the solution of the second equation, it would be necessary to construct the form of the interaction tensor, such that $\nabla^\nu \overline{K}_{\mu\nu} =$. But why do this? To have the second, induced metrics ($\overline{g}^{int}_{\mu\nu}$, $\overline{g}^{ext}_{\mu\nu}$) and construct the corresponding geodesics? To compare them with what observational data? None! Therefore, the fact of not being able to define this interaction tensor does not constitute an objection to the model, contrary to the criticisms formulated in ref. [24]. To clarify this, the Janus model will need to account for fluctuations in the CMB, the hobbyhorse of ΛDCM supporters. This is underway. It should be noted that the alleged agreement with the ΛCDM model, requiring the adjustment of six parameters, is undermined by the dissonance between the required value of the Hubble constant and that deduced from observations.

XIII. THE IMPLICATIONS IN THE FIELD OF QUANTUM MECHANICS

They were immediate. Indeed, the acceleration of cosmic expansion, when attributed to negative mass, implies the existence of negative energy states. However, these have been a priori excluded from quantum field theory by attributing exclusively to the space inversion operator P the nature of a linear and unitary operator and to the time inversion operator T , on the contrary, the nature of an antilinear and antiunitary operator ref.[33]. Incidentally, the probability of existence of a state is E/m. By considering the inversion of energy without considering that of mass, those who have considered exploring this path have immediately denied themselves access to it. The mathematician Nathalie de Debergh, reacting to this remark in ref.[13], published articles (ref.[34–36]) showing that by freeing themselves from this constraint, the Dirac and Schrödinger equations generate these states. We conjecture that the inclusion of negative energy states constitutes one of the keys to the quantification of gravitation, without being the only one. The gap separating general relativity and quantum mechanics lies in the fact that the former manages continuous functions and even ignores the particle reality. There exists a world which, in a way, is situated halfway between the two, that of the kinetic theory of gases, where we start from a particle reality by introducing a probability of presence, in the form of a distribution function f, but in fact by giving up on locating them. When quantizing the electromagnetic field, the reaction of the vacuum is taken into account, considered as a mixture of particles and their antiparticles, with opposite electrical charges. In plasma physics, the evaluation of collision cross sections for a Newtonian field leads to infinite values. It is by taking into account the reaction of the surrounding plasma, subjected to the field of a particle, that the screening effect manifests itself, canceling the field beyond the Debye distance. There is a form of similarity between these two manifestations of infinity. From the perspective of quantizing the gravitational field, the gravitational vacuum should be considered a mixture of virtual masses of both signs. We are not in a position to produce a structured and coherent scheme, only to point out the potential importance of integrating negative energy states into quantum field theory. Another angle is to replace the Ricci scalar R with a curvature probability fR, associating it with a distribution function f. Another idea is to consider a geometric translation of the Heisenberg uncertainty principle by considering a "position-velocity" sector made up of finite elements such that the elementary hypervolume is:

$$\Delta x\, \Delta y\, \Delta z\, \Delta v\, \Delta v\, \Delta w = h^3 \tag{65}$$

Another aspect: the introduction of additional dimensions (in even number). Chapter 5 of reference [19] presents an example of geometric compactification by fibration. Finally, and this is the part of the work that we are developing ref.[37], we can make space-time more complex, for example by replacing the Minkowski space with a Hermite space, with metric:

$$ds^2 = (dx^0)^* dx^0 - (dx^1)^* dx^1 - (dx^2)^* dx^2 - (dx^3)^* dx^3 \tag{66}$$

With the real Gramm matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{67}$$

We have as an isometry subgroup, the complex Lorentz group, defined by:

$$L^*\, G\, L = G \tag{68}$$

By adding to it a complex space-time translation vector C we obtain the complexified Poincaré group:

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \tag{69}$$

By constructing the action of the group on the dual of its Lie algebra we obtain the action of the group on its space of moments (M, P) where M is an anti-Hermitian matrix and P the complex extension of the Energy-momentum four-vector.

$$M' = L\, M\, L^* \,+\, 2\, C\, P^*\, L^* \tag{70}$$

$$P' = L\, P \tag{71}$$

These are just tools. But, despite the currently haphazard and fragmentary nature of the approach, we are convinced that taking negative energy states into account could reveal new theoretical aspects as well as previously unsuspected aspects of the world of physics.

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[1] Jean-Pierre Petit. Twin universes cosmology. *Astrophys. Space Sci.*, 226(2):273–307, 1995.

[2] D.Weaire and R.Phelan. A counter-example to kelvin's conjecture on minimal surfaces. *Philosophical Magazine Letters*, 69:107–110, 1994.

[3] Yehuda Hoffman, Daniel Pomarède, R. Brent Tully, and Hélène M. Courtois. The dipole repeller. *Nat. Astron.*, 1(2):0036, jan 2017.

[4] J.P. Petit H. Zejli. Nature of the dipole repeller. *Russian Journal of Earth Science*, 2023.

[5] P. Midy J.P. Petit and F. Landsheat. Twin matter against dark matter. In *Where is the matter?* International Conference on Astrophysics and Cosmology, 2001.

[6] H. Bondi. Negative Mass in General Relativity. *Rev. Mod. Phys.*, 29:423–428, Jul 1957.

[7] A.D. Sakharov. Violation of CP Invariance, C Asymmetry, and Baryon Asymmetry of the Universe. *Zh. Eksp. Teor. Fiz.*, 5:32, January 1967.

[8] A.D. Sakharov. Baryon asymmetry of the universe. *Zh. Eksp. Teor. Fiz.*, 76:1172–1181, April 1979.

[9] A.D. Sakharov. Cosmological models of the universe with reversal of time arrow. *Zh. Eksp. Teor. Fiz.*, 79:689–693, September 1980.

[10] K. Finn L. Boyle and N. Turok. Cpt-symmetric universe. *Phys. Rev. Lett*, 121(251301), 2018.

[11] Jean-Marie Souriau. *Structure des systèmes dynamiques : maîtrises de mathématiques.* Collection Dunod Université. Dunod, 1970.

[12] J.-M. Souriau. *Structure of Dynamical Systems : A Symplectic View of Physics.* Progress in mathematics. Birkhäuser, 1997.

[13] J.-P. Petit and G. d'Agostini. Negative mass hypothesis in cosmology and the nature of dark energy. *Astrophys. Space Sci.*, 354(2):611–615, 2014.

[14] Jean-Pierre Petit and Hicham Zejli. Study of symmetries through the action on torsors of the janus symplectic group. *Reviews in Mathematical Physics*, 37(1):2450054, 2025.

[15] Jean-Pierre Petit. The missing mass problem. *Il Nuovo Cimento B*, 109:697–710, 1994.

[16] W. Boy. Über die curvatura integra und die topologie geschlossener flächen. *Mathematische Annalen*, 57:151–184, 1903.

[17] Jean-Pierre Petit. Une représentation analytique de la surface de boy. *Comptes Rendus de l'Académie des Sciences Paris*, (t.293):269–272, 1981.

[18] J.P. Petit. *Le Topologicon.* Progress in mathematics. Belin, 1981.

[19] Jean-Marie Souriau. *Géometrie et Relativité.* Ed. Jacques Gabay, 1964.

[20] Thibault Damour and Ian I. Kogan. Effective Lagrangians and universality classes of nonlinear bigravity. *Phys. Rev. D*, 66:104024, Nov 2002.

[21] S. Hossenfelder. Anti-gravitation. *Phys. Lett. B*, 636(2):119–125, 2006.

[22] S. Hossenfelder. Bimetric theory with exchange symmetry. *Phys. Rev. D*, 78:044015, Aug 2008.

[23] T.Damour. Sur le modèle cosmologique janus de jean-pierre petit. Site Internet de l'Institut des Haute Etudes de Bures sur Yvette, 2019.

[24] T.Damour. Incohérence physique et mathématique du modèle cosmologique janus de jean-pierre petit. Site Internet de l'Institut des Haute Etudes de Bures sur Yvette, 2022.

[25] H.Zejli J.P.Petit, F.Margnat. A bimetric cosmological model on andreï's twin universe approach. *The European Physical Journal*, 84(1126), 2024.

[26] J.P.Petit. Cosmological model with variable velocity of light. *Modern Phys Letters*, (A3):1527, 1988.

[27] G. D'Agostini J.P.Petit, F.Margnat. Alternative to inflation. variable constants regime in primeval era. e-print HAL Id: hal-04919593, 2025.

[28] A. Benoit-Lévy and G.Chardin. Introducing the dirac-milne universe. 537(A78), 2012.

[29] W.B. Bonnor. Negative mass in general relativity. 21:1143–1157, 1989.

[30] G D'Agostini and J.-P. Petit. Constraints on janus cosmological model from recent observations of supernovae type Ia. *Astrophysics and Space Science*, 363(7):1–7, 2018. https://doi.org/10.1007/s10509-018-3365-3.

[31] K Lodha. Extended dark energy analysis using desi dr2 bao measurement. e-print arXiv:2503.14743v1, 2025.

[32] Jean-Pierre Petit, Gilles d'Agostini, and Nathalie Debergh. Physical and mathematical consistency of the Janus Cosmological Model (JCM). *Prog. Phys.*, 15(1):38–47, 2019.
[33] S. Weinberg. *The Quantum Theory of Fields: Volume 1, Foundations.* Cambridge University Press, 2005.
[34] N. Debergh and J-P Petit. On some reasons why the time reversal operator could be unitary. *Physics Letters A*, 422:127791, 2022.
[35] N. Debergh and J-P Petit. On spacetime algebra and its relation with negative mass. *Reports on Mathematical Physics*, 91:165–181, 2023.
[36] N. Debergh, J.-P. Petit, and G. D'Agostini. On the poincaré algebra in a complex space-time manifold. *Journal of Modern Physics*, (12):218–228, 20231.
[37] F. Margnat J.P. Petit, H.Zejli. Positive and negative real, complex, imaginary masses and energies. e-print HAL Id: hal-04733998, 2024.