

# On the Accelerated Expansion of the Universe

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**Abstract**—If we look from a quantum perspective, the most natural way in which the universe can be created is in entangled pairs whose time flow is oppositely related. This suggests the idea of the creation of a universe-antiuniverse pair. Assuming the validity of this hypothesis, in this paper, we show that the universe expands in an accelerated manner. The same reasoning holds for the anti-universe as well. This idea does not require any form of dark energy as used in the standard cosmological model  $\Lambda$ CDM or in modified theories of gravity.

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## 1. INTRODUCTION

The study of type Ia supernova in 1998 by two independent projects, the Supernova Cosmology Project and the High-Z Supernova Search Team revealed an astonishing result that the universe is not only expanding but it is expanding at an accelerating rate [1]. The accelerated expansion of the universe is thought to have begun since the universe entered its dark-energy-dominated era roughly 5 billion years ago. The currently accepted theory of gravity, General Relativity, accounts for this accelerated expansion by introducing a small and positive value of the cosmological constant  $\Lambda$ . Alternative explanations, such as quintessence (see [2] for a review), are also hypothesized as the explanation for cosmic acceleration. Some explanations beyond four dimensions also exist, such as the Dvali–Gabadadze–Porrati (DGP) model [3] which argues that gravity behaves as 5D at large distances while reducing to that of usual 4D gravity at short distances. However, the precise nature of dark energy remains unknown. Alternative explanations are promising but have their own shortcomings, as was argued in [4] that the self-accelerating solution of the DGP model [5] contains a ghost mode. Very intriguing arguments are presented in [6–9] that the universe should exist in pairs with their time oppositely related. In this letter, we show using recently conjectured Quantum Focusing [10] that if the universe exists as a universe-antiuniverse pairs, it must expand in an accelerated manner. We begin our discussion by reviewing the Quantum Focusing Conjecture (QFC).

### 1.1. Quantum Focusing Conjecture

In this section, we review the formulation of the Quantum Focusing Conjecture (QFC).

**1.1.1. Generalized entropy for Cauchy-splitting surfaces.** Generalized entropy was originally defined in [11] in asymptotically flat space as the area  $A$  of all black hole horizons, plus the entropy of matter systems outside the black holes:

$$S_{\text{gen}} = S_{\text{out}} + \frac{A}{4G\hbar} + \text{counterterms}. \quad (1)$$

A rigorous definition of  $S_{\text{out}}$  can be given as the von Neumann entropy of the quantum state of the exterior of the horizon:

$$S_{\text{out}} = -\text{tr} \rho_{\text{out}} \ln \rho_{\text{out}}. \quad (2)$$

The GSL was introduced to keep the second law of thermodynamics intact when matter entropy is lost in a black hole. Bekenstein conjectured that the GSL [11] survives: the area increase of the black hole will compensate for the lost matter entropy, so that the generalized entropy will not decrease.

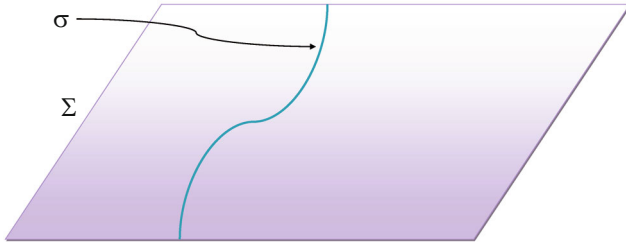
The notion of generalized entropy can be extended beyond the context of causal horizons [10]. Let  $\sigma$  be a spacelike codimension-2 surface that splits a Cauchy surface  $\Sigma$  into two portions. By picking any one of the two sides of  $\sigma$  arbitrarily, we can define an entropy restricted to one side of  $\sigma$  as  $S_{\text{out}}$  (Fig. 1).

**1.1.2. Quantum Focusing Conjecture.** It conjectures [10] that the quantum expansion  $\Theta$ , where  $\Theta$  is given by:

$$\Theta = \theta + \frac{4G\hbar}{\mathcal{A}} S'_{\text{out}}, \quad (3)$$

( $\theta$  is the classical expansion, and  $\mathcal{A}$  is the width of a null congruence along its generator), cannot increase

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**Fig. 1.** A Cauchy surface  $\Sigma$  is divided into two parts by a surface  $\sigma$ .  $S_{\text{out}}$  is defined as the entropy restricted to any one side of the splitting surface  $\sigma$ .

along any congruence, which is valid for quantum states too:

$$\frac{d\Theta}{d\lambda} \leq 0, \quad (4)$$

where  $\lambda$  is an affine parameter. The evolution of the expansion  $\theta$  along the congruence is determined by the Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma_{ab}\sigma^{ab} - R_{ab}k^ak^b, \quad (5)$$

where  $R_{ab}$  is the Ricci tensor,  $\sigma_{ab}$  is the shear, and  $k^a$  is the (null) tangent vector to the congruence. This gives the QFC as:

$$\begin{aligned} 0 &\geq \Theta' = \theta' + \frac{4G\hbar}{\mathcal{A}}(S''_{\text{out}} - S'_{\text{out}}\theta) \\ &= -\frac{1}{2}\theta^2 - \zeta^2 - 8\pi G\langle T_{kk} \rangle \\ &\quad + \frac{4G\hbar}{\mathcal{A}}(S''_{\text{out}} - S'_{\text{out}}\theta), \end{aligned} \quad (6)$$

$\zeta$  is the shear. A special choice of the congruence for  $\theta = \zeta = 0$  gives the Quantum Null Energy Condition (QNEC)

$$\langle T_{kk} \rangle - \frac{\hbar}{2\pi\mathcal{A}}S''_{\text{out}} \geq 0. \quad (7)$$

### 1.2. Relative Entropy and the Bekenstein Bound

The Bekenstein bound is given as

$$S \leq 2\pi ER, \quad (8)$$

where  $R$  is the radius of the boundary of the system, and  $E$  is the total energy. Casini [12] showed that the above bound can be written in the form

$$S_V \leq K_V, \quad (9)$$

where  $S_V$  is a localized entropy, and  $K_V$  is a localized energy. Using  $K = -\log \rho_V^0 - \log(\text{tr} e^{-K})$  and  $\text{tr} \rho_V = \text{tr} \rho_V^0 = 1$ , this can be written as

$$\text{tr}(\rho_V \log \rho_V) - \text{tr}(\rho_V \log \rho_V^0). \quad (10)$$

This is simply the statement of positivity of the relative entropy  $S(\rho_V || \rho_V^0)$  between the state  $\rho_V$  and the vacuum state  $\rho_V^0$ , and thus the bound holds. For an object in a half-space that is far from the circumscribing boundary, (9) reduces to (8). We will use this limit in the next section.

## 2. TOWARDS ACCELERATED EXPANSION

A measure of distinguishability between any two states of an arbitrary quantum system is the relative entropy. It is a quantity of particular significance in quantum information theory. Its illuminating use in the gravitational context was put forward by Casini [12]. Consider a null surface  $N$  that is split into two parts by a codimension-2 plane  $\sigma$ . For states restricted to the part of  $N$  in the future of  $\sigma$  together with a portion of null infinity, the relative entropy is related to the energy-momentum tensor  $T_{ab}$  as [10]

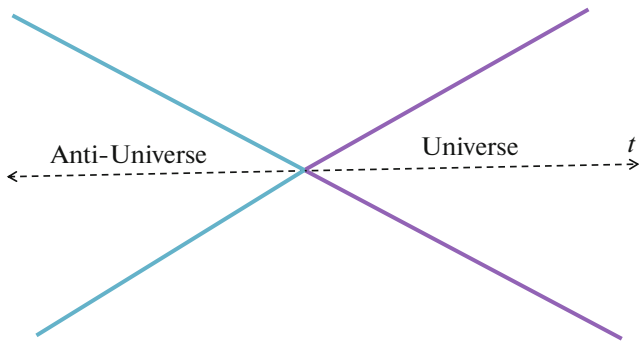
$$S''(\rho || \rho^{(0)}) = \frac{2\pi\mathcal{A}}{\hbar} \langle T_{kk} \rangle - S''_{\text{out}}, \quad (11)$$

where  $\rho^{(0)}$  is the vacuum state, and  $\rho$  is some other state.  $T_{kk} = T_{ab}k^ak^b$ , where  $k^a$  and  $k^b$  are null vectors. Equation (11) essentially tells us that the second derivative of the relative entropy with one of the states being the vacuum state is nonnegative. This can be easily verified by noticing that the right-hand side of (11) is simply the statement of QNEC (7). The relative entropy, in turn, can be written as the statement of the Bekenstein bound as

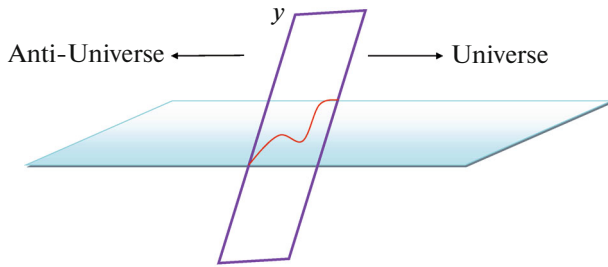
$$S(\rho || \rho^{(0)}) = K_V - S_V, \quad (12)$$

where  $K_V = \text{tr}(K\rho_V) - \text{tr}(K\rho_V^{(0)})$  with  $\rho_V$  and  $\rho_V^{(0)}$  being reduced density matrices defined on a half-spatial plane  $V$  (whose causal development is the Rindler wedge). More precisely,  $V$  is the cut of the Cauchy surface  $t = 0$  defined by  $z \geq 0$ .  $S_V = S_{\text{out}} - S_{\text{out}}^{(0)}$  is the vacuum subtracted entropy, where  $S_{\text{out}} = S(\rho_V)$ , and  $S_{\text{out}}^{(0)} = S(\rho_V^{(0)})$ .  $K$  is the modular Hamiltonian that acts on the right Rindler wedge.

We now apply this result to the universe as a whole. Since the arguments presented hold locally (for a Rindler horizon), the extension to the entire universe requires the global existence of a causal horizon. This is similar to the existence of the Unruh effect for a local Rindler horizon to that of the Hawking effect for a global event horizon of a black hole. This global extension can be achieved by considering a universe-antiuniverse pair, in which case the surface at  $t = 0$  naturally acts as a causal horizon (Fig. 2). The universe runs into the future ( $t > 0$ ) while the anti-universe runs into the past ( $t < 0$ ). The universe-antiuniverse pair is classically disconnected



**Fig. 2.** A diagram showing an entangled universe-antiuniverse pair. The universe runs into the future ( $t > 0$ ) while the anti-universe runs into the past ( $t < 0$ ). The surface at  $t = 0$  is the entangling surface. We choose the half-space to be the region given by  $t > 0$  (the universe of the universe-antiuniverse pair). The modular Hamiltonian  $K$  is restricted to act on this half-space. Therefore, for this setup, we can apply both Casini's idea and the QNEC.



**Fig. 3.** A Cauchy surface (horizontal blue surface) is divided into two parts by  $\gamma$  (shown by a red curve).  $\gamma$  is a section of the null surface  $y$  (at  $t = 0$ ).

but quantum-mechanically entangled with the surface at  $t = 0$  being the entangling surface.

In this case, we can define an entanglement entropy  $S(\rho_V)$  and a vacuum entanglement entropy  $S(\rho_V^{(0)})$ , where  $\rho_V$  is the reduced density matrix of the universe defined on the cut of the Cauchy surface by  $\gamma$  (see Fig. 3) obtained by tracing out the degrees of freedom of the anti-universe. The modular Hamiltonian  $K$  acts on the half-space defined by the region  $t > 0$  (the universe of the anti-universe pair). Thus, the presence of a universe-antiuniverse pair is required to define the entanglement entropies.

Moreover, the pair constitutes the whole space with the universe causally disconnected from the anti-universe, lies in the half-space where  $K$  is restricted to act. These two key insights allow a global extension of the above results to the entire universe. Note that the entropy  $S_{\text{out}}$  refers to the entropy on one side of the Cauchy surface, but by unitary time evolution is equivalent to states restricted to a null surface  $N$  [10, 13]. So, consider a Cauchy surface

which is split into two parts by a spatial surface  $\gamma$  which lies on the null surface  $y$  at  $t = 0$  (see Fig. 3). Therefore, states on the Cauchy surface are unitarily equivalent to the states restricted to the null surface  $y$ . In this case, we make use of the fact that the vacuum entanglement entropy on null cuts is stationary ([10–15]). Therefore, we can now apply (11) and (12) to the universe as a whole. Using (12), (11) becomes ( $S_{\text{out}}''$  cancels on both sides, and  $S_{\text{out}}''^{(0)} = 0$  since the vacuum entropy is stationary)

$$K_V'' = \frac{2\pi\mathcal{A}}{\hbar} \langle T_{kk} \rangle. \quad (13)$$

Applying now (13) to the half-space of the universe, we first make the assumption that the universe respects the Null Energy Condition (NEC), so that  $\langle T_{kk} \rangle > 0$ . Thus, (13) becomes an inequality as

$$K_V'' > 0. \quad (14)$$

The affine parameter  $\lambda$  can now be taken to be the cosmic time  $t$ , and the total energy  $E$  of the universe is constant. Considering the circumscribing radius of the universe to be  $R$  (can be thought of as an imaginary sphere of radius  $R$  circumscribing the universe), then, for the half-space, we have (following Casini)  $K_V \equiv 2\pi ER$ . Therefore, we can finally write (14) as

$$\ddot{R} > 0. \quad (15)$$

This equation tells us that the second derivative of the circumscribing radius is positive, which means that the universe is expanding in an accelerated manner, all by itself! (provided the universe respects the NEC) and does not require any hypothetical form of energy such as dark energy (which is the cosmological constant of  $\Lambda$ CDM). The same reasoning can be applied to the anti-universe by considering it as the half-space defined by the region  $t < 0$ , so that it also expands in an accelerated manner. Therefore, in general, both the universe and anti-universe are expanding, which is consistent with [8].

### 3. CONCLUSION

In this letter, we showed that if the universe exists as a universe-antiuniverse pair, then it expands in an accelerated manner. The same reasoning can be extended to the anti-universe if we take it to be a half-space defined by region  $t < 0$ . An important condition that is required to hold is the Null Energy Condition (NEC). A familiar statement exists in the study of black holes where the horizon area is a nondecreasing function of time, which is the famous Hawking area theorem. It is interesting to note that Hawking's area theorem also requires the Null Energy Condition (NEC) to hold. This suggests that the study of causal horizons reveals a deeper understanding of the

universe. If our approach is correct, then we need no dark energy to explain the present acceleration of the universe, and we may need to modify the notions of our standard cosmology.

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### CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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