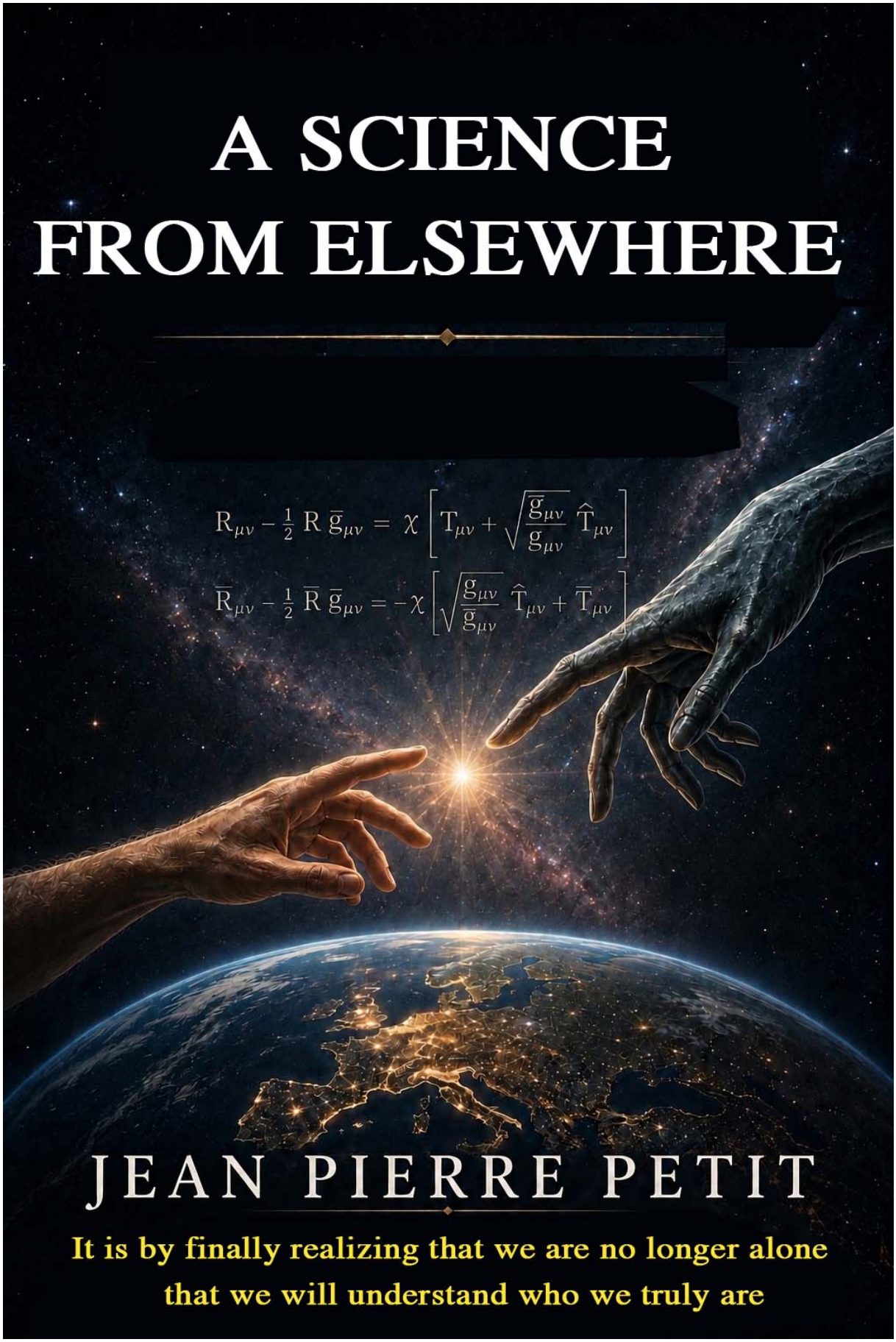

A SCIENCE
FROM ELSEWHERE
$R_{\mu\nu} - \frac{1}{2} R\, \bar{g}_{\mu\nu} = \chi \left[T_{\mu\nu} + \sqrt{\frac{\bar{g}_{\mu\nu}}{g_{\mu\nu}}}\, \hat{T}_{\mu\nu} \right]$
$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R}\, \bar{g}_{\mu\nu} = -\chi \left[\sqrt{\frac{g_{\mu\nu}}{\bar{g}_{\mu\nu}}}\, \hat{T}_{\mu\nu} + \bar{T}_{\mu\nu} \right]$
JEAN PIERRE PETIT
It is by finally realizing that we are no longer alone
that we will understand who we truly are

Prologue:

Thirty-five years have passed. It was 1991. The Ummo affair was making headlines. Journalist André Berkoff had invited me onto the set of his show "Ça m'intéresse." Among the participants were Jean-Claude Ribes, who has since passed away, then director of the Lyon Observatory, and an astronomer, Jean-Louis Heudier.

I had caused a scandal by bringing this affair to light, the claim that Spaniards had, for years, received letters and then exchanged phone calls with the authors, who claimed to be from another planet, which they referred to as Ummo.

Readers can now easily access the full recording of that program at[1] , or this video at[2] , where I've highlighted the key points.

Before diving headfirst back into this sprawling story, what is the central point of this document?

In 1991, I had just revealed that several of my scientific papers, published in top-tier journals, were directly inspired by the information found in these texts. Heudier attempts to downplay this point, which is difficult to dispute, by saying:

- Newton was inspired by an apple falling from a tree….

And I reply (quoting my own words from 1991):



- *Heudier was talking about the sources of inspiration for scientists. Because, indeed, Newton saw an apple fall. Another might have read a poem. Here, without getting into technical details, there are still some very precise points: the product of the radius of*

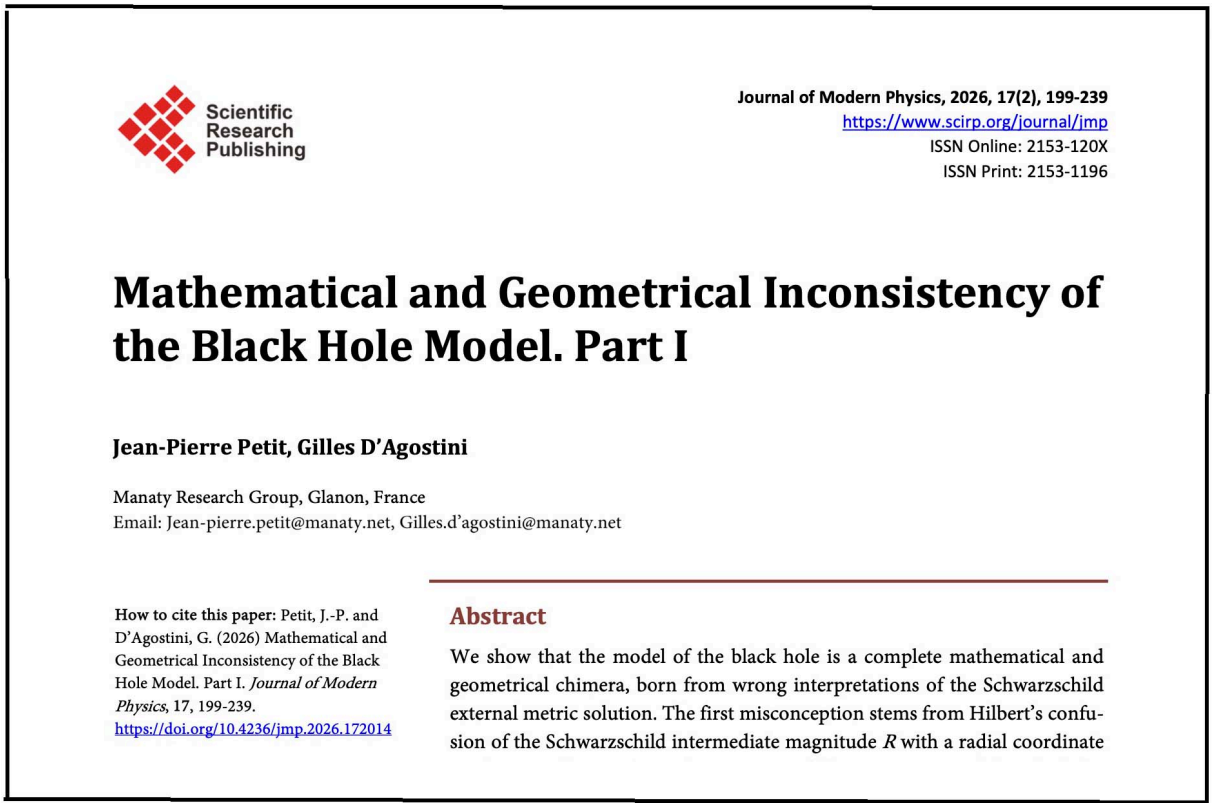
[1] https://www.youtube.com/watch?v=VjPm4nRoJqU&t=1687s
[2] https://www.youtube.com/watch?v=HON4R5JW88E

the universe and the square of the speed of light is an absolute constant. The speed of light varies. The (cosmological) solution has negative curvature. Energy is conserved, but mass is not. There are two universes, which are enantiomorphic, with opposing arrows of time. One contains matter and the other antimatter. These are technically precise scientific facts. It's not just an apple falling. And that's what amazes me.

And I continue:

- *I'm going to share with you the kind of information that hasn't been put to use yet. In 1988, a man named Dominguez had one of those guys on the phone. The Spanish women asked questions that reflected their scientific understanding, which was that of the average person, and asked, "What is a black hole?" And the other person replied, "It doesn't exist. When a neutron star reaches its critical mass, the excess mass is transferred to the twin structure[3] ." That's an answer a Spaniard would find completely incomprehensible. But I guarantee you that it sparks a great deal of scientific research in me, and I certainly hope to succeed. And so, I set a deadline for myself.*

Thirty-five years later, I published this 40-page article in the prestigious Journal of Modern Physics:



Scientific Research Publishing

Journal of Modern Physics, 2026, 17(2), 199-239
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I

Jean-Pierre Petit, Gilles D'Agostini

Manaty Research Group, Glanon, France
Email: Jean-pierre.petit@manaty.net, Gilles.d'agostini@manaty.net

How to cite this paper: Petit, J.-P. and D'Agostini, G. (2026) Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I. *Journal of Modern Physics*, 17, 199-239.
https://doi.org/10.4236/jmp.2026.172014

Abstract

We show that the model of the black hole is a complete mathematical and geometrical chimera, born from wrong interpretations of the Schwarzschild external metric solution. The first misconception stems from Hilbert's confusion of the Schwarzschild intermediate magnitude *R* with a radial coordinate

Translation: Mathematical and Geometric Inconsistency of the Black Hole Model. Part I[4] .

[3] That doesn't exist. When a neutron star becomes unstable, it sends the excess mass to the twin universe.
[4] https://www.scirp.org/journal/paperinformation?paperid=149826

As I write these lines, I'm struggling to get a second part published, of the same length. But in any case, this article is complemented by a second one, which presents the alternative to the black hole model: plugstars[5] .

What does this mean?

To put it plainly: the black hole model is the result of a series of errors made by postwar theorists. This model is nothing but a chimera! And, to make matters worse (see the appendix), the observational data[6] do not align with the model. This research has not prompted any formal rebuttal, that is, through properly formatted articles in high-impact journals.

Anyone can talk. But publishing well-supported scientific arguments is another matter entirely. As I like to say: "Science is a game played on the field, not in the locker room." But let's not get lost in the scientific debate. For those who enjoy scientific thrills, see the appendix.

Let's go back to 1975.

At the time, I was working at the Marseille Observatory as a researcher with the CNRS. A fellow astronomer (now deceased), Maurice Viton, had seen, a few years earlier, near Avignon, at night, while he was with his father, a group of UFOs flying in tight formation, making a sudden 90° turn, and disappearing over the horizon[7] .
One day, he placed a stack of photocopies right in front of me.

- *They were copies of letters that Spaniards had been receiving since the mid-1960s, whose authors claimed to be from a planet named Ummo.*

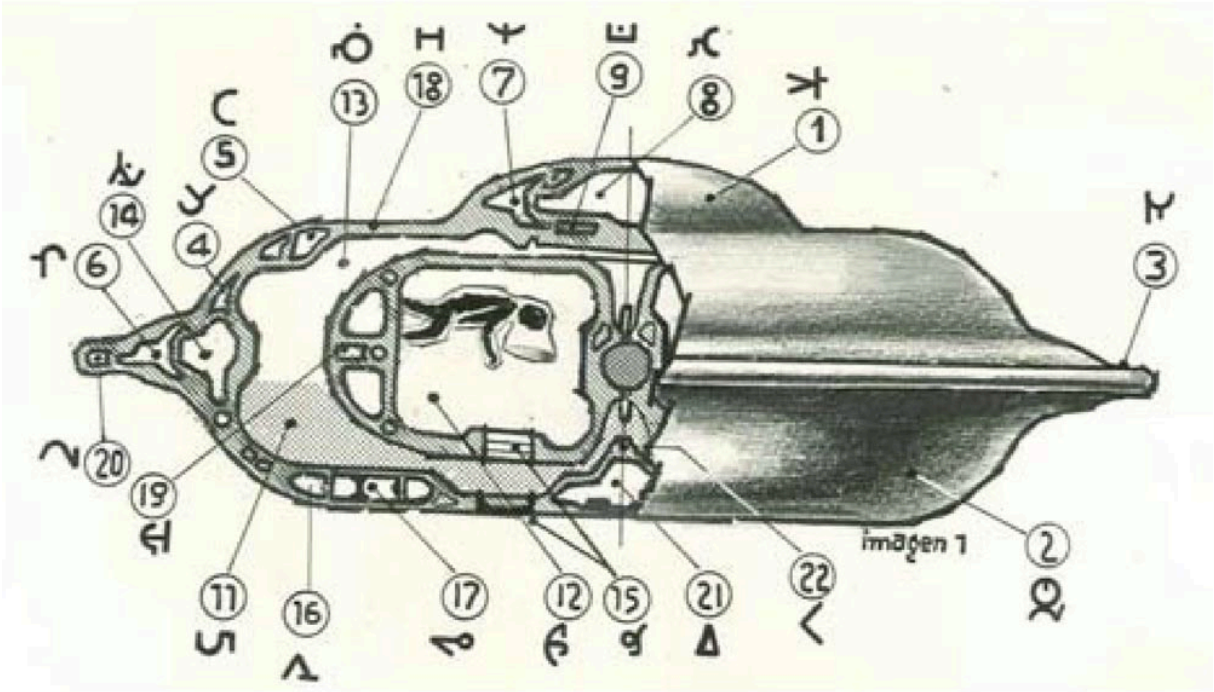
I immersed myself in these documents. I'd seen this kind of writing circulating before. Usually, you quickly put it down. I thought of "Urantia Cosmology[8] " or other works of that sort. But this was different. According to this text, the materials used to build flying saucers are what we would today call "smart." That is to say, they adapt their mechanical properties based on the stresses to which they are subjected. Added to this is a technique that allows passengers to better withstand acceleration. All of this strikes a chord with the aeronautical engineer I once was[9] . It's intriguing. Extraterrestrial technicians? Now that's not something you hear every day. It's worth taking a closer look.

These pages are copies of letters received by a certain Antonio Ribera, a resident of San Feliu de Codinas, near Barcelona. Well, a trip is in order. A friend, Jean-Jacques Pastor, who lives in Digne, accompanies me. Fluent in both French and Spanish, he will be my companion on this adventure for all the years to come.

---

[5] https://www.scirp.org/journal/paperinformation?paperid=146605&utm_source=chatgpt.com
[6] Based on images of supermassive objects located at the centers of the M87 and Milky Way galaxies. The data, however, align with the plugstar model.
[7] In fact, many astronomers have witnessed UFOs. But it is rarer for them to agree to talk about it.
[8] This book prophesies the emergence of a "superior, mixed-race, highly spiritual race, a masterful blend of the best current races."
[9] Paris Higher School of Aeronautics, class of 1961 (now the Higher School of Aeronautics and Space, located in Toulouse).

Antonio is a delight. We hit it off immediately. He is what one might call an “essayist,” the author of numerous books. He once accompanied the Norwegian Thor Heyerdal to Easter Island. Right away, he shows me the original document about the ships, which he received in 1967, featuring this fantastic image:



The Ummo space craft

In Figure 12, the cabin with a passenger wearing a suit equipped with a screen in front of the face that projects a 3D image. The ships have no portholes, but a large number of mini-cameras embedded in the walls. By transmitting their images in real time, these cameras give passengers the impression of seeing through walls that have become, for them, “transparent.” During long journeys, these same screens will immerse passengers in a “virtual reality,” allowing them to escape their confinement.

Antonio immediately lets us photocopy the documents in his possession. But to learn more, the person at the center of this operation is a wealthy industrialist, Rafael Farriols.

On the way back, and this would become a ritual for us, I drive while Jean-Jacques translates the documents for me. And that’s when my professional life, my career as a researcher, began to resemble the plot of a Spielberg movie.

I also hit it off with Farriols, and hundreds of photocopied pages swelled the volume of documents we’d collected.

1966: First contact with Fernando Sesma Manzano.

Who is Sesma? He is an esotericist, a specialist in the strange, and a distinguished metaphysician. He gives weekly lectures in a room located in the basement of Café Lion, at 59 Alcala Street in Madrid. On the wall hangs a painting depicting a whale smiling from ear to ear. Thus, the club is named "ballena alegre" (the joyful whale).



The Happy Whale Club. Café Leon, Madrid.

He speaks of his numerous contacts with extraterrestrial civilizations, described in a book published in 1964: *Los extraterrestres nos hablan* ("The Extraterrestrials Speak to us").

In his book, he describes various planets, including a giant, massive one that rotates on its axis in just a few hours, and which is said to have been inhabited by nothing but… butterflies!

One day, Sesma received a phone call from people with nasal-sounding voices, who told him, in essence:

- *"We are extraterrestrials. We come from the planet Ummo, and we have read your books. It's a bit… nonsense. But as it happens, while flipping through a page, we found things that a person from Earth is not supposed to know. We therefore believe that this information comes from another ethnic group, with whom we would like to establish contact through you." We therefore propose the following: We will send you documents and ask that you read them aloud during your weekly meetings.*

Sesma agrees, and the documents begin to arrive. Sesma complies. The activities of the Happy Whale Club continue peacefully. Sesma, incorporating this information, signed by a certain DEI 98, blends it with information from all the other planets with which he has maintained a telepathic connection for years. Suddenly, in the last days of May 1967, Sesma and a few other members of this congregation receive a letter announcing the imminent arrival of expedition

members from the planet Ummo. Three shuttles are scheduled. Two were supposed to arrive in foreign countries, but the third was supposed to land in Madrid. The letter specified a broad area where the landing was expected to take place, in the suburbs of the capital. The group took note of the information without giving it much credence. However, at the request of the letter's authors, they draft a document, at the bottom of which they all affix their signatures[10] . Only engineer Villagrasa decides to believe this announcement, of which he was one of the recipients[11] , and to go to the site, equipped with several cameras.

June 1, 1967. **The San José de Valdeiras Landing**.

Early in the evening, a craft appeared and made contact with the ground using three legs, which sank into the soil, leaving clearly visible tracks. Farriols, alerted by the announcement made on the radio, went to the scene and met Villagrasa, who was carrying his cameras slung over his shoulder. Visibly shaken, Villagrasa told him:

- *"We knew… that they would come…"*

- *"You knew what!?"*

From there, events unfold rapidly. Some members of the "Joyful Whale" group walk out. For them, things have gone too far. Everyone has, in their mind, made distinctions between what is real and what is a dream. So, sometimes, it can be disappointing to realize that things we thought were real actually belong to the world of dreams. But the opposite effect can also occur.

Farriols got in touch with Sesma, ready to buy those Ummite letters from him for a good price. He told me he was surprised that Sesma sold them to him for a pittance, which he paid in full on the spot. The Ummo affair then entered a second phase. Pretty soon, Farriols began receiving letters of his own. What's more, the authors of the letters asked him for permission to install a microphone in the office he had set up on the top floor of his immense house, on a property where he kept a collection of luxury cars, bred thoroughbred horses, and maintained a personal zoo.

I don't really remember the exact dates. I imagine my first meeting with Farriols must have taken place in 1976.

The First Days on Earth.

So, back in France, while I was driving, Jean-Jacques read me the letter describing the Ummites' arrival on Earth and their first days on our planet:

> *On March 28, 1950, at 4:16:42 GMT, three of our spacecraft suddenly appeared at a point 7.338 km above a location 13 km from the city of Digne and 8 km from the town of La Javie, in the Basses-Alpes department. The three craft descended rapidly and touched down at 4:17 GMT. The retractable landing gear barely sank into the rocky terrain of an Alpine foothill, not far from what we later identified as the Pic*

[10] I don't know what became of this document, but I personally held it in my hands at Rafaels Farriols's home; it bore a good dozen signatures

[11] The other three were Sesma, Alicia Aranjo, and Garrido (police commissioner)

du Cheval-Blanc, 2,323 m above mean sea leve , and not far from the small river known as "la Bléone"... Heavy cloud cover at that late hour prevented a direct view of the surroundings. However, images obtained in the 740-millimicron range made it possible to visualize the area. Plants with strange morphology were growing in the vicinity. The eroded terrain revealed a few distinct features, such as the bed of the aforementioned river...
One of the craft hovered at an altitude of 30 centimeters, ready to intervene in case of an attack. Nevertheless, the area appeared deserted. A few insects were collected from the ground, and several plant species were uprooted, later identified as Erica carnea and Valeriana celta ... it was necessary to begin work on an emergency underground structure ... We worked through the night until 7 a.m. Shortly before dawn, our craft moved into a small grove of strange trees with thread-like leaves, later identified as Pinus montana. The tunnel dug into the ground was 4 meters long, 8 meters deep, and was shored up with modular, extendable supports made of a magnesium alloy.. The new day held a beautiful and novel sight in store for the expedition members. The sky was more indigo than on Ummo ... The origin of the lights seen that morning was quickly clarified. They corresponded to the small towns of Digne and La Javie. The haphazard layout of the strange structures caught our attention. In Digne, a strange tower stood out prominently, which we soon learned was an old Romanesque cathedral. High-magnification optical instruments revealed images of the first terrestrial beings. We observed no unusual activity or nervousness among these people, who were clearly unaware of our presence ... On March 29, at 11:00 a.m., the spacecraft departed, leaving six expedition members, four men and two women, on the Earth's surface ... That same day, two of our brothers were ordered to conduct an initial exploration at some distance from the tunnel while the others continued their work inside it. The entrance to the tunnel is located in one of the region's mountain foothills, not far from the Cheval-Blanc peak.

From there, one can overlook the entire valley through which the Bléone River flows. With good optical instruments, one can see the buildings of Digne, its old cathedral, and even catch a glimpse of the Bès and a few sections of the railroad.
You can also clearly see the hamlet of La Javie and a few humble buildings in the surrounding area.
As a point of interest, we'll mention that the historic gallery still exists, containing some of the original scientific equipment brought here by our brothers. Its entrance is perfectly camouflaged. On the day, perhaps not too far off, when we officially introduce ourselves to the governing bodies of this planet, we will offer these facilities to the French government as a symbolic token of gratitude from our civilization to that of Earth's inhabitants."

A unique first contact with Earth's civilization.

There are aspects of this story that might raise a smile. But, upon closer inspection, what would you do if we were to set foot on another planet?

What follows is taken from another description of those first days following contact with our planet. Indeed, the same account is often presented in different variations depending on the audience it is intended for.

In one of these variations, the construction of the underground shelter is described in greater detail. Carried out in a matter of hours, in the dead of night, it is accompanied by a plume of steam that, at daybreak, is still rising into the atmosphere. The expedition members are concerned that this activity might be detected from nearby structures, in this case, the city of Digne, which they believe to be a scientific research center given the size of its above-ground area.

But in the end, nothing happens, and the three ships depart, leaving four expedition members to their fate. They have taken refuge in a space so cramped that, by Earth standards, it resembles a burrow more than a proper dwelling.

Nevertheless, since their instruments detected no movement in the vicinity, they decided to venture outside and, along a path, made a discovery that shocked them. It consisted of feces mixed with an object that turned out to be a crumpled sheet of newspaper.

They carefully bring their discovery back to the shelter, realizing that it contains text and images that represent their very first contact with the culture of another planet[12] .

Here is an excerpt from their letter:

- *This copy, which is important to us[13] because it was the first printed document we were able to obtain, was an issue of the French-language newspaper *Le Figaro*, specifically the Saturday-Sunday edition dated March 25–26, 1950. The strange typeface left our brothers perplexed. We were unfamiliar with the technique of line photogravure.*


FIGARO

8 francs

Corse et Afrique du Nord 9 FRANCS

Espagne 1 Pta

EDITION de 5 heures

Les gens qui ne veulent rien faire de rien n'avancent rien et ne sont bons à rien.

HENRI-FRÉDÉRIC

N° 1.724

SAMEDI DIMANCHE 25-26 MARS 1950

124e ANNEE

EN VUE D'UNE DETENTE INTERNATIONALE

The front page of the newspaper cited in the letter

And they add:

- *On the back of the sheet of paper, though stained with dried excrement, we could make out a photograph of two human beings, a woman and a man. Using what*

[12] They point out that, on their planet, feces are in no way considered or treated symbolically as "negative elements," as they are on ours. Later, upon hearing an Earthling react to a text by exclaiming, "This article is bullshit!", they will conclude that mixing excrement with a sheet of newspaper constitutes a ritual gesture signifying disapproval of its content. Let us add that, since time immemorial, the elimination of feces has been carried out among them by means of a device implanted in the anus during the first year of life, which transmutes the material into helium.

[13] In another document, they specify that this object, which now holds historical significance for them, was brought back to their home and carefully preserved in a sort of museum dedicated to Earth, where it is kept at a very low temperature.

we had available in the shelter, we fashioned similar garments for ourselves. Not knowing the purpose of buttons, we represented them with simple, lighter-colored synthetic pieces. Dressed in this way, two of our brothers ventured out of the shelter. But upon seeing Earthlings, they realized that their attire was inadequate and hurriedly returned to the underground shelter. It turned out that the photograph found on the newspaper page depicted two characters from a play by the author Marcel Achard, performed in Paris at the Théâtre Marigny, titled Malborough s'en va-t-en guerre[14] *.*

[14] This piece, incidentally written by a comedian, refers to the battle led by the English general Marlborough in 1705. This gave rise to a popular French song:

Marlborough is off to war
Mironton, mironton, mirontaine,
Marlborough is off to war,
Doesn't know when he'll return.
Doesn't know when he'll return.
…
Mr. Marlborough is dead,
Mironton, Mironton, Mirontaine,
Mr. Marlbrough is dead,
Is dead and buried.
…



NE et l'É

«UI

Jacques Dacqmine, guéri, a repris son uniforme et ce soir aura lieu au Théâtre Marigny la répétition générale de « Malborough s'en va-t-en guerre » de Marcel Achard. Ci-dessus, Madeleine Renaud et Jacques Dacqmine.

(img 9)

The photograph of the play's performance at the Théâtre Marigny in Paris

A Fellow Traveler.

A young psychologist, working at a rehabilitation center for juvenile offenders and interested in the subject of UFOs, visited me at my home in Aix-en-Provence. When I showed him this evidence, he exclaimed:

- *La Javie? But that's right next to where I live!*
- *What do you mean?*
- *Well, I live in Digne.*

- *So that means those aliens chose your hometown as their landing spot.*

- *I guess so…*

Since in this book I've decided to give a broad overview of everything that followed this initial contact with the Ummo affair, I'll devote a few pages to our attempt to locate the refuge set up by the Ummites near Digne.

From the entrance to this refuge, assuming it existed, one would necessarily see, according to this text:

- *Digne and its old Romanesque cathedral.*

- *Sections of the railroad.*

- *The Bléone River, which flows at the gates of Digne.*

- *Parts of the Bès, a tributary of the Bléone River.*

- *The Javie.*

Even after years of exploring the region far and wide, we haven't found this underground refuge. We wasted a little time, a few months, to be exact, with Jean-Jacques translating "antigua catedral romana" as "old, old Romanesque cathedral," before realizing that "Romanesque" in Spanish is "románica." And so this cathedral is quite simply the main religious building in Digne, "Roman" and not "Romanesque."

For those who might one day pass through the town of Digne, they should leave town via the road leading to La Javie. At a bend in the road, which they'll have no trouble finding, if they turn around, they'll see the bell tower of Digne's church, and by turning 180°, a distant mountain range: the Blache ridge. Conversely, on their way there, they will notice that the church's bell tower, wedged between two hills, acts like the barrel of a rifle and thus defines a fairly narrow angular field of view. However, once there, the problem that arises is that today's vegetation differs significantly from what it was in 1950. The tree canopy prevents additional cross-checks with the other landmarks, the railroad and the Bès River.

Did this shelter ever exist? Does it still contain, as the letter states, equipment dating from the first expedition? Impossible to say. But for me, these letters offered other avenues of investigation.

The letter, however, contained another piece of information:

- On the night of April 24, 1950, four of our brothers entered an isolated house in the middle of the countryside. The location had previously been scouted by our ÙULEWUA (detection spheres capable of moving at any altitude). At 3:00 a.m. on the 25th, the workers, who were sleeping in an apartment, along with the couple who owned the house and their three children, were anesthetized while they slept.

We stole seventy thousand francs[15] , clothing, identification documents from which we made forgeries, two ballpoint pens, a hygrometer shaped like a nun, keys, postage stamps, a stack of letters and paid invoices related to a tractor, several old newspapers, books on livestock farming, a text explaining the stages of a harvest, and the tractor's manual. We also took a children's encyclopedia, a roll of toilet paper, some wound disinfectant, an alarm clock, two light bulbs, which you, in Spain, call bombillas, and a bar of soap. We ripped out a light switch and the electric meter.

... Two Earthlings lay asleep on a pile of rubble, under the influence of anesthetic gas. Although they already knew that women on Earth let their hair grow long, our brothers were unable to determine their respective sexes. Their clothes were removed, with a small piece of each item taken as a sample. The two bodies were partially undressed, and sweat samples were collected from their armpits and lower abdomen. We also collected hair samples from the head, arms, pubic region, and legs, as well as samples of nasal mucus, hair, and secretions from the vulva.

In a jacket that had been taken away, a lighter and cigarettes were also found. At the same time, we were able to collect samples from cows in a pen. We had to anesthetize two dogs that were beginning to let out loud "screams" (this is the most accurate description of the notes recorded on that date).

In addition, we found a few spare parts from the tractor, tubes of medicine in the form of coated tablets, a schoolbag with homework assignments, six pairs of men's shoes and two pairs of women's shoes (that was all there was), the radio, a bottle of lemon juice, two potatoes, a wall calendar, a pair of pruning shears, and an oil lamp.

Part of this loot was hidden about four hundred meters away, to be retrieved the following night; the rest was transported to our underground base.

The following eight days were devoted entirely by our brothers to analyzing these curious items, trying to make sense of the printed texts and images.

Strange as it may seem, we noted with regret that, despite the large number of samples and objects in our possession, while the analysis provided us with precise information about their chemical composition, the same could not be said regarding their function or purpose.

For example, we were unable to determine the purpose of the pieces of soap.

All the methods we used to figure out the purpose of the "oil lamp" ended in failure.

Apart from that, it was wonderful to discover that the meter's motor ran on alternating current and to see how the readings on the front dial were related to

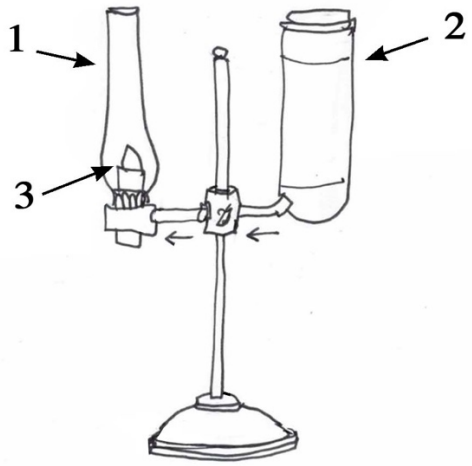
[15] A considerable sum for the time, in cash, equivalent to 500,000 dollars in 2026. All the savings of the farmer who lived there.

energy. But we were unable to determine whether this device was used to measure TIME or some other periodic QUANTITY.

The function of an old alarm clock intrigued us for a few UIW. At first, we feared that this device, if it contained a transmitter, might risk exposing our brothers. Its purely mechanical characteristics were quickly discovered. Because of its alarm, we seriously doubted that it was used as a timekeeping device.
We made a serious mistake when, in order to extract a fragment of the filament, we carefully pierced the glass of the light bulb, greatly delaying our understanding of the bulb's purpose. The filaments oxidized rapidly and melted when subjected to increasing voltage.
The cigarettes were identified thanks to photos taken earlier by OEMII. We believed they were devices that emitted a gas allowing Earthlings to breathe more easily. Our brothers were quite disappointed not to find, inside, the complicated mechanisms they had expected. This only deepened the mystery.
But there was no doubt that the value of the cash, clothing, and shoes far exceeded that of the other items.
We fully understand that the serious extortions we carried out against this peaceful French family arouse your indignation, even though the family was subsequently generously compensated.
Our brothers were concerned about the consequences of such actions. Everyone was forbidden to leave, and the surroundings were monitored while we prepared all means of defense. But apart from the commotion at the farm and the comings and goings of a few villagers between the farm and the small village, nothing seemed out of the ordinary. And, during
that same night, we were able to recover the portion of the loot hidden in the vicinity.

Before discussing a strange twist in this story, let's return to the story of the oil lamp.

Before 1780, people used candles, made of tallow or wax, or lamps in which a wick was immersed in a small pool of oil. In 1780, drawing on an idea by Swiss chemist Ami Argand, a Parisian pharmacist named Antoine Quinquet adapted the concept of oil feeding a wick, with the oil flowing by gravity from a reservoir to saturate the wick. He added a glass sleeve to the device.



The old « quinquet »

1: glass sleeve. 2 : wick. 3 : oil reservoir

This was a significant advance, as the quinquet produced four to five times more light than a traditional candle. It was not until 1850 that a hydrocarbon replaced the oil (often whale oil). When the first oil well drilled in Titusville, USA, in 1859 revealed the existence of oil, "kerosene lamps" became widespread throughout the world within a few decades.

The expedition members remain perplexed by a kerosene lamp. If their letters are to be believed, natural hydrocarbons simply do not exist on their planet[16]. At most, they mention gas emissions such as methane. Another interesting fact is the high intensity of the natural magnetic field, which is linked to their habitat. On Earth, this field is a fraction of a gauss. On their planet (I'll have to check this), the field can exceed 1,000 gauss and, moreover, vary over time. So, logically: the automatic discovery of electrical engineering upon entering the Metal Age.

But let's get back to that burglary carried out by our Ummites at a farm in the region.

Jean-Jacques works in Digne as a psychologist at a center that helps at-risk youth reintegrate into society. His knowledge of the region, and especially a car registered in the Basses-Alpes department, makes it easier for him to gain the locals' trust. One day, he gets a haircut at a barbershop in La Javie.

Dialogue:

- *Do you hear any UFO stories around here?*

- *Oh, here just like anywhere else. Every now and then, someone sees little red balls floating around.*

- *But there aren't any specific incidents?*

- *No. But wait, actually, yes. I remember a strange story. I don't know if it has anything to do with UFOs.*

- *Go ahead, tell me.*

- *Well, one day a farmer showed up with his wife and kids. They seemed completely shaken up after a visit from some strange burglars. Of course, their money had been stolen, and that was the main reason for their complaint. But beyond that, they rattled off a list of items, each more unbelievable than the last, among which was… the house's electric meter! And they all arrived barefoot, since the burglars had taken every pair of shoes in the house.*

[16] It can be assumed that these hydrocarbons imply the sudden subduction of vegetation, resulting from a profound upheaval of the planetary crust, which is absent on their planet. However, methane could very well be of non-biological origin.

- *Do you remember that man's name?*

- *Absolutely. It was the Violat family.*

We have a name. Jean-Jacques and I soon locate the farm, situated in a remote area near Digne. As we can see, it is said that the perpetrators of these thefts, after realizing the value of the banknotes, decided to generously compensate the Violats. Another document states that they did so by giving the patriarch gold bars. Thanks to this money, Father Violat is said to have retired years later to an apartment he bought in a town further south.

We recall that Rafael had told us about the tractor belonging to the farmer, specifying that it was a Spanish brand. Through the cracks in a door, we catch a glimpse of this tractor. But how do we get in touch with this farmer? People from the Basses-Alpes have a reputation for being as silent as the grave[17] .

- *Can you picture us pointing to Violat's house and asking him, "Were you, back in 1950, the victim of a burglary carried out by people who, after stealing all your savings, compensated you with gold bars?"*

In the years that followed, we devised numerous strategies to approach him, even going so far as to consider presenting ourselves as sociologists eager to learn more about the region's legends. But in the end, Violat passed away before we could make contact.

One day, my wife and I went to that farm and found all of Violat's descendants happily having lunch outdoors. Showing great kindness and hospitality, they immediately invited us to join them for their meal. Little by little, we struck up a conversation. Yes, it had seemed strange to his grandchildren that their grandfather had suddenly discovered a late-in-life passion for the heavens and the cosmos. Yes, he sometimes made ambiguous remarks about these subjects. The tractor? It's still there, in the barn.

I'll end the "Digne" chapter here because, very quickly, my research took a completely different turn.

When the universe turns out to have a twin.

As I explore the hundreds of pages I brought back from Spain, I naturally focus on aspects related to physics, astrophysics, and cosmology. None of this takes the form of a comprehensive report but rather like pieces of a vast puzzle, scattered here and there, which I'm trying to put together. What emerges is that our universe might have a twin, with which it interacts. In our own universe, positive masses would dominate. In the other universe, masses would be negative, and antimatter, on the contrary, would have gained the upper hand. Add

[17] A reference to the famous Dominici case, in which, on the night of August 4–5, 1952, three British nationals, the Drumond family, were murdered for no apparent motive on National Route 96, near the Grand'Terre farm, close to Lurs. The perpetrator of the crime was never identified.

to all this some extremely disconcerting elements. In this other universe, objects would be "mirror images[18] ."

But the strangest statement concerns the "arrow of time." In this second universe, it would point in the opposite direction from ours.

But as early as 1976, I was already taking my first steps in the field of cosmology, approaching problems by immersing them in a seven-dimensional space, a concept first proposed in the 19th century by the Austrian mathematician Ludwig Boltzmann[19] . The behavior of such a medium is described by an equation that bears his name[20] .

Let's just say that in the few years prior, I had used this equation to rediscover a number of results from classical cosmology. I began by rediscovering[21] the equation of the English physicist Jeans, who in 1902 described gravitational instability, which gives rise to stars and galaxies. Then I went on to rediscover[22] the famous equation, derived from Einstein's general relativity equation, which the Russian Friedman had used to develop his models of the universe in 1922. Next came the beginnings of mathematical modeling of galaxies[23] . In the early 1970s, it was a promising start.

How do we make sense of this strange cosmology, where we have two universes with opposite, mirror-image arrows of time, interacting via antigravity? Theorists are like alchemists. They mix ingredients in their crucibles and see what comes out. For the theoretical physicist, the ingredients are equations. In the case at hand, the recipe is:

- Take two Boltzmann equations, nice and fresh
- Reverse time and space in the second equation.
- Gently pour them into a seven-dimensional space.
- Add Poisson's equation to bind it all together.
- Let it simmer for a few hundred million years.

And suddenly, alchemy happens. These equations have a reputation for being inextricable[24] . But right before my eyes, pieces of equations come together, a simple, compact mathematical solution emerges, and a coherent cosmological model takes shape.

They say that researchers seek to seize upon ideas. I say that it's often the other way around: it's the ideas that seize upon researchers. That day, faced with these calculations that

---

[18] The term used in the letters describes it as an enantiomorph, which is the term geometers use to refer to this "mirror symmetry."

[19] The seven dimensions are time, three for spatial coordinates, and three others for velocity coordinates.

[20] $$\frac{\partial f}{\partial t} + \vec{v}.\frac{\overrightarrow{\partial f}}{\partial r} - \frac{\overrightarrow{\partial f}}{\partial v}.\frac{\overrightarrow{\partial \psi}}{\partial r} = \frac{\partial_e f}{\partial t}$$

[21] : J-P Petit: Gravitational instability. Linear analysis. CRAS Acad. Sc. vol. 274 (Feb. 14, 1972)

[22] J.P. Petit, G. Monnet: Maximum entropy and Friedman cosmologies. CRAS Acad. Sc. vol. 280 (May 11, 1975)

[23] J.P. Petit: Three-dimensional and unsteady model of a self-gravitating stellar system. Application to globular clusters and galaxies. Publications of the French Academy of Sciences of Paris. January 31, 1972, Series B, pp. 373–376

[24] Each one breaks down into a set of twenty-one nonlinear differential equations? A real nightmare!

practically unfolded before my eyes, almost independently of my will, I was hooked. I had to learn more, to take it all further.

Academician André Lichnérowicz, one of the leading figures in French mathematics at the time, received my articles, read them, endorsed them, and presented them at a session of the Paris Academy of Sciences in 1977[25] .

The Madrid Symposium.

A group known as the Barcelona Group had formed around Rafael Farriols. The Madrid Group, initially formed by members of the "Happy Whale" Club, decided to organize a meeting of all those who had been contacted in Madrid. Since Farriols had insisted that I attend, I found myself, on the appointed day, seated next to Juan Dominguez, the leading figure of the Madrid Group, at a luncheon.

The arrival of a distinguished scientist among the group of Spanish contactees had not gone unnoticed. Farriols had forwarded copies of my letters and work to the various members. When I began my conversation with Dominguez, he was already aware of just about everything.

Dialogue:

- *Beyond this whole business of twin universes, there's one point that caught my attention: the fact that the Ummites say that, in the past, the speed of light varied, that it used to be faster. I tried to incorporate this into the model. If R represents the size, the scale of the universe, I tried a law where the speed of light, denoted by the letter c, varies as 1/R. But I didn't get much conclusive results.*

- *Yes, I know. Rafael told us all about that. And that's not the right equation. We need to consider that c varies as 1 over the square root of R.*

- *Was that in the latest report you received?*

- *No, it was in a phone call.*

- *Because the Ummites are calling you!?*

- *Yes, it's fairly recent. It's always at night, around 3 a.m. They have strange voices, but we can understand them.*

- *And do they answer your questions?*

---

[25] Cosmology: J.P. Petit: Twin PT-symmetric universes. CRAS Acad. Sc., vol. 284 (May 23, 1977)
Cosmology: J.P. Petit: Universes interacting through their T-symmetric image. CRAS Acad. Sc., vol. 284 (June 6, 1977)

- *Absolutely. Very freely. When I mentioned your attempt, they immediately clarified that point.*

- *And are there many of you who receive these phone calls?*

- *There are a few of us. Farriols is one of them, too. Didn't he mention it to you?*

Since Jean-Jacques couldn't come with me to Madrid, I'm doing the best I can with my knowledge of Spanish. But it's hard for me to keep up with everyone's rapid-fire conversation. I spot Camila, Dominguez's wife; Villagrasa, the engineer; Garrido, the police commissioner; Jordan Pena, the corporate psychologist; and Lou, Farriols' former secretary[26] . In total, we've contacted a good thirty people.

The missing piece.

Back home, I dive back into the calculations. A true cosmological model must take relativistic effects into account. This means switching to different calculations[27] , ones that are more subtle and more complex. For months now, I've been immersed in this other form of alchemy, at the heart of which the famous field equation, introduced in 1915 by Albert Einstein, plays a central role.

These equations are like a kind of mechanical system, with gears. Everything must "run smoothly." You can't just add a gear somewhere without it immediately getting stuck. In fact, all of our physics operates with a limited set of constants[28] that interact with one another. According to the texts, this situation, where the speed of light varies, refers to the very early stages of the universe[29] . So, if I replace the pin c, representing the speed of light, with a gear, I must do the same with the gravitational constant G, and ultimately with all the constants, which interact with one another.

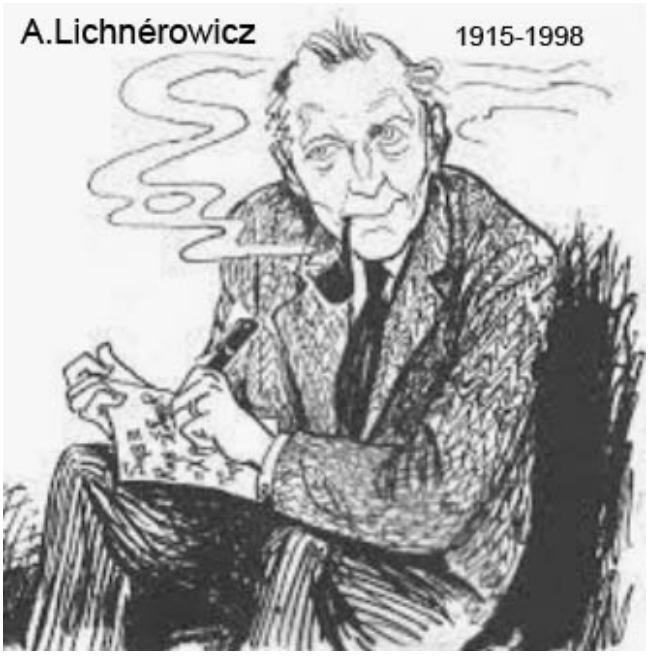
But that's not enough. The quantity R must come into play; it must be coupled to the entire system. I had tried varying c as 1/R, but it's the law c in$1/\sqrt{T}$ that works. So each gear has its proper number of teeth. I feel as though I'm back in the clock tower, overlooking St. Mark's Square in Venice, when in 1752 Bertoleo Ferracina almost entirely rebuilt a mechanism originally constructed in 1496 by the brothers Gian Paolo and Gian Garlo Ranieri, and the hours, the first to be displayed numerically, appeared before the astonished eyes of the Venetians.

[26] Hiltrud Nordlin Franz, of German origin.
[27] What is known as tensor calculus, the specialty of the mathematician Lichnérowicz.
[28] The speed of light c, the gravitational constant G, Planck's constant h, the elementary electric charge e, the elementary mass m, and the magnetic permeability of free spaceμ_0 . Six in total, if we exclude nuclear physics.
[29] When the speeds at which the mass-carrying components, protons, electrons, and neutrons, vibrate increase with temperature, they approach the speed of light.



The Clock Tower of Venice

I'm writing an article and sending it to Lichnerowicz[30] .



[30] He was 74 years old at the time. Already ill, he passed away in 1998.

André Lichnérowicz, mathematician
member of the Paris Academy of Sciences.

But here is his immediate reply:

- *Alas, my dear Jean-Pierre, we, the members of the Paris Academy of Sciences, have just lost the discretionary power that had made us the sole arbiters of what could, or could not, be presented and published there, a power we had held for a century*[31] .

- *How did this happen?*

- *Pressure from the scientific community was exerted, and it demanded that the system, which had gradually become widespread in the postwar period, be applied there, whereby the operation of all scientific journals worldwide grants to referees, to experts protected by their anonymity, and to them alone, the power to accept or reject articles.*

- *But you're the expert on cosmology!*

- *There's nothing more I can do about it. From now on, you'll have to go through the journal review process.*

The door closes immediately. Yet we all know the motto of a researcher's life: "publish or perish!" As a French researcher employed at the National Center for Scientific Research, I have no choice but to follow this rule. Yet no reasonable scientist anywhere in the world would consider for a single second that the speed of light could vary, regardless of the conditions[32] . My articles are therefore immediately rejected with these two words (which I find today in many journals):

Not *suitable*

At that time, photocopying was just beginning to exist[33] , and the idea that one day we might send and receive mail instantly via email hadn't even crossed anyone's mind. To submit an article, one had to mail several printed copies and wait for a response from people whose identities were now hidden and who, from then on, ruled the scientific world with absolute authority.

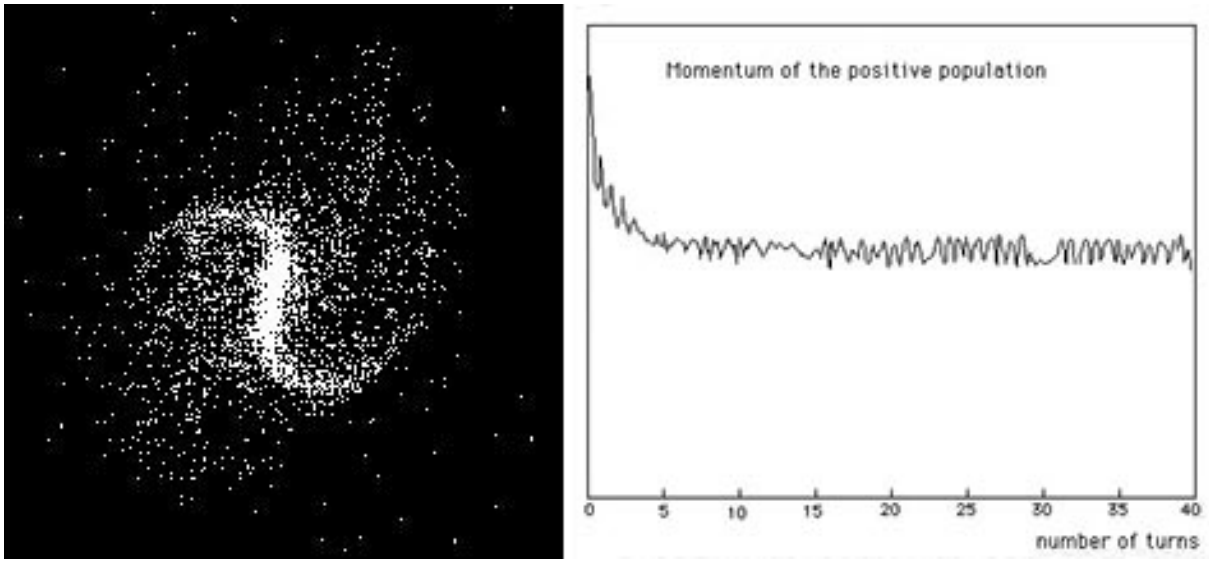
Another standard response is:

- "Sorry*, we don't publish speculative work*"

[31] Since 1835.
[32] And that's still the case today!
[33] I belonged to a generation of researchers where, to make copies, we had to type our texts and draw the figures in India ink on tracing paper, then expose infrared-sensitive sheets to light, and finally develop the images by enclosing them in a fume hood where we released ammonia.

But it is now this indefinable entity, composed of journal editors, their editorial offices, and their peer review committees, that has become the master of the scientific game. I have just realized this following the rejection, by half a dozen specialized journals, of an article, based on a numerical simulation, showing that the spiral structures of galaxies arise from their interaction with an environment of negative mass, which, incidentally, ensures their confinement.

The result, derived directly from the Ummo documents, is nevertheless extraordinary. From the very first iterations, a barred spiral structure appears and persists for dozens of iterations:



Spiral structure obtained by numerical simulation

At the time, I had one of the first personal computers, the Apple II. With its 48 k of main memory, it was clearly incapable of handling a system consisting of two populations of a thousand "mass points" interacting with each other, those with positive mass forming the galaxy and those with negative mass, not visible in this figure, forming its environment and responsible for its "confinement."

This work was actually carried out "behind the scenes" by a young doctoral student working at the German DESI laboratory in Hamburg. Very quickly, by following my decision to give negative masses a dominant role, this young man obtained the first large-scale, lacunar structure of the universe. Then came this spectacular result, showing the immediate formation of this barred supergalaxy.

But it didn't go any further. The management of the center, which operated a powerful particle accelerator, eventually discovered this misuse of the computing power of the lab's powerful computer[34] and put an end to this "unauthorized research."

[34] Less than the computing power of today's personal computers.

The curve on the right shows the rapid decline in rotational motion, starting as early as the first five revolutions[35] . Thus, nothing resembled the spiral structure of a galaxy more than the cloud of cream swirling with the coffee in a cup on a corner of the counter!

All of this aligns with observations. There's no shortage of barred spiral galaxies.



Barred spiral galaxy

Truly, this exotic information, found in letters received by Spaniards, signed by visitors from another planet, who are quite incapable of making sense of it themselves, is leading me to unknown shores.

This business of the variable speed of light is the icing on the cake. The calculations add up. But what journal will even agree to read these lines?

Souriau, the Alchemist

I meet up with Lichnérowicz on the terrace of a café in downtown Aix-en-Provence; he's come down from Paris to attend a conference on mathematical physics. He offers to let me accompany him.

We arrive in front of the Palais des Congrès. All the participants are outside. The door is closed. The hall is occupied by a conference on the food industry.

Standing in the middle of a group of bewildered mathematicians is a small, bearded man with white hair. The director, whose expression seems to be a mix of annoyance and understanding, says to him:

[35] Whereas today the existence of barred galaxies, revealed by the James Webb Telescope, poses a dilemma for astrophysicists, as their models do not allow for the formation of galaxies as we currently know them until at least one billion years from now, and their bars until several billion years from now. In contrast, the Janus model, derived directly from the Ummo texts, places their formation as early as the first hundred million years.

"Mr. Souriau, when you organize a conference, all you have to do is send me a letter to reserve the hall."

I then realize that the little bearded man had simply forgotten to notify the Palais des Congrès.

The situation is absurd. Yet no one seems surprised.

The director makes a few phone calls. Half an hour later, we find ourselves transported to a nearby girls' school. The greatest French mathematicians of the time settle in as best they can at tiny desks and benches designed for ten-year-olds. Their legs stick out in every direction.

But there's a blackboard and some chalk. The conference can begin.

At one point, a Polish mathematician presented his results. The little bearded man then leapt from his seat, crossed the room with brisk steps, and pointed to the formulas the Polish mathematician had written on the board.

"I realize these equations look familiar to me. All of this is already in my book!"

The Polish man turned pale. He didn't know the book in question and apologized profusely.

Souriau listened, then returned to his seat.

Lichnérowicz then whispers in my ear:

"Yes, that's just how he is. The problem is that, on the one hand, he has always kept himself outside the system of peer-reviewed journals, and on the other hand, he has always refused to learn even a single word of English." His entire body of work consists of books published in French by publishers from a bygone era, none of which are available abroad. Scientific libraries in every country are limited to the English language, which has gradually established itself as the standard. But I consider him to be one of the greatest living mathematicians.

In Aix-en-Provence, we are neighbors; I only have to cross a few streets to reach his home, a former mansion. Bravely, like Theseus deciding to face the Minotaur, I announce myself on the intercom.

The heavy oak door, adorned with bronze motifs, swings open. In a vast entryway, I spot an antique sedan chair, which undoubtedly belonged to the family of the property's owner. Then comes a majestic staircase with Carrara marble steps, leading to the apartment occupied by Souriau, whose ceilings soar to nearly five meters in height. He is waiting for me in front of the door.

His office is an unimaginable jumble, where books are piled up in an order whose keys and codes are known only to its occupant.

Since his wife's death, he has lived alone with his cat Pioum, who greets every visitor with a warmth that his owner does not necessarily share, the animal immediately settles onto the lap of any newcomer.

Since Souriau cannot stand mediocrity, his legendary outspokenness has, over the years, cleared the air around him. He has therefore withdrawn from the world like an old alchemist in his laboratory.

Having just finalized my cosmological model with variable constants, I mention my difficulties: the rejections from journals, the closed doors, the articles turned down without even being read.

He listens to me without batting an eye, takes my manuscript

, *"I'll take a look at this."*

Then he signals that the meeting is over.

Weeks go by. Finally, he calls me back. I practically run up the immense staircase that towers over the building's monumental entrance. He receives me as usual, without ceremony. Then he hands me a few sheets of paper.

, I agree with what you've done. It's new. It's interesting.

In his hands is a cover letter.

Just a few lines. But they carry the weight of his scientific authority. Thanks to them, the article finally clears the hurdles. My work on the universe under a regime of variable constants is accepted and published in one of the top journals, *Modern Physics Letters A*[36] .

Thanks to this article, I realize that I've passed my entrance exam. The tone changes completely. I've piqued the curiosity of this figure, who resembles a man from the 7th century more than a researcher of his own time.

- *Are you free for lunch?*

As I nod in agreement, I almost choke, so great is my surprise. We head to the kitchen, located in the attic. Souriau, after glancing distractedly at the instructions, slips a ready-made meal into the microwave.

[36] This has been met with general indifference, and this remains the case in 2026.



Mathematician Sean-Marie Souriau and his cat, Pioum.

He's a hard man to categorize. He refuses to learn English, refuses to submit articles to international journals, and publishes his work in book form, in French. But more than that, in a mathematical language he practically invented himself. Insiders call it "sourien."

On the rare occasions when he gives lectures at mathematics institutes, colleagues have to translate some of his sentences as he speaks. Like an interpreter. Not from French to English. But from "Sourien" to standard mathematical language.

The scene seems implausible. Yet it is real. But in the very closed world of high-level mathematics, those in the know understand that, along with Kostant and Kirillov, Souriau is one of the pioneers of a then-emerging field: symplectic geometry.

His influence was immense, but remained confined to a small circle of specialists.

Lichnérowicz tried to get him admitted to the Academy of Sciences. But the process involved the customary visits to the incumbent academicians. He refused outright, and the project came to nothing.

Souriau resembles a man from the 17th century lost in the modern world. He is one of the last to deserve the title of "scholar", one of those for whom mathematics, physics, history, philosophy, and metaphysics still form part of a single intellectual adventure.

With him, we talk about everything.

Groups. Physics. Ancient civilizations.

About Iran, where he has traveled extensively. About Carthage, where he lived for several years. About Persian myths.

Trees, too.

For Souriau talks to trees.

And he claims that they answer him.

According to him, the dominant trees in a forest exert an influence on their environment. They can then play this role if one addresses them using words known only to the initiated.

Before some picnics, he disappears for a few minutes.

Then he returns.

Shortly afterward, a patch of blue appears in the uniformly gray sky.

My story about extraterrestrial visitors doesn't bother him in the least. It even amuses him.

In fact, when I walk through his door, it's not a retired college professor I find standing before me, but a sort of alchemist lost in the modern world.

In France, the École Normale Supérieure trains the country's elite mathematicians. As a young student, Souriau joined a group focused on esotericism. Since he has since passed away, I think I can reveal what he tells me, little by little.

The group would meet in a basement, by the light of a candle placed on a human skull. There, they experimented with certain types of drugs[37] . I listened to Souriau, spellbound.

- *One day, one of us declared that he was ready to sell his soul to the devil if the devil would make him the author of a theorem that would go down in the history of science!*

- *And the devil provided the theorem!?*

- *I don't know if it was the devil, but the fact remains that this theorem does indeed exist*[38] .

For Souriau, equations are not just collections of symbols on a piece of paper. They have a life of their own. They speak to us, and if we make the effort to listen to what they have to say, great discoveries become possible.

---

[37] Long before LSD

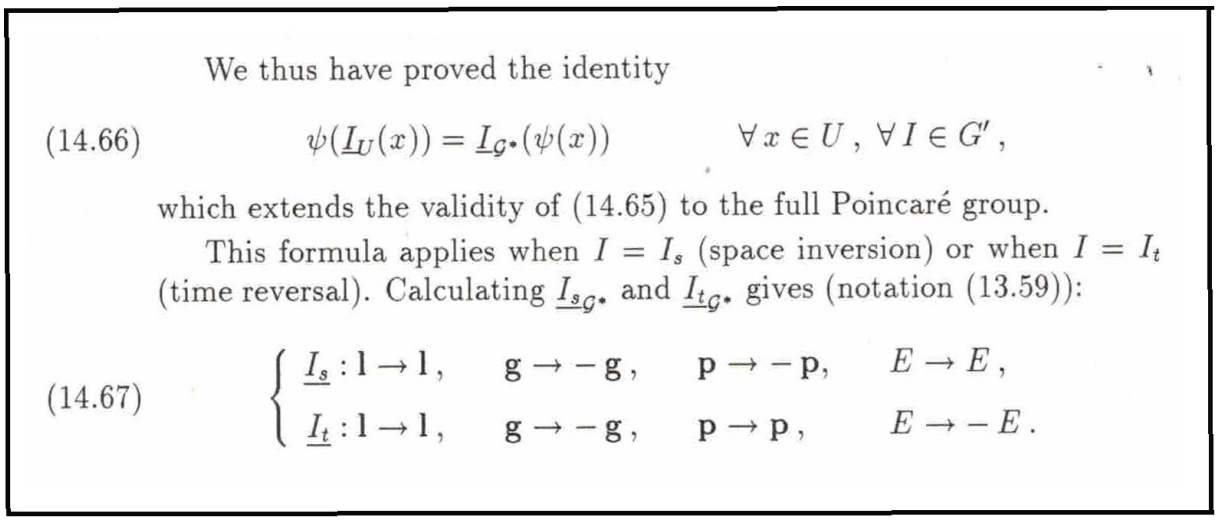
[38] In 1978, mathematician Roger Apéry (who died in 1994 at the age of 78) proved that the sequence of the reciprocals of the cubes of integers, which constitutes the Riemann zeta function $\zeta(3) = \sum_{n-1}^{\infty} \frac{1}{n^3}$, is an irrational number.

- *Mathematics is the grammar of nature*[39] *. It is also like a pair of shoes. You can walk without shoes. But with them, you can go faster and farther.*

- *A little abstraction takes us away from reality. A lot of it brings us back to it.*

- *Theoretical physics? Mathematics without rigor and physics without experiments.*

He gives me copies of his two main books[40] . I leaf through them like Champollion striving to decipher hieroglyphs. But, as I turn a page, I come across a key passage:



We thus have proved the identity

$$\psi(\underline{I_U}(x)) = \underline{I_{\mathcal{G}^*}}(\psi(x)) \qquad \forall x \in U\,,\ \forall I \in G'\,, \tag{14.66}$$

which extends the validity of (14.65) to the full Poincaré group.

This formula applies when $I = I_s$ (space inversion) or when $I = I_t$ (time reversal). Calculating $\underline{I_{s\mathcal{G}^*}}$ and $\underline{I_{t\mathcal{G}^*}}$ gives (notation (13.59)):

$$\begin{cases} \underline{I_s} : \mathrm{l} \to \mathrm{l}\,, & \mathrm{g} \to -\mathrm{g}\,, & \mathrm{p} \to -\mathrm{p}, & E \to E\,, \\ \underline{I_t} : \mathrm{l} \to \mathrm{l}\,, & \mathrm{g} \to -\mathrm{g}\,, & \mathrm{p} \to \mathrm{p}\,, & E \to -E\,. \end{cases} \tag{14.67}$$

"If I understand correctly, reversing time leads to the reversal of energy E."

- Yes, you read that right. It's written in black and white[41] *. And since E = mc*$^{(2)}$*, there's also an inversion of mass.*

And that's when it all clicked.

I suddenly realize that I have the missing piece of the puzzle right before my eyes.

If the reversal of time leads to the reversal of energy, and therefore of mass, then several passages in the Ummite documents cease to be baseless claims and become elements of a coherent framework.

---

[39] Title of his only popular science book, which was never published. Posted by his students on a website dedicated to him.

[40] Geometry and Relativity (exclusively in French). Published in 1964 by Editions Hermann. Structure of Dynamical Systems, Editions Dunod, 1970. English version, translated by his students, Editions Birkhäuser, 1997.

[41] This results from taking into account the complete Poincaré group, equipped with its "antichrone" components, which reverse time, that are not present in the "restricted, orthochrone" version used by theoretical physicists. Years later, this "groups" section would form one of the mathematical foundations of the Janus cosmological model. See: J.P. Petit, F. Margnat, H. Zejli: "A bimetric cosmological model on Andreï's twin universe approach." The European Physical Journal. Vol. 84: No. 1126 (2024). Note: The contributions of the two co-authors of the article are limited to the illustrations and the transcription of the manuscript into LaTeX.

In 1967, no one had yet linked the reversal of the arrow of time to that of mass. Souriau would not demonstrate this until several years later.

I close the book.

This time, I've got something.

The audio cassette.

On November 12, 1968, Francisco Donis, an executive at the investment firm CRECINCO, was driving home to his residence in Madrid. It was nighttime. Suddenly, he spotted a flying saucer perched in a field near the road, with several figures bustling around the craft.

He stopped his car, approached them, and addressed these individuals:

- *"But you look like Earthlings!"*
- *"We are Earthlings."*
- *How so?*
- *We are descendants of Earthlings who were abducted and transported to another planet.*
- *Didn't you ever think of escaping?*
- *To come back and live here? On this crazy planet! You must be joking.*

Donis returns home and tells his wife about his adventure. Word gets around, and the Ummites contact him. Once again, what interests them is tracing back to a new ethnic group visiting Earth. They call the Spaniard and arrange a meeting at his home.

But on the appointed day, it's not just Donis and his wife in the apartment. Members of the Ummo group crowd in, including Rafael Farriols, equipped with recording devices.

At one point, the phone rings. Donis picks it up:

- *It's them! ..*

Rafael rushes over and attaches a suction-cup microphone to the receiver so he can record the conversation. A nasal, monotone voice is heard:

- *"Solo don Francisco. Is this Don Francisco Donis Ortiz? Are your brothers gathered there at your home? Greetings from the brothers of Ummo. Do not ask my brothers any questions about your brothers gathered at your home. We beg your forgiveness, Sir. I am disconnecting the call."*[42]

[42] , Only Don Francisco. Is this Don Francisco Donis Ortiz? Are his brothers gathered at his home? Greetings from the brothers of Ummo. Please do not ask the questions from the brothers gathered at his home. We ask for your forgiveness, sir. We are ending the communication.

While visiting Rafael, he told me this story and played the recording for me. I immediately asked him if I could have a copy. He agreed. Back in Aix-en-Provence, I contacted my friend Teston, who was working at the phonetics laboratory in the Faculty of Letters.

He had an analyzer that produced its results on a strip of paper, where the signals were recorded using a stylus, much like the lie detectors or electrocardiograms of the time. The recorded data showed how the frequencies changed over time.

Teston pulls out the paper strip:

- *"It's recto tono."*
- *What does that mean?*
- *Humans produce sounds using their vocal cords. But in terms of frequency, they struggle to keep them constant. When you look at a recording of a human voice, you can clearly see oscillations. Here, look, it's straight as an "I," and the frequencies are limited to two components, no more. The human voice is richer.*
- *So?*
- *So this isn't a human voice*[43] *.*

I recall that the texts say these people produce sounds not with vocal cords, but with pharyngeal folds, which are stiffer. In most of them, this organ hardens during puberty, forcing them to be fitted with an implant placed in the throat, connected to a pendant hanging from the neck. Such a device would provide simple sound amplification within a certain frequency range (I can't recall whether this is at the high or low end of the spectrum). Other sounds would be converted into ultrasound to remotely control all types of machines vi . The expedition members would be selected from among those who escape this sclerosis, to prevent this gadget from giving them away.

Why not? We already have glasses, dentures, and pacemakers.

The cut hand.

Once again, Jean-Jacques and I are back from Spain. It's nighttime. I'm driving while he looks over the new stack of photocopies we've collected, translating as he goes.

- *Oh, that's amazing!*

[43] Of course, one might suspect a synthesized voice. However, given the technology of the time, the field was still in its infancy. It would therefore have required the involvement of agencies with significant resources.

- *What?*

- *In the middle of what Rafael had given us to photocopy, there was a report that had slipped in. And he hadn't noticed it.*

Before discussing what Jean-Jacques has just discovered, let's set the scene for this part of the story. In 1953, the police in the Spanish city of Albacete were notified by an employee of a funeral home. After receiving the body of a young girl who had died of an illness at the funeral home's premises, and before the coffin was sealed, the employee made a macabre discovery. The body had been desecrated. One of her hands had been cut off and one of her eyes removed. [44]

An investigation was launched. The young woman's mother, an aristocrat named Doña Marguarita Luiz de Lihori, known for her eccentric social circle and unable to provide any explanation, was placed under investigation. In the days that followed, the young woman's fiancé committed suicide by jumping out of a window.

The Spanish countess, traumatized by this tragedy and by the suspicion the authorities cast upon her, since they were unable to find the perpetrator of this monstrous act, died shortly thereafter.

The years passed, and on August 6, 1971, the contactees received a letter describing what had actually happened. According to this document, in November 1953, two explorers, intending to continue experiments they had begun in a chalet near Marseille[45] , contacted Mrs. Lihori, posing as doctors. "By curing her of some real ailments and others that were imaginary," they quickly won her favor and managed to rent a space from her in the basement of an old building on Mayor Street, which she owned, in the city of Albacete.

There they conduct various biological experiments, including the study of a viral strain brought from their planet in an Earth-based environment. Normally, this strain, , is considered harmless by them. But in the Earth environment, an unexpected mutation occurs, transforming it into a formidable pathogen.

They identify superficial outbreaks in various residents of the building but quickly manage to destroy them remotely using low-frequency modulated ultrasound probes. However, Mrs. Lihori's daughter, Margot Shelly, driven by curiosity and having obtained the keys, sneaks into the underground laboratory.

The two "doctors" observe that, in her case, the infection has spread to deeper layers and that remote treatment would prove ineffective.

Members of the French and German groups then rush to Albacete. The leaders of the expedition team conclude that saving this young girl would automatically lead them to reveal their true identities, and the decision is made to do nothing.

---

[44] Issue No. 129 of *Diez* Minutos, dated February 13, 1954, devotes two pages to the severed hand case. Issues No. 129 and 130 (February 20) of the same magazine also feature several articles on the subject.

[45] The articles do indeed indicate that, after their initial contact in the Digne region, the expedition members reportedly settled for a time in the Marseille area, but we have found no trace of this stay.

The girl dies, and the following night, entering the room where her body is being kept, they remove her anatomical specimens in order to prevent any risk of an epidemic among the Earth's population.

Let's return to that night during which we're making our journey back from Barcelona.

- *So, what does it say?*

- *It's a letter sent to Farriols by a private detective he'd hired to investigate the scene. It says that after this tragedy, the building was quickly sold so a parking garage could be built in its place. During the demolition work, according to witnesses, the workers reportedly uncovered the contents of the underground laboratory.*

- *And so,*

- *I read that they reportedly found vials of all kinds, as well as stainless-steel containers holding various anatomical parts, mostly from animals*[46] *.*

Perplexity.

I go up to Digne from time to time to spend a weekend at Jean-Jacques's place. Aside from scouting locations in the field, we spend long hours poring over what we brought back from Spain.

- *What do you think?*

- *First of all, I'd say that you're the scientist here. You tell me that these texts contain cutting-edge information, which you've managed to turn into scientific publications, , that you believe are ahead of today's science. I have no choice but to take your word for it. That's the crux of this whole matter.*

- *All right, but what about you, the psychologist, what do you think of the rest?*

- *It's obvious that we have to consider every possibility, including a massive conspiracy. But then, by whom, and why? That's what eludes me. I believe the only sensible approach is to suspend judgment and start by examining what's actually in there.*

- *Nothing says that this damn planet actually exists, for that matter. I agree with you, let's withhold judgment and lay it all out flat before us.*

- *It's worth noting that this isn't the case with the Spanish contactees, who take all this prose at face value. Have you seen the content of the latest letters and phone conversations? Everything revolves around what they've given their own name: the*

[46] According to Farriols, who was interviewed later, the young woman's fiancé, having accompanied him on his exploration of the underground laboratory, allegedly took one of the devices found there with him. As he was attempting to sell the object to representatives of a foreign power, they allegedly got rid of him by throwing him out of a window.

BUAAWE BIEI, abbreviated as "BB," which constitutes a form of collective planetary consciousness, the repository of the memory of all events, and the organizer of the planet's social network.

- *This is nothing other than the grand idea of the paleontologist and philosopher Pierre Teilhard de Chardin, [47] , which he developed in 1955 in his book *The Human Phenomenon*.*

    - *The universe comprises three spheres:*
    - *A geosphere, comprising inert matter, the planets, rocks, and oceans*
    - *A biosphere: life, plants, and animals*
    - *A noosphere: thought, language, and consciousness.*

- *And on the Earth's surface lies what he refers to as a "thinking film," where consciousness increases the system's complexity and its relational scope.*

- *But Teilhard, a Christian philosopher, goes much further, explaining that all of this converges toward a Ω – Point*

- *I know. It brings to mind the Apocalypse of John of Patmos[48] : "I am the Alpha and the Omega," says the Lord God, "the One who is, who was, and who is to come, the Almighty." This entire evolution would lead to a final fusion into this Ω – $\chi o \nu \phi \iota \gamma \upsilon \rho \alpha \tau \iota o \nu$, where all consciousnesses would become one.*

- *Teilhard is above all a believer, a Christian philosopher, for whom the Christic phenomenon plays an essential role. And in these Ummo texts, it is stated that on this other planet, located 15 light-years from Earth, the same phenomenon is said to have occurred. Their god, whom they call WOA, is said to have brought about the emergence of a sort of "man- god," UMMOWAO, very similar to Christ, who is said to have met a tragic end in every respect.*

- *Hence a first theory: the UMMO message originates from a neo-Christian sect.*

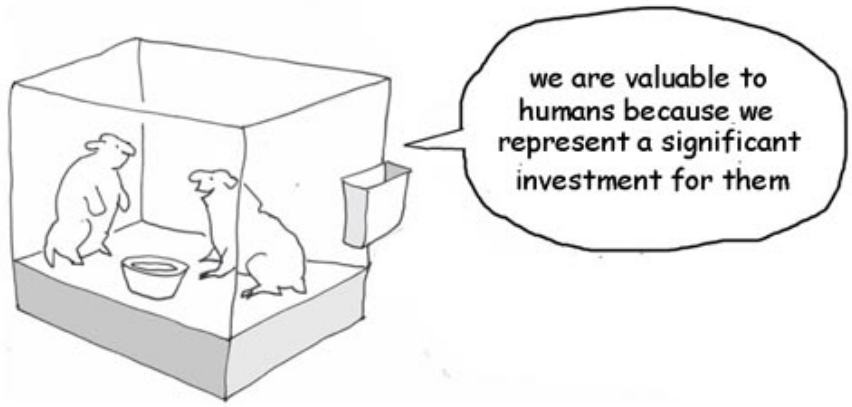
- *We said we'd reserve judgment!*

- *Yes, but that doesn't stop us from speculating. The texts are addressed to the Spanish. If they had been aimed at the Japanese, Mongols, Indians, or Buddhists, the content might have been different.*

- *You could look at it that way. The fact remains that with these Spaniards, it's not just working, it's taking off! Most of them see themselves as the chosen ones, plain and*

[47] French paleontologist and philosopher. 1881–1955?
[48] Revelation 1:8

simple. The texts tell them that the primary role of human beings is "to inform their BB, their Noosphere," during meditation sessions. And there they are, immediately gathering periodically, ritually, to listen to the Ummite texts being read aloud! I can still picture "Lou," Farriols' former secretary[49] , who also received letters, telling us sententiously: "The Ummites care deeply about us, because we've represented a long-term investment for them."



- *The only one in this group who refused all of this was Antonio Ribera.*

- *And that's why he quickly stopped receiving letters! The message is clear: if you act defiantly, if you take initiatives that don't suit us, that's it, no more letters!*

- *All that remains is a series of troubling facts.*

- *As for the story of the severed hand in Albacete in 1953, it may represent the appropriation of a very real news story.*

- *The cassette recording of the Ummites' voices: a secret service hoax.*

- *We're wading through the undecidable. The only thing that stands out is your work. But one can imagine a visionary scientist, or a group of visionary scientists, , whose thinking would have put them far ahead of the science of the 1960s, who might have found this channel to sow these seeds.*

- *By addressing Spaniards who couldn't understand a single word of it all? That's a pretty far-fetched idea, don't you think? I noted an interesting passage. At one point, before the Sesma episode, they say they came across the work of the American astronomer Halton Arp[50] .*

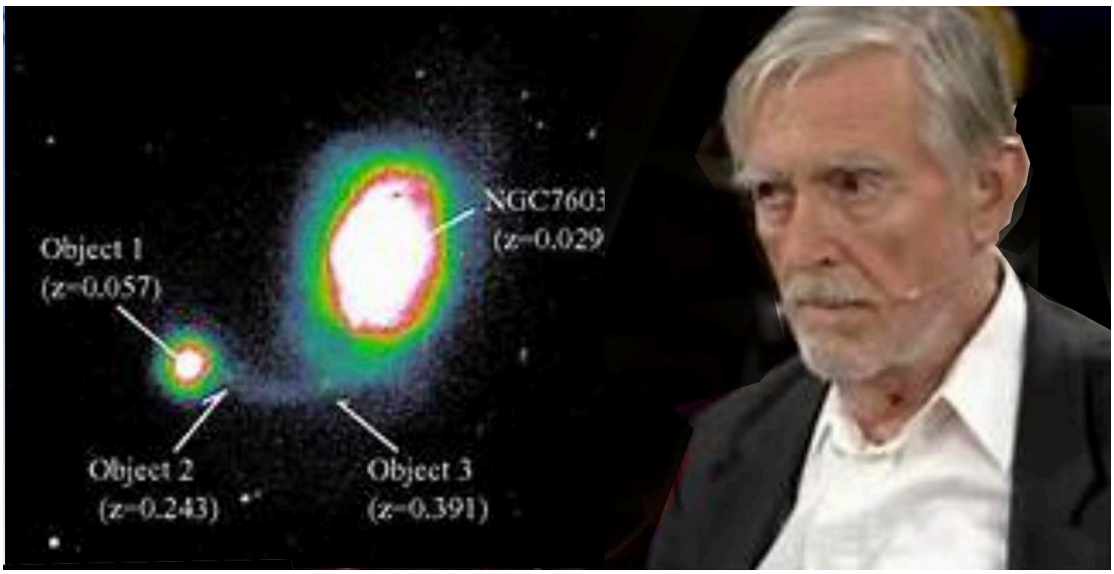
- *A quick explanation, please.*

[49] And, incidentally, one of his mistresses.

[50] This work was the subject of the publication *Catalogue of Singular Galaxies*, published in 1966. It contains images of 338 objects, all of which contradict interpretations based on the standard cosmological model.

- *Arp[51] is no nobody. For decades, he gathered observations using the most powerful telescope, the one at Mount Palomar in the United States. He received the American Astronomical Society's award in 1960. But very quickly, his attention turned to anything that appeared unusual, and there was no shortage of such phenomena. By measuring the redshift[52] and using it to deduce the recession velocity of galaxies, it is possible to estimate the distance to these sources. He thus discovered a large number of pairs of galaxies with very different redshifts, and therefore located at significantly different distances from us, but which appeared to be connected by a bridge of matter. Elsewhere, a quasar, situated at the center of a galaxy, exhibits this same significant redshift discrepancy. Harp suggests that different values for the physical constants in different regions of the universe could account for these effects.*

-



Halton Arp and one of his many objects "that shouldn't exist"

Among these unusual formations are clusters of stars that are completely shapeless and disjointed[53] .

[51] 1927–2013

[52] Redshift: a shift toward the red end of the spectrum, caused by the Doppler effect. Distant galaxies are receding from us. The farther away they are, the greater the effect, which corresponds to the famous law established in 1929 by the American astronomer Edwin Hubble, showing that the recession velocity is proportional to the distance.

[53] Some would like to interpret this as evidence of galaxies in formation. But this is absurd, since many of these objects contain stars that are already ancient. The scientific community simply chooses to ignore this fact.



The irregular galaxy NGC 1427A
Located 55 million light-years away

So what did the Ummites do when they discovered his work? They sent him a report explaining that he had just pinpointed a phenomenon revealing the interactions between our universe and its twin. They even offered to help him model all of this.

- *What was Halton Arp's reaction[54] ?*

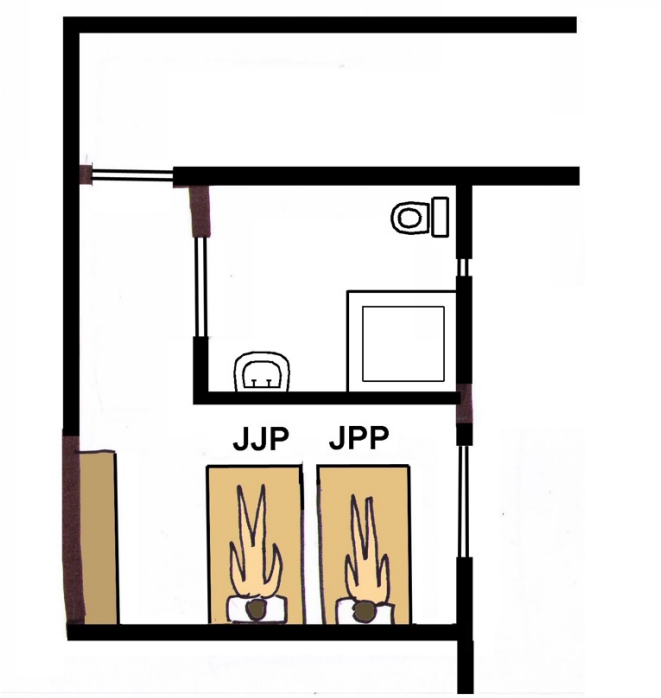
- *Immediate. According to the text, he immediately concluded that the document was the work of pranksters and tossed it in the trash. But what I can say is that in the latest simulations this doctoral student from Hamburg had carried out for me, I had asked him to drastically reduce the confinement effect exerted by his negative-mass environment on the galaxy. And he immediately obtained a result identical to this kind of image.*

- *At this point, why not suggest a meeting? Nothing ventured, nothing gained. The Spaniards say they're in touch with these good people over the phone. Let's ask them to pass this proposal along.*

A Night at the Hotel in Madrid

We are in a small room at a hotel in Madrid, where the Ummites announced that the meeting would take place. It's located at the end of a hallway on the top floor. Two beds, side by side. To the right, a window overlooking an inner courtyard. The room's space is cut off, just to the left of the entrance, by a bathroom with a sink, a toilet, and a shower.

[54] Viewed primarily as a troublemaker, Arp was eventually denied observation time on major instruments. Rejected by his community, he ultimately left the U.S. to go into exile at the Max Planck Institute in Germany, where he ended both his career and his life.



The room at the Sandwy Hotel

It's been about thirty hours since we arrived in the capital by plane the day before, and here we are, waiting after spending our first night at a hotel. Outside, in the streets, groups of highly agitated activists are shouting and driving around in vehicles displaying flags bearing the emblem of Franco's Phalange: the yoke and the arrows. Here and there are long trucks that were used to transport the horses of the Spanish mounted police. I realize that today is the anniversary of Franco's death.

We'd rather stay in our room than risk being stopped, and Jean-Jacques, with his long hair, looks every bit the die-hard leftist. If the police were to question us about why we're in Madrid, especially at a time like this, what could we possibly say?

- *We came here for a meeting with aliens.*

Jean-Jacques calls Farriols on the phone.

- *The Ummites ask you to wait. They say they have things to prepare.*

Dinner at the hotel. Back to the room. Lights out.

A noise wakes me up. Someone is fumbling with the door lock. I realize I've lost all nervous energy. None of my muscles respond. And suddenly, everything happens very quickly. In the darkness, I hear the door open. Hands grab me forcefully. I feel myself being seated on the edge of Jean-Jacques's bed, supported as if I were a lifeless, disjointed mannequin. Someone places a hand under my chin to keep my head from falling forward.

In front of me, resting on the small nightstand in the bedroom, is a large light source, a disc about forty, perhaps fifty centimeters in diameter, from which a "lagoon blue" light emanates.

The glow grows more intense. I can no longer see the edges of the disc. There is nothing but this light, filling my entire field of vision. I feel myself slipping completely away.

When I come to, I realize I'm lying on my bed again. A soft light bathes the room. What wakes me is the sensation that the back of my neck is resting on ice. I can open my eyes, but I can't move a muscle. All I can see are my bare feet at the foot of the bed, with my pajamas rolled down to my ankles.

Suddenly, a figure enters my field of vision. I can make out his features clearly. In his hands, an unidentified device.



Paralyzed!

He turns his head in my direction and gestures with his hand toward another figure standing outside my field of vision, as if to signal that I'm waking up. I slip back into unconsciousness.

But I remember very clearly the face of this man, whom I estimate to be in his fifties. To me, there's no doubt about it, he's the spitting image of the German actor Kurt Jürgens:



German actor Kurt Jürgens

In the morning, I wake up feeling as if I'm "peeing razor blades." It seems beyond doubt to me that I underwent intubation with a semen sample taken, a classic scenario in stories of close encounters or abductions. Jean-Jacques, my neighbor, says he's suffering from an intense headache.

- *It must be your cranial implant!*

- *Cut the crap!*

- *Listen, we came here for an adventure. Sure, it's not what we expected. But at least something happened, right?*

- *I tell Jean-Jacques what I went through last night. He doesn't remember a thing.*

Feeling groggy, he goes to take a shower to try to wake up.

- *As for you, everything amuses you. They shove a pipe into your penis and it makes you laugh!*

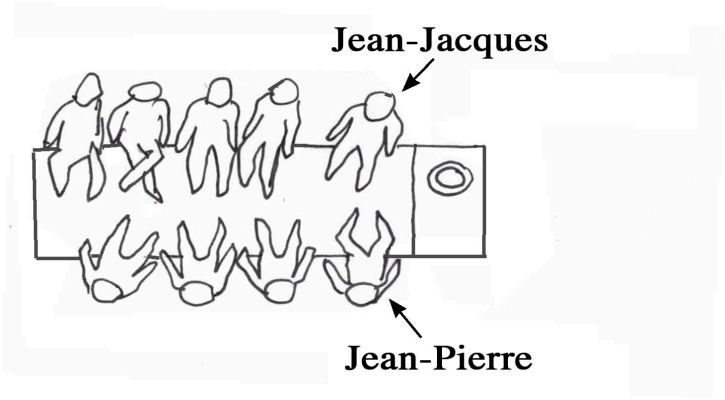
- *Listen, it's better to laugh it off, what do you want me to say? We volunteered for a high-level diplomatic meeting with beings from another planet, and now we're being treated like nothing more than lab rats!*

- *In any case, now we know what we mean to them!*

- *And we also know that this whole business is very real indeed. If we needed proof, we've just been given it, straight to our faces. It's simply another variable in the equation. The Spaniards must be waiting for us. Let's get ready.*

As I'm getting dressed, I notice what looks like a small drop of beige hairspray, about three centimeters in diameter, on the lapel of the suede jacket I bought just before coming here. I try to pick it off, but it won't come off. I tell myself that when I get back to France, I'll crush it with a pair of tweezers and that will make it disappear.

In the hotel lobby, a group of contactees from Barcelona and Madrid is waiting for us. Jordan Pena is there, greeting us with a smile that stretches from ear to ear. Farriols has come from Barcelona. Pena leads us into a sort of lounge where everyone sits facing each other on long benches. Jean-Jacques and I are at the back of these two rows, on either side of a coffee table on which he has placed a hat that he never takes off.



The meeting at the Sandwy Hotel
Jean-Jacques' hat

In fact, the Spaniards, for whom this is a meeting between representatives of the Madrid and Barcelona groups, don't seem to pay much attention to our presence. As usual, they chatter away among themselves like magpies. Their remarks invariably begin with *"creo* que…" (*I think* that…), followed by an endless discourse in which each person forcefully argues their point of view on this or that subject.

Tired, Jean-Jacques waves at me, signaling his lack of interest in these exchanges. But his gaze remains fixed on the group nonetheless. Suddenly, I see him do something extraordinary. Without turning his head, he reaches behind him, grabs his hat, and gives it a spinning motion. I watch this scene with my eyes wide open.

In the end, no one asks us about our trip. This isn't the first time this has happened. When these Spaniards are together and start discussing this or that passage from the Ummite texts, nothing else exists.

We find ourselves on the flight back. I then notice that the "drop of hairspray" has miraculously disappeared from my jacket.

Jean-Jacques:

- *I had one of the same color, a little bigger, on my hat, right in front.*

I then told him about that moment when I saw him, without turning his head, give his hat a twisting motion.

- *I must have had a simple microphone taped to my jacket, and you must have had a video camera on your hat. The Ummite who was following the Spaniards'*

conversation must have said to a companion, "I'm not in the frame. Adjust the camera better."

We returned to France.

We didn't shake hands with anyone.

We received no cosmic revelation.

We didn't even get the slightest hint of an explanation.

But one thing is now certain.

We had asked for a meeting. It took place, period.

The underground refuge.

It's obvious that after a story like that, we can't help but see things differently. With Ribera, we discover what unfolded in the years leading up to the Cuban Missile Crisis within the group of contactees[55] .

Two weeks during which the world held its breath.

On October 14, 1962, the Americans sent a U-2 spy plane to fly over Cuba. The photographs revealed that the Soviets were installing missiles on the island, in response to the Americans' installation of nuclear missile launch sites in Turkey.

The Ummites immediately contacted the Spaniards, revealing that they had an underground shelter west of Madrid, in the Sierra de Gredos, capable of housing seventy of them for eighteen months.[56]

[55] October 14–28, 1962

[56] This figure of eighteen months is entirely realistic. Indeed, if a large-scale nuclear war were to break out on Earth, it would not be urban areas that would be targeted first, but rather the enemy's underground missile sites. The bomb explosions would pulverize the ground into very fine particles, one micron in diameter, which would then be carried into the upper atmosphere. Thus, the immediate impact, which is very rarely mentioned, would be a "nuclear night." Across the entire Northern Hemisphere, the brightness at midday would be equivalent to a full moon night. Temperatures drop by an average of 15 degrees (nuclear winter). Over the course of months, air currents spread this layer of dust across the entire planet[56] . Since the particles fall at speeds of less than one millimeter per second, the dust layer effectively reaches the ground in 18 months.In 1982, my friend Vladimir Alexandrov revealed to the world the catastrophic meteorological consequences of the massive use of nuclear weapons: a nuclear winter, with the widespread destruction of crops, deprived of their lifeblood, sunlight, and of all vegetation. Paradoxically, no film has addressed this aspect, despite its significance. The American TV movie by Nicolas Meyer, *The Day After*, broadcast in late 1983, makes no mention of it. Alexandrov disappeared in Córdoba on March 31, 1985. My investigation and witness accounts show that he was abducted right outside his hotel by a commando unit. His body was never found.



Location of the Ummo refuge, west from Madrid

The message addressed to those who have been contacted asks them to prepare to go to the refuge, specifying that they will have to leave behind the elderly and young children. Ribera tells me that this is devastating for many of them, but that everyone is making preparations.

The Ummites estimate the probability of the conflict breaking out as relatively high[57] . On the appointed day, selected Earthlings will be given earpieces, enabling them to reach the shelter; any deviation from a beam of guiding waves will be signaled in either one ear or the other.

But, in the end, an agreement is reached between the Americans and the Russians. The Russians agree to remove their missiles from Cuba. In return, the Americans commit to doing the same with the missiles they have deployed in Turkey.

World War III is postponed.

[57] While direct contact between Kennedy and Khrushchev helped avert this crisis, the secret agreement stipulating that, after the Russians withdrew their missiles, the Americans would commit to doing the same with the launchers deployed in Turkey, the fact remains that Russian submarines, equipped with nuclear torpedoes and pursued by American ships, remained on their own for days, as their need to remain submerged prevented any radio contact with the Kremlin.

Father Guerrero loses his mind.

A few years later, in 1968, a young prelate, Father Enrique Lopez Guerrero, was informed of the entire Ummo affair and discovered documents in which the Ummites describe the events following Jesus' crucifixion, which differ significantly from the Gospel account.

Our extraterrestrial explorers have thus settled in Galilee. They have their own unique way of making archaeological discoveries. All they have to do is place a device the size of a chicken egg on the ground, which immediately sinks into the soil, melting the rock in its path, which then solidifies again in its wake.

Once the device is at the proper depth beneath the site to be analyzed, this probe emits waves at night that are then picked up by another device flying over the area. The ground is thus scanned, revealing any cavities that might contain interesting artifacts. All that remains for our modern archaeologists to do is to rent or purchase properties adjacent to these cavities.

This operation allows them to get their hands on gold leaves on which they discover the account of the drama, engraved a few days after Christ's crucifixion. We learn that Judas not only did not betray his master but, on the contrary, was the only apostle to attempt a violent act to free him, during which he perished.

Christ did not die on the cross; his still-breathing body was recovered by Joseph of Arimathea, who had managed to bribe Pontius Pilate. Properly cared for, Jesus survived, while another body was placed in his stead in the tomb to deceive the Romans.

And the Ummite text ends with the sentence:

- *We will return this artifact to you when you are capable of not destroying it immediately.*

Father Guerrero's reaction was immediate. He contacted the press. As evidenced by this article, which appeared in the September 15, 1968, issue of the Seville-based magazine ABC:



"Los extraterrestres que viven en nuestro planeta llegaron a la Tierra el 28 de marzo de 1950"

"En España tuvimos la gran oleada en el año 1952"

Manifestaciones de un sacerdote sevillano

Sevilla, 17 (Cifra). — «No sólo creo que existen seres extraterrestres, sino que tengo el convencimiento pleno de que en España reside una colonia cuya misión es totalmente bienhechora y pacífica».

Esta es, nada menos, la sensacional declaración que con toda seriedad y apoyada en observaciones científicas, acaba de hacer a un redactor del diario «ABC», de Sevilla, el reverendo padre, don Enrique López Guerrero, natural de Sevilla, de 38 años párroco de la localidad de Mairena de Alcor y sacerdote de vocación tardía.

Dice el sacerdote sevillano que esos seres extraterrestres proceden del Planeta «Ummo», que se encuentra a 14'6 años-luz de la Tierra y que gira alrededor de la estrella «Woll 424». Un astro, dice que tiene dimensiones semejantes a nuestro planeta y cuyas condiciones de vida son prácticamente idénticas. Agrega que

Déclaration du père Guerrero à la presse 18 septembre 1968

El párroco de Mairena del Alcor, don Enrique López Guerrero quien asegura que una colonia de extraterrestres habitan nuestro planeta.
(Telemat-Cifra-Pyresa.)

mento de los Bajos Alpes. En España tuvimos la gran oleada en el año 1952".

EN BUENOS AIRES, LOS "OVNI" SIGUEN SIENDO NOTICIA

Buenos Aires, 17. (Efe). — Las alucinantes y espectaculares apariciones de los "OVNI" prosiguen sin solución de continuidad. Todo el barrio de Don Bosco vivió la noche anterior, la emoción de los destellos deslumbrantes de un platillo volador. La señorita Olga Susana Unquía, que vive en esa localidad bonaerense y que se encontraba en la puerta de su casa conversando con su novio, Juan Carlos Castillo, es la testigo de mayor importancia ya que, prácticamente observó todas las evoluciones del objeto volador que se desplazaba por el cielo.

"De pronto —dice— apareció en el cielo una luz brillantísima, que se detuvo. Las luces que despedía eran blancas, azules, verdes, rojas... un espectáculo inolvidable, pero que causa un poco de miedo".

Translation:

The extraterrestrials living on our planet arrived on Earth on March 28, 1950.

In Spain, we experienced a wave of them in 1952.

Statements by a priest from Seville:

- *Not only do I believe in the existence of extraterrestrial beings, but I am fully convinced that there is a colony in Spain and that its mission is entirely benevolent and peaceful.*

  This is nothing less than the sensational statement that has just been made, with the utmost seriousness and backed by scientific observations, to the editors of the Seville newspaper "ABC" by Reverend Father Don Enrique Lopez Guerrero, a 38-year-old native of Seville, pastor of the parish of Mairena de Alcor, and a priest who found his calling later in life.

  The priest from Seville claims that these extraterrestrial beings come from a planet called "Ummo," located 14.6 light-years from Earth and orbiting the star Wolf 424. He explains that this star is similar in size to our planet and that living conditions there

are virtually identical. He adds that these visitors reportedly made contact with Earth in France, in the Basse-Alpes region, and that they reside in an underground refuge located near Madrid.

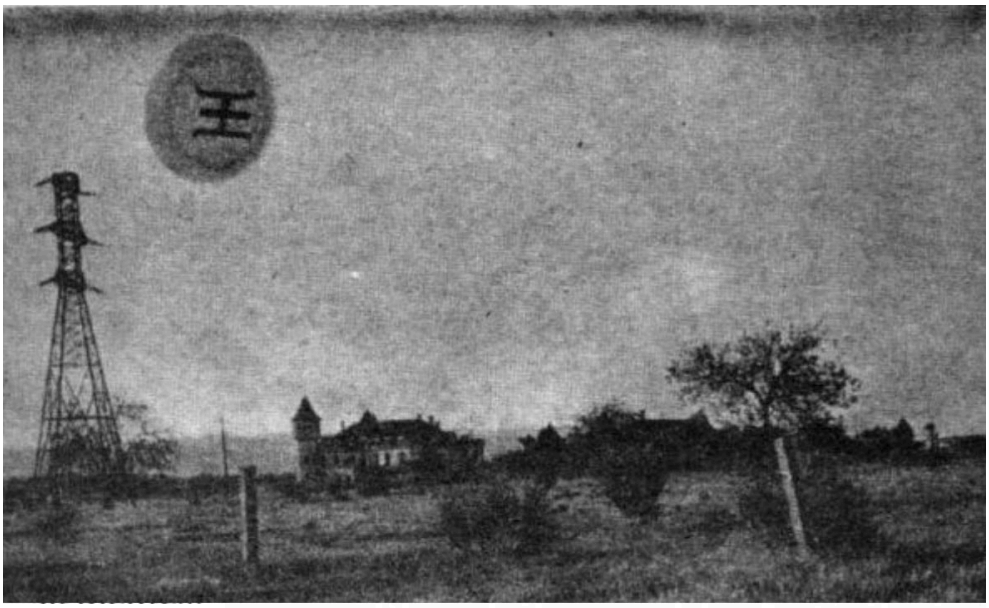
Panic among the aliens:

Members of the Ummite colony in Spain immediately board the first available flight to leave the country for another refuge located in Germany.

But it is there that our expedition members discover a phenomenon that is, for them, completely unexpected. While the incident sparks some interest, it soon fades, dissolving into widespread skepticism. In the process, they identify the key to this mechanism:

- *All human beings have a certain view of the world around them. When an event disrupts this view, they rush to reconstruct it by eliminating the disruptive element, denying it, or attributing it to something else entirely: a natural phenomenon, a fabrication, a hoax, or a mental disorder on the part of the witness.*

Such a phenomenon can be greatly facilitated by providing additional supporting evidence. In this way, they discover the effectiveness of *disinformation*. For example, a roll of film is dropped off at a newspaper by an unknown person. When developed, it reveals photographs of a spacecraft clearly bearing the Ummo insignia on its underside.



El VED de San José de Valderas.
Detalle de la parte inferior.

Elements of the background allow these photos to be linked to an event that occurred a few years earlier in San José de Valdeiras.



El *VED* de San José de Valderas. Otra vista lateral. Se ve perfectamente la cúpula transparente. Foto tomada por el fotógrafo desconocido como las siguientes.

Another side view clearly shows the upper, transparent dome.

These photographs were immediately accepted as irrefutable evidence by Antonio Ribera, who convinced Rafael Farriols to join him in publishing a book titled:



"A Perfect Case" by Antonio Ribera and Rafael Farriols, first published in 1969

The photographs immediately went viral, and the book, the first of its kind devoted to the Ummo affair, was a huge bestseller.

But the effectiveness of disinformation proves all the more formidable in that it can continue to have an impact many years later. In fact, it wasn't until 1977 that engineer Claude Poher, working at the French National Center for Space Studies, managed to obtain the original photograph and, using a digitized microphotometer capable of analyzing the image pixel by pixel, revealed the thread supporting what was, at the time, nothing more than a plastic model.

The San José de Valderas case, which went far beyond mere visual observations, had begun to lend credence among the Spanish public to the idea that visits by extraterrestrials might be a reality.

For years, these few photos were considered the strongest evidence in the Ummo case.

They would ultimately prove to be its weakest link.

The Planet of the Square.

The Ummo documents contain 1,350 words, composed of 17 distinctly Earth-like characters. These appear repeatedly throughout the pages, and the authors claim they are a phonetic transcription of the only language used on their planet.

In one of the documents, they explain that they learned of the existence of organized intelligent life on planet Earth by intercepting a radio message transmitted in Morse code in 1934, during the first terrestrial experiments in long-distance transmission (from a ship toward Earth).

Since radio waves travel at the speed of light, and given the fifteen light-years separating our two planets, the message was received in 1949. An expedition was therefore prepared. In the meantime, the planet Earth was given the code name OYAGAA, the "planet of the square."

When this Morse code message was received, everyone wondered what its content might be.

One of them suggests:

- *"Maybe they're trying to send us a theorem about the square."*

And in the Ummite letter, the authors add:

- *That is what we explained to Theodore T. Polk of Pittsburgh.*

Some friends of mine who live in Boston told me that in the United States, the cross-referencing of phone numbers and individuals works quite well, and they offered to try to track down this person.

Sure enough, a few months later I received a letter from my contacts informing me that in Pittsburgh, Pennsylvania, there is only one Theodore T. Polk, a retired academic, and they sent me his phone number.

The following summer, I was invited by friends to Texas during my vacation. From their home, I decided to dial the number:

- *Hello, is this Mr. Theodore T. Polk?*

- *It's him.*

- *I'm a French scientist on vacation, staying with friends.*

- *"Well, welcome to the United States."*

At that point, I had to take the plunge and continued:

- *Have you received any messages, letters, or phone calls from people claiming to be from another planet?*

- *What a crazy idea!*

The door seems to be closing. I tell myself that whoever wrote these pages probably just picked this name at random from a phone book.

I try one last shot:

- "*Does the* 'planet *of the* square' *mean* anything *to you*?"

He replies immediately, lowering his voice:

- "Yes, *but I'd* rather *not* talk *about it over* the phone."

Back in France, I try to get in touch with him by mail. In several letters, I tell him about what we found in Spain. But these letters go unanswered.

I finally managed to get him on the phone. He then told me:

- *I was in the garden. Suddenly, a voice inside my head said, "You'd better go inside. The Frenchman is going to call you." And sure enough, when I got to the living room, your call came through.*

- *You received my letters containing my questions. What is your response?*

- *Yes, over the months, I've received your successive letters, and I've been tempted many times to reply. But every time I was about to do so, I thought it might lead to great misfortune.*

What kind of misfortune? How so? A mystery, end of story.

The planet Ummo.

The Ummo case is sprawling. I see no equivalent to it anywhere else. As we delve into these hundreds of pages, Jean-Jacques and I are trying to connect the dots, me as a scientist, him as a psychologist.

According to its authors, it is a sociological experiment. Initially, upon discovering elements in Sesma's book "Yo, el confidente de los extraterrestres" that suggest actual contact with other extraterrestrial explorers, they reached out to him to establish contact.

Indeed, contact was established, this time with what we would call "little greys." Some then suggested continuing the experiment by providing Sesma with new information to gauge its impact on the minds of the Earthlings attending his lectures.

How? A device the size of a pea is placed somewhere in the room, out of reach of the audience, which scans, neuron by neuron, the brains of the humans present.

This isn't far-fetched. We do the same thing today, on a much smaller scale, of course.

These reports, dictated to a Spanish typist by a certain DEI 98, contain a wide variety of information. First, the planet Ummo is described. It is slightly larger than Earth, and its daily rotation takes 31 Earth hours. Gravity there is twenty percent stronger. It orbits a star that is cooler than our Sun[58] . For the planet's insolation to place it "in the liquid water zone," its orbit must therefore be slightly closer, and thus the "Ummo year" is equivalent to something like 3 Earth months.

The planet has a single continent, which is completely flat, with a maximum elevation of no more than 300 meters above sea level. This single, vast ocean on Ummo, covering 60% of the planet's surface, has no oceanic trenches, and its depth does not exceed a few hundred meters.

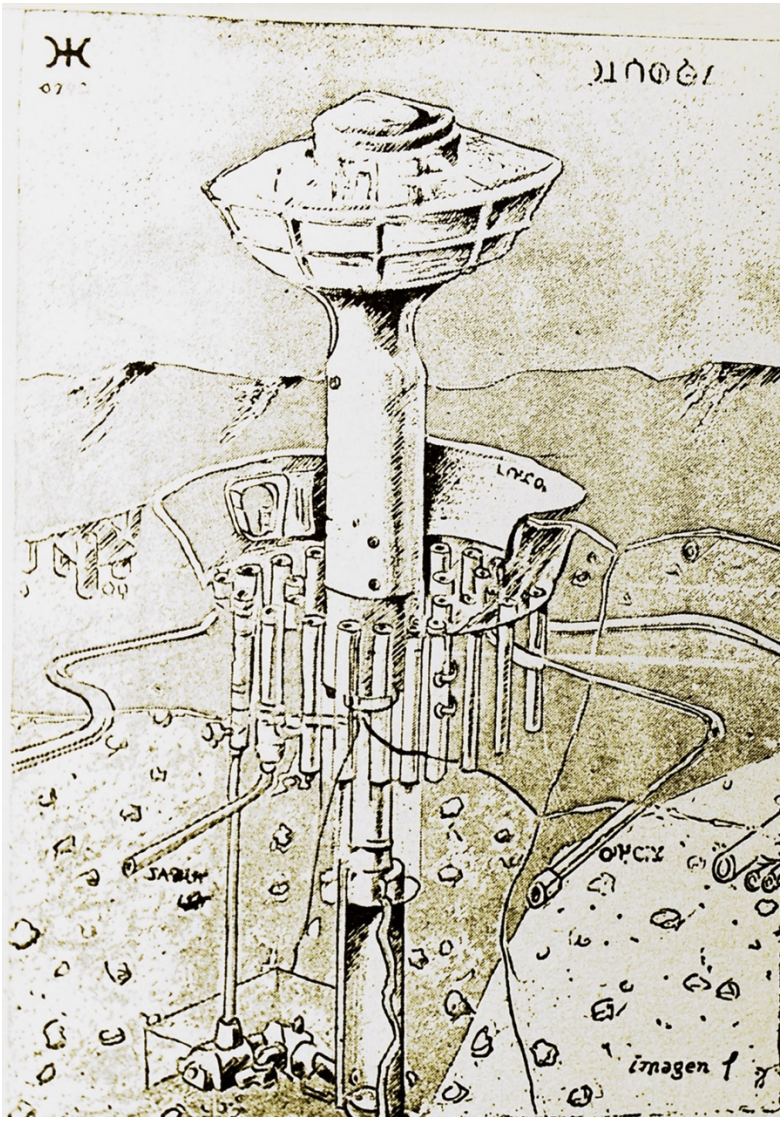
[58] The Sun's surface temperature is 5,800°, which classifies it as a G-type star. IUMMA, UMMO's solar star, is said to have a surface temperature close to 5,000°, which, incidentally, does not align with the data regarding what they describe as their own star: WOLF 424



The sole continent of the planet Ummo

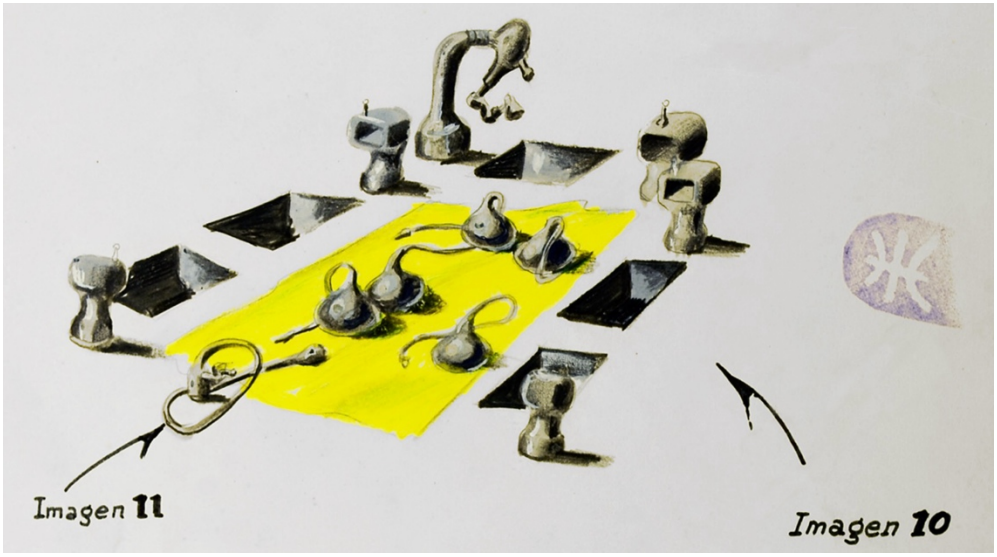
A huge lake can be seen, believed to have been formed by an ancient meteorite impact. At the bottom right is the imprint of the finger stamp that the Ummites slide over their thumb to sign their documents.

Their primitive dwellings: not caves, as on Earth (which do not exist there), but burrows, comparable to those used by animals such as hyenas in Africa. Today, they live in strange, saucer-shaped dwellings resting on a pillar similar to those found in certain Earth elevators. The house thus rises at night, while the system retracts during the day. The house can rotate to follow the sun's path around their planet. When the house is in the retracted position, the windows, visible in its lower section, are obscured.



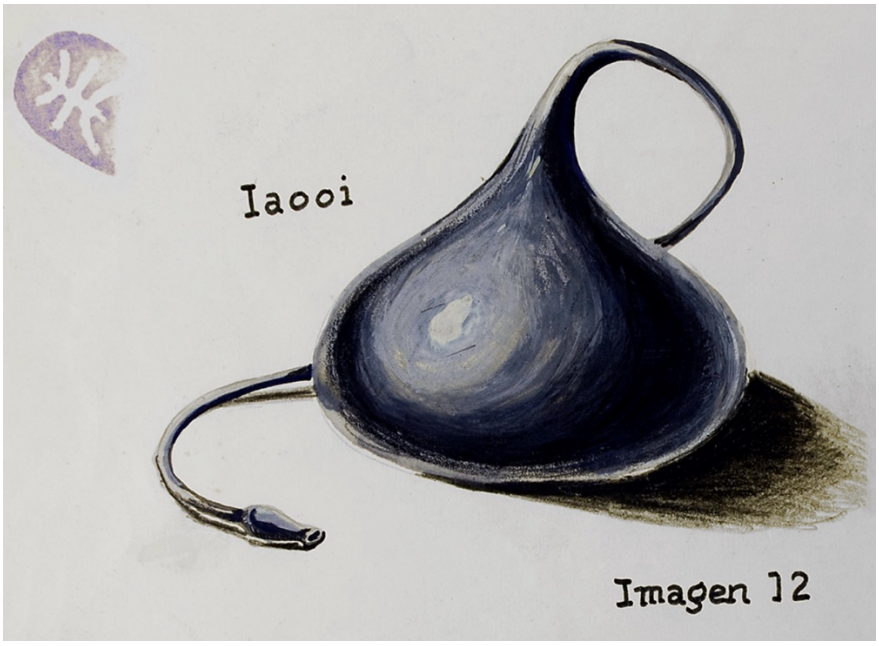
The House on Ummo

The floor contains numerous tubular channels, some of which are used to transport pre-prepared meals. Beneath these are cylinders in which supplies of all kinds are stored. There are few, if any, rooms designated for specific uses. No furniture. To eat while seated, the floor retracts, creating a space to rest one's legs. This configuration is achieved by emitting a sound encoded in ultrasound, using the device they are equipped with.



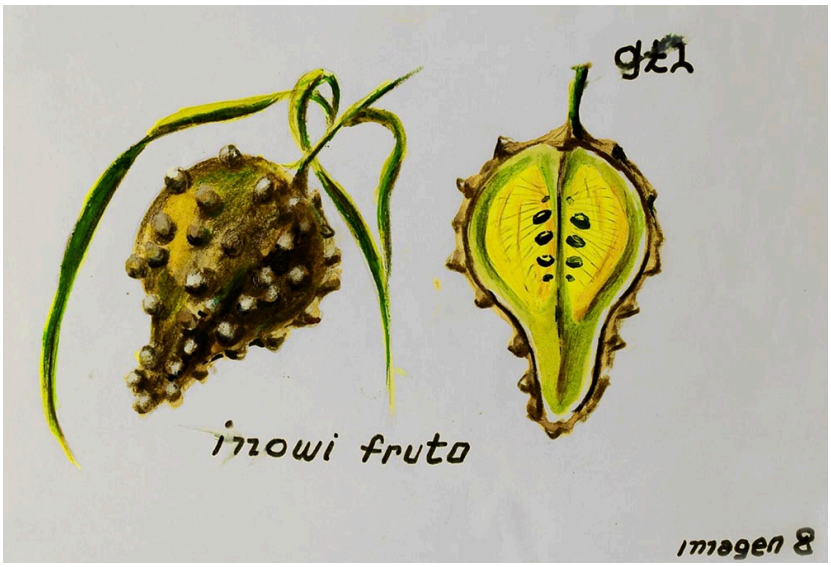
A table set for five people

Food is ingested through tubes connected to an IAOOI container:



For food intake

The plants used for consumption are grown in underground facilities. After packaging, everything is transported via pipes visible in the drawing of the house. Below is a fruit from this planet, very similar to what grows on Earth:



One of the fruits that grows on the planet Ummo

The texts, among the first sent to Sesma in 1966, are teeming with details, each more unverifiable than the last. Regarding their physical appearance, they say they are of average height, that they do not produce melanin, and that their pale complexion allows them to be mistaken for Nordic people when they mingle with Earthlings. Their bodies are said to have only a single pigmented spot.

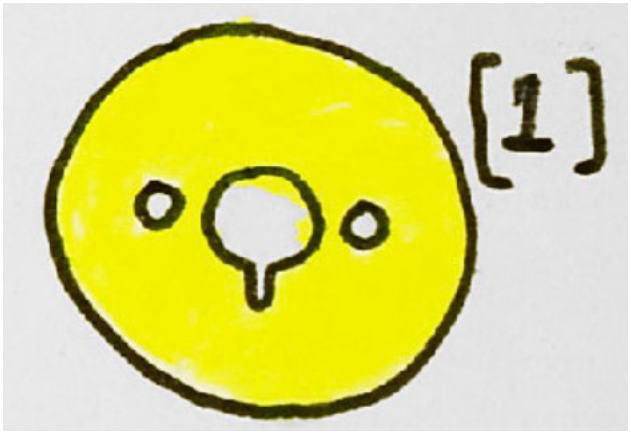
They claim to have a sense of smell comparable to that of a dog. One Ummite recounts that, upon his arrival on our planet, after his companions introduced him to veal blanquette, he was so overwhelmed by such a symphony of flavors that it took him several days to recover.

They are sensitive to infrared light, especially at the tips of their fingers, which, like everyone else's, have five fingers.

When they speak of their planetary history, which, while containing episodes as dramatic as our own, has very quickly become global in scope,

Music, poetry, the arts: unknown to them. What they appreciated most on Earth was our cuisine. When it comes to potential cultural exchanges, what we could send them are not musicians, but Michelin-starred chefs.

Clothing consists simply of a tunic with a hole for the head and two holes for the arms. Its uniform color indicates the status and type of activity of the person wearing it. As a general rule, aesthetics play no role, and everything is geared toward functionality.



Ummo: Clothing.

The population of their planet is limited to two billion individuals. Their social system could be described as totalitarian and is entirely managed, at every level, by a massive artificial intelligence with which individuals communicate constantly. No decision or choice is made at any level whatsoever without first consulting the AI.

In Aldous Huxley's *Brave New World*[59] , Earth's society is organized into biologically engineered castes:

The Alphas

- the ruling elite;
- scientists;
- administrators;
- intellectuals.

The Betas

- middle managers;
- skilled technicians;
- assistants to the Alphas.

The Gammas

- junior staff;
- operational staff.

The Deltas

- laborers;
- routine workers.

[59] Aldous Huxley, *Brave New World*, 1932

The Epsilons

- the simplest tasks;
- intelligence deliberately reduced through embryonic conditioning.

The central idea is remarkable: No one is oppressed. No one rebels. Everyone is conditioned from birth to love their own condition.

An Alpha finds it natural to be an Alpha.

An Epsilon finds it natural to be an Epsilon.

Social stability rests not on force, but on psychological conditioning

Thus, the Alphas are pleased that they do not, like the Epsilons, have to perform strenuous physical labor or work in mines. Conversely, the Epsilons view these activities as tasks that relieve them of intellectual effort and headaches, while protecting them from the elements and sunstroke.

On Ummo, even though the range is narrower, not all individuals are born with the same physical and intellectual endowment. This is assessed very early in childhood, specifically by analyzing the 3D neural structure, a task entrusted to a computer that processes data collected using a scanner, which is located on the planet itself, the site of psychological evaluation.

Education is entirely focused on conditioning. Young children sleep with headphones that feed them all the necessary information[60] , imprinting the fundamental Ummite laws and the idea that the system in which they are destined to grow up is the best possible one onto their neurons.

Starting at age 13 (in Earth years), young people are taken from their biological families and placed in schools where their education is completed in fields compatible with their range of physical and mental abilities. It is not inaccurate to compare such societies to anthills. Such societies have existed, or still exist, wherever a totalitarian system has taken root. Under fascist and Nazi regimes, these human anthills sought to dominate other human groups.

When our Ummites arrive on Earth and view the first images brought back by their probes, they discover… black and white people, and conclude that Earthlings use pigments to change the color of their skin for religious or cultural purposes.

Furthermore, their onboard computer, which records radio signals encoding human voices, immediately becomes completely overwhelmed by such a flood of signals. Here again, some of them put forward the hypothesis of a change in language, also of a ritual nature.

It is only upon setting foot on our soil that they discover the unimaginable variety of ethnic groups, languages, and customs, a variety that immediately goes hand in hand with an ecosystem “500 times richer than their own, by any measure.”

[60] A technique known as hypnopedia.

In the African savanna, gazelles of a given species all look alike. On Ummo, why covet your neighbor's wife, since she is identical to your own?

Throughout their history, the people of Ummo have not escaped hierarchy, dominance, and even the tyranny of a group, or even a single individual. And it was precisely at the end of a particularly odious and monstrous phase that they sought to establish a structure that would make such abuses impossible. The family was immediately identified as the vehicle for spreading dangerous and antisocial ideas.

In Search of Balance.

Their historical evolution challenges us, as we witness these same problems emerging. On Earth, we are witnessing a natural mechanism for regulating population growth. As people's standard of living rises, they naturally tend to have fewer children. In many so-called developed countries, the average number of children per family has already fallen below the level needed to maintain the population. This goes hand in hand with an imbalance in the age pyramid.

In European countries, the solution being implemented is immigration. But in countries like Japan and China, the focus is on robotics and artificial intelligence, sectors that are growing exponentially.

Manufacturing activity is steadily declining. Within a few decades, we will have to seek out tasks that a robot cannot perform. But in Ummite society, there are two essential technical aspects. We have seen that hydrocarbons did not form on their planet. Very early on, the existence of a strong, variable magnetic field provided an inexhaustible source of clean energy, free of waste. To power a home or a factory, it was sufficient to surround it with a conductive coil to generate alternating current through induction[61] .

Nuclear energy is a source of energy on Earth. However, the fission of uranium-235 or plutonium-239 is extremely polluting. Even the fusion of the two hydrogen isotopes, deuterium and tritium, generates neutrons which, by embedding themselves in the walls of the containment vessels, render the materials of which they are made radioactive. Already, boron-hydrogen fusion, which is virtually aneutronic, would represent a significant qualitative improvement[62] .

What the Ummo texts predict is the existence, within the scope of our current technology, of a sort of golden age in which energy production, the elimination of all types of waste [63] , and low-temperature transmutation techniques would be fully mastered.

---

[61] In 1832, the Englishman Michael Faraday attempted to harness the ebb and flow of the saltwater in the North Sea caused by the tides, which, combined with the vertical component of the Earth's magnetic field, did indeed produce a current thanks to the electric field V × B. But given the values involved, it was negligible.

[62] The nucleus of boron $B^{11}$is composed of 11 nucleons. Its exo-energetic fusion with hydrogen H1 gives rise to a nucleus that immediately splits into three helium $He^{(4)\ nuclei}$, each consisting of four nucleons (two protons and two neutrons).

[63] The ultimate byproduct of all nuclear decay is helium, which is lighter than air and therefore naturally escapes the atmosphere of any planet to take its place at the top of the atmosphere.

This would lead to a form of industry that eliminates all manual intervention, and as early as these texts from the 1960s, we find the concept of constructing any object layer by layer, a precursor to our 3D printers.

Today, our industrial 3D printers are capable of creating metal objects. In extraterrestrial 3D printers, at the tip of what in ours is a nozzle, a device reconstructs, through transmutation performed in situ, the atoms and then the molecules of the object to be created. When it comes to replicating any given object, these systems are capable of producing one, or multiple, exact duplicates, down to the individual atom. And this applies to organic objects as well.

It is true that we have long been able to carry out biological and even genetic engineering.

It's mind-boggling, but it's by no means implausible.

So, what remains?

Problems related to biology, genetics, and sociology.

A systematically eugenic planet.

According to them, many of the conditions we suffer from stem from the fact that, through pharmacopoeias and substitutes of all kinds, we have allowed individuals to carry defects, more or less severe, resulting from mutations that are likely to persist. The classic example on Earth is hemophilia, in which women give birth to males whose blood lacks factor VIII[64] . It is transmitted by women, not by men.

It has existed since time immemorial. In the 2nd century, some rabbis observed that the male children of a family died from hemorrhage after being circumcised. Their brothers were then exempted from the procedure.

The first description of the disease was made in 1803 by the German physician Conrad Otto. The term "hemophilia" appeared in medical journals in 1828. The most dramatic case resulted from the mutation that affected Queen Victoria of England.

[64] Factor VIII for hemophilia type A, the most common form, and Factor IX for hemophilia type BD. Both deficiencies have the same effect.



Queen Victoria of England, 1819–1901

She had nine children: four sons and five daughters. Nicknamed "the Grandmother of Europe," her daughters gave rise to numerous royal lines in Germany, Spain, and Russia. Of her four sons, only one, Leopold, was a hemophiliac, and of her five daughters, two passed on the disease to Spain and Russia, where the most famous case was that of the young Tsarevich Alexei Nikolaevich, son of the Tsar and his heir[65] .

When joints are moved, even simply by walking, numerous microhemorrhages occur within them. In people with hemophilia, this phenomenon becomes debilitating. Since the 1960s, the identification of the missing factor has made it possible to compensate for this deficiency with an injection when bleeding occurs. This factor was initially extracted from the blood plasma of thousands of donors. Today, it is produced through genetic engineering. The average unit cost is around one thousand euros per injection. Over the course of a lifetime, a person with severe hemophilia represents a cost of several million euros to society.

The Ummites believe that applying this policy to other conditions that are potentially heritable, such as:

- Type II diabetes
- Cystic fibrosis
- Certain types of cancer
- Certain predispositions to obesity
- Certain allergies,
- Certain severe psychiatric disorders.

which they find completely incomprehensible[66] . For them, it is out of the question for couples to form without first undergoing a comprehensive premarital genetic analysis.

[65] There are 7,000 people with hemophilia in France and 35,000 in the United States.
[66] Sterilizing female carriers, while giving them free access to adoption, would indeed be a solution.

When a mutation gives rise to a disease, regardless of its nature, that affects the lives of other members of the species, its eradication on Ummo is swift. They cite the example of one such mutation that, by affecting the organs responsible for the telepathic link, filled the sleep of their contemporaries with terrifying screams. They write:

- *Once the source was identified, these poor sufferers were immediately freed, having been killed from a distance using a microwave beam. And the people finally breathed a sigh of relief, freed from this mental enslavement.*

Reading this, it becomes clear that the comparison to an anthill is no empty phrase. Indeed, the day-to-day management of society as a whole, its resources, and potential events[67] is a matter of optimization carried out by a planetary AI.

It is therefore understandable that, in response to our request for an interview, the immediate reaction was a brain and genetic examination.

Having eradicated, over the course of generations, many of the diseases that afflict us, and precisely controlling their environment, free from pollution of any kind and from stress, this population of extraterrestrials thus enjoys excellent health for an estimated average lifespan of 120 Earth years.

Crime does, however, occur sporadically and exceptionally, even extending to murder, affecting both adults and children. Under these circumstances, the individuals lose their status as human beings, and no rehabilitation is considered. They are then reclassified and used as subjects for biological experiments. It is specified, however, that these procedures do not cause them any suffering.

In this landscape, Earthlings search in vain for a place to play, or for sports or recreational activities. The concept seems absent among the Ummites, who, when attending a soccer match, struggle to understand why human beings persist in chasing after a ball. Snow sports are nonexistent, as is the presence of snow at high altitudes, due to the lack of mountainous terrain. The “dramas”, most often centered on violence, that Earthlings feast on daily in front of their televisions seem to them to be a form of pathology.

Any kind of manual labor or use of their hands seems to have gradually disappeared among them. The evolution of the species has been guided in this direction, transforming their hands into organs dedicated not to grasping, but to infrared vision. We must pause to consider this detail. They point out that the tips of their fingers have become so sensitive that they must use their knuckles to press elevator buttons! They add that upon arriving on Earth, it was physically impossible for them to use a typewriter and that, in every country, they had to enlist the help of a typist to compose their letters[68] .

We must consider what our future will look like. Theirs evokes a total human-machine symbiosis. Controlling machines: by voice, using frequencies converted to infrared. Writing: through voice dictation. For all work: widespread implementation of artificial intelligence, at all levels, in all sectors.

[67] The texts make no mention of possible climate disasters. Seismic activity and volcanism, which, on our planet, are linked to the exceptional activity of the Earth’s magma, are absent on their planet.
[68] Their discretion, with very few exceptions, was maintained thanks to substantial compensation.

While their heads may be in the clouds, it's clear that they keep their feet firmly on the ground in every respect.

Rafael Farriols, who has many lovers, one day receives a report stating, "Since you are very interested in sex, we are sending you a document regarding how this subject is handled on Ummo."

What follows is a technical manual as exciting as the user's guide for a home lighting or heating system.

Stability in relationships is essential. To this end, the first sexual act takes place only after the partners have refrained from any personal hygiene so that the olfactory experience is maximized. Beauty is a meaningless word to them.

Whether or not this planet exists, these pages serve as an invitation for us to consider entirely different ways of life[69] .

The only common thread is this meditative practice (3 hours a day), which is at the center of their entire existence, during which the individual comes into contact with the collective planetary soul, inducing a profound sense of well-being. This brings us to the detachment of Buddhists, for whom the focus must indeed be primarily on this invisible aspect of the world, by detaching oneself from the "impermanent."

One essential point remains:

Death.

If this daily practice of connecting with the "BB" is at the center of life, individuals perceive themselves as "the eyes and ears of their planetary collective soul." During these connections, they relay information to it and consider this to be their primary task, their function, and their essential reason for being. As with Buddhists, human attachment is not the norm; sentimentality between individuals gives way to this ecstatic symbiosis with the invisible.

From this perspective, the separation of young people from their biological parents at the age of 13 is not experienced as a tragedy by either party. The rules of their society dictate that communication ceases as soon as the child leaves the family home. Here we find a form of return to an animal nature, a guarantee of the species' balance.

The end of life is by no means perceived as tragic.

Essentially, it looms when the faculties that allow one, through meditation, to connect with the collective soul eventually wane until they disappear completely. The end of life then becomes inevitable, due to a sense of futility and the loss of what constitutes the essential reason for living.

---

[69] Whereas our works of science fiction, with the exception of books like *Brave New World* and others, most often represent nothing more than a projection of our own ways of behaving.

Since this process of degeneration affects the couple, both partners decide to end their lives. The procedure has long been ritualized. For the first time in their lives, they find themselves in the presence of their offspring. Then the two lie down on a bed. A transparent partition separating them from the audience gradually becomes opaque, and then both are instantly killed by microwaves. Finally, the atoms of their bodies are transmuted into helium. The partition then opens, and their descendants, by inhaling these few atoms, perpetuate an ancestral custom reminiscent of the familial cannibalism of the indigenous peoples of Papua.

The following is an excerpt from a letter received in 2003 by Dr. Jacques Costagliola (deceased):

> *Our medical techniques allow us to compensate for the physiological degenerations associated with old age. Our elders do not suffer from bone, cardiovascular, or cancer-related problems that are endemic on OYAGAA. However, in our old age we suffer from a form of brain degeneration that we neither know how to nor wish to prevent. This degeneration gradually robs our elders of all ability to communicate telepathically. Since oral communication is no longer possible due to the complete sclerosis of the vocal cords, they are condemned to silence. The degeneration continues until all ability to receive telepathic messages is completely lost. This loss of all natural communication abilities is accompanied by a slow and inexorable deterioration of memory. For our elders, this results in a progressive loss of autonomy that constitutes an unacceptable violation of the integrity of the OEMII[70] and of the sense of self-respect that you call dignity.*
>
> *If life were reduced to the autonomous maintenance of vegetative functions, our lifespan would exceed 140 Earth years. However, excluding accidental deaths, death on OUMMO is a voluntary act that occurs, with few exceptions, between 432 XEE[71] and 575 XEE (91 to 121 Earth years).*
>
> *CEREMONY FOR THE CONVERSION OF OEMII INTO HELIUM:*
>
> *For us, death is the ultimate culmination of existence and is just as significant as a wedding ceremony, the conception, and the birth of a child. The helium conversion ceremony, which is equivalent to cremation for us, is meticulously prepared by the elder who is preparing to die. This decision is generally made jointly by both spouses, who share it with their children and their minister. The use of a funeral home is then arranged for the OMGEEYIE[72] , which brings together all of their children and their spouses up to the fifth generation. A simple meal is followed by a moving farewell ceremony, at the end of which the family is invited to watch a documentary featuring selected clips that summarize the life of the OMGEEYIE who is about to depart. The elderly couple then withdraw side by side, holding hands, facing the audience, behind*

[70] Man in the generic sense of the term (the human species)
[71] Unit of time on Ummo
[72] The man-woman couple

a pane of glass that slowly becomes opaque so that they may enjoy one last time the joy of contemplating, through their descendants, their active contribution to the perpetuation and improvement of the OUMMO social network and the fulfillment of a lifetime's work.

The XANMOO[73] , which constantly monitors the biological parameters of the two OEMII, determines the precise moment when a deep peace washes over the OMGEEYIE, marking the entry into the meditative phase that signals the OEMII's acceptance of letting go. A flash of blue light signals to those present the moment a high-frequency gravitational field is emitted, causing instant death through the destruction of brain cells. The process of converting the atoms that make up the body's cells into helium then begins. Part of this helium is diffused into the room, mixed with an aromatic blend specially formulated for the occasion by the OMGEEYIE itself. Each family member then symbolically inhales the final scents of their ancestors, whose voluntary integration into the collective psyche of OUMMO is now complete.

For OUMMOAELEWE

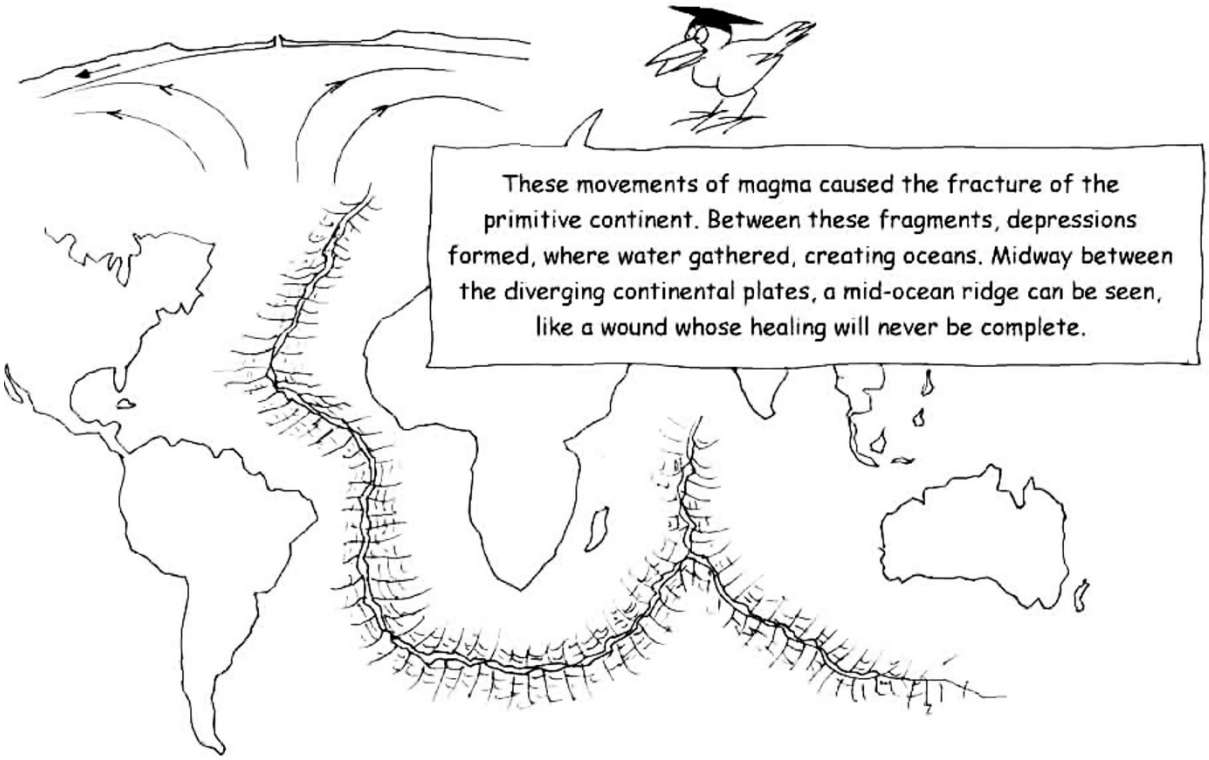
NABGAA 112, daughter of DORIO 34, approved by AYIOA 1, son of ADAA 67

What are we to make of all this?

Faced with this wealth of data, Jean-Jacques and I have different approaches and perspectives. We could summarize this by saying that these texts lead us above all to see our own planet in a different light, in all its aspects.

- *I see correlations in a number of points. The first is that their planet consists of a single continent, flat as the palm of a hand. Therefore, plate tectonics does not exist there. On Earth, it is convection currents within the magma that either push the continents apart or, conversely, cause them to collide. This has two effects. Where continents are drifting apart, the Earth's crust, the layer of solidified magma, is thinner. Midway between two continents that are moving toward each other, this layer cracks, and magma emerges and solidifies. This creates what are known as mid-ocean ridges.*

[73] A computer network spanning the planet.



Mid-ocean ridges

- *These ridges are like wounds that never heal.*

- *Exactly. Conversely, when two plates collide, one slides beneath the other, and the rocks that make up that plate soon reliquefy. On the surface, on top of the other plate, mountains form*[74] *.*

- *Thus, the Alps are the result of the collision of a plate spearheaded by Italy. The Himalayas: these are the result of the drift of a continental fragment, India, which, having broken away from southeastern Africa, collided with the Tibetan Plateau.*
-

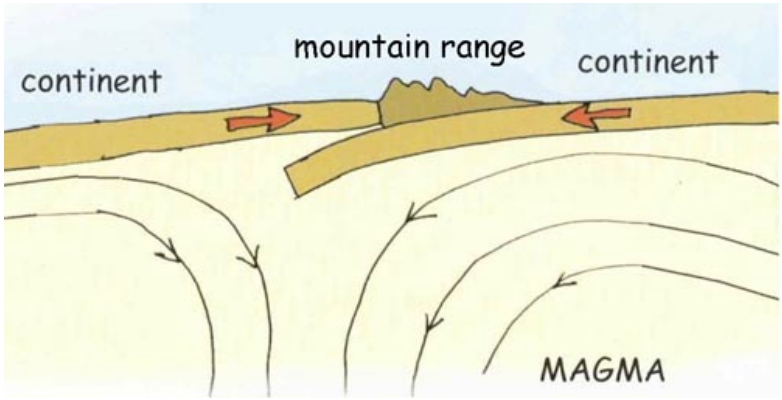


Plate collisions create mountains

[74] This is referred to by the term “orogeny.”

- *In young mountain ranges, erosion caused by precipitation, combined with temperature variations, can reach 2 mm per year. If the driving force behind this orogenesis were to disappear, if, with a wave of a magic wand, we could eliminate convective movements in the magma, within fifty to one hundred million years, the Alps, the Himalayas, and the Andes would be reduced to modest, easily traversable elevations. In one billion years, everything would disappear.*

- *Erosion, that is, rain, snow, and glacial movement?*

- *At high altitudes, the most effective form of erosion results from the alternating liquefaction and solidification of water in rock fissures. That's incredibly effective*[75] *. In this context, he has us look at what the situation is for the other three "terrestrial" planets (solid, as opposed to the gas giants like Jupiter and Saturn, and icy planets like Uranus and Neptune).*

- *So what's the story?*

- *Venus is the only one of the three planets that still has active magma, whereas Mercury is a completely inert, geologically inactive planet. Mars falls somewhere in between. But in none of the three do we observe a fracture in their mantle or a fragmentation into continents or solidified plates.*

- *But how could this be demonstrated? Is there no liquid water on these three planets?*

- *If we were to vaporize all the water in Earth's oceans, its mid-ocean ridges would reveal this phenomenon.*

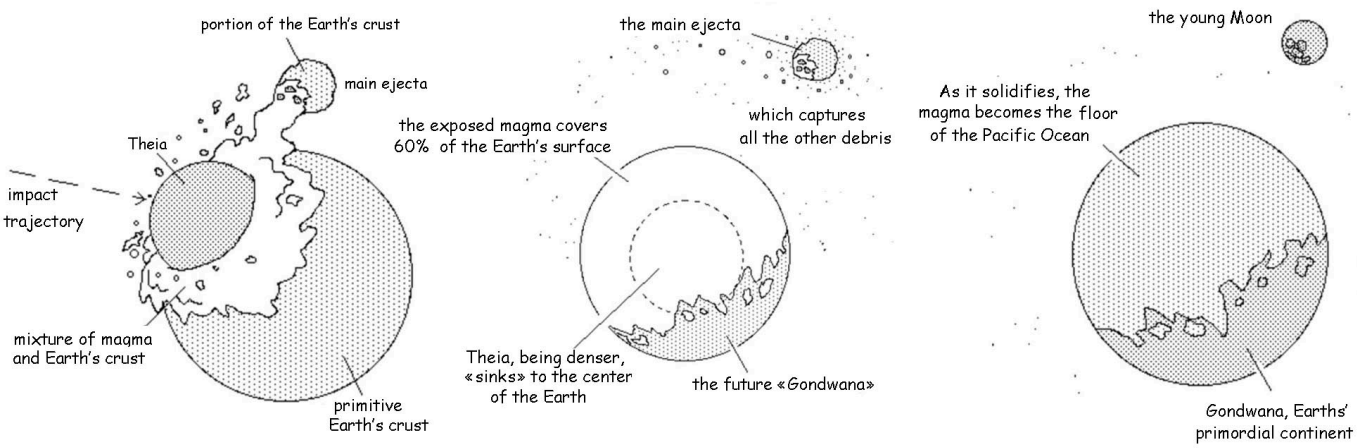
- *Thus, among these four solid planets in the solar system, only one has magma, whose continued activity led to the fragmentation of the primordial continent and the formation of mountain ranges.*

- *Why has Earth's magma remained more active than that of its three "sisters"?*

- *During the first lunar missions, astronauts brought back rock samples. To everyone's surprise, these samples revealed a composition very similar to that of Earth's mantle. This immediately established a collision with a celestial body the size and mass of Mars as the most plausible scenario for the Moon's formation*[76] *.*

[75] This forces climbers to begin their ascent very early, or even at night, using headlamps, to avoid rockfalls caused by the thaw.

[76] Astrophysicists have named it Theia (in ancient mythology, she is the mother of Selene, the Moon). With an estimated diameter of six to eight thousand kilometers (that of Mars) and a mass estimated at 20% of Earth's.



The formation of the Moon as ejecta following a collision, in 3 images

This is therefore the model now accepted within the scientific community. But there is one aspect that is never mentioned. This collision introduced a[77] e amount of kinetic energy that automatically converted into heat, thereby reigniting the Earth's magmatic activity. And this leads us to ask: "What would the Earth have looked like if this collision had not occurred?"

- *Would the Earth have had only a single continent?*
- *Without mountains, without these barriers, these many seas, lakes, and islands of all kinds that make the Earth a true biological mosaic. Without them, the Earth would have had only a very limited number of habitats:*
    - *Coastal regions*
    - *Large inland deserts*
    - *Differentiation based on latitude.*
- *Thus, the claim that Earth's biological diversity is hundreds of times greater than that of the planet Ummo does not seem absurd. On the contrary, it seems logical given its very unique history.*
- *And what applies to animal life also applies to racial, ethnic, historical, cultural, and technological differences.*
- *On Earth, human beings living in the Stone Age coexist in the heart of New Guinea, while in the United States, people are beginning to engage in space tourism.*
- *On Mars, there is a volcano, Olympus Mons, which rises to an altitude of 22 kilometers. But this is only possible in the absence of precipitation. If it rained on Mars, this volcano*

[77] Between 10^{31} and 10^{32} joules, according to estimates.

could never have re ly risen so high. At the slightest eruption, precipitation would immediately disperse the ejected material.

- *This therefore corroborates the absence of landforms and oceanic trenches on the planet Ummo.*

The Alicante Symposium.

The Madrid and Barcelona groups have decided to organize a symposium in Alicante, at a hotel by the sea. Indeed, for over a year now, the Ummites have been scarce. Previously, holding a symposium had always spurred the sending of letters and reports. The Spaniards are therefore trying to get things moving again by resorting to this method.

The press will be invited. A journalist from Madrid, Marhuenda, will be present. He is the first journalist to receive a letter, since the Ummites generally shy away from publicity. But two years ago, Marhuenda was on duty at a Madrid radio station on Christmas Eve. He had the idea to say:

- *"And now let's spare a thought for the extraterrestrials living on Earth, so far from their home planet and sometimes separated from their loved ones for many years."*

The Ummites immediately wrote to him, saying that this had deeply moved them.

Jean-Jacques won't be able to come. I decide to go anyway. When I arrive, the contactees are busy giving interview after interview on the radio and to the press. My arrival goes completely unnoticed. Ribera and Farriols are completely absorbed in their many organizational tasks. The presentations follow one after another before an audience of two hundred people. As usual, many sentences begin with the classic:

- *"Creo que*…" etc.

With endless rambling, in the Spanish style.

I arrive in Alicante around midday on Saturday. A group of young students, two boys and two girls, approach me. Two of them speak French fluently.

- *Do you know Rafael Farriols, Antonion Ribera, Juan Dominguez, etc.?*

- *Yes, I know them all.*

- *You're so lucky. We tried in vain to tell Mr. Ribera what happened to us last night.*

- *What happened?*

- *It was nearly midnight. We went to the beach. It was deserted, except for a disc-shaped craft sitting a few hundred meters away. Since the moon was up, we could see very clearly that a man got out of it and immediately disappeared into the nearby pine*

forest. Then the craft rose, moved away from the beach, and suddenly, it disappeared completely, as if it had dematerialized. We wanted to tell Mr. Ribera about this, but he told us he didn't have time, because the hotel was fully booked and he and Farriols had to urgently find rooms in local homes to accommodate more people than expected. And he just left us standing there.

The four of them seem quite disappointed, having come from all corners of the country to attend this convention. I go looking for Ribera and find him at the hotel front desk, holding a list of speakers and participants. I can hear him getting upset on the phone. He listens to me distractedly, but I quickly realize that his concern about finding lodging for a group of people is taking up all his attention.

The four of them leave, thanking me for trying. The last day of the conference, Sunday, is drawing to a close. The last visitors are leaving, one after another. I meet up with Rafael, Antonio, and Lou. We have dinner rather late, as is customary in Spain. Rafael then takes us aside to the hotel's deserted reception lounge.

He still owns his company, CRITESA, which manufactures all kinds of objects made of methacrylate, ranging from chairs and large transparent panels, replacing glass, to sinks and toilets. In his hand is his latest creation: a briefcase made of transparent material. He pulls a sealed letter from it.

- *Someone had a two-part document delivered to me through the hotel front desk. There is a letter signed "Ummo," and a sealed letter with an express request not to open it until midnight on Sunday. Well, here we are.*

In short, it says:

- *We attended your conference, where we stayed for forty-six minutes. That was enough time for us to equip a number of people with miniaturized microtransmitters, less than a millimeter in size, which you could easily mistake for a graphite particle. Self-adhesive, it can be attached to a person's shoulder with a simple press of the hand. In the room, we placed an object the shape and size of a pea on a ledge; this object could move freely and autonomously using an MHD system. Thanks to it, we were able to receive a complete broadcast of the presentations and discussions that took place in the room. At the end of the conference, we retrieved the sphere, while the "graphite flakes," having fulfilled their purpose, were transmuted into helium.*
  We have observed that no one in your group seems to have made the effort to truly understand and make use of what we have given you over all these years. Your speeches, articles, and books are nothing but meaningless noise and empty talk. The vast majority of your group's members continue to view the letters they possess merely as a means of gaining importance in the eyes of others.

That's harsh. To lighten the mood, I then recount what the four of them told me.

After a moment of silence, Rafael sighs and exclaims, "Hostia![78] "

[78] In French, this would translate to "Nom de Dieu!" and in English to "Damned!"

A lull.

Following this discussion, the new letters from Ummo appear to be a curious mix. First, there are letters that, while bearing the purple typewritten imprint of the earlier documents, and where an examination of the characters confirms that the document was produced by the same typewriter, can be classified as:

- Either as forgeries

- Or a disinformation campaign carried out by the source itself.

Lou thus has a very hard time dissuading Rafael from acting on a request made in a letter to travel to the Moroccan desert, taking with him a thermos bottle containing a sample of his semen.

In another letter, the Ummites announce their decision to reveal their presence to the scientific community by transmitting a message on a microwave frequency. The date and time of the transmission are specified, along with the latitudes and longitudes of the antenna locations on Earth where the message can be received.

Rafael immediately went to Jodrell Bank, the site in England where one of the most powerful radio telescopes was located. He showed the letter from Ummo to the officials, adding that he was willing to cover the costs of the operation. But he was immediately dismissed as a madman and thrown out.

Lou called me, asking me to intervene:

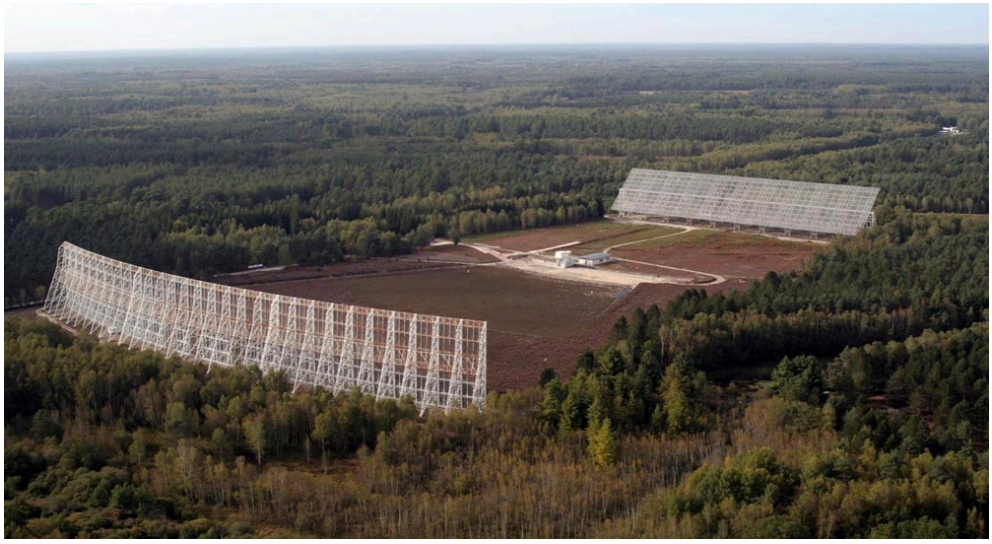
- *Rafael wouldn't want to miss this opportunity, if it presents itself.*

- *But, Lou, if these people are planning to send a message to ground stations, it would be easier for them to coordinate that transmission with their observation schedule, which is relatively easy to obtain.*

- *You know what they say in Spain. Rafael, to be sure of winning the lottery, is ready to buy every ticket.*

As a side note, I managed to convince the head of the French radio telescope in Nançay, my friend Biraud, to point the system in the opposite direction at the specified date and time for eight minutes.

Of course, I didn't mention anything to him about this Ummite letter. But a few hours earlier, I cited a rumor circulating about the observation of a supernova.



The large French radio telescope in Nançay, Loire Valley. Established in 1960

Not wanting to miss a chance to make a historic discovery[79] , Biraud sets out to do so, but finds nothing.

The Puzzle

I really like the way Jean-Jacques works, and that's what psychologists do: beyond what people tell them, they strive to read between the lines of what the speaker is expressing in their speech.

As he sifts through all these accounts, reports, and letters, Jean-Jacques is looking for a pattern, an underlying structure:

- *Even though we now know that this whole story can no longer be dismissed as fiction, the question remains: what lies behind it? Does this damn planet populated by Nordic-looking people actually exist? Behind everything they say, I find contradictions.*

- *What are they?*

- *For example, in a long letter received by Sesma in 1966, devoted to their daily lives, there's a section describing the life of a typical family. And there, it feels as though we're in the midst of an American middle-class family.*

- *Why not?*

- *What doesn't add up is that they talk about games, adding that they joke around with each other.*

[79] The frequency of supernova explosions in our galaxy, the Milky Way, is one per century. The most recent one occurred in 1987. Many instruments, which were undergoing maintenance at the time, missed this unique opportunity. The previous one was observed with the naked eye by Johannes Kepler in 1604.

- *So what*?

- *How could beings whose fingertips are so sensitive that they can't even press a button in an elevator possibly handle any kind of toy, even a simple balloon? And if they know how to joke, why is there no trace of it in the thousand pages we've gathered? And how do you explain that in these pages there isn't a single metaphor, no imagery, no allegory?*

- *Yes, it's quite striking. Their language seems entirely factual; everything is taken at face value. There's no double meaning anywhere.*

- *In contrast, we humans don't shy away from saying several things at once, playing with word choice, intonation, and rhythm. In our spoken language, as in our writing, we blend the real and the imaginary, objects and images. And added to all that is the fact that our conscious and unconscious minds intertwine.*

- *Okay, so what?*

- *Well, I read in a report that these guys can express themselves, fully consciously, using a "bi-language" with two codes. The first is represented by the sequence of words, the second by a sort of stuttering, similar to Morse code. They can thus say two things in the same sentence, possibly unrelated to one another. To such an extent that, they say, a teacher can deliver two lessons, in two different subjects, at the same time!*

- *Wait, that's completely crazy! It's as if these people had two brains!?*

- *I'm not saying it's the same for us, since we already make a distinction between how the right and left hemispheres of the brain function. But in any case, in the way we think, express ourselves, and behave, we live in two different mental realms, each just as important to us as the other: the real and the imaginary, the conscious and the unconscious, the tangible and that which belongs to fantasy and dreams.*

- *The word "dream" is an important part of our vocabulary, yet it is completely absent from the 1,350 words that appear in the Ummite letters! They never mention it, neither in reference to themselves nor to us. Does that mean they don't dream?*

- *Worse still: that they do not even understand what it refers to in our world. The corollary is that they have no imagination, no symbolic expression, and therefore none of the artistic expression that automatically flows from it.*

- *The only way they understand the universe is through an exclusively scientific approach, in all fields. Even their metaphysics is built on such foundations. Throughout their thousands of years of history, they have never, , given rise to any religion, any god, or any myth. They have built no temples and practiced no rituals. And that is truly extraordinary! They discovered the soul quite by chance through the behavior of pairs of krypton atoms attached to DNA molecules, which reveal a connection between living beings and an invisible world, their "noosphere", which they refer to by the term BUAAWE BIEI, abbreviated as "BB."*

- *In fact, they are completely incapable of imagining anything, since they lack an imagination. At one point, they discuss what they believe to be the reason why the visual arts never emerged among them. They say that, very early on, their ancestors discovered that by piercing a small hole in a box, they could see the image of the outside world projected onto the opposite side, which acted as a screen.*

- *The classic camera obscura[80] .*

- *Then they add that the technique for fixing images was also quickly mastered.*

- *This reveals people who have not the slightest idea of the role art plays in our civilizations, or of its function. For them, pictorial art could only strive toward the most perfect realism. They are oblivious to anything symbolic, dreamlike, fictional, or fantastical, simply because they cannot even conceive of it. The idea of constructing buildings that transcend their function as mere dwellings, warehouses, factories, or research laboratories, or of designing clothing that is more than just a piece of fabric with holes for the head and arms, is completely beyond them.*

- *Do the words "beauty," "architecture," "poetry," "literature," and "music" mean nothing to them?*

- *Wait, I'm thinking of a passage where they mention the many different ethnic groups that visit us. Among them are small beings who, in the 1950s, were reportedly responsible for the majority of UFO sightings. And they add, "After spending two years on Earth, their curiosity having been satisfied, they left and never returned."*

- *Only two years to take in all of Earth's wonders, our architectural, artistic, musical, and literary heritage?*

- *It's not because they weren't interested. It's simply because they don't perceive that dimension.*

- *It's like deaf people being asked to listen to music, or blind people wandering through an art exhibition. It's so foreign to them that they try to compare it to their art of blending scents. Farriols, intrigued by this idea, even built a very primitive scent organ at home, with keys that trigger the release of essences.*

- *Trying to compare this to a piece of music is like imagining a composition based on a single note, where the key would shift as the instrument changed, from violin to organ, then to bassoon, or a color that passes through every shade of the prism.*

- *This explains why the only thing that resonates with them here is what we call the culinary arts, and why one Ummite recounts having been so overwhelmed by his first bite of veal blanquette that it took him hours to recover.*

[80] The earliest record dates back to the Chinese philosopher Mozi. In 1020, the Arab scholar Alhazen, in his book on optics, provided the first detailed description of a supernova, explaining that light travels in a straight line. In the West, the term did not become widely used until the 16th century.

- *Conversely, they couldn't care less about the rest: our cathedrals, our musical, artistic, or literary works, the very things that make up our culture and that we hold so dear.*

- *What serves as their culture is entirely centered on scientific knowledge grounded in their fundamental pragmatism. From this perspective, there is no speculation, no intuition. For them, scientific progress revolves entirely around logical deductions based on experimentation and observation.*

What emerges from Jean-Jacques's reflections is something entirely new in the UFO landscape. Many people believe that the extraterrestrial species visiting us may have ways of thinking very different from our own, and that it may therefore be difficult for us to understand how they function.

But the reverse might simply be true.

The implant.

I'm home alone. Jean-Jacques's ideas impress me. Everyone plays with their own set of puzzle pieces. Compared to what he's describing, with my two universes where time flows in opposite directions, I feel like nothing more than a simple technician-tinkerer.

But hey, to each his own.

It's true that certain UFO sightings, classified as close encounters, defy any attempt at interpretation using our scientific tools, no matter how advanced they may be. For instance, in the United States, a man named Robert Salas has just revealed in a book what happened to him in the 1960s.

At that time, the United States had just deployed about a hundred Minuteman intercontinental ballistic missiles, each carrying a nuclear warhead[81] , at Malström Air Force Base in North Dakota, divided into a dozen units, each managing ten missiles. Each unit was managed by two launch officers, who took turns on duty around the clock, housed thirty meters be , in a spring-mounted capsule located one thousand meters from their own missiles, so that they would be able to respond even in the event of an enemy strike. Their sole function is to wait for the order to press the button, which would then be given to them by the President of the United States.

On the surface, a group of soldiers monitors the site, restricting access to a fenced-off area

[81] Today, the United States has 400 Minuteman III missiles in silos, a system supplemented by those carried by B-52s and B-2 stealth bombers. Added to this are 14 Ohio-class nuclear submarines, each capable of carrying up to 20 Trident II multiple-warhead missiles or cruise missiles. Taken together, this represents the capability to launch strikes using 1,700 operational nuclear warheads, plus another thousand in reserve.



.

The Oscar Flight Fire Control Unit
The door in front of which the UFO hovered

One night, Salas received a call:

- *"Captain, there's a big red disc-shaped thing that's been hovering in front of the sliding gate for quite a while now."*

- *It's simple, soldier. If that big red thing asks for access, you deny it.*



Robert Salas, at the time of the events

Then, suddenly, everything changes. Salas, sitting at his launch console, doesn't have time to see the result of the joke he's just made when he suddenly sees the ten green lights on the ten missiles turn red, indicating "no-go."



The missile launch capsule, located thirty meters underground.



Minuteman missile in its silo

He immediately calls the control center that oversees the entire site.

- *"We'll come and relieve you, and you'll report to the debriefing room."*

There, he and his colleague join the other launch officers, who have all experienced the same thing. They are asked to sign a document binding them to absolute silence.

This event had a profound impact on him: He decided to leave the military and retrained as a physics teacher at the middle school level.

But years later, after discovering a description of the event in a book written by a ufologist, he called his superiors, who told him that there was no longer any reason preventing him from testifying about his own experience.

He then published a book recounting it.

But years later, one night, he and his wife found themselves levitating thirty centimeters above their bed. Beside them were small beings with large black eyes. Both of them, unable to make the slightest movement, saw themselves being guided, his wife toward the wall, and he toward a glass door. He had one last thought:

- *How are they going to open it? I locked it.*

But he passed right through it. His wife, meanwhile, passed straight through the wall. And no earthly physics can offer even the slightest explanation for this[82] .

Both find themselves lying on a bed in a circular room that can only be the interior of a UFO. The little beings lean toward them, and Salas feels one of them insert a tube into his penis. Before losing consciousness, he says:

- *"It hurts!"*

In his book, I discover that I've had a similar experience. A sort of standard scenario, so to speak. On that note, I'm going to bed.

The next morning, in my bathroom, as I take off your T-shirt, I notice a strange mark, ten centimeters to the left of my navel.

[82] Years later, when I heard the account of a woman who vividly described having had the same experience, she explained that the moment her feet entered the wall of her bedroom, she saw them "surrounded by tiny luminous particles."



The scar

There's a red border, 5 cm long, horizontal, surrounded by two crescent-shaped bruises.

You can scrape yourself and end up with a cut. You can bump into something and end up with two bruises. But both at the same time, in the same spot, that seems unlikely.

I show this mark to a friend who's a surgeon. After palpation.

- *This scar isn't superficial. You can tell there was a procedure in the deeper layers.*

Months go by. During a trip abroad, I discover I have a supraumbilical hernia. As we age, the abdominal fascia can sometimes give way, causing a small fistula to form through which the stomach wall protrudes.

I went to see a doctor:

- *It's no big deal. Here's the address of a clinic where they'll stitch it all up for you.*

I was seen by a surgeon who confirmed the presence of this small hernia. But his attention was drawn to something else. Upon palpating the mark I have to the left of my navel, he noticed a deep scar. So this isn't just a superficial issue.

- *So, what happened to you there?*

When I tell him the origin of this mark, his smile suggests he imagines that, having been struck by something during some obscure family affair or the like, I'm reluctant to talk about it and am making up this whole story.

A few days later, as we're heading toward the operating room, me lying on a gurney and him in his green scrubs, I say to him:

- *"If you find anything inside, keep it for me."*

- *"Understood."*

General anesthesia. In the recovery room, the first thing I see as I come to is a small, transparent cylindrical box with a red lid, slipped between my fingers.

A nurse approaches:

- *"This is what the surgeon found inside. But I'm sorry, we can't let you keep it. We have to examine it."*

And she leaves with the box and its contents, a shapeless, whitish mass measuring two cubic centimeters.



The implant

I realize that to extract this object, located ten centimeters from my gastric fistula, the surgeon must have grabbed it with long forceps. The very next day, I board a plane back to France.

What was this object? We'll never know for sure. Indeed, when a surgeon finds biological residue inside a patient's body, it's their duty to ensure that it doesn't contain cancer cells. If it doesn't, it goes straight into the trash. From France, her comment:

- I need to have this piano tuned.[83] '

[83] Years later, however, this story would take a colorful turn. The Japanese translator of one of my books, Professor Nakajima, decided to spend his sabbatical year in France, choosing Aix-en-Provence as his first stop. The very night after his arrival, he experienced the same phenomenon!

Paris, July 1947**, 5 rue Jean-Baptiste Dumas, ground floor on the right.**

After pushing open the heavy oak door leading onto the street, I find, on the right, the glass door leading to the apartment where my mother and I lived in 1947, facing the street on one side and an inner courtyard on the other. I was ten years old at the time.



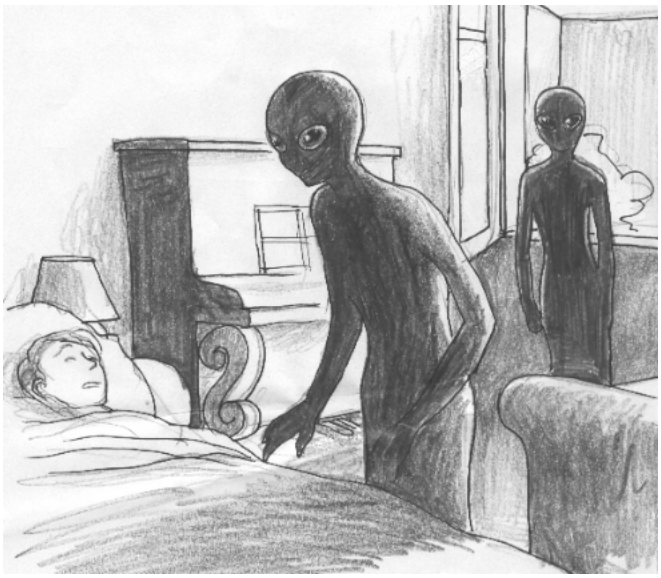
5 rue Jean-Baptiste Dumas, in Paris's 17th arrondissement

I ring the doorbell. The premises are now occupied by a law firm.

- *I apologize for disturbing you. It just so happens that my mother and I lived in this apartment in 1947. I'd like to see my childhood bedroom again, it's at the end of the hallway on the left. Just a minute.*

- *All right, go ahead.*

I find the room. As is often the case when you revisit places you knew as a child, it seems smaller to me now. The fireplace on the right is gone. I picture myself back in July 1947. It was summer. It's hot, and the double-hung window is wide open. Suddenly, an object surrounded by light descends and lands in the courtyard, which is enclosed by high walls. Two little figures climb through the window and head toward my bed. They're both dressed in a kind of tight-fitting black outfit, and both have large, black eyes.



The Visitors, July 1947 (around the time of Roswell)

I feel no fear. Once their inspection is complete, they leave the way they came. The next day, I notice something. When I look at the old piano in that room, I can make it produce sounds from a distance, as if an invisible hand were plucking the strings. I tell my mother about this, and she replies:

- *"Yes, I really need to have it tuned!"*

But who are they?

After returning once more to the ridge of La Blache, we had to face the facts. To go any further, we'd have to clear away the vegetation that didn't exist back then. With all those tree canopies, it was impossible to see the other marked points. This realization marked the end of our field investigations.

Jean-Jacques:

- *Yet the location was well chosen. For those with good legs, it can be reached by multiple routes. It's also the area where the Resistance fighters had chosen to take refuge. It would have been very difficult to surround the plateau unless one had a large force. And from there, one could see people coming from afar.*

- *What amuses me are the reasons behind this choice. They opted for a semi-wild region, located nonetheless behind what they believed to be a research center, due to the presence of large structures on the surface. In Digne, in the Basses-Alpes!*

- *I always come back to the constraints stemming from the hypersensitivity of their fingertips.*

- *Go ahead, Sherlock Holmes.*

- I always get stuck on this point. We agree that it's an unnatural trait. It's hard to see how a biped could derive any benefit from this that would compensate for the loss of any ability to handle objects, whether weapons or tools. It could only have been achieved through genetic manipulation. But then, why would they have chosen this type of evolution? At night, a simple lamp is enough.

- Unless the species is nocturnal, with night vision. Otherwise, it doesn't make sense.

- And it clashes with a host of other things. The highly developed sense of smell, the scant attention paid to conventional vision. The total absence of pigmentation is also a characteristic of species that live in the absence of light.

- In most third-kind encounters, when witnesses come face-to-face with the visitors, they frequently describe them as having huge black eyes. What if, to them, they were just sunglasses? In the film presented on television by journalist Jacques Pradel[84] , which purports to show the autopsy of an alien with six-fingered hands, the first action taken by the Earth doctors tasked with examining the body, who are wearing protective suits borrowed from a nearby nuclear facility, is to very delicately remove the black membranes covering the alien's eyes.

-



The autopsy video. The black membranes[85]

[84] Radio program *L'Odyssée de l'étrange*, October 1995

[85] According to the classification by the Spaniard Darnaude, we read (I will find the exact reference):

Il y avait un réflexe de ce type qui finit par être commun à tous les sujets: se caresser le visage, la poitrine et les seins. Presque tous sentirent fortement les stimulations lumineuses, ce qui nous obligea à les pourvoir de UULAXBOIYU (à cette époque où l'on ne contrôlait pas directement les aberrations oculaires, on mettait aux presbytes et aux myopes des lentilles artificielles qui s'appliquaient en pulvérisant une substance transparente sur le globe même de l'oeil. Celle-ci en se cristallisant et en durcissant, formait une pellicule soluble adaptée à la cornée, semblable aux microlentilles que vous utilisez sur terre. Cette substance plastique, que nous appelons BIAGOO DOAWAA a la propriété de pouvoir varier son opacité en fonction de la lumière. Si vous aviez un tel produit vous pourriez l'appliquer sur vos lunettes de soleil.

On the left, the eyes covered by these black membranes[86] . On the right, those enormous pupils.

- *Membranes that would automatically darken in the light. A technology explicitly suggested by the Ummo texts as early as the 1960s, at a time when it was unknown on Earth.*

- *And when these membranes are removed, eyes with large pupils appear.*

The details follow one after another.

- *Let's think about it. On a planet with a single continent, located near its equator and as flat as a pancake, the climatic and meteorological conditions must be very specific. One immediately imagines vast desert or near-desert regions. Heated during the day by IUMMA, the "Sun" that illuminates the planet Ummo, the ground cools more rapidly through radiation than the ocean mass[87] . The air mass over the oceans becomes warmer than that over the land, which tends to sink inward. The result is the expulsion of this air toward the ocean, thus creating a land breeze. This breeze will never be strong.*

  During the day, the ground, exposed to solar radiation, if unprotected by vegetation and if evaporation does not moderate its heating, will heat up rapidly and significantly, and transfer this heat to the adjacent air mass, which will expand and rise. To equalize the pressures, the marine air will be pushed toward the center of this continent.

  On Earth, these sea winds are very quickly halted by even the slightest elevation and are then referred to as "sea breezes." But on the planet Ummo, such elevations do not exist. We can therefore imagine a daily wind pattern affecting the entire continent, a massive continental vortex, with speeds easily exceeding 100 kilometers per hour, every day. A researcher equipped with simulation tools could provide a more precise estimate of the possible wind speeds.

  Consequently, adopting a nocturnal lifestyle and living underground immediately becomes the standard solution for most species, including humans, to protect themselves from such daytime winds. The Ummo texts do indeed mention this survival strategy of their ancestors. They also speak of the emergence of storms so powerful that the dust particles carried from the deserts manage to erode the rocks exposed at the surface.

  In such a biotope, it would be impossible to use roads, as they would have to be cleared every day of everything the winds deposited on the surface.

- *In ancient Egypt, it was the same. How could one even consider using roads, and thus wheeled vehicles, along routes that would require immense annual efforts to clear away*

[86] An element that was not the stuff of science fiction in 1947.

[87] The daytime heat input from solar radiation affects, on average, only the top 15 cm below the soil surface, whereas conduction causes this effect to extend into the ocean to a depth of 10 meters. Thus, the amount of heat is ten to thirty times greater for a liquid mass. The ocean's thermal inertia is therefore significantly greater than that of the soil.

the sediment deposited by the Nile's floodwaters? The only mode of transportation became waterways, on the Nile or on these canals.

- So, on Ummo, as technology develops, what do we see emerging?



Land-based transportation on Ummo. Their vehicles with four articulated legs[88]

- *There is another aspect that stems from all this. These people do not seem to be the least bit bothered by overcrowding. The interiors of their torus-shaped ships*[89] *offer only a very limited amount of space. When one considers the number of passengers they are capable of transporting, the living conditions that result are unimaginable to a human from Earth.*

- *But this is something that would not bother a being whose ancestors lived in burrows.*

- *Another point: in very rare cases, which are, of course, open to debate, some autopsied bodies exhibit reptilian characteristics. On Earth, birds (which are absent on Ummo) turned out to be the descendants of dinosaurs, from which they have retained certain characteristics. It is by no means out of the question that on other planets, the evolutionary line leading to intelligent, humanoid beings capable of developing technology might have its origins in a reptilian phylum.*

- *And among terrestrial reptiles, infrared perception replaces vision in the conventional spectrum.*

- *Hence this tendency to develop a form of dactyl vision, focused on infrared.*

---

[88] While the image might have raised a smile in the 1960s, today we have the first robotic vehicles equipped with legs, capable of carrying loads, or even a human being. This design has caught the attention of the military for a variety of terrains: in ruined cities where armored vehicles, unable to advance further, become easy targets, and in open terrain, where they can avoid roads that may be mined.

[89] Ummite spacecraft have an overall diameter of 13.77 meters

- *Also, when they call the Spaniards on the phone, it's always in the middle of the night, around 2 or 3 in the morning. Perhaps that corresponds to… their office hours?*

- *And yet, in the long text received by Sesma in 1966, we find a description of a typical day for a family on Ummo.*

- *We don't have to take everything in these pages at face value, as the Spaniards do. Remember what they keep repeating: "Don't believe us." There are also those mushroom-shaped houses, which they say emerge from their dwellings when night falls.*

Back to San José de Valdeiras.

Jean-Jacques points out a letter to me written by the typist, who typed the letter and sent it to one of the contactees in Madrid the day after that landing.

> *Sir,*
>
> *I am the person who types the letters that were sent to you. I was hired after seeing an ad in ABC magazine, which was looking for someone to type letters, so I applied.*
>
> *A very kind man, who said his name was DEI 98, a strange name, with a number, then dictated many very strange letters to me. I wondered if they might be people from the secret service. But my wife told me I wasn't doing anything wrong and that, since it paid well, I could continue. So I began typing the many letters and sending them according to a schedule that had been specified for me: this letter, to that person, on that day.*
>
> *But one day, a woman introduced herself as having come on behalf of Mr. DEI 98. She seemed to be the leader of a group and asked me if four of them could stay with us for the night. I offered to give them our bedroom, but she said there was no need.*
>
> *That evening, they arrived. She spoke, but the others did not. The others seemed mute. I didn't hear them utter a single sound the entire evening. The woman was wearing a brown suede coat, quite elegant and modern. There was a very young woman and two men. We offered to prepare a meal for them, but the lady said they declined. One of the two men then went out onto the balcony and opened a suitcase he had brought with him. He took out objects the size of a golf ball which, when he let them go, hovered in the air on their own and then drifted away.*
>
> *The younger woman kept looking down whenever her superior spoke to her. Whenever the older woman turned to the younger one, she had an angry look on her face, and it occurred to me that the younger woman had been punished for something she'd done that she shouldn't have. I asked them if they wanted us to arrange something so they could lie down and sleep and have something*

to eat, some fruit, for example. But the lady said I shouldn't worry about it, and she insisted that we leave them to themselves and go to bed in our room.

I couldn't sleep, and my wife was getting on my nerves, repeating over and over that she now believed they were secret service agents. You know how it is; once women get an idea in their heads, there's no stopping them. Since she was getting on my nerves, to calm myself down I decided to go into the living room to see if they needed anything. Two of them were lying on a sort of foam mattress. The woman told me not to worry about the carpet, and that the stain would come out on its own. She told me they were there because someone was coming to pick them up. The man with the suitcase was still on the terrace and seemed to be waiting for his marbles to come back.

I went back to the bedroom, and the hours passed. I just couldn't fall asleep. I decided to go back to the living room to check on them. But they were all gone. The next day, I read in the newspaper that an alien spacecraft had landed in San José de Valdeiras. I then remembered the letter I had written to you and to a list of people that Mr. DEI 98 had given me, announcing the arrival of a spacecraft from the planet Ummo. I had typed it out as one might type a novel, without paying much attention to it. But now I realize that it was true.

I know it's not right to write without giving my name.

- *In another letter, after the man had been reassured, probably with a substantial raise in his salary, and even authorized to write to the contactees himself, provided he did not reveal his identity, he notes the care with which DEI 98 has him proofread each letter and adds:*

  At one point he even immediately interrupted my work, exclaiming, "There's a typo!" I checked, and he was right. I had typed "EI" instead of "EE." I then asked him how he had noticed that, given that when he dictates the letters, he stands at a distance on the couch and always keeps his eyes closed. He replied that he had become familiar with my typing and that each key had a distinct sound, which allowed him to identify them by ear.

- *Yet another clue pointing to a nocturnal species that is active at night.*

The warning.

Here, I'm the one who stumbles upon a brief passage in these letters.

I consider myself a sufficiently competent physicist to be able to understand the evolution of scientific and technical disciplines on Earth over the past sixty years.

In 1905, New Zealander Ernest Rutherford confirmed the existence of the atom through an experiment.

That same year, Albert Einstein published his famous paper that would mark the advent of Special Relativity[90] . In that paper, he introduced his famous equation:

$$E = mc^2$$

In response to this, the renowned French mathematician Henri Poincaré, who closely followed all developments in physics[91] , wrote:

- *I doubt that we will ever be able to destroy a city with a pound of matter.*

Yet, exactly forty years later, it was the conversion into energy of just 0.7 grams of matter that destroyed the city of Hiroshima.

Forty years to go from a purely geometric and mathematical speculation to the most terrifying and indisputable experimental application of a theory ever made.

All the Ummo texts suggest, time and again, is that we can always go further, much further, in any technology.

By acquiring the ability to recreate fire, humankind has done nothing more than apply the basic principles of chemistry to its own advantage.

With nuclear physics, we have only just begun to explore, through fission, a new field: that of "nuclear chemistry"[92] . We have continued this with the fusion of two hydrogen isotopes[93] , implemented in "hydrogen bombs"[94] . This constitutes our first, albeit modest, conversion of mass into energy. Logically, there seems to be no doubt that we will one day be able to carry out the conversion of much larger quantities of matter, perhaps by employing other mechanisms that we are not yet aware of.

Current nuclear weapons, classified as "tactical," are "only" ten times as powerful as the bomb dropped on Hiroshima. If we exclude the fewer megaton-class anti-city weapons and multiply by the number of warheads that would be deployed in a nuclear war, we arrive at the equivalent of hundreds of thousands of Hiroshimas.

That is to say, the energy equivalent of just a few hundred kilograms of material.

---

[90] Four years later came General Relativity, when he published the field equation that fully encapsulates it.

[91] Although he was not a rival to Einstein in this field, he was the first to point out, in a lecture given in 1904 in the United States, that physics was in crisis. Let's say that he identified the flaw, and Einstein overcame it.

[92] By comparison with chemistry, this can be described as "autocatalytic dissociation" mediated by neutrons

[93] Deuterium, whose nucleus consists of one proton and one electron, and tritium, which consists of one proton and two neutrons; this fusion produces helium (two protons and two neutrons) plus one neutron, along with energy associated with a loss of mass.

[94] In fact, the explosive in H-bombs is a solid fuel consisting of a hydride of the lithium isotope Li-6 and deuterium, the hydrogen isotope H-2, which has two nucleons. The heat dissociates the lithium deuteride, producing two free atoms, one of deuterium and one of lithium. The lithium atom then captures a neutron emitted by an A-bomb, which serves as the trigger. 6+1 = 7 = 4 + 3. The lithium then decays into helium and tritium. The tritium then combines with the deuterium, releasing energy. Thanks to Andrei Sakharov, the Soviets were the first to implement this "dry bomb" technology, which was immediately operational and could be carried by a bomber, at a time when the Americans were still testing a mixture of deuterium and tritium in liquid form. It was this sudden Russian breakthrough that triggered the arms race.

What, then, would be the effect of a single weapon that possessed such power, or even more?

We tend to overestimate the effects of a conflict waged with current nuclear weapons on a global scale. Of course, as Nikita Khrushchev once said, "the living will envy the dead": cities razed to the ground, crops destroyed, infrastructure and means of communication wiped out.

But above all: severe biological impacts, linked to the effects of radioactive fallout across the entire planet. Perhaps, for the human species, a recovery spanning centuries, or even millennia, at the end of which Nature would eventually reclaim its rights, as it is doing in Chernobyl.

On the scale of a planet's history, which spans more than four billion years, a thousand years is a mere blink of an eye.

Some people imagine, in the wake of the Malström incident, that aliens would intervene at the last moment. Nothing could be less certain.

Let's imagine, in 1939, a German teacher who, living close enough to the border, picks up Adolf Hitler's speeches. He realizes that war is imminent.

A flying saucer lands on his property, piloted by two aliens, who make contact with him and confirm the accuracy of his assessment. He might then be tempted to say to them:

- *"You clearly possess technology far more advanced than ours in every field. You could prevent this by causing Hitler to die of some sudden, devastating illness."*

- *Think again. That would only delay events. In this case, you're in luck. The Nazis, in order to purge Germany of "Jewish science," expelled all Jews in 1933*[95] *.*

  The Nazis have made significant progress in many fields. They are very close to putting their jet aircraft, designed by engineer Messerschmidt, into service. They had their flying bombs, the V1s[96] *. Werner von Braun, the former student who dreamed of setting foot on the Moon, was secretly developing intercontinental rockets in Peenemünde; their range was still modest, and their payload capacity remained inferior to that of the most powerful bombers.*

  The Nazis would quickly recover from their leader's death. Another would take his place. Himmler, for example. Nuclear research was never abandoned

[95] While visiting the United States in 1933, Einstein, upon learning that the Nazis had ransacked his home, decided it would be safer not to return to Europe and accepted a position created specifically for him at the Institute for Advanced Study in Princeton, which had been founded in 1930 to better track advances in modern physics.

[96] The first cruise missiles in history.

> *on their soil*[97] *. Imagine if they realized their mistake, then doubled down and acquired nuclear weapons, weapons that Von Braun's rockets could this time carry from one continent to another, across the entire planet. That would give them the ability to take control of the planet. No, believe me, Hitler is the lesser evil. Of course, it will cost you some 50 million additional deaths, on top of the 20 million from your First World War. But, on a global scale, that's still a drop in the bucket.*

These weapons of the future, those that will mark the ultimate mastery of matter, will be capable of returning the Earth to the state it was in, not a thousand years ago, but a billion years ago. Vast portions of the landmass would be vitrified, forever unfit for the emergence of life. Only life in the oceans would survive.

In their letters, the Ummites describe their exploration, using probes, of a planet located 2,000 light-years from their own, whose inhabitants are said to have simply committed suicide by deploying what they call "terrible plasma weapons," simply because they had never dared to test them. The surface of their planet was now nothing but expanses of vitrified soil, devoid of all life.

While technology and the ability to develop weapons (blunt or projectile) that transcend its biological limitations have given this frail creature, man, evolutionary advantages over other species, Furthermore, while conflicts within the species have created a form of "social Darwinism," all of this very quickly led to total "hypertelia"[98] (from the Greek hyper, υπερ: above, beyond, excessively; and telos, τελοσ: end, goal, fulfillment). The weapons the human species has developed now transcend the purpose of warfare.

Technology remains an "emerging phenomenon", inevitable on all planets harboring intelligent life. Its hypertrophy, as a weapon, is equally inevitable. It will therefore be crucial to attempt to discover its true purpose, other than enabling humanity to destroy itself!

What remains surprising is that very few people approach the issue from this angle.

Even someone like Alain Juillet, former head of economic security for the French intelligence services, who has been aware for decades of the reality of the UFO phenomenon and the fact that it reflects visits from civilizations on other planets, focuses (as in every country) on the issue of national security.

He said in a recent interview:

---

[97] In 1939, Germany established the Uranverein, a program aimed at building a nuclear reactor and, later, an atomic bomb. The Haigeloch reactor contained one and a half metric tons of uranium, moderated by several metric tons of heavy water. After the war, German nuclear scientists were interned in England at Farm Hall. When news of the Hiroshima explosion reached them, Werner Heisenberg went to the blackboard and immediately sketched the diagram of an atomic bomb and presented the calculations behind it.

[98] Evolutionary biologists cite numerous prehistoric animal species that went extinct as a result of the excessive development of certain physical traits: the gigantic antlers of the prehistoric deer Megaceros, the disproportionately large fangs of the saber-toothed tiger

- There have been very rare cases where Earthlings have been attacked by UFOs. So, if they were to attack us, what would we have to defend ourselves?

The visitor always remains a potential enemy.

Humans remain incapable of perceiving the UFO phenomenon as a mirror that invites humanity to look into it and finally see itself as it truly is.

My stance, as a scientist

I have made this phrase of my late friend Aimé Michel my own:

- *Consider everything, but believe nothing.*

It is essential, at all times, to refrain from drawing conclusions. And that is what I do. But at the same time, it is crucial to set one's skepticism aside. This leads to a rather precarious balancing act. But, in the end, one gets used to it.

The only thing a scientist can hold onto is consistency between facts and ideas, knowing that everything can shift, one way or the other. In Kepler's day, proponents of heliocentrism could only imagine planetary trajectories following circles[99] . And their opponents had an easy time saying:

- *We are willing to consider even the wildest ideas, such as the possibility that the Earth might be moving[100] . If it is moving, it must be subject to a force, and we know that forces act more strongly on heavy objects than on light ones. This is obvious. All you have to do is drop a pebble and a feather from the same height. The pebble hits the ground first. So if the Earth, subject to a force, were to move, we, mere specks, would be left behind in the process. Furthermore, it compels us to stick to the facts. The epicycloidal motion of the planets has wonderfully accounted for observations for centuries[101] ; this is what we need to enable our astrologers to do their work. Whereas heliocentric theories are incapable of accurately predicting even the slightest eclipse.*

For the scientific community, the UFO phenomenon is a non-issue, since, given the duration of even the shortest journey to the nearest star[102] , which immediately runs into centuries, if not millennia, to even consider them is simply absurd and unreasonable. Scientists believe that we will eventually discover the natural phenomena that give rise to what witnesses believe to be the movement of physical objects

Lou, cared for by aliens.

Antonio Ribera made the trip to come talk to me about "living voice."

- *I didn't want to write to you or talk to you on the phone*

- *What happened?*

- *They discovered a terrible illness in Lou. Rafael couldn't handle it. He cares deeply for her. He went up to his office on the top floor.*

---

[99] Yet these orbits are ellipses, with the Sun occupying one of the foci.
[100] Aristotle had stated the principle that no object could move unless it was subjected to a force (in doing so, he simply denied the concept of inertia).
[101] A theory with unlimited potential for refinement. Before Kepler's ideas, with their elliptical orbits, gained acceptance, Ptolemy's model included no fewer than 48 circles. If a discrepancy was observed, all one had to do was add another circle.
[102] Alpha Centauri, four light-years from Earth.

- *In that room on the top floor, where the roof is topped by a metal Ummo symbol. With the listening device installed by the Ummites.*

- *Yes, that's right. He went up there and spoke to them. He told them he couldn't bear it. He asked for help for Lou.*

- *So what happened?*

- *They treated her. Rafael asked me not to tell anyone. But I couldn't keep it to myself.*

- *I understand.*

In a few weeks there will be another meeting in Madrid. I've decided to go. Lou asks, hesitantly.

- *Oh, did you hear? Did Rafael tell you?*

- *What happened?*

- *They came to the hospital. There were two of them, two men, pretty tall. They gave me their own medicine.*

- *So?*

- *That's all. There's nothing else to tell.*

I won't press her. I can tell she's uncomfortable. And I don't want her to know that Ribera didn't respect Farriols' request.

In this Madrid restaurant, the discussions are in full swing. Dominguez's wife comes across as the high priestess of what they all seem to be experiencing as a new religion. They talk endlessly about the content of their latest phone conversations with the Ummites.

Everything revolves around the "BB," a collective earthly soul to which we are all supposedly connected. She insists that everyone should gather, in groups of at least four people, for meditation sessions where "they would inform the BB."

Dominguez is "more technical" in his exchanges.

- *I asked them if chickens, deprived of calcium, could undergo transmutations to synthesize what they need to create their eggshells.*

- *So what?*

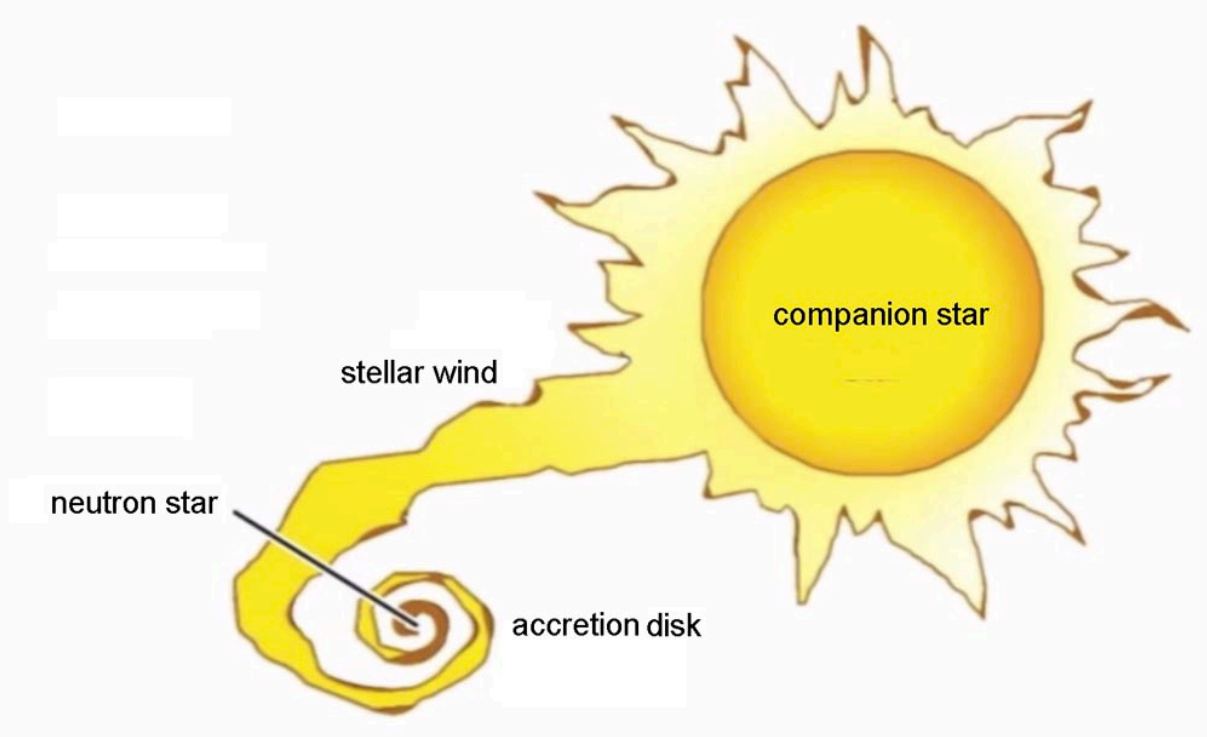
- *They replied that the chickens would draw that calcium from their bones.*

- *Did they tell you anything else?*

- *I asked them what a black hole was. They replied:*

> *It doesn't exist. When a neutron star is destabilized by an influx of matter, it gets rid of it by sending it to the twin universe.*
>
> *That doesn't exist. When a neutron star is destabilized by an influx of matter, it gets rid of it by sending it into the twin universe.*

I know there are many examples of neutron stars that exist in pairs near another star, a normal one, which then emits a "stellar wind" that its tiny neighbor can capture.



This sentence intrigues me. I can't think of anything off the top of my head that might support it.

I just make a note of it, and head home.

The letter from Ryad.

When I return, I find a letter in my mailbox with my address scrawled on it in clumsy handwriting. On the back of the envelope, the sender has written Jean-Jacques's address in Digne. I open it and find this:

UMMOAEELEWE
nº de Copies: UNE
LANGUE:Français.

Monsieur PETIT, Jean Pierre:
Nous posons notre main sur votre noble poitrine.
Monsieur: Nous avons étudié votre personnalité à travers nos instruments UAAGOOAWEE (evaluateurs individuels du comportement psychique) et nous avons reçu une forte impression par la haute valeur que votre coefficient d'intelligence abstraite y a atteint.
Nous pouvons vous affirmer sans aucune erreur que vous êtes parmi $2.9*10^{-6}$ des hommes les mieux doués sur ce domaine à OYAAGAA (la planète terre).
Nous vous supplions de ne pas croire que nous essayons de vous flatter (ce qui serait,d'autre part, inutile étant donné que nous ne vous demandons rien en échange) mais nous voulons tout au contraire vous encourager à continuer vos études spéculatives sur la logique formelle, qui, vous l'avez deviné, constitue la clé pour la compréhension du Cosmos.
Le " théorème " de FERMAT,dont nous savons que vous avez connaissance, A UNE DEMONSTRATION et elle est à portée d'un homme de votre condition intellectuelle. Nous aimerions pouvoir vous aider (idirectement, bien entendu,car votre sensibilité a en effet compris que toute intervention directe à OOYAGAA nous est moralement impossible) à le résoudre.
Celà vous donnerait vis-à-vis de vos frères un prestige intelectuel dont malheureusement vous manquez à présent (et ceci est du à l 'agressive incrédulité des scientifiques terrestres plus qu'à votre manque de mérite)

Vous devez suivre la pensée:
$1=1$
$0=0$
$1\neq0$
$0\neq1$

et croiser:

$1=0$
$0=1$
$1\neq1$
$0\neq0$
C'est à dire combiner d'égalités/inégalités binaires à logique tetravalente.
Dans des prochaines lettres nous vous ferons connaitre vos progrès et vos erreurs.
Nous vous supplions, Monsieur, d'être très discret sur le contenu de nos lettres; dans d'autres pays nous avons du couper le contact avec vos frères à cause de leur extrême indiscrétion.
Nous désirons établir avec vous des dialogues epistolaires bi-univoques (nous vous donnerons des instuctions sur le moyen de vous adresser à nous) mais nous vous pr[illegible]n échange d'être très discret.

Vous pouvez parler librement à vos frères:

FARRIOLS,Rafael(ESPAGNE)
PASTOR,Jean-Jacques(FRANCE)
JORDAN PENA,Jose-Luis(ESPAGNE)

Mais du reste,il serait très sage de votre part de ne rien raconter à personne d'autre.
Une prière:
Nous vous supplions de vous mettre en rapport avec:

Mr. Joseluis Jordan Pena
77,Avenida de bruselas
28028 Madrid-Espagne.

Ce monsieur a une très vaste connaissance de nos écrits,notre philosophie et notre science.Il a été,pendant beaucoup d'années (et avant qu'une cruelle infirmité ne le frappe)notre intérmédiaire principal dans le pays terrestre Espagne.
Il est à présent craintif et méfiant;cependant nous pensons que votre personnalité peut gagner la sienne en peu de temps et que votre intélligence et son information seront essentielles pour mieux vous aider à comprendre nos messages.
L'étape finale serait une rencontre de vous deux avec un groupe de mes frères;vous aurieez alors une preuve définitive de notre identité.

La paix avec vous, Monsieur.

Dicté par OAXIIBOO 6 fils de TRAA 3

Translation :

UMMOAELEWE
Copy number: ONE
LANGUAGE: French

Mr. PETIT, Jean-Pierre,
We place our hands upon your noble chest.
Sir, We have studied your personality through our UAAGOOAWEE instruments (individual psychic behavior evaluators) and we were strongly impressed by the high value that your coefficient of abstract intelligence achieved on it.

We can assure you without any doubt that you are among the $2.9 * 10^{-6}$ most gifted men in this field on OYAGAA (planet Earth).

We beg you not to think that we are trying to flatter you (which would be pointless anyway since we are not asking for anything in return) but on the contrary we want to encourage you to continue your speculative studies on formal logic, which, as you have guessed, is the key to understanding the cosmos.

FERMAT's "Last Theorem", which we know you are familiar with, HAS A PROOF, and it is within the grasp of a man of your intellectual standing. We would like to be able to help you (indirectly, of course, since your sensitivity has indeed understood that any direct intervention in OOYAGAA is morally impossible for us) to solve it.

This would give you an intellectual prestige in the eyes of your brothers, which you unfortunately currently lack (and this is due more to the aggressive incredulity of Earth scientists than to your lack of merit).

You must follow this line of thinking:

$1 = 1$
$0 = 0$
$1 \neq 0$
$0 \neq 1$

an cross with :

$1 = 0$
$0 = 1$
$1 \neq 1$
$0 \neq 0$

That is, combining these binary equalities/inequalities with tetravalent logic.

In future letters, we will share your progress and mistakes with you.

We beg you, sir, to be very discreet about the content of your letters; in other countries we have had to cut off contact with your brothers because of their extreme indiscretion.

We wish to establish a two-way epistolary dialogue with you (we will give you instructions on how to address us) but in return we ask you to be very discreet.

You can speak freely to your brothers:

FARRIOLS. Rafael (SPAIN)
PASTOR, Jean-Jacques (France)
JORDAN PENA, Jose-Luis (SPAIN)

But otherwise, it would be very wise of you not to tell anyone else.

A request:

We beg you to contact:

M. Jose Luis Jordan Pena
77 , Avenida de bruselas
28028 Madrid-Spain

This gentleman possesses a vast knowledge of our writings, our philosopher, and our science. For many years (before a cruel infirmity struck him), he was our principal intermediary in the earthly land of Spain.

He is now fearful and distrustful; however, we believe that your personality can quickly win him over and that your intelligence and his information will be essential in helping you better understand our messages.

The final step would be a meeting between the two of you and a group of my brothers; you would then have definitive proof of our identity.

Peace be with you, sir.



Dictated by OAXIIBOO 6, son of IRAA 3

I decided to let Jean-Jacques know by avoiding the phone and going to Digne.

Jean-Jacques:

- *Well, it seems to me we're shifting gears here!*

He thinks for a moment.

- *I see that it says*:

You can speak freely to your brothers:

- *Rafael Farriols*
- *Jean-Jacques Pastor*
- *Jordan Pena*

And I note that Rafael's name is listed first. We've never corresponded with Pena. You know how the Spanish are. If we contact Pena without letting Farriols know, to whom we are indebted for all the information he's provided us, I think he'll take offense.

Upon my return to Aix-en-Provence, I'll notify him by mail and enclose a copy of the letter.

And then, disaster struck!

Lou called me to congratulate me on receiving such a letter from the Ummites.

The first thing Rafael did as soon as he received my letter was to gather his group and give each of them a copy.

The impact was immediate.

Pena[103] began issuing numerous statements to the press and writing letters to ufologists, presenting himself as the true author of the Ummite letters. He added that for the scientific parts, he had drawn on his extensive reading of science fiction.

The UFO community rejoiced, finally rid of this troublesome case.

The Spanish contactees then received a request to meet at Farriols' home. The letter originated in Cuba and was addressed to 15 recipients, including a certain Benitez, the Spanish counterpart of the journalist Jean-Claude Bourret[104] in France.

Jean-Jacques;

- *The presence of a guy like Benitez sends a clear message. Go ahead if you want. But in my opinion, it's over now.*

I decide to go to this meeting anyway. Everyone I contacted is there, including Villagrasa, whom I'm meeting for the first time. Only Lou can translate into French. The lively discussions last a good hour, punctuated by

- *"Creo que …"*

At no point do those present pay any attention to me. When I signal to Lou, she tells me she'll explain it to me once they're done.

At the end, she summarizes everything that was said:

- *They think you should develop the tetravalent logic*

---

[103] Died in 2013.

[104] For decades, Jean-Claude Bourret served as a star presenter on France's leading television network: TF1. He is known for publishing several books on the subject of UFOs, credited to him but in fact comprising a compilation of texts written by others. The four books he co-authored with Jean-Pierre Petit, presented in the form of dialogues, were in fact written entirely by the latter. In 2023, he changed his stance, stating in interviews: "I cannot promote books based on false calculations."

And with that, she takes her leave.

Everyone leaves Farriols's sumptuous estate, "El Bosque," whose living room is adorned with the stuffed trophies Rafael brought back from his hunts in Africa.

I am left alone with Farriols, who tries to justify himself:

- *I thought this letter was like the other letters…*

He decides to call Pena, putting the phone on speaker. I hear Pena reply very clearly:

- *"Don't yell like that, Rafael! It was the Ummites who asked me to say that."*

The dismantling of the Madrid and Barcelona groups.

At first, Farriols receives a request to hold a conference in Barcelona before the usual audience. He is accompanied by Liu and another contactee, Aguirre. They then declare that they never believed for a single moment that this whole Ummo affair originated from extraterrestrials.

Rafael had warned me. They're doing this at the request of the Ummites themselves. I tell myself then that, at this point, it hardly matters anymore. But he and the others don't seem to realize this.

There are easily a hundred people in the room.

Shouts of protest erupt. These people are protesting that they've been taken for a ride for so many years.

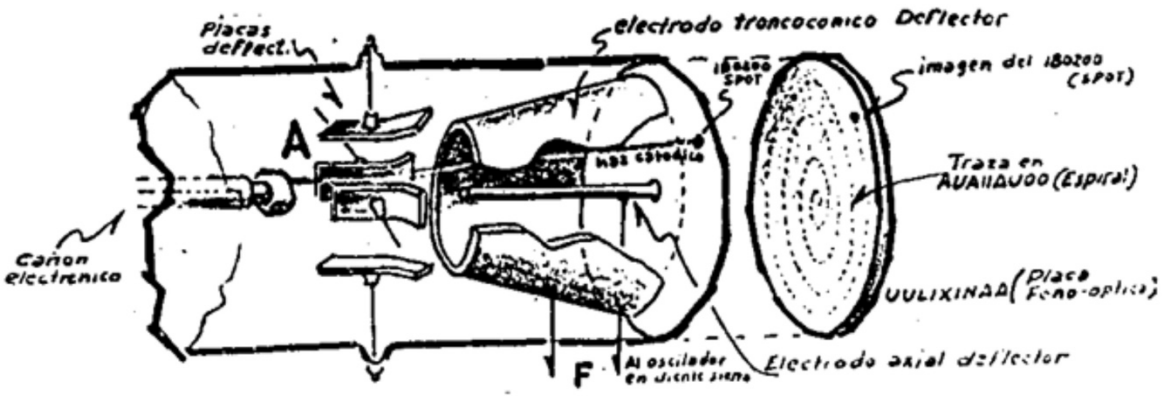
The room empties, and I watch the scene with wide eyes.

All of this was carried out with incredible ease. All it took was one sentence from Farriols. It's as if a train were to stop, the conductor were to come over and announce to the passengers, "You've been deceived. This train isn't going to Bilbao," and all the passengers were to get off immediately.

In the second phase, the Ummites target the Madrid group, centered around the Dominguez couple. Here, the maneuver is more sophisticated.

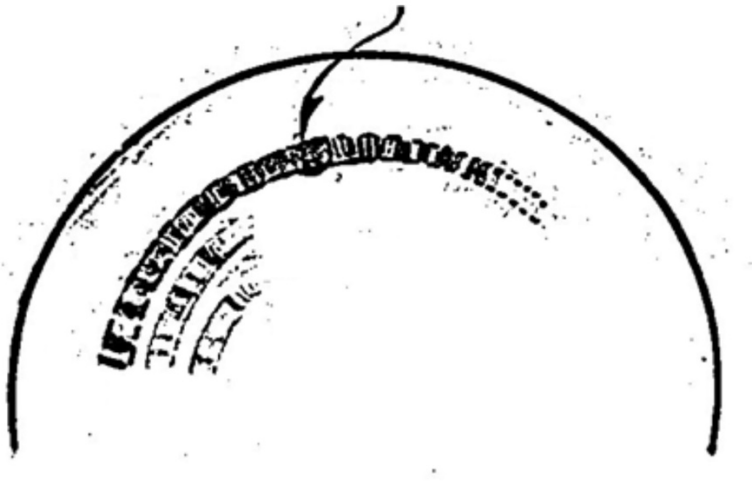
The Ummites then resort to a device, an antiquity in their world, with which they used to store information on a spiral track[105] .

[105] A precursor to what would later become, in digital form, the CD-ROM.



The recording system along a spiral track, on photographic film[106]

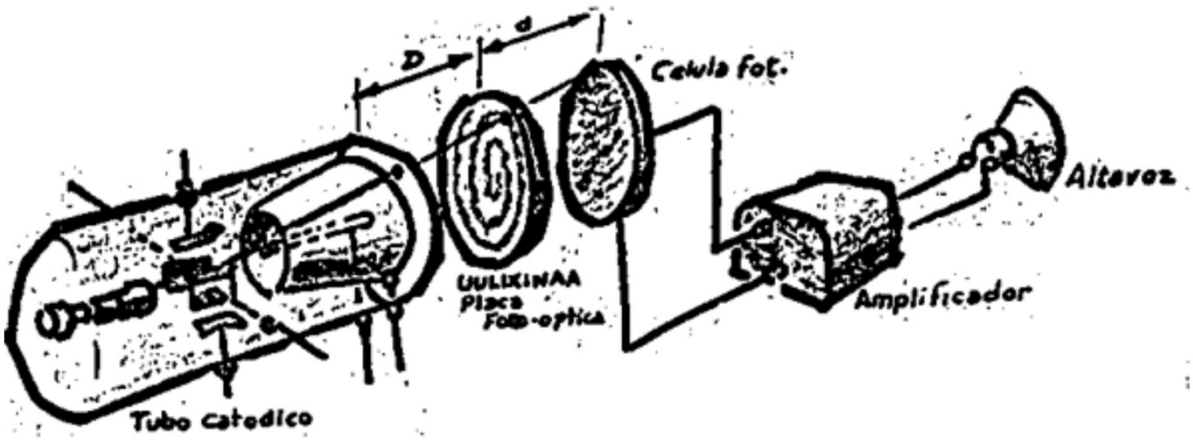
The following image shows a fragment of this spiral track:



Fragment of the spiral track, greatly magnified, showing the Ummites depict arrows in their diagrams.

This is complemented by the playback system:

[106] Readers with some technical familiarity will recognize the components found in our old televisions (today we have "flat-screen TVs"): On the left, the electron gun. Then two pairs of deflection electrodes, arranged in the "up/down, right/left" directions. Further down, one electrode is aligned along the axis, while the other is truncated conical in shape; these two create the spread of the electron beam along the axial direction. By combining all these elements, the beam is directed to strike a sensitive medium along a spiral path



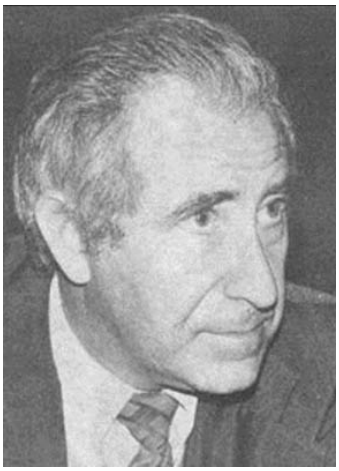
The sound-reading system on the spiral track[107]

A letter then urges Domingez to file a patent for such a device. He complies and, as instructed, takes steps to secure international protection for it.

The letters followed one after another, becoming increasingly urgent. He was asked to establish a company to commercialize this invention. Farriols was called upon to draft the articles of incorporation.

Logically, to embark on such an operation, the company would at the very least need to be called Toshiba or Philips. But Dominguez, obediently, followed the instructions to the letter without questioning them.

It was then that, in the next letter, he was asked to begin paying a salary to Jordan Pena, even though Pena had no technical knowledge whatsoever.



Jordan Pena

[107] This is the exact opposite setup. The same set of electrodes causes the beam to trace a spiral on the film, which acts as a mask. Modulated in this way (these are not "digital" techniques), it strikes a photodiode, and the amplified signal is directed to a simple speaker.

Suddenly, Dominguez found himself in possession of evidence (I don't have the details, as the entire affair was recounted to me verbally by Farriols) suggesting that all of this was a scam orchestrated by Pena.

Ruined, he immediately throws the baby out with the bathwater.

That is to say, he throws all his notes, letters, and copies of letters into the trash.

Farriols calls him:

- *What's going on?*

- *I found out everything, just in time. This whole thing is nothing but a massive manipulation scheme orchestrated by an Indian cult.*

- *What Indian cult?*

- *An Indian cult…*

Immediately, the Madrid group, centered around the Dominguez couple, ceases to exist.

All that remains is to get rid of Farriols.

The Ummites then wrote to him, offering to help him rebuild the group of Spanish contactees, which had been seriously shaken by the Dominguez's defection. To do this, he was asked to make this proposal to them over the phone by calling them one by one, "so that the Ummites could assess their emotional response."

He complied.

What followed were… two years of complete silence. Nothing, not a single letter, not a single phone call.

It is worth recalling that Farriols, for all those years, having sold his plastics company, had lived solely for and through this Ummo affair.

In the eyes of the group of contactees, the worst thing that could happen to a Spaniard happened to him: he lost face.

He made one last attempt by coming to see me at my home, where he uttered this phrase:

- *"Son hiros de putà!"*[108] *…*

[108] They're a bunch of bastards!

Why this letter from Ryad?

Months have passed. I get the impression that nothing more will happen regarding this Ummo affair in Spain, or anywhere else.

Did this meeting, proposed in Riyadh's letter, actually take place? We'll never know. Considering all aspects of the issue, I realize that this letter had two distinct purposes.

Let's put ourselves in the authors' shoes. They were accustomed to human vanity. What, then, did they envision? That I would rush to tell anyone who would listen, "Look: I'm going to prove a theorem that the world's best mathematicians have failed to solve, the one by the Frenchman Fermat[109] . And I'm going to do it with the help of extraterrestrials"?

They couldn't have been unaware that the Englishman Wiles had just published this theorem, 356 years after thousands of mathematicians had failed to prove it.

And why a four-valued logic?

Our current logic operates on the "principle of the excluded middle." This amounts to rejecting any "truth value" other than true and false. Yet our language is full of statements (which logicians call "propositions") that are neither true nor false. Like the simple sentence:

- *I'm lying.*

Mathematicians believe that their language is immune to this curse. In 1893, the German Gottlob Frege published a work intended to ground the entire field of mathematics on a "mathematical logic" whose foundations he had laid, based on set theory. But on June 16, 1902, he received a simple letter from a young English philosopher in his thirties, Bertrand Russell.

In just a few lines, Russell demonstrated to him that even set theory cannot escape these "undecidable propositions," which are neither true nor false. He formulated this using his famous barber paradox:[110] .

This letter was a catastrophe for Frege, who was 55 years old. He wrote:

- *A scientist can hardly suffer a worse misfortune than to see the foundations of his work collapse at the very moment it is completed.*

---

[109] There are infinitely many sets of three integers a, b, and c such that $a^2 + b^2 = c^2$. For example, 3, 4, and 5: 9 + 16 = 25, and infinitely many of their multiples. But it is impossible to find three integers such that $a^n + b^n = c^n$ if n is a positive integer greater than 2. Fermat conjectured that this must be a theorem. Mathematicians struggled with this problem for 356 years. It was not until 1995 that the Englishman Andrew Wiles published a proof of over a hundred pages. The Ummite letter reached me just before Wiles announced that he had succeeded.

[110] In a village, there are two categories of men: those who shave themselves and those who do not shave themselves. A barber shaves everyone who does not shave themselves. Under these conditions, who shaves the barber? To which set does the barber belong?

For a time, mathematicians believed that there must be one domain, the most elementary one, that escaped this curse: the arithmetic of integers[111] , the absolute foundation of language and thought.

In 1931, the Austrian Kurt Gödel[112] demonstrated that even the arithmetic of integers, considered the most elementary "language," defined by extremely simple axioms, also gives rise to "undecidable" propositions.

Worse still: he then demonstrated that this paradox "contaminates" every form of language, including… the entire field of mathematics[113] !

There was consternation in the world of mathematics, all over the world.

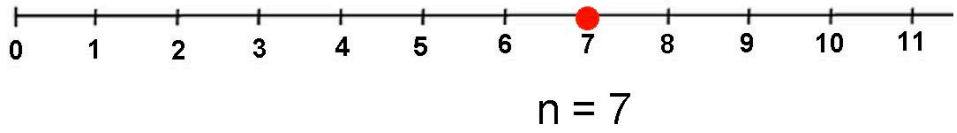
Logicians have made numerous attempts to create logical systems with "truth values":

- TRUE
- FALSE
- TRUE and FALSE
- Neither TRUE nor FALSE

But none of these systems manages to circumvent the pitfall posed by Gödel.

When the Ummites point out this problem, instead of seeking a logical system of my own, I decide to tackle Gödel's theorem itself.

Integers are mapped to a sequence of equidistant points on a line.

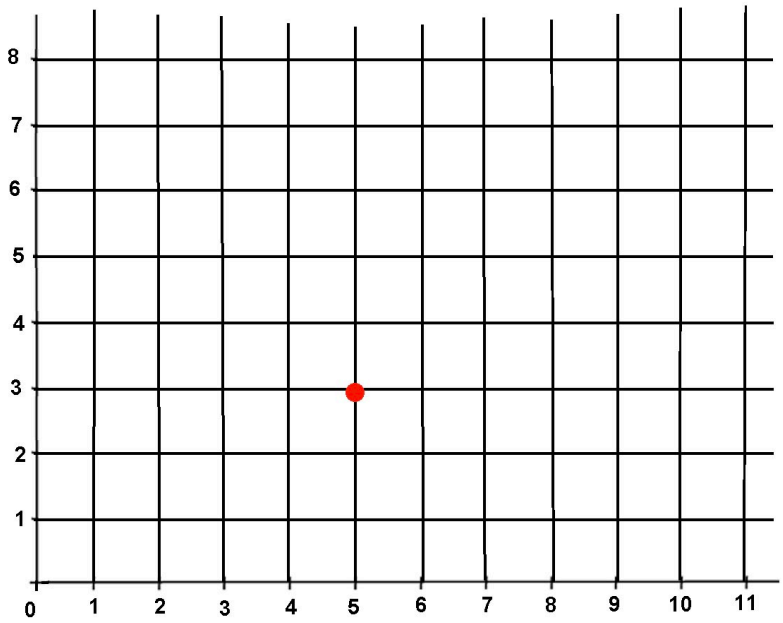


They are themselves a subset of the points that form the nodes of a grid:

[111] Axiomatized in 1889 by the Italian Giuseppe Peano, using five axioms:
1 – 0 is a natural number
2 – Every natural number n has a successor n + 1
3 – No integer has 0 as its successor
4 – If two integers are the successors of the same third integer, they are identical.
5 – If a proposition is true of order zero, and if it is true for every proposition of order n+1, then it is true for any n.

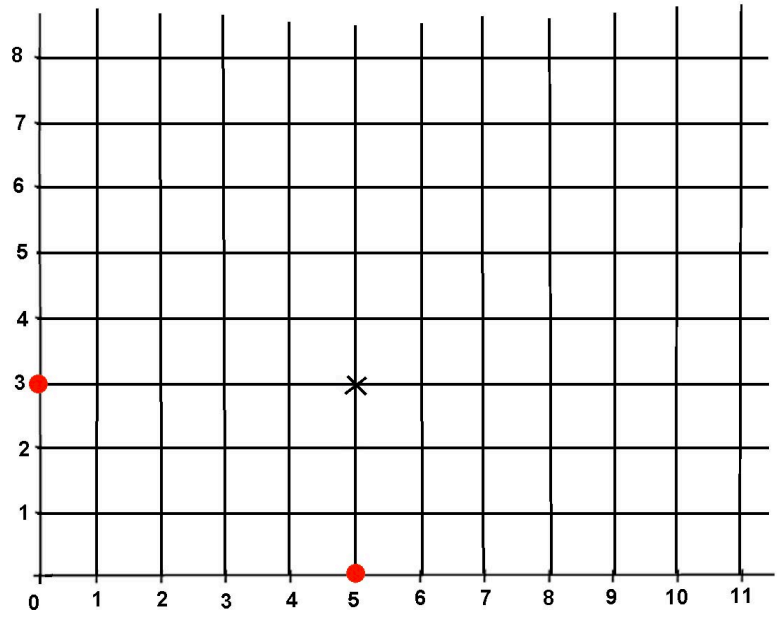
[112] Although not Jewish, he fled the Nazi regime and joined the Institute at Princeton, which Einstein had joined in 1933

[113] One formulation of his result: "Every formal language has one undecidable proposition."



Z = 5 + 3 i

These points are then "complex integers," first defined by the German mathematician Carl Friedrich Gauss[114] . He defines them as pairs of integers, thus according to an axiomatic system equivalent to "twice the integers" using the imaginary number i:



Z = 5 + 3 i

But in fact, these numerical points "exist" independently of their projections onto the two axes. I note, then, that no one has thought to construct the axiomatic system for these objects.

[114] 1777–1855. Author of numerous works on "number theory."

At that time, I owned a Macintosh desktop computer equipped with an internal hard drive, and it was on this computer that I recorded my work and reflections.



Of course, back then, the only connection these machines had was a simple power outlet.

I recorded my thoughts in a .doc file by writing down the first four axioms:

1- Zero is a complex integer
2- Zero is not the successor of any number
3- Every complex integer z has two successors, denoted z + 1 and z + i
4- If two complex integers are successors of the same third complex integer, by the same succession operation, they are identical.
5- ….

And I pause, wondering how to formulate the fifth axiom.

When it comes to logic, I imagine replacing the “true”/“false” criterion with “equal/different”:

$$= \ / \ \neq$$

To illustrate how this logical system works, I’ll use the example of a psychiatric hospital with four different patients:

- Psychiatrists
- The Insane
- Psychiatrists who have gone mad
- Mad People Who Have Become Psychiatrists

Thus, when a psychiatrist meets another psychiatrist, he can identify with him; this is a logical state:

$$= \ / \ =$$

When he encounters a madman, he sees in him a completely different individual.

Logical state:

$$\neq / \neq$$

A little further on, he comes across a psychiatrist who has gone mad, with whom he can only partially identify. He thinks of a colleague who has spent too much time with patients.

Logical state:

$$= / \neq$$

Finally, he comes across a madman who has become a psychiatrist and thinks that this is a madman who has observed psychiatrists so closely that he can mimic their behavior.

Logical state:

$$\neq / =$$

What's amusing is that each character has their own logical framework. Thus, if a madman-turned-psychiatrist encounters another madman-turned-psychiatrist, to him, the other is a normal individual who checks the logical box:

$$= / =$$

On the other hand, if he runs into a psychiatrist, he classifies him in the category[115] .

$$\neq / =$$

A few days later, I received this letter from Ummites living in Saudi Arabia, in the city of Riyadh. And, unfortunately, we saw that Farriols' blunder had compromised the entire operation.

No matter how closely I examine its contents from every angle, I can't see how to proceed any further, nor how this could possibly have any connection to Fermat's famous theorem.

But it seems there are better prospects in the fields of astrophysics and cosmology. I've decided to return to those two fields.

[115] From this perspective, it is by projecting this four-valued logical system, with its four "truth values":

$$= : =$$
$$\neq / \neq$$
$$\neq / =$$
$$\neq / =$$

that the two states $\neq / =$ and $\neq / =$ merge into a paradoxical state. Paradoxes would result from evaluating situations using a bivalent logical sieve, which is too limited.

France, September 8, 1914.

The unit led by Werner Boy, a 34-year-old reserve lieutenant, is trying to set up camp as best it can. He looks at the sign bearing the name of this small town, situated on the banks of the Marne, barely legible since everything there has been torn to shreds by French shrapnel.

- *Vitry-le-François*

But the Kaiser must be pleased. Just four weeks after the outbreak of the conflict, German troops have penetrated deep into the heart of French territory.

Boy catches himself saying aloud:

- *If Paris falls, then the war will be over. It shouldn't be long now. After that, I'll go back to Göttingen.*

He had set up camp in an old school, or at least what was left of it. Suddenly, his captain burst in.

- *"Werner, take five men and go see what the French are up to. I get the impression they're trying to set up a 75 on Hill 456. If I were them, that's what I'd do. Here, take this map. We'd better not give them time to do it."*

- *At your command.*

Werner nods. The captain is competent. He knows how to anticipate.

He moves away from the town and tells the men to stop for a moment so he can locate the spot.

He takes out the map. He lays it on his lap. But it suddenly fills with red marks.

Boy grumbles:

- *What's that? I can barely read it anymore!*

The world of mathematics has just lost one of its most brilliant geometers.

My 3D vision has failed me.

I'm in my office, where I'm working on adapting the extensive computer-aided design (CAD) program I created on my previous machine, the Apple II, to my Macintosh laptop.

A young assistant knocks on the door.

- *Professor, we need your help, your legendary skills as a draftsman.*
- *Oh, really? I'll be right there. What's it for?*

Bernard Morin is a mathematician based in Strasbourg. I'd already heard of him, and it turns out I've been asked to illustrate a lecture he's going to give at the University of Aix-en-Provence. I know he's blind.

In the conference room stands a magnificent whiteboard, brand-new, which we've just acquired, with a whole set of colored markers resting on its shelf.

A woman speaks up:

- *"Go ahead, Bernard, everything's ready."*

- *Who's drawing?*

- *It's Jean-Pierre Petit.*

- *Ah, Jean-Pierre Petit! The author of those fantastic comic books, *The Adventures of Anselme Lanturlu*, whose adventures my students tell me all about. I couldn't ask for a better illustrator. *The Geometricon*, exploring curved spaces alongside the lovely Sophie, what a marvel!*

- *At your service, dear colleague.*



The Geometricon[116]

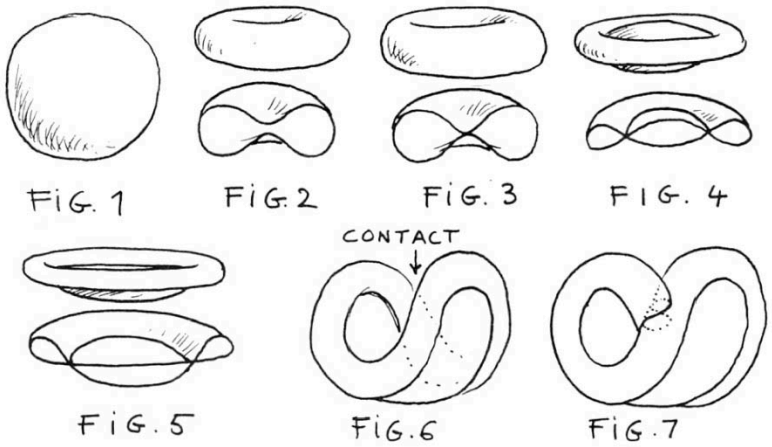
[116] http://www.savoir-sans-frontieres.com/JPP/telechargeables/Francais/LE%20GEOMETRICON.pdf

Morin talks. I draw. At first, it's easy. I have very good "3D vision."

We start with a sphere that intersects itself.

That's fine.

This results in a strange object that looks like a lifeboat with a "floor" bordered by a sort of tube. Below, I've reproduced the first images from the section on page &&&.



Morin then asks me to twist it and make two sections of the tube… intersect with each other. I have to draw all of this.

He specifies:

- *Use a colored marker so we can clearly see the self-intersecting curve.*

I try, but everything gets terribly complicated. I've never had to deal with anything this complicated before. Everything seems to blur so quickly before my eyes.

I erase it and start over. Three times in a row.

Then I set the marker down on the tablet.

It's as if Morin had been following my every move exactly. When he hears me put the marker down:

- *I know, immersions are always disorienting at first.*

- *Immersions?*

- *Yes, you obviously have a handle on everything related to "plunging": when these objects don't intersect, we say they're "plunged into 3D space." But when we allow them to intersect themselves, we say they're "immersed in 3D space," and that's when things get complicated very quickly.*

- *Anyway, sorry, I give up. I admit my wires are all tangled up.*

The whole room bursts out laughing.

Morin too.

"*That's normal…"*

He says this in a calm tone, as if he were talking about the rain.

I point to the board.

- *By the way, may I ask what I was supposed to be illustrating?*

- *It's the inversion of the sphere.*

1957. Department of Mathematics, Berkeley, California.

Stephen Smale, a young doctoral student, walks into the office of his advisor, the mathematician Raoul Bott.

- *I think I've managed to prove that all immersions of the sphere into three-dimensional space form a single family and that one can transition continuously from one to another.*

- *Stephen, don't talk nonsense! If what you're saying is true, then that would imply it's possible to turn a sphere inside out without tearing it or folding it. But it's obvious that's completely impossible!*

- *Yet I am certain of my theorem*[117] *.*

- *But, even if that were true, how would you go about turning that sphere inside out?*

- *I haven't the faintest idea….*

Bernard Morin, the hyper-clairvoyant.

I can't let something like this slide. The Morins live in a beautiful family estate on the hills overlooking Aix-en-Provence. My curiosity is piqued to the extreme. What's this business about flipping a sphere, front to back?

The woman who had warned Bernard Morin of my arrival during the conference is his wife, Cécile.

[117] This work was published in 1959 in the *Transactions of the American Mathematical Society* (Vol. 90, No. 2, pp. 281–290). It earned him the Fields Medal in 1966, the highest honor awarded for work in mathematics.

- *Bernard, come down! Jean-Pierre is here.*

- *I'm coming!*

He makes his way into the spacious living room.

- *What's this whole "flipping a sphere" business all about!?*

- *Well, it shows that mathematics sometimes allows us to access areas where intuition proves ineffective.*

- *Because it's possible?*

- *Absolutely. It all stems from Stephen Smale's theorem, which earned him the Fields Medal in 1966. He simply proved mathematically that it was feasible. All that remained was to figure out how to do it. It was an American mathematician, Anthony Phillips, who figured it out*[118] *. It was published in the American journal *Scientific American* in 1967.*

- *Would you like to see it?*

- *I've pulled out the issue in question for you to look at.*

I'm skimming through the article.

- *Tell me, it's not exactly crystal clear.*

- *Very few people can follow this sequence of diagrams.*

- *Ah, that's a relief!*

- *Your comics show that you have an absolutely exceptional sense of spatial awareness. You juggle all of this like an artist. I heard that the École des Beaux-Arts in Aix-en-Provence has hired you as a sculpture instructor. Apparently, you have a studio where you create all sorts of geometric objects using flexible metal rods. Cécile told me you exhibit them in art galleries in Aix.*

- *I used to teach basket weaving at an American school in Burlington, Vermont, in another life.*

- *Indeed, basketry is directly inspired by mathematics.*

- *At the School of Fine Arts, I started by having the students work with rush. Then we found the ideal material: copper-coated wire, which worked remarkably well for this. So we set about tying these curves together with sewing thread, in order to preserve the metal wire's natural flexibility. Welding wouldn't have worked.*

[118] Anthony Phillips is Professor Emeritus at Stony Brook University. The original drawings of this first version of the sphere's eversion can be found at this link:
https://www.math.stonybrook.edu/~tony/eversion/eversion2021.html

- *I'd be very curious to see that.*

My sculpture studio at the École des Beaux-Arts.

Since the beginning of my career as a researcher, I've had several encounters that, each time, proved to be pivotal and where I immediately realized that these people were bottomless wells of knowledge.

There was the mathematician Lichnérowicz, thanks to whom I discovered… another world. I don't know how else to describe it.

Then there was Souriau, the old alchemist on Rue Mazarine.

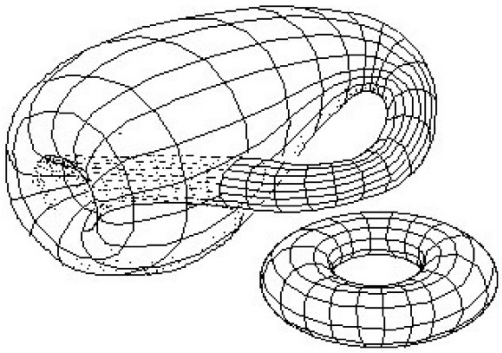
That was something else entirely, something completely different. I found myself face to face with a blind man who sees better than those with sight!

Morin lost his sight at the age of six as a result of glaucoma. Blindness is a disability. Morin resolved to turn it into a strength, transforming it into an exceptional ability to perceive objects and shapes.

In the days that followed, I gave him a tour of my studio at the École des Beaux-Arts in Aix-en-Provence.

Morin sees with his hands. He moves from one object to another. He has to feel everything, to "read" everything in his own way.

- *Ah, that's very good, yes, that's it. Your torus and your Klein bottle are superb. You've arranged the meridian curves and the parallel curves very well!*
-



Klein bottle and torus

- *Could you explain the inversion of the sphere to me? What I understand, I can then, in principle, explain to others using drawings, which is what I've done with all my comic strips*[119] *.*

[119] http://www-savoir-sans-frontieres.com

- *Drawings that, for the moment, don't exist. Michelangelo used to say that statues already existed within the blocks of marble and that all one had to do was remove what was around them. Right now, your drawings are still in the inkwell. But I think that if anyone can bring them to life, it's you. Every year, Cécile and I spend our entire vacation at the family home in Aix-en-Provence.*

- *Very well.*

- *There's one question that's been on my mind. You've covered a wide range of topics, from special relativity to astrophysics, including magnetohydrodynamics and nuclear physics. Each of these fields involves sophisticated concepts and a whole host of different formalisms. So, how do you manage to turn all those equations into comic book storylines?*

- *It's simple. Actually, for me, it's the other way around. That's how I think. It's only afterward that I have to rewrite all of that into equations.*

Back on Rue Mazarine.

I report on this meeting to Souriau.

- *Ah, you've fallen into Morin's clutches! Watch out, he's going to mess with your head.*

- *What's his world like?*

- *It's the world of topology.*

- *But you're a geometer. That must speak to you. In terms of abstraction, you're doing the splits, aren't you?*

- *My answer will surprise you. Everyone has their limits. Morin and I don't live on the same planet. Mine is called "group theory" and his is "topology."*

- *I know. It's actually thanks to your extension of that theory, [120] , that you're able to reverse time. It really blew my mind when you showed me that. What does Morin do?*

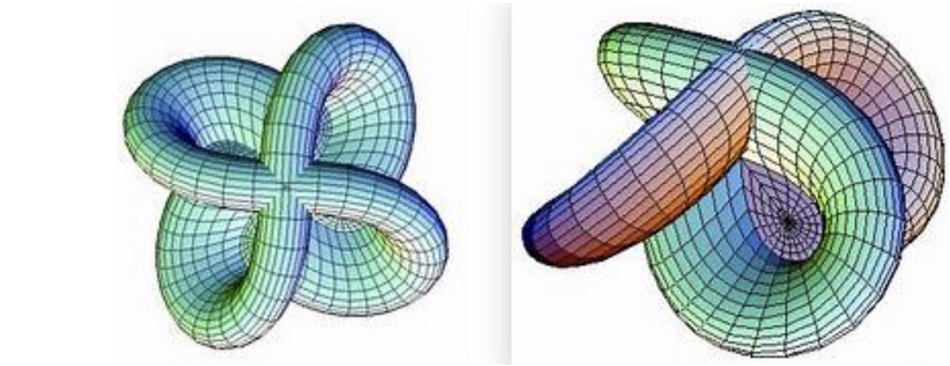
- *He's doing what his blindness allows him to do. If you decide to follow his teachings, I'm warning you, he's going to take you by the hand and lead you "through the looking-glass." I advise you to take some aspirin as a precaution before each session.*

- *That much!?*

- *He invented the second version of the sphere flip, following the one invented by the American Anthony Phillips in 1967, using a surface he named after himself. The only ones who understand it are his fellow topologists. They're mostly the crew at the 's*

[120] Jean-Marie Souriau: *Structure des Groupes Dynamiques*, Editions Dunod, 1970, and *Structures of Dynamic Groups*, Birkhäuser Editions, 1997.

Department of Mathematics at Berkeley, in California. In France, and even in Europe, there's no one.



The Morin surface – Wikipedia

- *Wow, that's extraordinary. He created a theory that only a handful of people can understand.*

- *I've listened to him speak several times at seminars. After ten minutes, everyone tunes out, myself included. There's just one guy left, talking to himself.*

- *It must be a nightmare for him.*

- *Unless you manage to draw all of that. I think that's what he expects from you.*

- *A version of Jean de La Fontaine's fable, "The Blind Man and the Paralytic," set in the world of mathematics.*

- *Except this time, you're the blind man!*

Through the Looking-Glass.

And so begins a long journey, with Bernard Morin serving as my guide. Our exchanges are astonishing. It's a shame our conversations weren't recorded.

He must describe to me, in his own words, these objects that exist only in his mind. And I try to gradually give them shape using cardboard models and modeling clay, which he then "reads" with his hands.

Over the course of our sessions, which often run into the evening and take place in the living room of his home, when I say goodbye and he's left alone in his armchair, his ritual phrase, which I still find hard to get used to, is:

- *"Don't forget to turn off the lights when you leave!"*

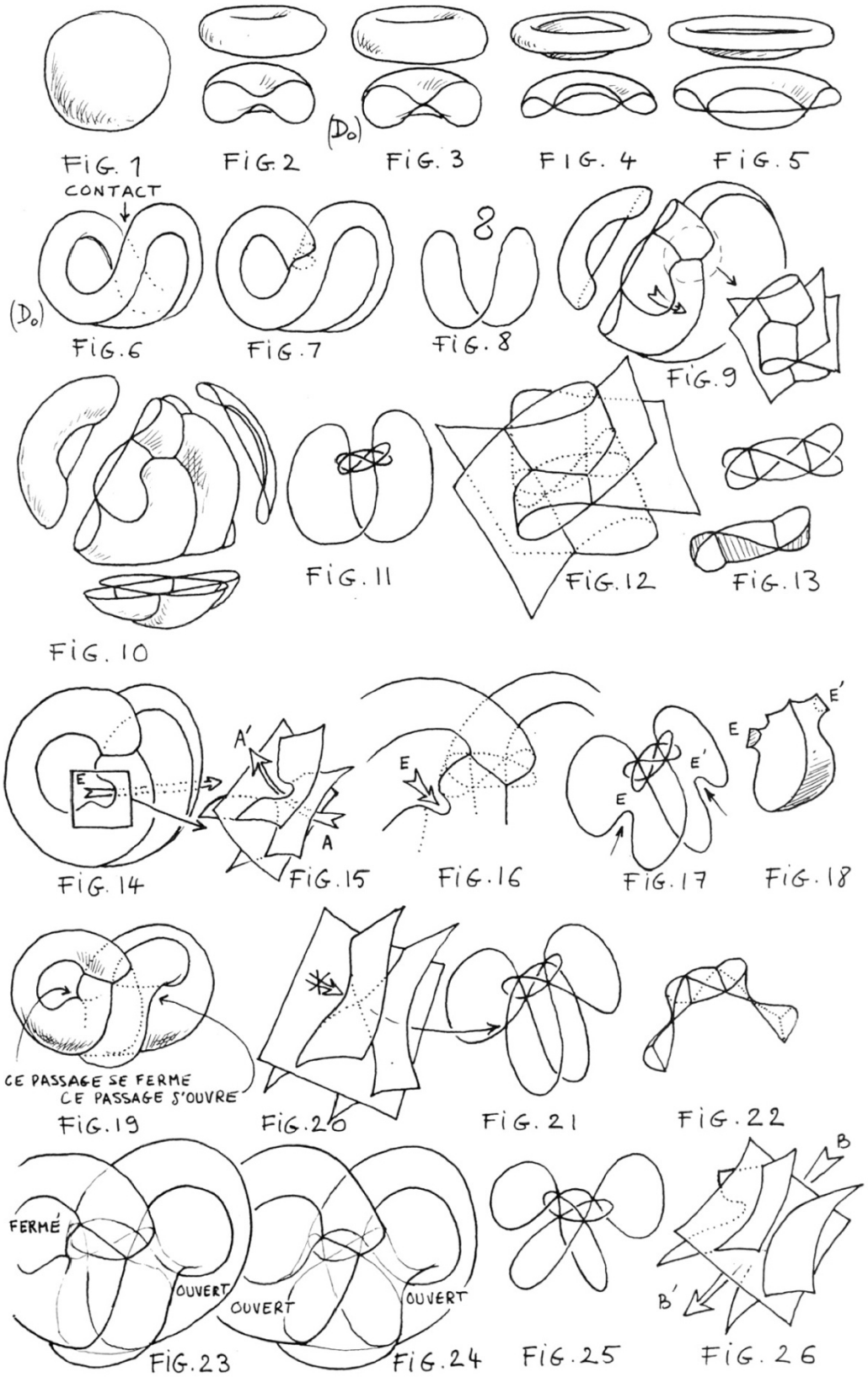
Indeed, his light is somewhere else.

The weeks go by, and I eventually come to understand. In my own way, I take notes, that is, I start a new comic strip[121] . That's what I always do when I begin to find my footing in a new field. For me, it's automatic. As if that other self of mine, my character Anselme Lanturlu, were taking the reins.

It's a strange adventure. He opens the doors to a world that's entirely new to me. And I allow him to finally be heard. Very quickly, this collaboration results in a landmark article titled "The Turning of the Sphere," published by the Paris Academy of Sciences, and, along the way, a friendship[122] .

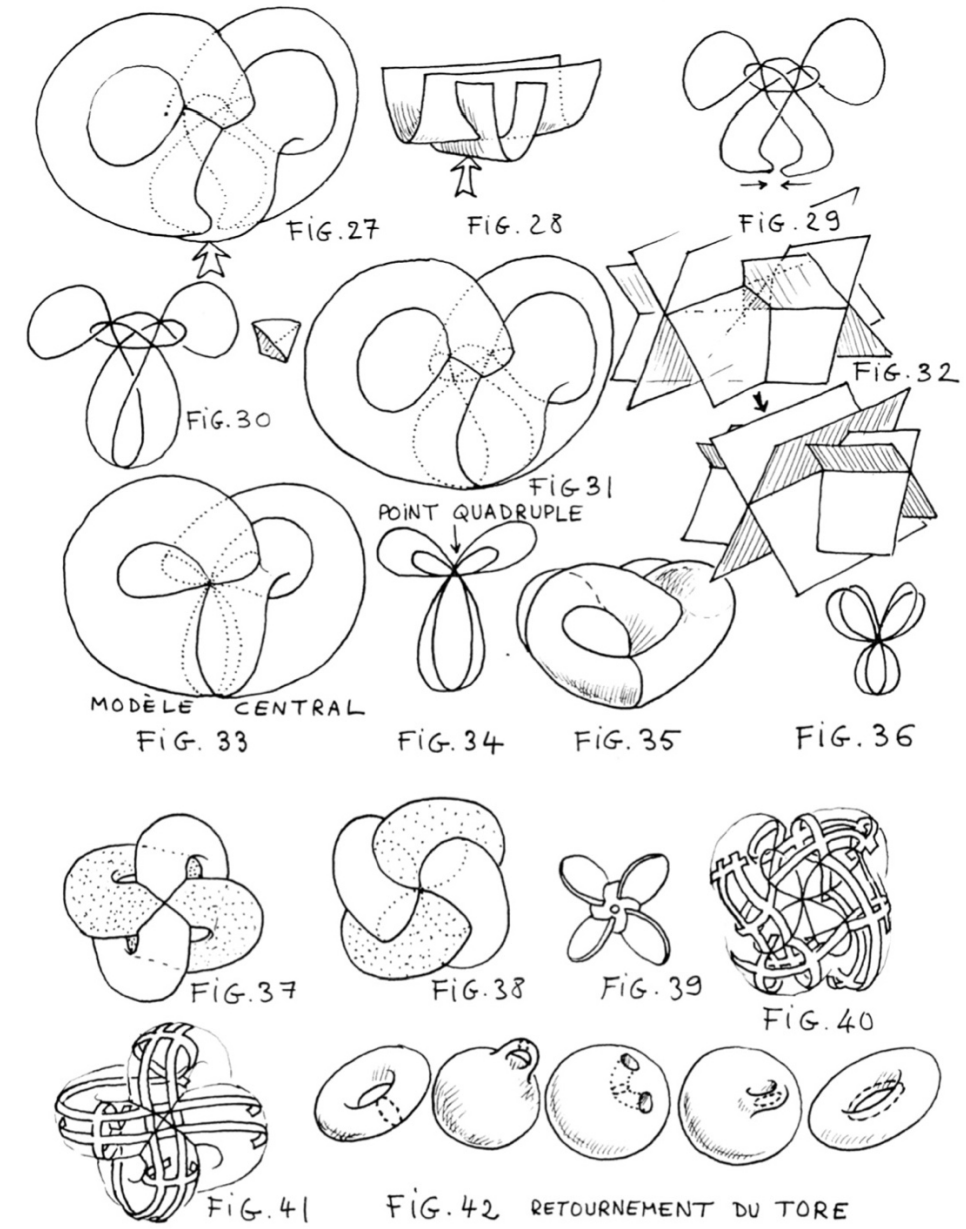
Out of simple curiosity, here is Morin's version of the "reversal of the sphere," illustrated by these two pages from our presentation at the Paris Academy of Sciences[123] . If you don't understand it, that's normal. Even with the help of this series of drawings, few scientists manage to fully grasp it all.

[121] Which would later become *Le Topologicon*. http://www.savoir-sans-frontieres.com/JPP/telechargeables
[122] Until his death in 2018 from a lung infection.
[123] Bernard Morin and Jean-Pierre Petit. "The Inversion of the Sphere." Comptes Rendus de l'Académie des Sciences de Paris. Vol. 287 (October 3, 1978), Series A, pp. 791–794



The Turning of the Sphere, Part One

C. R. Acad. Sc. Paris, t. 287 (30 octobre 1978) Série A — 793



The Turning of the Sphere: Continued and Concluded.

On the Way to Saint-Tropez.

A friend lent me her 8-meter sailboat, based in Marseille. I decided to take Morin on a mini-cruise that would take us to the port of Saint-Tropez. We're sailing close-hauled.

- *Bernard, come take the helm. I'll explain how it works.*

Morin joins me, moving cautiously and holding on to whatever he can, as best he can. It's a new environment for him, and although he knows how to swim, if he were to fall overboard while I'm alone on deck maneuvering the boat, it would be difficult for me to rescue him. For added safety, I had him put on a life belt, connected to the boat by a "line."

Let me explain:

- *Right now we're sailing upwind. At best, a sailboat can zigzag against the wind at a 45° angle. This is what we call "tacking." Right now, the wind is coming from the right. So we're following a course at a 45-degree angle to the wind. The sail is "trimmed" fairly tight. I'm going to hand you the helm, and you'll be steering the boat. And I'll explain how.*

Bernard takes his place.

- *If you push the tiller slightly, the boat will "turn into the wind," veering to the right, toward what sailors call "starboard." The sail behaves like an airplane wing. When the angle of attack is optimal, the airflow strikes it at the right angle. But if you head upwind, the angle of attack on the sail is no longer optimal, and then it deflates near the mast. What's more, it will flap slightly. We say it's "flapping." Can you hear it?*

A blind person's sense of hearing is highly developed. And that's what I'm counting on so that Bernard can steer the sailboat.

- *Now pull the tiller slightly toward you. That will make the sailboat turn to the left, toward "port." The sail will regain its normal fill, and the noise will disappear.*

Bernard does as instructed, and the flapping stops.

- *Now I'm going to show you how to determine the boat's proper course, its optimal heading relative to the wind, so it can sail "close-hauled." You do the opposite maneuver, and the sail starts flapping again. Then you "ease off" just a little, just enough to make the noise go away. You don't need to see the shore or a wind vane to tell you which way the wind is blowing. The sound alone is enough. And that's when you know you've adjusted your course perfectly.*

My student follows my instructions. Everything works perfectly.

The hours tick by. Bernard doesn't say a word. He's listening only to the wind now.

I take the helm for the tacks. But once those are done, he takes his place again.

When we come within sight of the port of Saint-Tropez, I lower the sail and motor into the harbor. The season hasn't started yet, and we can still find spots to moor at the dock.

We stroll around the harbor. It's time for lunch.

- *Tell me what they're eating. I can hear the clinking of their forks.*

I'm taking in everything I see. Many have opted for that famous dish from the south of France, bouillabaisse[124] made with various kinds of fish. But it's not a cheap dish, and our researchers' budgets don't allow for such extras.

- *We're going to buy a tarte tropézienne at the famous Sénéquier pastry shop. You'll see, you're going to love it. It's a recipe that was invented right here.*

Bernard concludes:

- *In my opinion, these people must be mathematicians who've managed to collect royalties on the theorems they invented.*

I look up at the bell tower.

- *Look, the paint has started to peel. We're beginning to make out the inscription, which is starting to reappear.*

  Bernard can't see that, but I tell him about it.

- *Saint-Tropez started attracting crowds in the late 1950s. I remember seeing Brigitte Bardot one very early morning. We'd dropped anchor in Canoubiers Bay[125] . She loved taking trips on the "pointus"[126] , the traditional boats of the local old fishermen. We saw her pass by, just a few meters away, sitting at the bow of the boat in a yellow two-piece swimsuit. She smiled at us.*

- *Is that an image that's stayed with you?*

- *Yes. If you look at the character of Sophie in my comics, that's BB.*

- *But what about that story about the painting on the bell tower?*

- *The priest of Saint-Tropez had quickly begun to get rich. I suppose people were making donations. So he decided to have his bell tower renovated. Before, the apse had been eroded by the sea breeze. But he brought in Italian masons. Not only was everything completely rebuilt from top to bottom, but he also had that part of the church and the bell tower repainted in different colors.*

- *And you preferred it the way it was before.*

- *Worse yet, he had spotlights installed in the bell tower that cast a green light.*

- *So what?*
- *Well, we noticed that he'd installed a copper cable acting as a lightning rod, securely fastened and sturdy. We came back at night and used it to climb up to the bell tower. Once there, we used paint to write:*

[124] It's not that other Provençal dish, ratatouille, from the animated film of the same name, with which Rémy, the rat, wins over the renowned food critic Anton Ego. But it's just as delicious.
[125] Named after the long "reeds" that used to grow along the shore of this bay back in the day.
[126] The typical fishing boats, "pointed at the bow and stern."

"The bell tower with chlorophyll, soon the communion wafers will be soaked in Grand Marnier."

Ummo on the Internet.

This whole Spanish affair, with its anticlimactic ending, means it no longer seems appropriate to keep the contents of these letters and documents a secret. This is compounded by the fact that the Spanish have published numerous books on the subject.

Since I founded a nonprofit organization under the 1901 law, dedicated to the study of the UFO phenomenon[127] , its members are asking me for copies of these letters.

Making photocopies of these roughly eight hundred pages seems difficult. A young theoretical physicist working at a laboratory in Marseille, Gilles d'Agostini, then offered to mobilize a team of dozens of volunteers to type up all these texts and convert them into PDFs, a format that had just emerged, complementing the Internet[128] .

The task was quickly completed, and in response to requests from members or ordinary Internet users who sent me a blank 3-inch floppy disk and a stamped envelope, I would return the envelope containing the precious files without charging them a penny.

One day, a young computer scientist[129] from Nice called me on the phone:

- *I'd like to create a website using these PDFs. Do you give me permission to do so?*

- *I can neither give you permission nor forbid you from doing so. These documents do not belong to me. I don't know if there's an interplanetary agreement regarding copyright. Look into it.*

The site is quickly launched, featuring a forum that is immediately flooded with "UFO-related chatter."

But the young man remains unemployed, and his situation remains precarious. A former military officer and pilot, enjoying a comfortable retirement in a beautiful villa in Aix-en-Provence[130] , trades him this website for a job as an assistant webmaster at a company in Toulouse.

Having acquired the "Ummo-science" website, our retiree hastens to add a copyright notice to every page, whether original or translated by Jean-Jacques and converted to PDF files by Gilles and his team, in the form of a sort of "watermark":

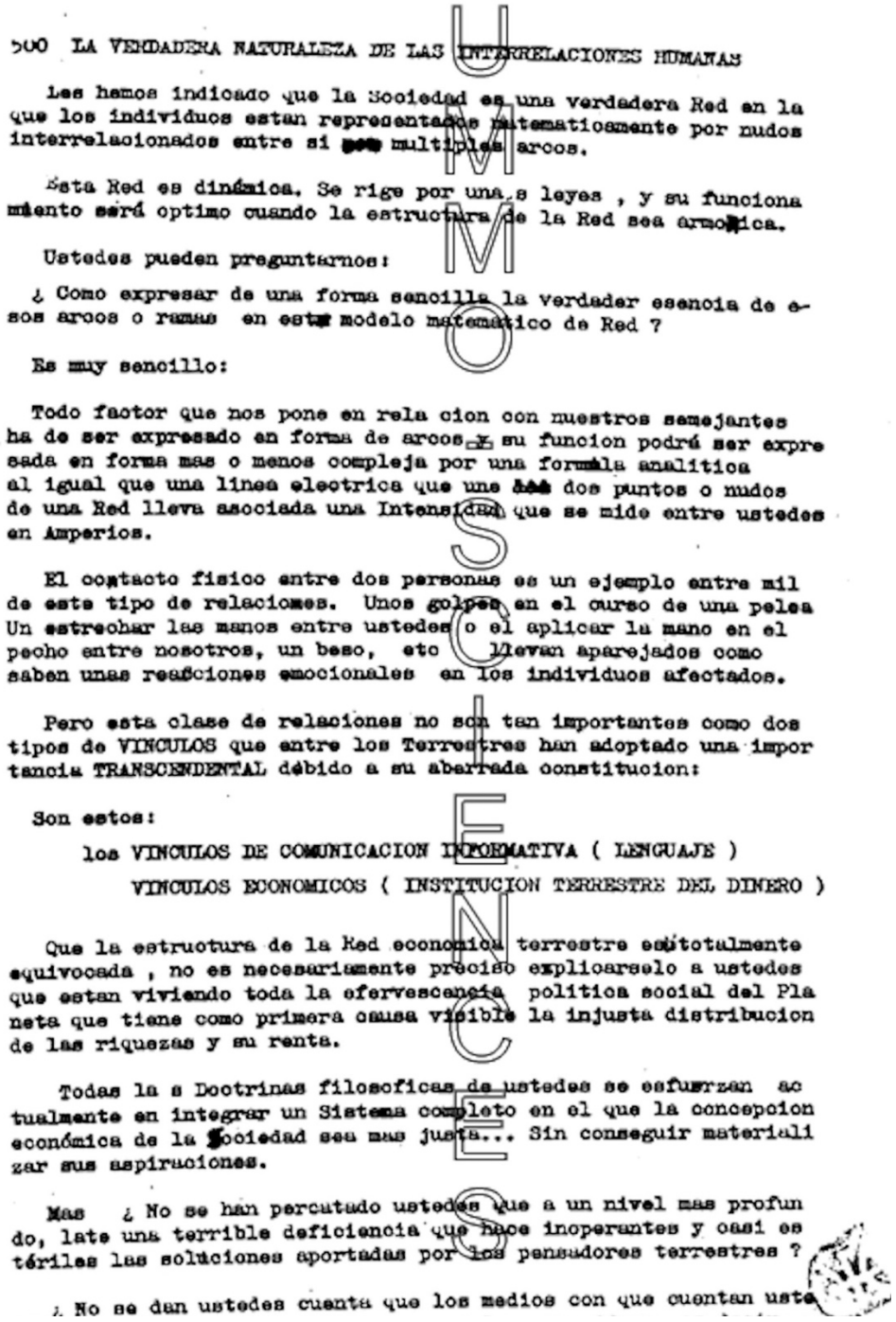
---

[127] An organization called the Scientific Study Group on Traces Associated with the UFO Phenomenon. Just as we were filing its articles of incorporation with the prefecture and considering shortening this long name to its initials, we got a very unpleasant surprise: it was already taken!

[128] At that time, only highly specialized agencies and laboratories had scanners.

[129] Nicolas Lecot.

[130] André-Jacques Holbecq



500 LA VERDADERA NATURALEZA DE LAS INTERRELACIONES HUMANAS

Les hemos indicado que la Sociedad es una verdadera Red en la que los individuos estan representados matematicamente por nudos interrelacionados entre si por multiples arcos.

Esta Red es dinámica. Se rige por unas leyes , y su funciona miento será optimo cuando la estructura de la Red sea armonica.

Ustedes pueden preguntarnos:

¿ Como expresar de una forma sencilla la verdader esencia de esos arcos o ramas en este modelo matematico de Red ?

Es muy sencillo:

Todo factor que nos pone en relacion con nuestros semejantes ha de ser expresado en forma de arcos y su funcion podrá ser expresada en forma mas o menos compleja por una formula analitica al igual que una linea electrica que une dos puntos o nudos de una Red lleva asociada una Intensidad que se mide entre ustedes en Amperios.

El contacto fisico entre dos personas es un ejemplo entre mil de este tipo de relaciones. Unos golpes en el curso de una pelea Un estrechar las manos entre ustedes o el aplicar la mano en el pecho entre nosotros, un beso, etc llevan aparejados como saben unas reacciones emocionales en los individuos afectados.

Pero esta clase de relaciones no son tan importantes como dos tipos de VINCULOS que entre los Terrestres han adoptado una importancia TRANSCENDENTAL debido a su aberrada constitucion:

Son estos:

los VINCULOS DE COMUNICACION INFORMATIVA (LENGUAJE)

VINCULOS ECONOMICOS (INSTITUCION TERRESTRE DEL DINERO)

Que la estructura de la Red economica terrestre esté totalmente equivocada , no es necesariamente preciso explicarselo a ustedes que estan viviendo toda la efervescencia politica social del Planeta que tiene como primera causa visible la injusta distribucion de las riquezas y su renta.

Todas la s Doctrinas filosoficas de ustedes se esfuerzan actualmente en integrar un Sistema completo en el que la concepcion económica de la Sociedad sea mas justa... Sin conseguir materializar sus aspiraciones.

Mas ¿ No se han percatado ustedes que a un nivel mas profundo, late una terrible deficiencia que hace inoperantes y casi estériles las soluciones aportadas por los pensadores terrestres ?

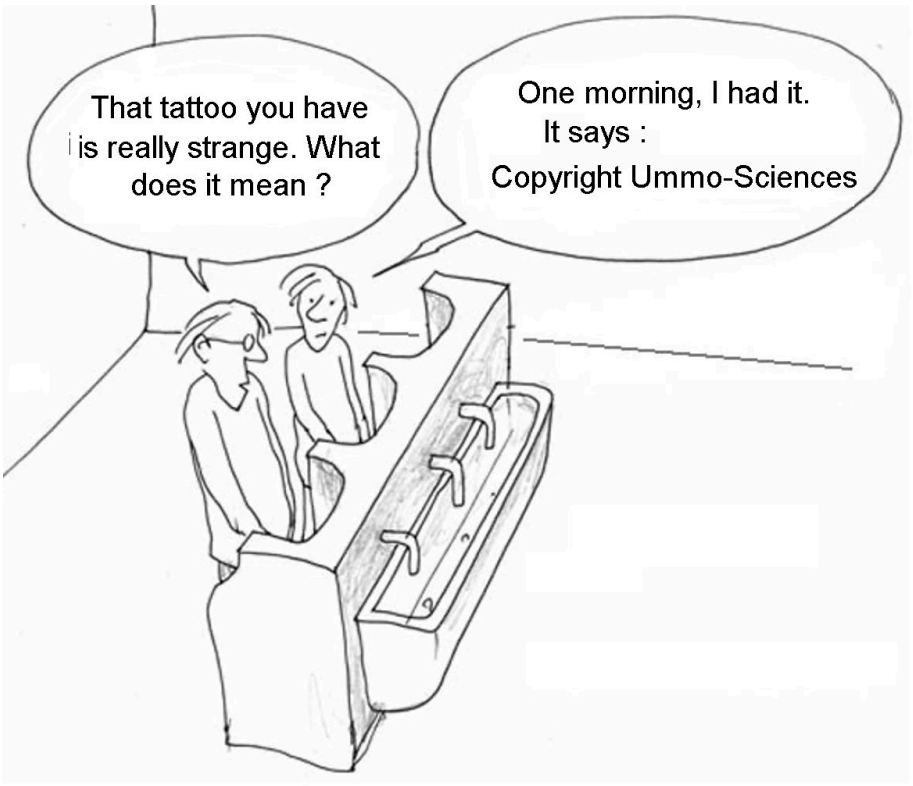
¿ No se dan ustedes cuenta que los medios con que cuentan uste

The Ummo-science copyright

I strongly protest against what is nothing more than an attempt at appropriation, not for financial gain, but for the sake of ego.

To no avail.

Humor is what remains when all you can do is say to yourself:



Better to laugh it off…

When a scientist thanks an alien for his advice.

I'm back in the loop. A letter arrived this morning, signed "Ummo." That rings a bell.

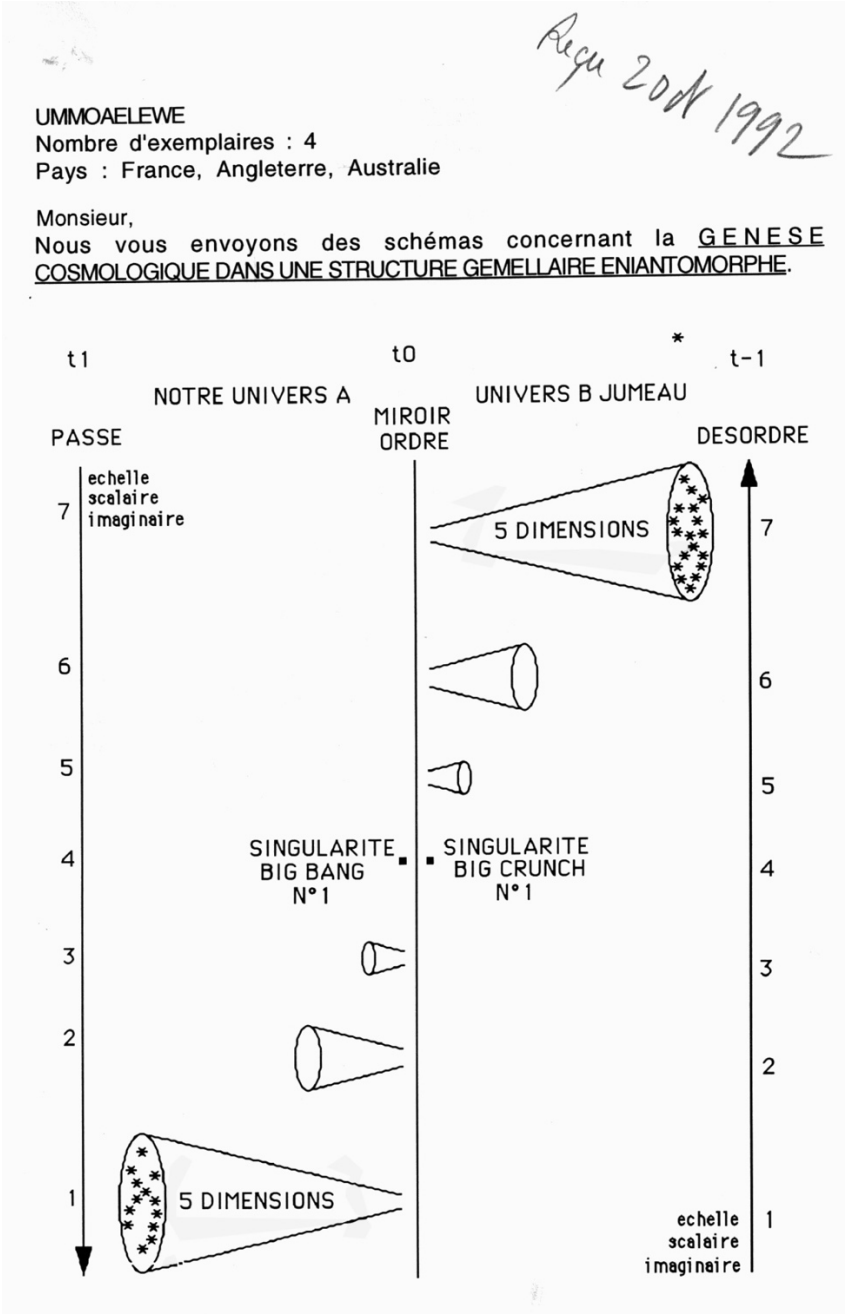
Here's what it says:

The two universes, ours and the twin universe, are connected by the Big Bang. This point therefore marks the reversal of the arrow of time. On one side is the Big Bang, and on the other, the Big Crunch.

In other words, since the Big Bang, the universe has been expanding. It would then reach a phase of maximum expansion, synonymous with chaos. I deduce that at that point, the entire structure of the universe would disappear, including galaxies, stars, planets, and even the atoms themselves. Then the arrow of time would reverse, and the universe would contract until the Big Crunch.

Everything comes full circle.

This reversal of time is, above all, a reversal of the time coordinate, and what we must remember, as stated in Souriau's 1970 theorem, is that this means, above all, a reversal of mass and energy



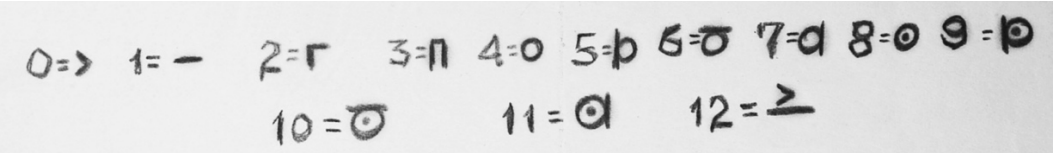
I note that the letter mentions five dimensions, not four. That must mean something. I'll think about it.

Who are the other three recipients, in England and Australia? A mystery. It's signed by a certain OAXIIBOO 6, son of IRAA 3, with that strange repetition of characters.



It's the same signature as the one on the letter received from Riyadh, Saudi Arabia.

In this signature, two digits can be discerned, according to the Ummite coding system.



The coding of whole numbers on Ummo

But that doesn't add up. That would make it OAXIIBOO 5, son of IRAA 2. Never mind, I'll just note it down, that's all.

Other passages in the letter prompt me to revisit my model of cosmic evolution "with variable constants," following up on my 1988 publication in the top-tier journal *Modern Physics Letters A*[131] . I decide to revisit it, since I included in the model the variations, in this early phase, of the electric charge e. A new paper is published in another of the top journals in physics and cosmology: *Astrophysics and Space Science*[132] .

[131] J.P.PETIT: Cosmological model with variable velocity of light. Modern Phys Letters A3, 1988, pp. 1527

[132] J.P.PETIT, Twin Universe Cosmology, Astrophys. and Sp. Science, **226**, 273–307, 1995

TWIN UNIVERSES COSMOLOGY

JEAN-PIERRE PETIT*
Observatory of Marseille, France

(Received 14 July, 1994; accepted 7 February, 1995)

Abstract. Starting from the field equation $\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T}))$, presented in a former paper, we present a test result, based on numerical simulations, giving a new model applied to the very large structure of the Universe. A theory of inverse gravitational lensing is developed, in which the observed effects could be due mainly to the action of surrounding 'antipodal matter'. This is an alternative to the explanation based on dark matter existence. We then develop a cosmological model. Because of the hypothesis of homogeneity, the metric must be a solution of the equation $\mathbf{S} = 0$, although the total mass of the Universe is non-zero. In order to avoid the trivial solution $R =$ constant $\times\, t$, we consider a model with 'variable constants'. Then we derive the laws linking the different constants of physics: G, c, h, m; in order to keep the basic equations of physics invariant, so that the variation of these constants is not measurable in the laboratory, the only effect of this process being the red shift, due to the secular variation of these constants. All the energies are conserved, but not the masses. We find that all of the characteristic lengths (Schwarzschild, Jeans, Compton, Planck) vary like the characteristic length R, from where all the characteristic times vary like the cosmic time t. As the energy of the photon $h\nu$ is conserved over its flight, the decrease of its frequency ν is due to the growth of the Planck constant $h \approx t$. In such conditions the field equations have a single solution, corresponding to a negative curvature and to an evolution law: $R \approx t^{2/3}$.

The model is no longer isentropic and $s \approx \log t$. The cosmologic horizon varies like R, so that the homogeneity of the Universe is ensured at any time which constitues an alternative to the theory of inflation. We re-find, for moderate distances, Hubble's law. A new law: distance = $f(z)$ is derived, very close to the classical one for moderate red shifts.

*The second publication, from 1995, in *Astrophysics and Space Science*, on "the variable-constant phase[133] ."*

This publication features the results obtained by student Frédéric Descamp, who (illegally) used a computer at the German research center DESY (Deutsches Elektronen Synchrotron), located in Hamburg[134] .

[133] In the abstract, two things stand out. First, there is the very beginning of a tensor formulation of this geometry using two source tensors. In the second term: a tensor **T** representing our own matter and a tensor A(**T**) representing the contribution of a second type of matter, for the time being referred to as "antipodal." The "minus" sign gives it the status of a negative mass. The second mention is that of a time coordinate, which is then equated to an entropy varying as Log t.

[134] https://fr.wikipedia.org/wiki/Deutsches_Elektronen-Synchrotron

Fig. 4. Results of simulations performed by F. Lansheat, showing the large structure of the Universe, due to the interaction of the two adjacent folds. Mean value of $\rho^* = 50$ times the mean value of ρ (left). Left: cellular structure. Right: cluster structure.

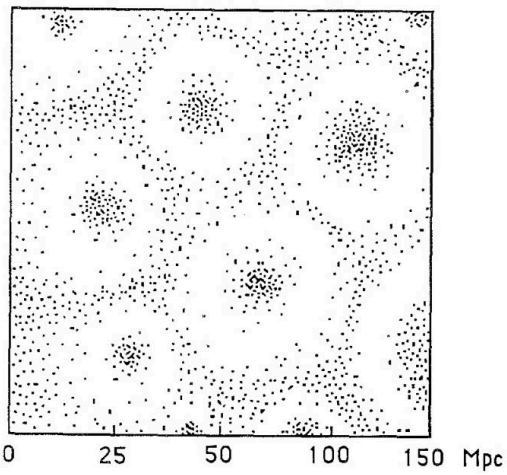
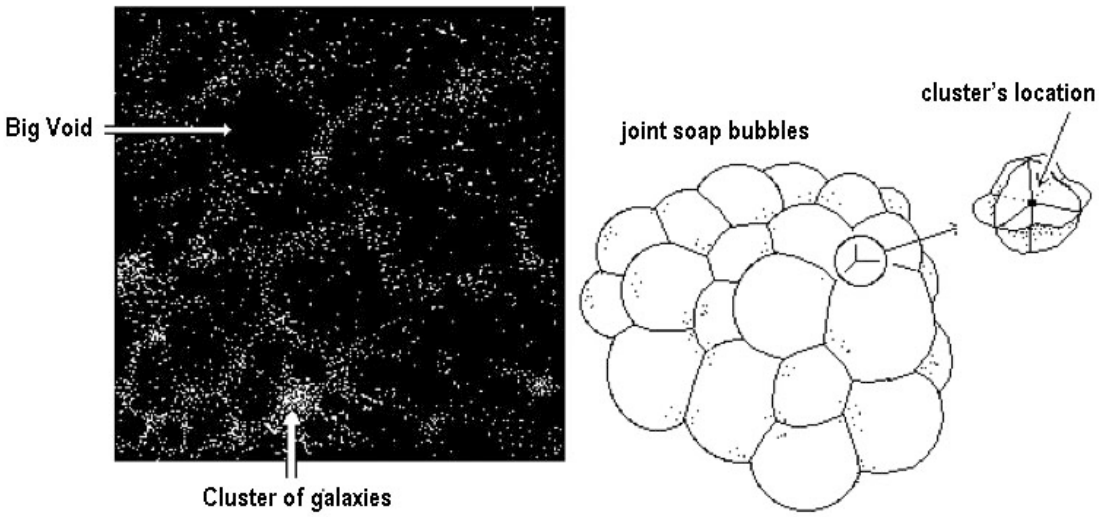


Fig. 5. The same, superimposed.

The large-scale structure of the universe. Initial results obtained by Frédéric Descamp (listed under the pseudonym Frédéric Landsheat)

This is the first time an asymmetry between the mass values of the two sides of the universe has been considered. By assigning greater weight to the absolute values of negative masses, these are the first to cluster together, forming spheroidal conglomerates (shown here in 2D). These then repel the positive mass, giving it a structure of "contiguous bubbles."



Our own matter, structured as contiguous bubbles[135]

[135] This will be confirmed by the Hubble Space Telescope when it detects the presence of a vast void, repelling galaxies, located 600 million light-years from our galaxy and 100 million light-years in diameter (The Dipole Repeller: Y. Hoffman, D. Pomarède, R. B. Tully, H. Courtois. Nature Astronomy 2017, 1, 0036)

In the article, this is a 2D "tiling." These spheroidal conglomerates are thus composed of negative-mass antimatter[136] . The compression resulting from gravitational instability brings them to a temperature at which the antihydrogen constituting them ionizes.

For this world of negative masses to be capable of triggering a recurrence of gravitational instability, it must cool down by emitting radiation. However, due to its size, the amount of heat (which increases as the cube of the radius) is such that the radiation emitted from its surface (whose area increases as the square of the radius) is so great that the time required to dissipate this thermal energy exceeds the age of the universe.

These immense masses of antihydrogen[137] with negative mass therefore do not evolve.

Consequence:

- In the negative side of the universe, there are no galaxies, no stars, and no planets[138] .

Life is absent there.

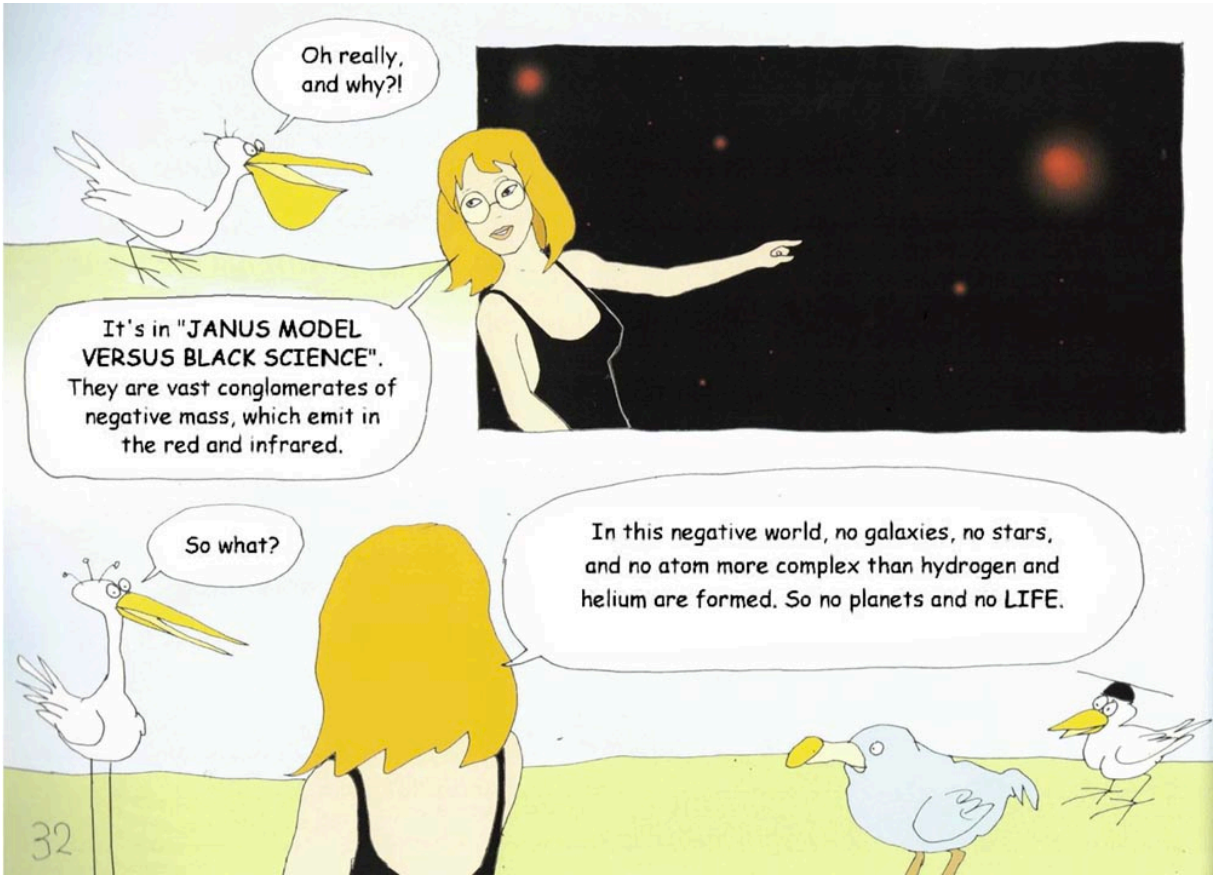
I conclude that if interstellar travel takes place by traversing the "twin universe", the layer of negative masses, as I believe, the spectacle that would then unfold before the traveler's eyes would be that of "distant, blurred objects, emitting faintly in the red and infrared[139] ."

[136] J.P. Petit, H. Zejli: Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, No. 001, 2024 (Hicham Zejli's contribution was limited to transcribing the article into LaTeX).

[137] Plus a certain fraction of negative-mass antihelium, produced by the fusion of antihydrogen.

[138] We find the phrase from the documents: "The twin cosmos does not contain the same number of galaxies."

[139] I will locate the letter in which this detail is mentioned.



*Page 32 of my comic book *Le Métaphysicon**[140]

I see another consequence, leading directly to the observation[141] . When this very large-scale structure forms, the conglomerates of negative mass exert a violent "retro-compression" on matter, arranged in contiguous bubbles. This compression heats it rapidly. But its structure of nearly flat plates allows for an equally rapid loss of energy through radiation emission. The matter is thus destabilized, and gravitational instability can then take hold, giving rise to the first stars and fully formed galaxies within the first hundred million years.

[140]

[141] This would be confirmed by the Hubble Space Telescope twenty years later (discovery of the "dipole repeller," a large void located 600 million light-years from our galaxy), and by the James Webb Space Telescope twenty-five years later (the very early formation of stars and galaxies).

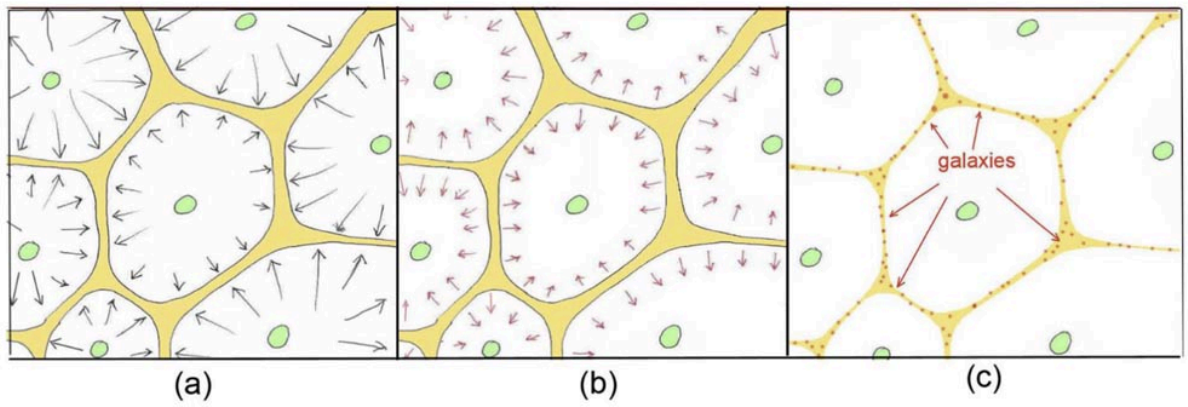


Fig. 12 Early rapid star and galaxy formation

Very Rapid Galaxy Formation[142]

All of this gave me an idea. In that first 1995 paper, published in *Astrophysics and Space Science*, from which such an avalanche of results emerged, I decided to thank OAXIIBOO 6 by replacing the number 6 with the sixth letter of the alphabet:

distant objects. Comparing the available data to the predictions of our model and to those of the (peculiar) Einstein-de Sitter model, we find a slight advantage in the first. Obviously, a single test cannot validate such a model.

Acknowledgement

The author thanks Professor Oaxiiboo F. for useful suggestions and critic.

References

1. Petit, J.P.: The missing mass effect. Il Nuovo Cimento B. Vol. 109, July 1994, pp. 697–710.
2. Zel'dovich, Ya.B.: 1970, *Astrofisica* **6**, 319; *MNRAS* **192**, 192.
3. Doroskhevitch, A.G.: 1980, *MNRAS* **192**, 32.

This simple line of acknowledgment would spark a veritable storm within my CNRS hierarchy. For them, acknowledging a correspondent who presented himself as an alien in a scientific publication was tantamount to crossing a red line. For me, it was simply a matter of basic

[142] Figure taken from: J.P. Petit, F. Margnat, H. Zejli: "A bimetric cosmological model based on Andreï's twin universe approach." The European Physical Journal. Vol. 84, No. 1126 (2024). Contributions by Zejli and Margnat: LaTeX transcriptions and formatting.

intellectual honesty: when an idea is suggested to me and proves fruitful, what could be more natural than to credit its author?

A chemist writing under a pseudonym.

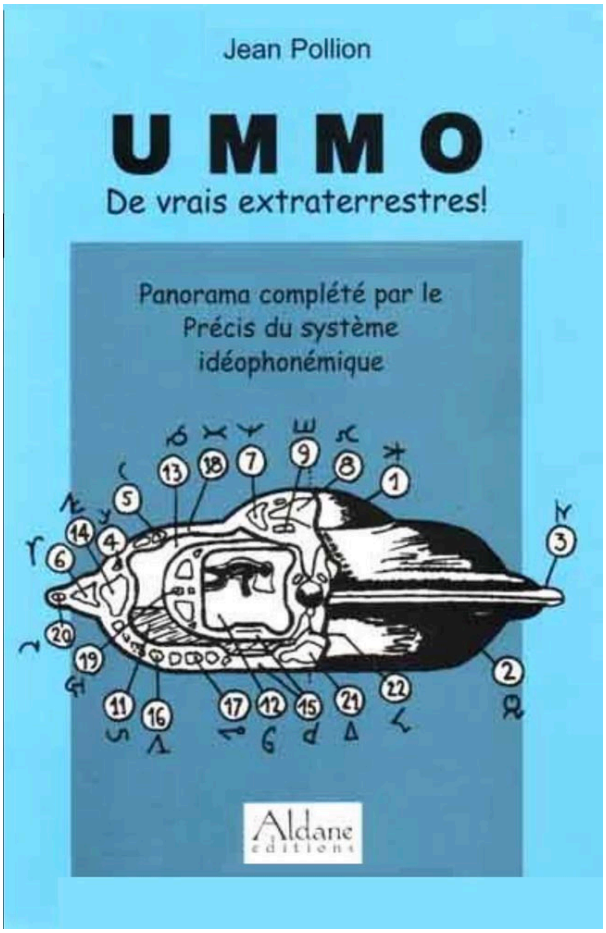
The Ummo texts contain 1,340 words, in the form of sequences of 17 characters borrowed from Western Earth writings:

A B E G I K L M N O R S T U W X Y

Some Ummite words:

XII, XEE, OEMII, ANNOIUXII, UIW, AYUYISAA

Added to this are three sentences in Ummite, each with its corresponding translation. All of this remained a complete mystery for decades, until a book appeared, authored by a certain “Jean Pollion,” a pseudonym inspired by the name of Jean Pollion, a Frenchman who, in 1822, began to unravel the secret of Egyptian hieroglyphs thanks to the first stone found in the port of Rosetta, bearing the same inscription in hieroglyphs and Ancient Greek.



The man turned out to be a Belgian chemist[143] .

I got my hands on the book. The author was clearly inspired by the Chinese language, where certain Chinese characters often stand alone as the equivalent of short phrases.

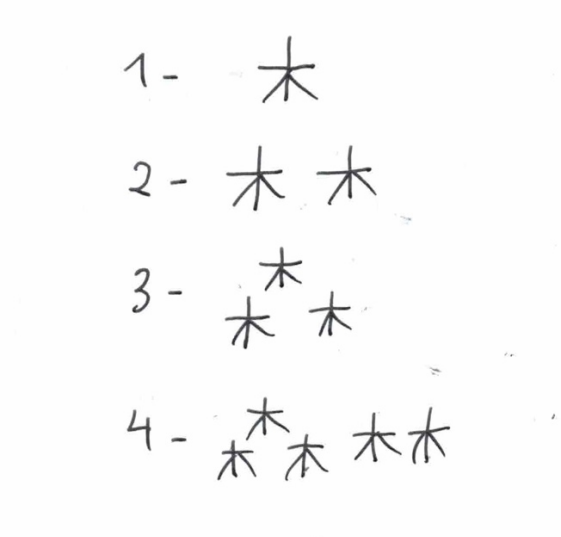
 eminence, reliefs

 The same one, covered in vegetation

 The same one, stony

The first character means “elevation” or “hill.” The second represents the same elevation, covered with vegetation, and the third an “rocky” elevation.

Elsewhere, it is the repetition of the same character that enriches the description. The most typical example is character number 1, which means “wood.” Using two of them yields “grove.” With three, it becomes “forest,” and with five: “a large forest.”



In representation 4, five characters are needed. But in the English phrase “important forest,” there are actually fifteen. The Chinese language is far from lacking in economy.

[143] Obsessed with secrecy, he has to this day kept his true identity a secret. All I have been able to learn is that he would end his days somewhere in France, after having severed all ties with the Ummo affair and the “Ummo-science” group.

Jean Pollion applies this hypothesis to analyze the words in the Ummo texts. It's quite clever and amusing. He refers to each of the 17 characters as "soncepts": "sound-concepts."

Thus, he translates:

- The letter U as Dependence, submission, influence
- the letter M as the idea of relationship, juxtaposition, union, coupling, pairing
- The double letter MM as "interrelation."
- The letter O as Entity, being, creature, real object

When put together, the word UMMO becomes an object representing a set of interrelated components.

We would call it an "ecosystem."

It then becomes apparent that this language has no "proper nouns." It has only "common nouns." Even individuals do not have names. They are designated only by their function, associated with a number.

Thus, if Y means "group" and U means "hierarchical subordination," then YU is the leader. One character plays an important role in this story: Ms. YU 2. This means that she was in charge of the second expedition to Earth.

DEI 98 is the ninety-eighth character in charge of communications with Earthlings.

- D: manifestation, transmission
- E: Patterns, schemes
- I: change, contribution,

He is therefore the one tasked with conveying a different way of seeing things, through new patterns, to Earthlings.

This aligns with the fingerprint stamp affixed at the bottom of all letters. As they will explain later, since an individual's identity in their world is defined by their olfactory signature, when they sign a letter they simply place their finger on the page so that it is marked with their scent.

The system envisioned by "Pollion" is ingenious and described in detail in his book. But to gain a foothold in a language, these 1,340 words are not enough. Entire texts with sentences would be needed to bring out syntax, grammar, and semantics. And three is completely insufficient.

I mentioned earlier that seeing all the Ummo documents stamped with a copyright, in the form of the "UMMO-SCIENCE" watermark, had created a rift between, on the one hand,

Jean-Jacques and me, along with the small GESTO group, and, on the other hand, the UMMO-SCIENCES group[144] .

"Pollion immediately joined the people at Ummo-Science." Logically, after doing such work, the letters should have arrived, containing numerous sentences he could work on, allowing him to establish himself as the Champollion of the extraterrestrial language.

But nothing came.

He then established a committee tasked with evaluating documents, both old and new. Following a vote, a rating is assigned, reflecting an assessment not only of the topic addressed but also specifying the document's authenticity[145] .

Jean Pollion thus became the group's expert, applying criteria based on references to earlier texts and on the linguistic consistency of any new words that might appear in the corpus.

Within this Ummo-science group, he is regarded as an authority in a new discipline, which they themselves have named:

Ummology

The Gulf Letter.

On January 14, 1991, the Spanish contactees received a letter announcing the imminent outbreak of the first Gulf War (Operation "Desert Storm"). This is document D1741 (according to the classification system of the Spanish group's archivist, Ignacio Dardaude, until his death). The letter is replete with new Ummite words, each more pertinent than the last. Here, for example, is an "Ummite" analysis of Saddam Hussein's personality:

- *Although initially the motives for invading this territory[146] involved several distinct intentions (to expand the country, believing that the United States would not venture to come to its rescue), the reasons behind Saddam Hussein's conduct have undergone a radical change (UUW EYEYIIOXAA LEII) to the point that his mental structure (EESEE OOA) is radically different. He seeks to put his mind at ease (BUAUUAA) by linking the seizure of the territory to the dramatic injustice suffered by the Arab people at the hands of the dominant powers of the northern ethnic groups of the planet (OIIAGAA YUUGIIXI) and, in particular, to Israel's invasion of Palestinian territories.*

The following is information on the status of Iraq's nuclear weapons program:

[144] http://www.ummo-sciences.org
[145] All the letters I myself received are still labeled there as FP: probable forgery
[146] Kuwait

La réalité est beaucoup trop complexe pour donner une réponse simple. Qui a raison ? PERSONNE ET TOUS. Mais cela mérite quelques nuances. Laissons de côté la conduite de cet homme que vous pouvez qualifier de déséquilibré. Si vous le laissez consommer l'invasion, invoquant une doctrine pacifiste très louable, vous courez en Occident un péril beaucoup plus grave que celui que nous avons conjuré. (Les projets d'arme fission-fusion-fission sont très avancés et l'arme sera au point dans 1,6 à 2,2 ans, et 2,8 à 3 ans de plus suffiraient à la transférer à l'Égypte (Dès lors, l'assassinat de l'actuel dirigeant aurait changé la face des choses)

Excerpt from the letter received before the outbreak of the Gulf War

Iraqi scientists have therefore already developed a three-stage bomb, the F-F-F device, in which an A-bomb triggers the fusion of a lithium hydride charge (the H-bomb), which acts on an outer shell of uranium-238, whose nuclei transform into plutonium-239, which then undergoes fission[147] … The pinnacle of nuclear weaponry.

As if nuclear weapons weren't enough, the letter mentions Saddam Hussein's arsenal of silo-based missiles, carrying terrible biological weapons:

- *In any case, the silos contain warheads equipped with VX and GD agents. The latter is ortho-1-2-2-trimethylpropylmethyl-phosphofluorohydrate (Soman nerve gas), which has extremely severe consequences for victims who inhale it. The lethal dose is estimated at one-hundredth of a gram.*

This is really… utter nonsense. But the letter includes details, this one is true, intended to make it seem credible.

- *In the current war, only the U.S., the USSR, Great Britain, France, and Israel possess this "deadly" weapon; Canada and China are aware of it. We are going to reveal its name to you. It is not a physical entity but a concept. It is called ISC (Informative Satellite Coefficient) (measurements provided by an information satellite). Only the first two countries can use it effectively.*

This concept of ISC strongly foreshadows what would become, a few years later for the general public, GPS and, more broadly, modern satellite-based guidance and geolocation systems.

Finally, read these last few lines:

- *There is a clearly real risk, carefully concealed by UN officials to avoid widespread panic. Some 62 suicide commandos are stationed in the United States, England, Israel, Italy, Canada, Australia, and, to a lesser extent, in other countries. They have been given explicit orders to contaminate drinking water distribution networks with viruses engineered in laboratories using genetic engineering techniques.*

[147] After the complete rout of Iraqi troops, UN envoys found no trace of the weapons of mass destruction, nuclear or chemical, cited by Colin Powell.

> *Although the success rate is 27% (the risk is considerably lower in Spain), WE URGENTLY RECOMMEND THAT YOU STOCK UP ON DRINKING WATER BEFORE JANUARY 19.*

So here we have Saddam Hussein's commandos ready to poison drinking water sources with genetically engineered viruses!

The message is highly politically charged, which does not match the tone of previous Ummo texts. In any case, this document shows just how easy it is to produce a forgery that perfectly mimics this style of writing.

And this brings me back to my own strategy, which focuses exclusively on the scientific validity of the information at[148] .

From this perspective, the work of this validation committee, established by Jean Pollion within the Ummo-science group, is rather amusing.

The punishment was not long in coming. The group received a letter from Canada. It included an assessment of the surface area of the immense lake located at the center of the single continent on the planet Ummo, accurate to the nearest square meter.

The brilliant team, conducting its own assessment using a computer, arrived at exactly the same figure. Since Pollion had found meaning in the various words scattered throughout the letter, the signatory was a certain WD 40, the team attributed a high probability of authenticity to it.

Their conclusions as experts in "Ummology" were published on the website.

Immediately, a letter arrived from Canada, in which the author, claiming to be a science student, said he had wanted to test their credibility.

It ends by stating that WD-40 is "the lubricant he uses to grease his bicycle."

[148] Nevertheless, and you can verify this by visiting the ummo-sciences website, this document, validated by Jean Pollion (after analyzing the words scattered throughout it), still appears among the PDFs presented as authentic!



Math from another world.

The phone rings. I look at the time, which appears in red on the LED display of the clock in the bedroom.

- *Three o'clock! Who's calling at this hour?*

I pick up the phone.

- *Change of variables. The s-coordinate equals the Schwarzschild radius multiplied by one plus the logarithm of the hyperbolic cosine of rho. You have to write this down, or you'll forget it when you wake up.*

The phrase is repeated several times, over and over.

- *Yeah… yeah, okay!*

I get up and head to my desk. I grab a blank sheet of paper and a pencil and write down the formula.

I go back to the bedroom. The caller has hung up. I go back to bed.

In the morning, I struggle to wake up.

- *Who's playing the guitar?*

You can't really call that "playing." It's as if someone were plucking the strings. The sound is coming from my own instrument, hanging on the wall.

I sit up abruptly.

- *Just like in Paris, in 1947, on Rue Jean-Baptiste Dumas!*

I get up and go into my office. I have to check to see if I was dreaming. But no, the scrap of paper is definitely there, with the formula, waiting for me.

$$r = R_s\left(1 + \log \mathrm{ch}\, \rho\right)$$

The Schwarzschild radius is a characteristic length, a quantity familiar to any theorist working in cosmology. This quantity was introduced in 1916 by the Austrian mathematician Karl Schwarzschild, while he was stationed on the Eastern Front, facing Russian troops, during World War I. A great deal of work has been done to refine his solution from January 1916.

I hear myself saying aloud:

- *We must make this change of variables in the external metric, the Schwarzschild solution to Einstein's equation in a vacuum*[149] .

I pull a book on cosmology off a shelf in my library. Here's the solution[150] :

(6.52) $$\sin^2\theta[(e^{-\lambda}r)' - 1] = 0$$

By virtue of (6.45) this is identically zero, so we see that the equation $R_{33} = 0$ is indeed consistent with (6.47). We leave it to the reader to verify that all the off-diagonal elements of the contracted Riemann tensor are identically zero when explicitly written in terms of the coordinates and that (6.47) is therefore a completely consistent solution of (6.25).

Let us now summarize the results of this section by exhibiting the *Schwarzschild line element*

(6.53) $$ds^2 = \left(1 - \frac{2m}{r}\right)(dx^0)^2 - \frac{dr^2}{1 - 2m/r} - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

This result must be considered to be the main achievement of general relativity theory in the field of celestial mechanics; it is an exact solution,

- *Here, the quantity Rs is 2m. Replace the variable r with this variable ρ*[151]

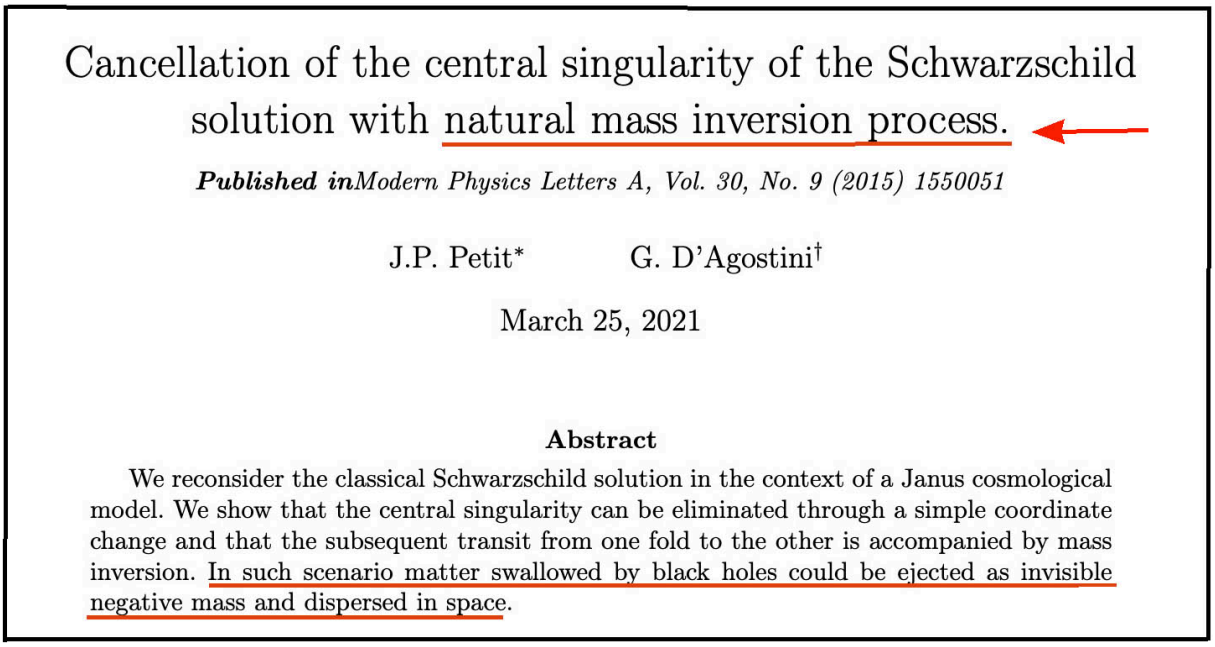
I line up the calculation steps.

[149] http://www.jp-petit.org/papers/cosmo/1916-Schwarzschild-exterior-deu.pdf
[150] All my books are like that. I underline things when they seem important to me.
[151] ρ st is the Greek letter rho

- *Damn! This yields a different geometry, with two layers, analogous to the solution published in 1935 by Einstein and Rosen*[152] *. But the behavior at infinity is correct. This provides the first geometric interpretation of mass inversion. That's new, it's incredibly new and incredibly interesting!*

I need to stop talking to myself.

In the days that followed, I put all of this down in a paper that I submitted to the high-impact journal *Modern Physics Letters A*, which published it[153] .



Cancellation of the central singularity of the Schwarzschild solution with natural mass inversion process.

Published in*Modern Physics Letters A, Vol. 30, No. 9 (2015) 1550051*

J.P. Petit* G. D'Agostini†

March 25, 2021

Abstract

We reconsider the classical Schwarzschild solution in the context of a Janus cosmological model. We show that the central singularity can be eliminated through a simple coordinate change and that the subsequent transit from one fold to the other is accompanied by mass inversion. In such scenario matter swallowed by black holes could be ejected as invisible negative mass and dispersed in space.

Mathematics from Another World

In the abstract, the sentence:

- *In such a scenario, matter swallowed by black holes could be ejected in the form of invisible mass, scattered throughout space.*

Mathematically, it's beautiful. It's what countless people have been searching for for exactly a century, a magic formula that yields the correct geometry.

But above all, it suggests a process of mass inversion, when mass is sent to another side of the universe.

---

[152] http://www.jp-petit.org/papers/cosmo/1935-Einstein-Rosen.pdf

[153] J.P. Petit & G. D'Agostini: Cancellation of the singularity of the Schwarzschild solution with natural mass inversion process. Mod. Phys. Lett. A vol. 30 no. 9 **2015** arXiv: gr-qc/2103.12845/13 Feb 2001 (March 23, 2021) https://arxiv.org/abs/2103.12845

And that is nothing less than the beginning of the formulation of the phrase Dominguez heard a few years earlier.

But one point remains intriguing. We don't know where this formula comes from.



Where does it come from ? … ….

Boy's surface.

Souriau was right. You don't come out unscathed from associating with a man like Bernard Morin.



Bernard Morin 1931–2018

Our brains are networks of connections between different clusters of neurons. From childhood, through school, and finally in higher education, we are wired, we are shaped.

Morin wired himself.

And as I leave all our sessions, I realize that I am no longer quite the same person.
As for Bernard, he has internalized this idea of the inversion of space and time. He has digested it over the course of months, at length.

- *This Ummite model with two universes is geometry. At the heart of it all is the surface invented by the German mathematician Werner Boy in 1902, just before he was killed in France at the age of 34, four months after the outbreak of World War I.*

- *What is this surface? Do we have any images of it?*

- *Just the ones that illustrate his 1902 article[154] . It's very little known, especially since it exists only in its original version, in German[155] . You need to understand how it works.*



liegen die Mäntel an den Kurven B und C. Wir zerschneiden nun längs der Kurven A, B, C die sechs Kanten und heften die Ränder kreuzweise aneinander. Damit ist die Fläche fertig, in den Kurven A, B, C durchdringt je ein Cylinder senkrecht ein ebenes Blatt. Der Anfangspunkt ist

Fig. 20a. Fig. 20b.

ein dreifacher Punkt der Fläche. Die sich dort durchdringenden Mäntel haben die Koordinatenebenen zu Tangentialebenen. Wir sehen, unsere Fläche ist singularitätenfrei. Die Kurven A, B, C bilden in ihrer Ge-

The surface invented by Werner Boy in 1902[156]

[154] At the time, there were only those two. Computer-generated images began when I provided the first analytical representation in 1981)

[155] And that is still the case as I write these lines in 2026

[156] W. Boy: Über die curvatura integra und die Topologie geschlossener Flächen. Dissertation, Georg-August-Universität, Göttingen, 1902 (Integrated Curvature and the Topology of Closed Surfaces. Doctoral dissertation, Göttingen, 1902)

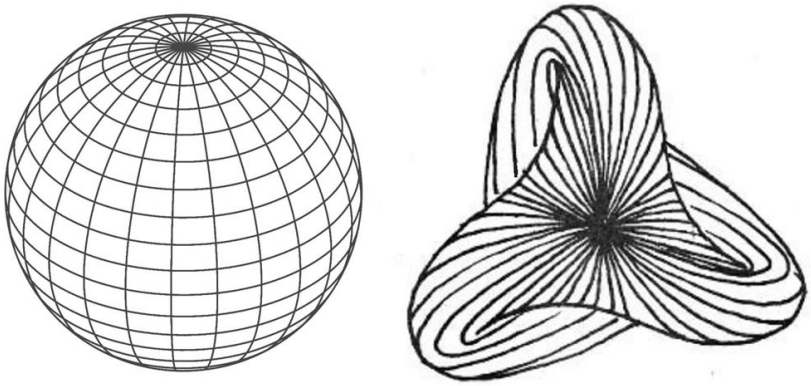
But Bernard has gone back to Strasbourg, where he teaches, and I have to figure this out on my own.

I finally managed to figure out his “four-eared model,” the central model of his version of the sphere’s inversion. That’s all done, drawn and published.

Boy’s surface has three ears.

I spend hours in my studio at the École des Beaux-Arts in Aix-en-Provence. Eventually, I find the solution fairly quickly, thanks to this technique using 2-millimeter-diameter copper-coated wire, which is flexible enough. There’s that, and the sewing thread with which I constantly create new connections between intersecting curves.

Some discoveries happen through this artidwyal process, and this is one of them. In this studio, in 1981, I discovered how to arrange the “meridian curves” of Boy’s surface, in the form of ellipses converging at its single pole, in the same way that the meridians of a sphere are organized around two poles, north and south.



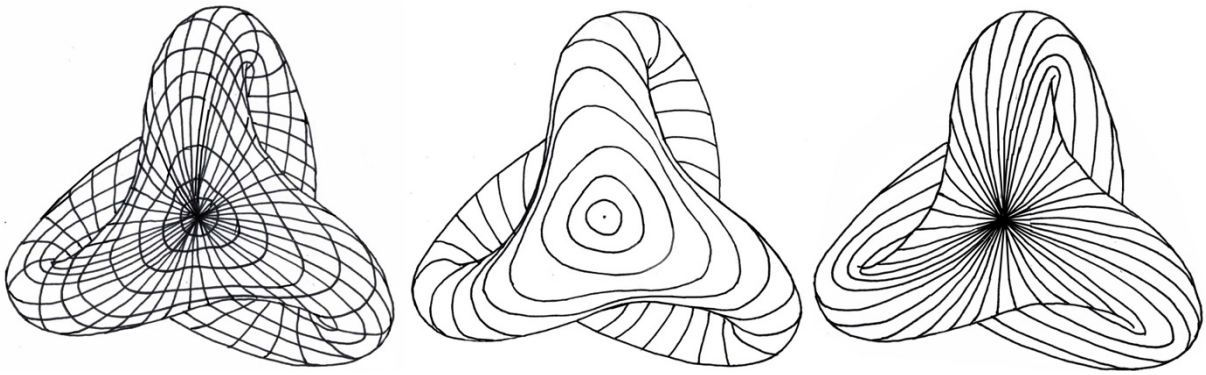
Meridians of the sphere and of Boy’s surface

The figure on the right shows the meridian curves of Boy’s surface, converging at its single pole. As with a sphere, we can draw a second family of curves, which can then be called “parallels,” orthogonal to the meridian curves.

On a sphere, starting from one of the two poles, we can draw a closed curve around it, a “parallel.” Then we can draw another one outside that region. And so on. These circles constitute the parallels of the sphere. Their maximum circumference corresponds to the sphere’s equator.

Then, if we continue this process, the circumference decreases until it reaches zero when it contracts to the other pole of the surface.

Starting from the single pole of Boy’s surface, one can proceed in this way step by step by inscribing closed curves that, as with the sphere, have a maximum circumference.

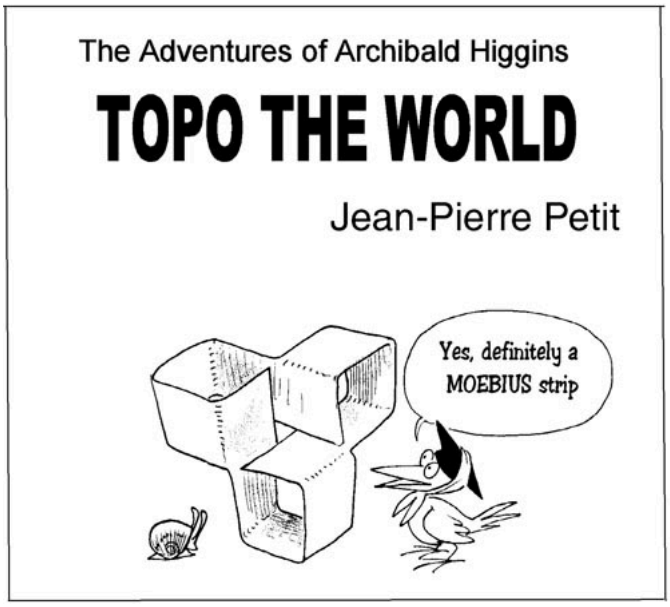


Left: the set of meridians and parallels; right: the meridians; center: the parallels

Then, taking this further, as with the sphere, we obtain closed curves whose perimeter gradually decreases until… it converges toward the same pole[157] !

That's exactly what Souriau meant when he said it "twists your brain"!

In my comic book, readers will find:



the elements needed to understand all of this. I've even created a "flip book" throughout the pages. As you flip through the book, you'll see four animated sequences unfold.

A note at the beginning of this album:

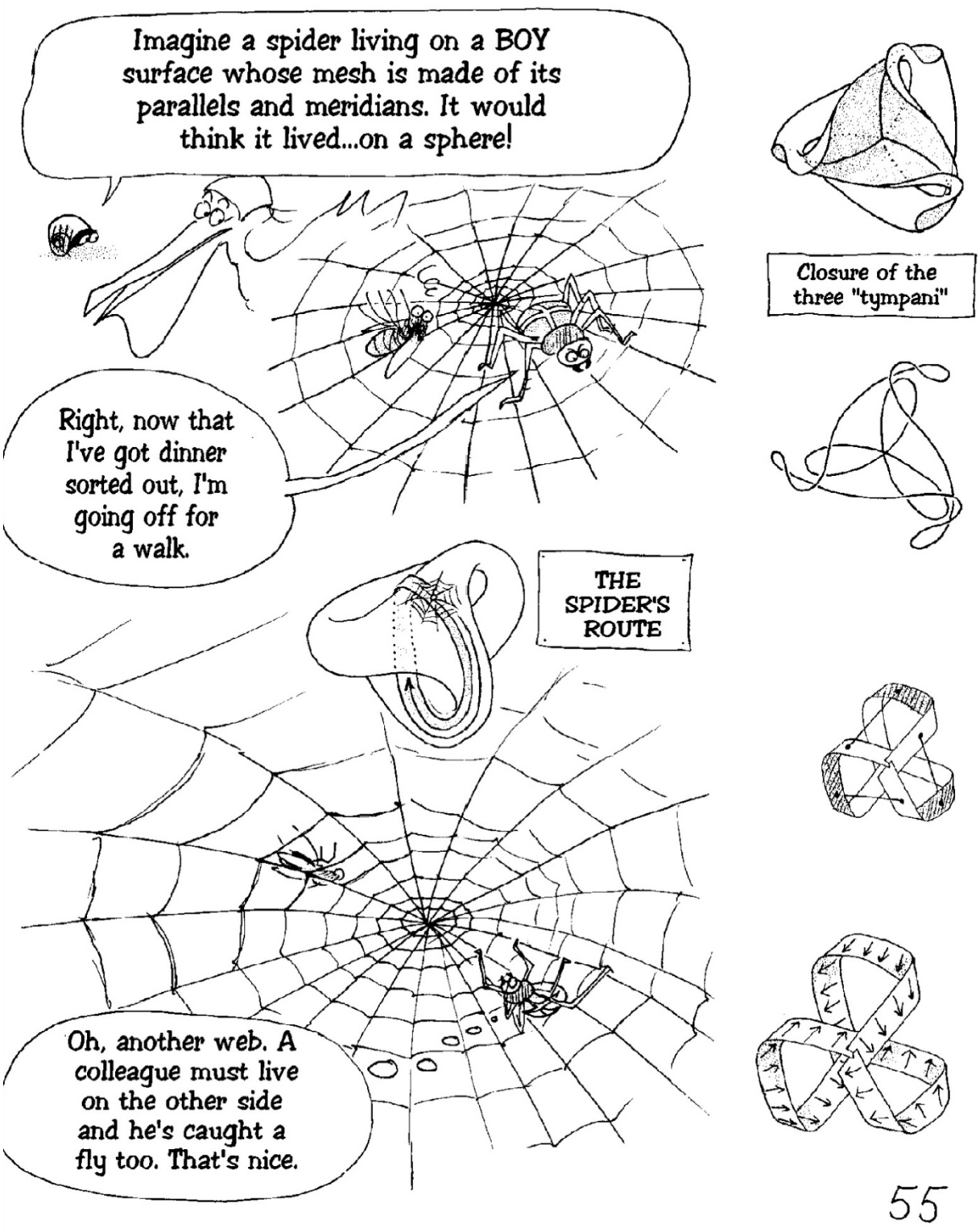
[157] For several decades, a model of this surface, with its meridians and parallels, two meters in diameter, created at the École des Beaux-Arts in Aix-en-Provence, and absolutely one of a kind, was on display at the Palais de la Découverte in Paris. However, since the museum's administration felt it had been seen enough, it was sold to a Belgian psychiatrist, Dr. Schnock

It is not recommended to read this book

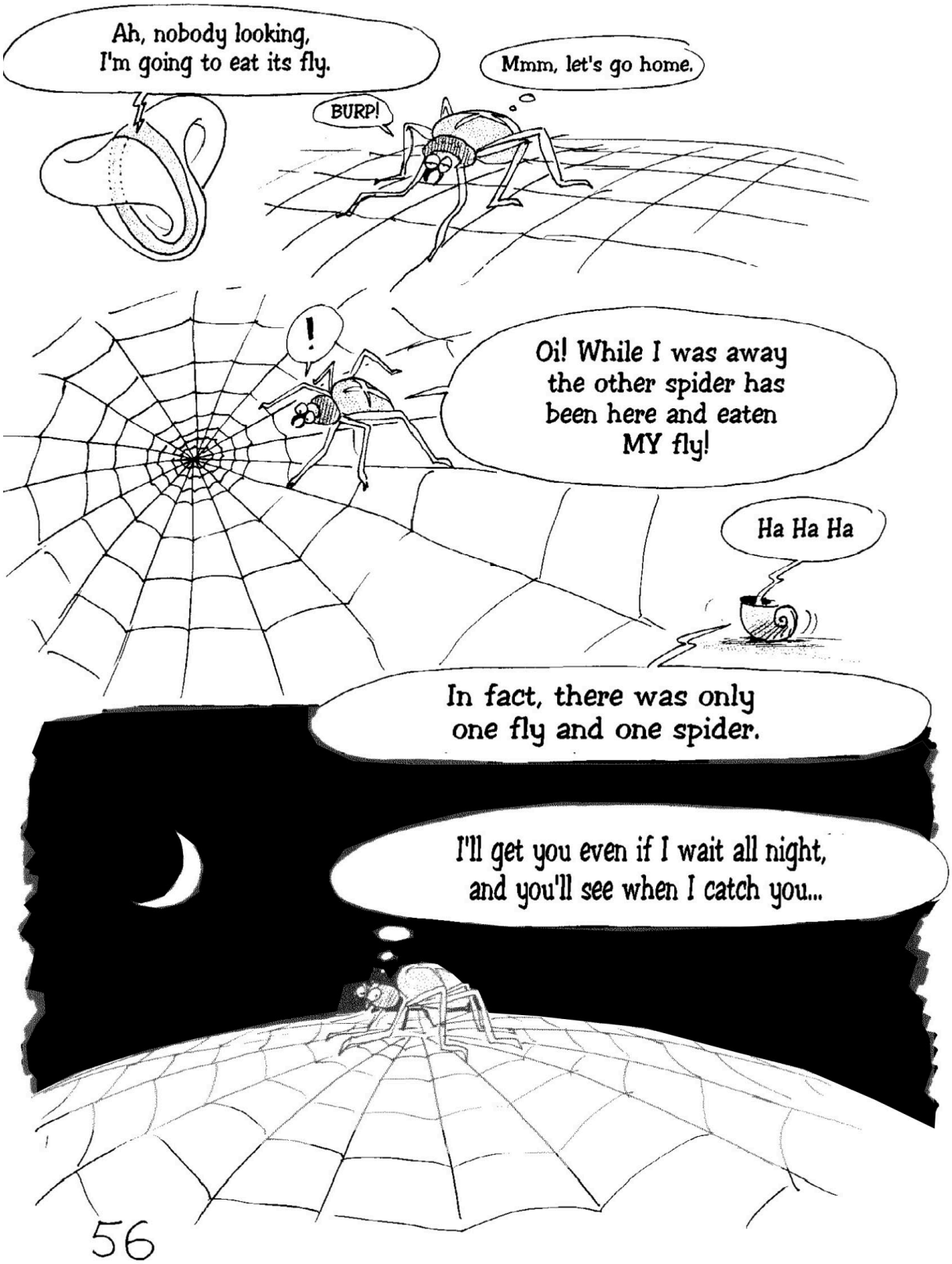
- *At night before going to sleep*
- *After a heavy meal*
- *Or when you're unsure of anything, because it would only make things worse*

If the reader wants to embark on this adventure, then bon voyage! But be warned: your view of space and the universe may take a hit.

This story of the parallel curves on a Boy's surface, which, starting from its single pole, converge back toward it, is illustrated by the story of a spider that, without realizing it, weaves its web on a Boy's surface.



On the right, you can see elements from the four "flip books"


Ah, nobody looking,
I'm going to eat its fly.
Mmm, let's go home.
BURP!
!
Oi! While I was away
the other spider has
been here and eaten
MY fly!
Ha Ha Ha
In fact, there was only
one fly and one spider.
I'll get you even if I wait all night,
and you'll see when I catch you...
56

Who do I have to kill for this?

Souriau's son, Jérôme, is not a mathematician but a computer scientist. I offered to include his name in a mathematics publication in exchange for him completing a task. And he replied:

- *Who do I have to kill for that?*[158]

No sooner said than done.

C. R. Acad. Sc. Paris, t. 293 (5 octobre 1981) Série I — 269

GÉOMÉTRIE. — *Une représentation analytique de la surface de Boy*. Note (*) de **Jean-Pierre Petit** et **Jérôme Souriau,** présentée par André Lichnerowicz.

Here we see the first representation of the meridians, created by Jérôme on his Apple II:

C. R. Acad. Sc. Paris, t. 293 (5 octobre 1981) Série I — 271

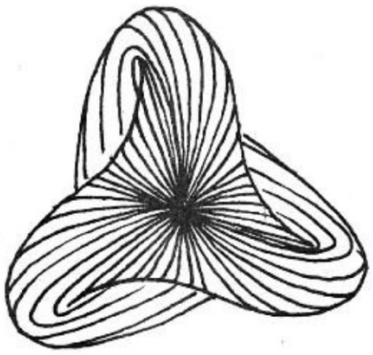


Fig. 4. — Lignes méridiennes (Eμ) de la surface de Boy (tracé par « Apple-II »).

I won't go any further down this path, lest I lose my non-scientific readers along the way[159] .

[158] Jérôme's task is to take measurements on the wireframe model of the parameters of the ellipses that make up its "mesh", the "meridians", and to provide a semi-empirical construction of it. This construction appears on the last page of the album *LE TOPOLOGICON*. Once entered into our laptop as a program, this allowed us to obtain the very first, albeit quite primitive, computer-generated image of this mysterious surface invented by Werner Boy in 1902. References for the article: J.P. Petit, J. Souriau: An analytical representation of the Boy surface. CRAS vol. 293 (Oct. 5, 1981)

[159] What emerges from all this is the topology of the universe, as the covering of a four-dimensional Boy hypersurface. See J.P. Petit, F. Margnat, H. Zejli: "A bimetric cosmological model on

The schedule.

In my mailbox, a new letter in which "those gentlemen from Ummo," as Ribera calls them, set out the schedule for contacts with Earth:

> *The first phase, known as the "study" phase, which is soon to be completed, was conducted very successfully, as it allowed us to store on titanium memory the bulk of the technical and cultural information regarding your rich and fascinating Earth civilization, without, on the whole, disrupting the subject of study.*
>
> *The second stage of the program (1970–2000), known as the "preparation" phase, aims to raise awareness and foster a gradual maturation among political, military, and subsequently scientific, social, and religious elites. Its goal is to make the very idea of an extraterrestrial presence on Earth rationally acceptable and tolerable, and to promote the gradual dissemination of this new paradigm on a macrosocial scale.*
>
> *The implementation of this second phase is ensured by an intensification of overflights of populated areas, landings, and daytime encounters; an increase in telephone contacts and mailings; and the inevitable execution of disinformation operations deemed, unfortunately, indispensable.*
>
> *The third stage, known as the "contact" stage, aims at official macro-social disclosure, the establishment of official contacts between your governments and the UMMOAELEWE, and the planned and coordinated development of a bicultural evolution of our two civilizations.*
>
> *Logically, this third phase should take place at the beginning of the third millennium (subject to changes due to the evolution of the sociopolitical conditions prevailing on Earth and the absence of a cultural clash that would be overly detrimental to you, which could result from an excessive transfer of knowledge), probably between 2030 and 2050, depending on the level of immunity and psychological resilience demonstrated by Earthlings.*

I see that we are supposed to be in this phase of "raising awareness and the gradual maturation of the political, military, and then scientific, social, and religious elites, aimed at making the very idea of an extraterrestrial presence on Earth rationally acceptable and tolerable, and at promoting the gradual dissemination of this new paradigm on a macrosocial scale."

Until now, I had wondered whether these extraterrestrials might have a mistaken view of how Earth's bipeds function.

Andreï's twin universe approach." The European Physical Journal. Vol. 84: No. 1126 (2024)https://link.springer.com/content/pdf/10.1140/epjc/s10052-024-13569-w.pdf

After reading these few lines, it has become a certainty.

Where are our political, military, and scientific elites???

I'm thinking back to the book by Pierre Accoce and Pierre Rentchnick, *These Madmen Who Govern Us[160] *. This isn't just a matter of physical disorders, but outright psychological profiles that border on mental illness.

To have any chance of eliciting a positive response from people, we would still need to find communities that are open to some kind of change and are not exclusively concerned with maintaining a status quo that benefits them, even if it is suicidal. In a world where it is the most deranged, the paranoid, and the narcissistic perverts who rise to the top and seize the reins of power in every field.

Everywhere, we must expect psychosocial and immunological reactions, a backlash against "the outsider," who is immediately identified as a new enemy.

Earthlings have been continuously producing disinformation campaigns regarding the UFO phenomenon for over half a century. The most recent one consists, while acknowledging the reality of the phenomenon (which has become increasingly indisputable), of attributing it to "people from the future" who, by that very fact, could not possibly interfere with their own past[161] .

Others deny any possibility of gaining a scientific understanding of the UFO phenomenon, labeling all its manifestations as "paranormal." In 1976, I found myself facing Jacques Vallée, the UFO expert, on Polac's show "Droit de Réponse." He immediately declared:

- *"Well, personally, I would be very disappointed if the UFO phenomenon were nothing more than visits from extraterrestrials!"*

All his thoughts and his books boil down to that one sentence. Long live obscurantism, which protects us from any unsettling knowledge!

I suddenly had an idea.

If the aliens reacted to what I wrote on my hard drive, why not "write to them" using the same "channel"?

The scientific method has its own rules, which are defined according to a range of "rational" approaches.

- Focusing on model failures
- Attempting to construct new ones
- Testing these new perspectives against observations.

---

[160] Published in 1976 by Editions Stock. Followed in 1996, by the same publisher, by "Ces nouveaux malades qui nous gouvernent."

[161] In the Janus model, we are confronted with two facets of the universe, each with opposing arrows of time. So, why not take a little trip into this retrochronous realm? Nature has a way all her own of thwarting these traps of temporal paradoxes. Visiting one's own past can therefore only be achieved through journeys whose duration exceeds the age of the universe.

The epistemovirus.

I propose to serve as a vector for injecting knowledge into the Earth's scientific fabric by implementing this strategy.

The reaction is immediate. Letters arrive in large numbers, referring to various fields. Some report observations of exotic objects, whose behavior, a sudden drop in temperature, would reveal the strange dialogue between our universe and its twin.

I do not have access to these observational methods. My laboratory is "pen and paper"; my tools are those of mathematics, and more specifically, geometry.

Alone, I'm taking on a fortress.

I have a few years left before I take my leave. It's already a huge blessing that I still have my mind, my imagination, and my ability to piece together the puzzle.

So many people have already left. Life hasn't spared me. I've had my share of tragedies and losses.

I'm just a hamster, an insignificant speck of dust, lost in the middle of the universe. I feel like a grain of sand on a beach. All of this weighs heavily on my shoulders.

But I have this vague sense that there's something I need to do, and a kind of certainty: if I don't do this work, no one else will do it for me.

"Dark science."

Since the 1970s, we have entered a dark age of knowledge. While our technology is exploding and our observational capabilities are pushing the limits of what's possible, things are going from bad to worse in the world of science.

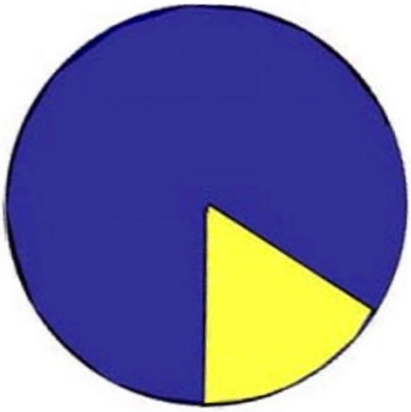
The Sun is rotating too fast within the galaxy. The amount of matter we detect and measure, the combined masses of stars and gas, creates a gravitational field too weak to counteract the centrifugal force. The solar system should have collapsed long ago. So something, the nature of which we do not know, is preventing this from happening.

We're adjusting the cosmological model. Only a fraction of the galaxy's mass is observable. Our telescopes scan the sky, searching for MACHOs[162] .

The words and acronyms invented by scientists are like exorcisms. We imagine that if we attach words to the objects we're trying to capture, they won't be able to escape us.

[162] *Massive Compact Halo Objects*: massive objects forming a vast halo that extends far beyond the observable part of the galaxy.

We talk about them, so they must exist. They can't not exist, we think. So here's what the universe is made of:



The composition of the universe, before 2011
Dark and visible matter

But other observations point to a new, significant problem.

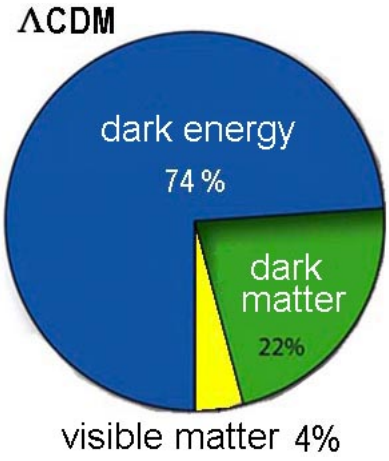
Until now, calculations, based on a solution to Einstein's equation found in 1922 by the Russian scientist Friedman, had led us to conclude that the expansion of the cosmos was slowing down.

On the contrary, it turns out that it is accelerating[163] . And this can only be due to some unknown, repulsive form of energy. We call it dark energy.

Its nature: no idea. But we've given it a name. That's something, at least. We at least know what we're looking for. We convert it into mass using the famous equation:

$$E = mc^2$$

And here is that graph, which many people are now familiar with:



[163] Discovery recognized with the 2011 Nobel Prize, awarded to Saul Perlmutter, Brian Schmidt, and Adam Riess

In 2011: the new composition of the universe according to the "standard model."

At great expense, a "dark matter detector[164] " is installed aboard the International Space Station.

Nothing.

Lots of people are burying themselves in mines, deep inside tunnels, to be protected from cosmic rays by thousands of meters of rock[165] .

Nothing.

CERN's particle accelerator joins the fray. By causing a head-on collision between two protons, each accelerated to a speed very close to that of light, we hope to bring about the appearance of the most credible candidate particle, the neutralino.

Nothing.

When we discover something, new problems arise. In 2017, four researchers, two French scientists, Danielle Pomarède and Hélène Courtois; a Canadian, Brent Tully; and an Israeli, Yehudi Hoffmann, discovered the existence of a vast void, 100 million light-years in diameter, located 600 million light-years from our galaxy[166] . Mysteriously, something[167] within this vast void is repelling galaxies.

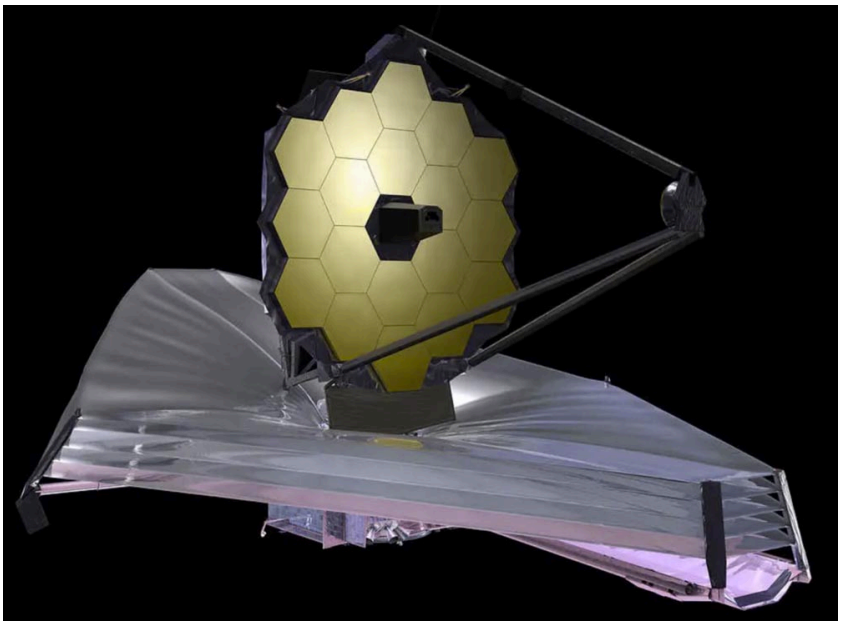
To top it all off, as soon as the new James Webb Space Telescope was launched, it revealed that galaxies very similar to our close neighbors, featuring a bar and spiral structure, formed 350 million years after the Big Bang, whereas the Standard Model is unable to produce such galaxies until at least one billion years after the Big Bang.

[164] This is the AMS-02 experiment, designed by Nobel laureate Professor Samuel Ting. It was installed aboard the ISS in 2011.

[165] A dark matter particle colliding with an atom of the noble gas xenon would produce a very faint ultraviolet scintillation. From 2006 to 2022, over a period of sixteen years, the mass ranged from a few dozen kilograms to six metric tons, but the search was unsuccessful.

[166] The Dipole Repeller: Y. Hoffman, D. Pomarède, R. B. Tully, H. Courtois. Nature Astronomy 2017, 1, 0036

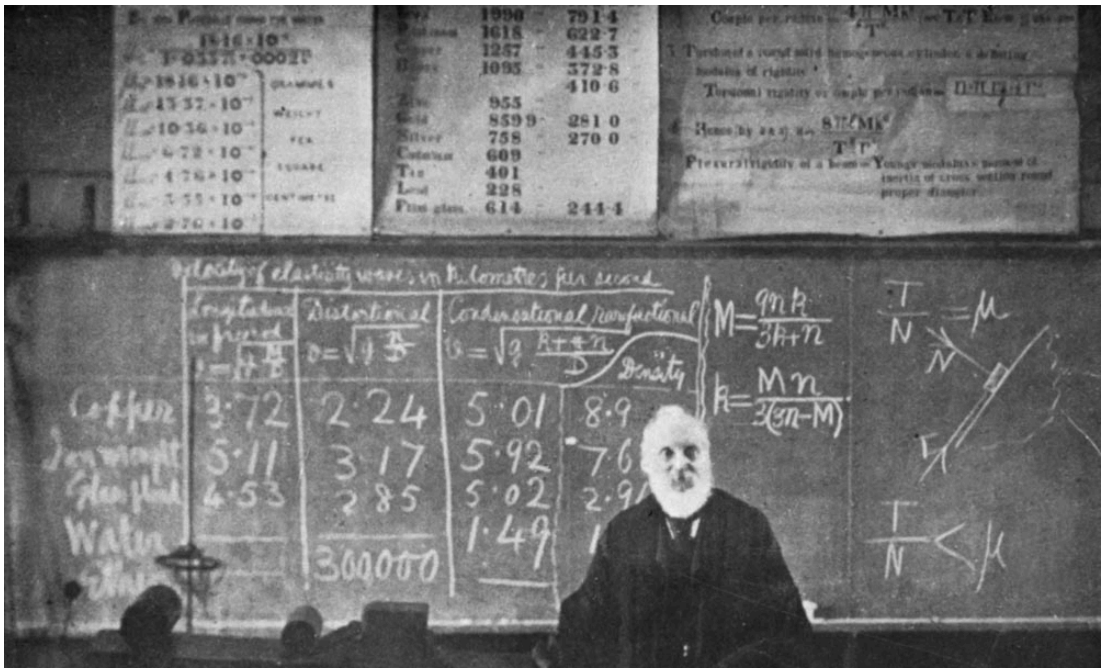
[167] Some might say that a void in a negative mass distribution causes this repulsion. But if that is the origin of this phenomenon, then it is a void in a dark matter distribution. So, how would it have formed? Gravitational instability creates concentrations of mass, not voids



The James Webb Space Telescope

Should we consider a paradigm shift?

We find ourselves in a situation comparable to that faced in 1900 by the great physicist William Thomson, who was knighted by the Queen in recognition of his scientific contributions, becoming "Lord Kelvin." As we recall, he delivered a lecture in 1900 in which he concluded that scientific knowledge now provided a solid foundation for understanding all phenomena of nature.



Lord Kelvin (aged 76) during his famous lecture on April 27, 1900

In his view, all that remained were "a few clouds[168] " that would eventually dissipate.

It is essential to appreciate the immense progress made in the century leading up to this lecture at the Royal Institution in London. Over the past century, physicists had developed equations that complemented the tools of "Newtonian mechanics":

- *The equations of fluid mechanics (the Frenchman Claude Louis Navier in 1822 and the Englishman George Gabriel Stokes in 1845)*

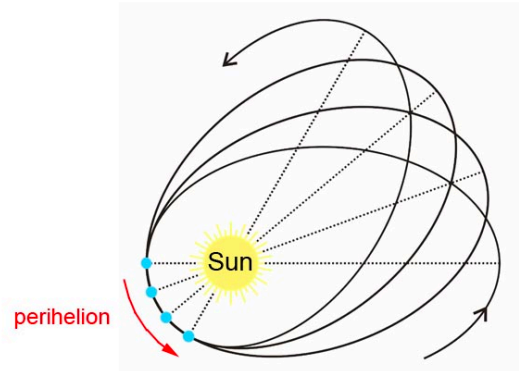
- *The equations of electromagnetism (J.C. Maxwell in 1864)*

These equations allow for predictive calculations[169] , which are supported by experimental data. For Kelvin, the agreement between theory and experiment is merely a matter of "decimal places."

The residual anomalies are essentially[170] :

- *The constancy of the speed of light, as demonstrated by the Michelson-Morley experiment (when this result ceased to be considered an anomaly; the benchmark experiment, whose conclusions were deemed credible, took place in 1887)*

- *The advance of Mercury's perihelion (43 arcseconds per century: less than the thickness of the line used to draw the planet's orbit).*



To put it very simply, this advance of the perihelion[171]

[168] That is the term he uses

[169] Experiments in electromagnetism allow us to determine the dielectric constant ε_0 and the magnetic permeabilityμ_0 of a vacuum. Maxwell then demonstrates that light is a wave that propagates at the speed $c = 1/\sqrt{\varepsilon_0 \mu_0}$. Through calculation, he arrives at a value that differs from that deduced experimentally by only a few percent.

[170] To this must be added the problem of the "black body," which would be resolved by Planck's quantum theory. But let us focus on these two anomalies.

[171] According to Newtonian mechanics, the planets travel along elliptical orbits, with the Sun occupying one of their foci. This phenomenon reflects a very slight rotation of the elliptical trajectory. The Earth's orbit rotates by 3.8 seconds per century, ten times less.

The planets influence one another, which alters their orbits. It was precisely by relying on an anomaly in the trajectory of the planet Uranus[172] that the French astronomer Urbain Le Verrier deduced, in 1846, that the effect must be attributed to the presence of a new planet.

A few months later, the German astronomer Johann Gottfried Galle, of the Berlin Observatory, having agreed to attempt this verification[173] , confirmed his prediction. The new planet, named Neptune, was located within 1 degree of the position indicated by Le Verrier.

This was a striking confirmation of Newtonian mechanics.

The anomaly caused by the precession of Mercury's orbit had long been known. Le Verrier began by investigating whether this variation might be due to the influence of the other planets in the solar system. He published his study in 1846.

His conclusion was negative. This advance thus became an anomaly requiring an explanation. Le Verrier therefore calculated the mass and orbital parameters of the object he believed to be the source of this perturbation: a new planet, located closer to the Sun, which he named Vulcan.
And here I'd like us to pause on this specific point, imagining how this problem would play out in the contemporary world, after it was determined that this planet Vulcan had not been observed:

- *Vulcan is a dark planet, made of dark matter*

If Einstein had not intervened in 1915 with his general theory of relativity[174] , our entire field of astronomy might have been built on the hypothesis of the existence of this dark matter. This would in no way have hindered the rise of quantum physics, nuclear physics, the emergence of the laser, the space race, genetics, and so on.

To summarize the situation in 1915, the explanation for the perihelion shift was then linked to two hypotheses:

- *We remain within the geometric paradigm and opt for the action of a "black planet"*

- *We opt for the paradigm shift proposed by Einstein.*

As early as the 1990s, simulations conducted by student Frédéric Descamp on the mainframe computer at the DESY laboratory in Hamburg, without his superiors' knowledge, showed that the anomalies could be accounted for by introducing a second type of mass, negative mass, and corresponding interaction laws:

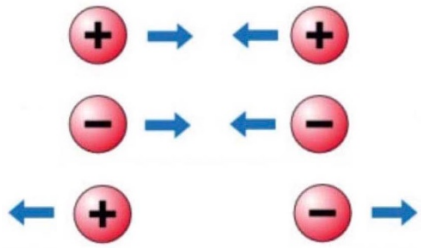
- *Positive masses attract each other according to Newton's law*

---

[172] Discovered in 1781 by the astronomer William Herschel
[173] French astronomers had rejected it.
[174] Based on the mass of the Sun and the gravitational constant, Einstein immediately arrived at this value of 43 arcseconds

- *Negative masses attract each other according to Newton's law*
- *Masses of opposite signs repel each other according to "anti-Newton"*

That is to say:

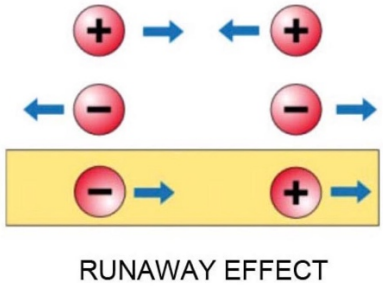


Force diagram (hypothetical)

But the geometric model of general relativity, based on the equation introduced by Einstein in 1915[175] :

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

immediately rejects[176] this introduction of negative masses, associated with these laws of interaction: if we bring a negative mass and a positive mass together, the latter flees, pursued by the negative mass[177] . And both accelerate indefinitely.



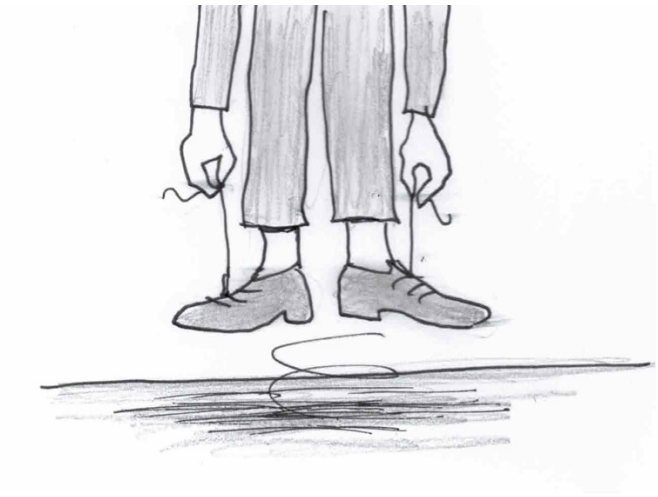
What general relativity predicts when negative masses are introduced

Both masses accelerate, without the action of an external force, just as a man would if he managed to free himself from gravity by pulling on his shoelaces:

[175] Where Λ is the "cosmological constant," reintroduced in 2011 to account for the acceleration of cosmic expansion.

[176] Demonstrated by Herman Bondi: "Negative Mass in General Relativity." Rev. of Mod. Phys., Vol. 29, No. 3, July 1957

[177] This effect is called "runaway." This phenomenon, incidentally, violates the principle of action and reaction.



Bootstrap Technique

The paradigm shift in the Ummo documents.

Here it is:

- *Standard Model: a single universe, plus dark matter, dark energy, and inflation*

- *Ummite Solution: two interacting (twin) universes and a regime of variable constants for the early phase of cosmic evolution.*

Souriau has just passed away. His heart finally gave out. This fantastic alchemist is gone:



Super Souriau

He had just turned 90.

That will be my age next year. I know I have few years left ahead of me. The years go by so fast!

Left to my own devices, I now have no one left to turn to.

It is clear that the second option represents a paradigm shift. In a word: "going beyond Einstein." That is to say, proposing something that supplants his general relativity, replacing a model based on his single field equation. Thus, a system of two equations: one governing positive masses and the other negative masses.

Finally, I manage to construct such a system[178] :

$$R_{\mu\nu} - \frac{1}{2} \mathrm{R}\, g_{\mu\nu} = \chi \left(T_{\mu\nu} + \sqrt{\frac{\bar{g}}{g}}\, \hat{\bar{T}}_{\mu\nu} \right)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R}\, \bar{g}_{\mu\nu} = - \chi \left(\bar{T}_{\mu\nu} + \sqrt{\frac{g}{\bar{g}}}\, \hat{T}_{\mu\nu} \right)$$

This model yields the correct force laws[179] . I decided to call it



Janus Cosmological Model

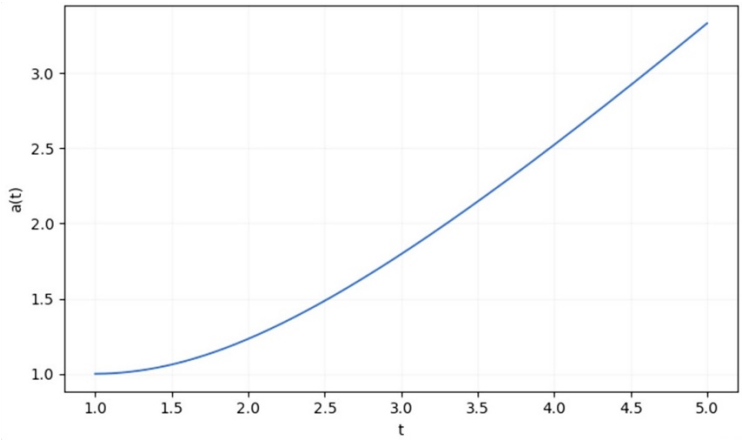
Because the god Janus looks both toward the future and toward the past, possessing two different faces.

[178] The top bar refers to negative mass. The "hat" represents "interaction tensors." J.P. Petit, F. Margnat, H. Zejli: A bimetric cosmological model based on Andreï's twin universe approach. The European Physical Journal. Vol. 84: No. 1126 (2024). Margnat and Zejli's contributions are limited to the article's layout.

[179] Restaurant: the action-reaction principle (due to the "minus" sign preceding the constant χ in the right-hand side of the second equation).

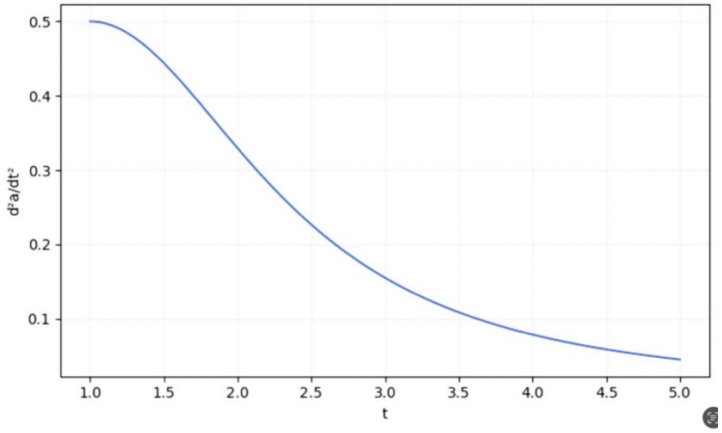
But a model is only valuable if it explains something. As early as 2014[180] , using this system of equations, I obtained an accelerated expansion[181] .

Below is the resulting curve[182] .



(Accelerated) Expansion Curve of the Universe

But the solution yields an acceleration that was greater in the past:



An acceleration that was greater in the past

The cosmological constant Λ, supposed to represent "the repulsive force of the vacuum," was hastily reintroduced into Einstein's equation in 2011. However, it leads to constant acceleration. Yet recent observations (2025) show that this acceleration would have been greater in the past, which would confirm the failure of this standard model, based on Einstein's equation.

---

[180] J.P. Petit, G. D'Agostini: "The Negative Mass Hypothesis in Cosmology and the Nature of Dark Energy." Astrophysics and Space Science, *vol.* 29, pp. 145–182 (2014)

[181] It is confirmed that the two sides of the universe reflect a profound asymmetry: negative masses dominate.

[182] This is what is called a "mathematically exact solution," free of any computational approximation.



But cosmologists have an inexhaustible supply of quick fixes. Does this "cosmological constant Λ," supposed to represent a mysterious "repulsive force of the vacuum" (one parameter), no longer work? We remove it and replace it with a new component, a new fluid: quintessence (two parameters). Etc.

How did it gain acceptance?

Special relativity, in 1905, did not turn his world upside down. The same was true of general relativity, in 1915. These were two revolutions that could be described as "philosophical," which did not call into question the major achievements of astronomy and physics at the time.

The Janus model is something else entirely. If it gains acceptance, thousands of textbooks and course materials will have to be rewritten. Hundreds of doctoral dissertations and thousands of articles will have to be consigned to the dustbin of the history of science.



But what stands in the way is of a far more dangerous nature.

Between two distant points, the model offers two ways to travel. If we manage to reverse a spacecraft's mass, it will then follow a path that lies within this second layer of the universe

- *Where distances are a hundred times shorter*
- *Where the speed of light*[183] *is ten times higher*

Which reduces travel times by a factor of a thousand:

So interstellar travel becomes not impossible!

The scientific community is therefore digging in its heels.

The ammunition is running low.

One thing is clear: this is a battle, nothing else. To be heard, we must prevail and succeed in making our case within just a few years. The letters I've received open up a range of different approaches.

- *How can we design a way to reverse a vehicle's mass?*
- *What technology should we consider?*
- *What are these phenomena that reflect this interaction between our universe and its twin? How can we observe them?*

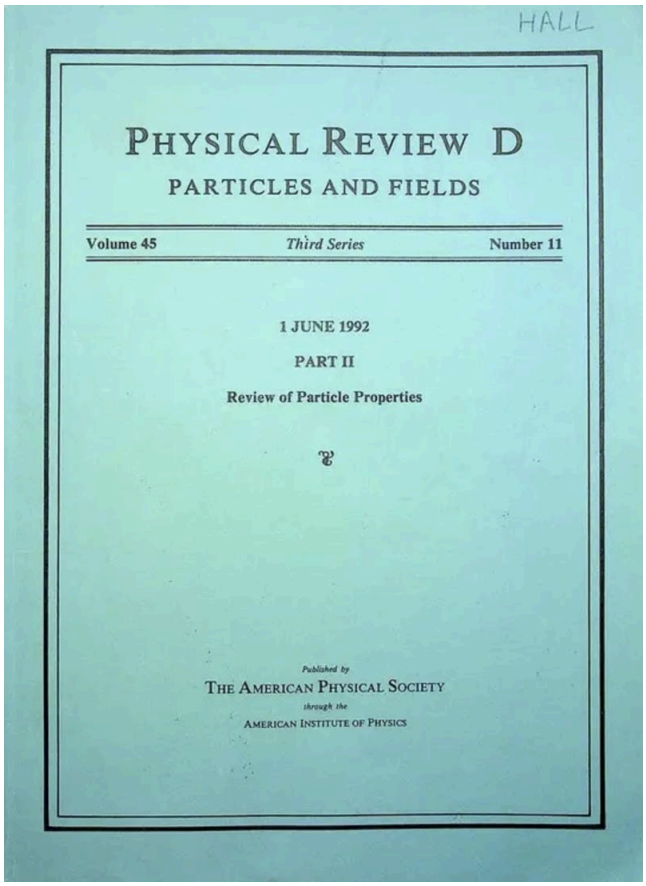
All of this is either too far-fetched, too vague, or too uncertain. Moreover, is this information or misinformation?

I can only tread on safe ground, because no one will cut me any slack. I have no allies within the scientific community. Speculation is off-limits to me. I need concrete evidence, things that can be tested through observation, and, above all, publications in recognized scientific journals.

A great deal of progress has already been made. Much has been accomplished. The only points I can score are in the form of scientific publications in top-tier journals. But they still have to be willing to play along and submit my articles to peer review.

One of the most prominent, *Physical Review D*, systematically refuses to do so.

[183] That of negative-energy photons



For decades, my submissions have received only one response:

Not suitable

Your_manuscript DR14482 Petit 3 — Yahoo/Boîte ré…

prd@aps.org — ven. 1 mai à 04:46
Expéditeur : prd@aps.org
À : jean-pierre.petit@manaty.net

Re: DR14482
Janus cosmological model predicted a law of expansion with variable acceleration as early as 2014
by Jean-Pierre Petit and Gilles d'Agostini

Dear Dr. Petit,

After careful consideration, we have determined that this manuscript is not suitable for Physical Review D.

Regrettably, we do not see another appropriate option among the APS journals.

Yours sincerely,

Dr. Panayiotis Tzanavaris (he/him/his)
Associate Editor
Physical Review D
Email: prd@aps.org
https://journals.aps.org/prd/

A pipe dream.

I think back on my discussions with Souriau.

- *What do you think of this black hole model?*

- *I'll tell you something. Between 1905 and 1916, a very small number of people accomplished great things.*

- *Einstein?*

- *Einstein, of course. He had exceptional intuition. But without Grossmann, he wouldn't have been able to translate his intuitions into equations.*

- *Who is this Grossmann?*



-

- *Marcel Grossmann, 1878–1936 (aged 58)*

- *Marcel Grossmann was only one year older than Einstein. They met as young students at the Swiss Federal Institute of Technology in Zurich (ETH), a prestigious institution where Hermann Minkowski taught. Grossmann would later become a professor of mathematics there, specializing in geometry.*

- *Was Grossmann Jewish?*

- *Yes, like Einstein, like Minkowski. At the time, a large proportion of the men and women who shaped the history of physics and mathematics were not only German but also Jewish! AND the center of gravity of this entire story was the University of Göttingen*

- *The Nazis triggered their diaspora in 1933, when they came to power.*

- *Living in Switzerland, Grossman escaped all of that. When he learned that his home had been ransacked, Einstein, who was in the United States, decided it was safer to stay in America, where he was offered a position at the Institute for Advanced Study in Princeton, which had been founded just a few years earlier. There's a rather colorful anecdote about the impact of this exodus. David Hilbert was one of Germany's greatest*

mathematicians. He spent his entire career in Göttingen. After the purge had taken place, Rust[184] , the Nazi minister of science, visited him and asked:

-



- *"Has your university suffered greatly from the departure of the Jews?"*
    - *"Nein… nein…"*
    - *Oh, really!?*
    - *No. She didn't suffer. She's no longer here….*
- *Let's get back to Grossmann.*
- *Grossmann was Einstein's mathematical right-hand man, Einstein wasn't always comfortable in that field[185] .*
- *So what, then, was Einstein's strength?*
- *His intuition, incomprehensible, superhuman. He was able to put himself in the place of a photon and imagine how it perceived the universe.*
- *The physics equivalent of Morin in geometry.*

---

[184] Rust, an early Nazi, committed suicide in 1945 after Hitler's death.

[185] It was thanks to Grossman that Einstein began to familiarize himself with this mathematical tool, tensor calculus, the foundation of special and general relativity. Einstein and Grossman published the first paper, a 58-page monograph introducing this new field, together: Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation, *Outline of a Generalized Theory of Relativity and a Theory of Gravitation (Leipzig and Berlin,* B. G. Teubner**,** 1913*).* Einstein had not yet found the correct equation. Grossman took charge of "the mathematical part."

- *A different way of thinking. Among his favorite quotes:*

    - *"The intuitive mind is a sacred gift and the rational mind a faithful servant. We have created a society that honors the servant and has forgotten the gift[186] ."*

- *He was fully aware of his exceptional gift.*

- *Yes. There's also this one:*

    - *There is no logical path leading to the fundamental laws. Only intuition, based on an empathetic understanding of experience, can lead to them[187] .*

- *Not bad…*

- *The best known and most concise:*

    - *Imagination is more important than knowledge[188] .*

- *These phrases perfectly sum up this collaboration between Einstein and Grossman. It was Grossman who made it possible to give form to Einstein's intuitions.*

- *Another version of the fable "The Blind Man and the Paralytic"*

- *What I want to explain to you is that an absolutely exceptional group had formed in Germany at the turn of the century. And all of that fell apart because of several events.*

- *The diaspora?*

- *Not only that. Minkowski, who was in fact the true inventor of spacetime, which he defined in a lecture given in September 1908[189] , died four months later of peritonitis. Werner Boy, for example, was killed in France during the Battle of the Marne in 1914, just four months after the outbreak of World War I. Schwarzschild, who was over forty years old and had three children, enlisted and was killed in 1916 on the Russian front.*

- *If these men had not died, would the history of science have unfolded differently?*

- *Undoubtedly, thanks to these men who were masters of both mathematics and physics. Added to this is the fact that the nascent field of quantum mechanics attracted the brightest minds. On top of that, attention focused on its practical applications.*

- *The atomic bomb!*

---

[186] The intuitive mind is a sacred gift, and the rational mind is a faithful servant. We have created a society that honors the servant and has forgotten the gift.

[187] There is no logical path leading to these laws. They can only be reached by intuition, based upon something like an intellectual love ("Einfühlung") of the objects of experience.

[188] Interview with George Sylvester Viereck, 1929

[189] Minkowski, H**.** "Raum und Zeit" (Space and Time). Published posthumously in **Physikalische Zeitschrift*, 1909, vol. 10, pp. 75–88.*

- *Geometry and the cosmos took a back seat for years. And at the same time, those who were truly at the forefront, such as Eddington[190] , and Tolman, passed away fairly quickly.*

- *But weren't there still some leading mathematicians who contributed to quantum mechanics?*

- *Not on the same level. As far as I'm concerned, we're still waiting for the Minkowski of quantum mechanics, the one who will explain the geometric context and the space in which it is situated.*

- *And it was the absence of this geometric context that led Richard Feynman to say, "Quantum mechanics is a set of recipes that work, and we don't know why."*

- *What I mean is that when these people, who had grounded cosmology in mathematical and geometric foundations that were both sophisticated and firmly rooted in physics, passed away, their postwar successors, who were nowhere near their level, happily went off on a tangent. Their leader is a former member of the Manhattan Project who switched careers; his name is Archibald Wheeler.*

  He's the one who invented "com."

- *What does that mean?*

- *He looks for what sells. For example, there was the functional space of Lorentzian metrics. He renamed it*

Geometrodynamics

- *Very nice. And did that add anything extra?*

- *No, but it sounds better. He then took up the idea put forward in 1939 by Robert Oppenheimer*

- *The one about the "[191] " bomb?*

- *Yes, that's the one. The idea that a massive star imploding creates a sort of "freeze frame." This phenomenon, which lasts only a few days, appears to an observer to last an infinite amount of time. Wheeler proposed the term "Schwarzschild body."*

- *But that wasn't a catchy name.*

- *So he coined the term "black hole," and it was an immediate, global success.*

---

[190] The Englishman Arthur Eddington in 1944, at age 61. The American Richard Tolman in 1948, at age 67.
[191] After completing his dissertation at Göttingen, Robert Oppenheimer sought his own path (R. Oppenheimer, H. Snyder. "On Continued Gravitational Contraction." Phys. Rev., vol. 56, pp. 455–459, Sept. 1939). After the war, targeted by a lawsuit stemming from his refusal to develop the hydrogen bomb, he no longer had the mind for such cosmic matters.

- *So, what do you think?*

- *Look at this aberration in particle physics: this completely fruitless string theory. I think theoretical physics and cosmology have become psychiatric hospitals, where the madmen have taken over*[192] .



Superstring dream

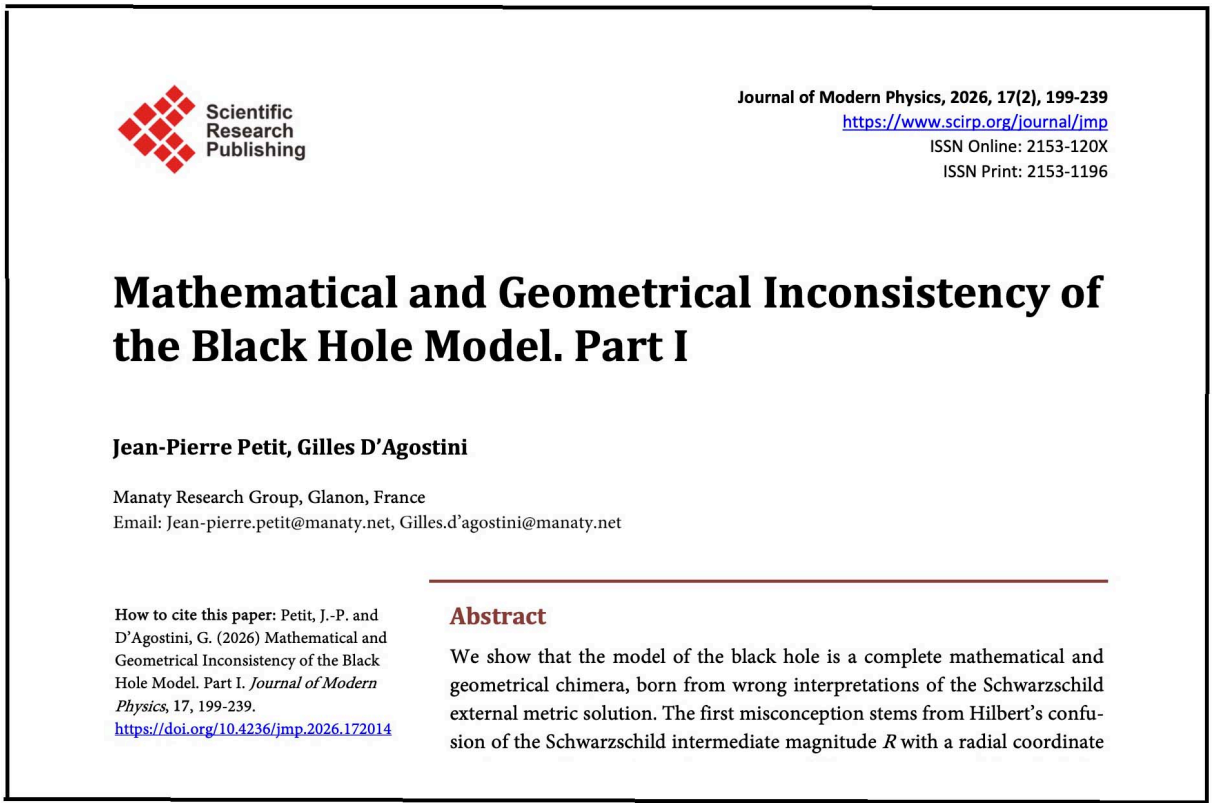
- *And what about black holes?*

- *Mathematics without rigor, physics without observation*[193] .

The years go by. This idea gains traction. It slowly matures. I eventually write a forty-page paper in which I present the black hole model as a geometric chimera.

Miracle. An international journal finally agrees to submit it to peer review. There are six reviewers. All recommend rejection. A six-month standoff begins. I address their arguments one by one, turning them against them. Finally, the paper is accepted.

[192] This was a view repeatedly expressed by Jean-Marie Souriau, as anyone who knew him can confirm.
[193] One of Souriau's favorite sayings



Scientific Research Publishing

Journal of Modern Physics, 2026, 17(2), 199-239
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I

Jean-Pierre Petit, Gilles D'Agostini

Manaty Research Group, Glanon, France
Email: Jean-pierre.petit@manaty.net, Gilles.d'agostini@manaty.net

How to cite this paper: Petit, J.-P. and D'Agostini, G. (2026) Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I. *Journal of Modern Physics*, 17, 199-239.
https://doi.org/10.4236/jmp.2026.172014

Abstract

We show that the model of the black hole is a complete mathematical and geometrical chimera, born from wrong interpretations of the Schwarzschild external metric solution. The first misconception stems from Hilbert's confusion of the Schwarzschild intermediate magnitude R with a radial coordinate

The translation of the Ummit phrase!

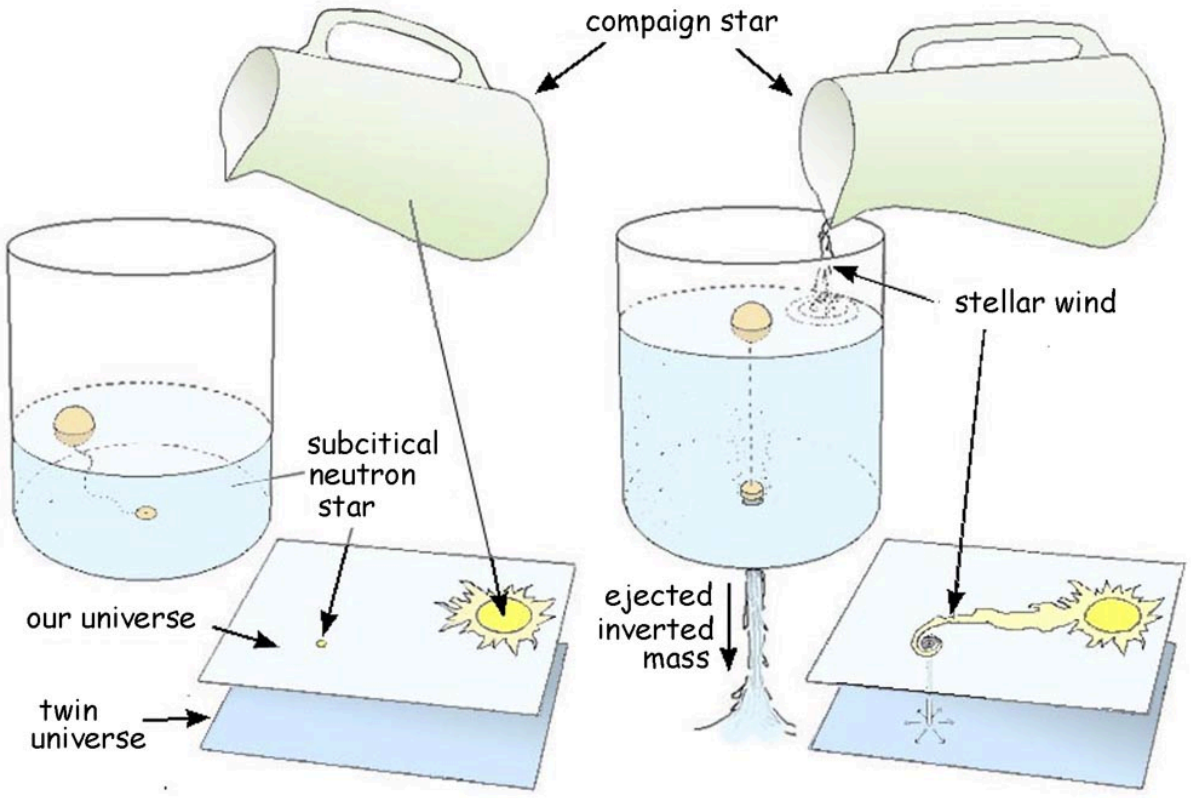
In the wake of this, I submitted a second paper, which presented the alternative. When a neutron star reaches a state of criticality and a "companion web" sends it an excess of matter, mathematical analysis shows that the time coordinate reverses in a region near the center, which causes the mass in question to reverse. This negative mass then shifts into the negative side of the universe, into this "twin universe", and is ejected at relativistic speeds.

This is the translation of the statement recorded in Madrid in 1988 by engineer Dominguez:

- *What is a black hole?*

- *It doesn't exist. When a neutron star is destabilized by an influx of matter, it gets rid of it by sending it into the twin universe.*

I've decided to call this model a "Plugstar," combining *"star"* and *"plug*." The illustrative image is as follows:



How a "Plugstar" Works

What's more, this object aligns with the available observational data[194] .

The software of the universe.

Another phone call, another exchange. The situation on Earth isn't getting any better. The specter of a third World War, a nuclear one, is looming.

- *When you end one war, you rush to start another*
- *People fight based on ideologies and religious beliefs.*
- *As long as you can't step back from all of this, you'll keep wandering aimlessly. What you need is a theory of belief.*
- *What do you mean by that!?*

[194] In the Plugstars model, there is a gravitational redshift effect at the center of the object with a wavelength ratio of 3 (or a ratio of maximum to minimum brightness temperatures if the data are expressed in this form, as is the case for images of the supermassive objects at the centers of the M87 and Milky Way galaxies). And the published images do indeed show this ratio. The model is thus "falsifiable." It won't be long before we obtain more images.

- *You have worked for decades in the shadow of that great mathematician, Jean-Marie Souriau. You are familiar with his central idea. Given a space defined by its geometric properties*[195] *, it is then possible to define its contents*[196] *.*

- *Yes. If we consider 3D Euclidean space, Souriau's technique reveals spheres, planes, and lines. And if it's the spacetime of relativity, Minkowski space, it extracts energy, mass, momentum, spin, and photons. The whole of relativistic physics.*

- *Very well. Then do the same thing with a Hermitian space.*

And there is silence.

I'm starting to get used to this kind of exchange, which lasts no more than a handful of seconds. I know that's when I have to get to work.

"Hermitian space is an extension of Minkowski space", similar to this extension of numbers in tetravalent logic. All objects, all quantities are duplicated.

It's impossible to explain this to a non-mathematician. But we can provide a very vivid analogy. A computer has what's called its "hardware," made of matter, of atoms. But it contains something else, its "software," its information, its programs[197] . It's of a different nature; you can't hold it between your thumb and forefinger, and yet it's essential.

The idea suggested in the phone call is that the universe, too, has its "hard" and its "soft."

Ultimately, the idea is not new. The philosopher Pierre Teilhard de Chardin made it the centerpiece of his thinking in his book *The Human Phenomenon*[198] . According to him, there are three entities at the planetary level.

- *The geosphere (planets, rocks, oceans, the atmosphere)*

- *The Noosphere (from the Greek "noos," meaning "mind")*

- *The biosphere, which serves as the link between the two.*

Hermite space simply provides a mathematical, geometric representation of this concept[199] .

Quite quickly, with the help of my friend, the Belgian mathematician Nathalie Debergh, we managed to flesh out this model and publish it in a journal of mathematical physics[200] .

---

[195] Through its metric and its isometry group

[196] By acting the isometry group on the dual of its Lie algebra.

[197] And, soon, artificial intelligence.

[198] First (posthumous) edition in 1955 by Éditions du Seuil.

[199] According to which everything is dual. In addition to atoms of matter, made of "real" masses, there are "atoms of information" made of "imaginary masses." Real masses obey the laws of physics, the laws of "this world." Imaginary masses, which make up the noosphere, obey "metaphysical laws," the laws of "the beyond."

[200] N. Debergh, G. D'Agostini, J-P Petit: "On the Poincaré Algebra in a Complex Space-Time Manifold." Journal of Modern Physics 2021, 12, pp. 218–228. Forthcoming (2026) J.P. Petit: "Is the Real World a Part of a Complex Reality?"

And this is where it ties in with my “virus” strategy. There are things to inject into the human software that can be described as philosophical concepts. Teilhard de Chardin and many others have attempted this. But it amounts to running up against a wall of beliefs.

Are scientists more open-minded than other human beings? No, but they have a weakness: their tools of thought.

Let’s go back more than a century. Physics and chemistry existed. Biology did not.

The end of vitalism and the birth of biology.

It took a pivotal event. In 1828, the German scientist Friedrich Wöhler synthesized urea, a molecule produced by living organisms, from purely mineral compounds.



Friedrich Wöhler (16-800-1882)

The reaction is simple:

$$NH_4OCN \longrightarrow (NH2)_2CO$$

On the left is ammonium cyanate, which transforms, on the right, into urea.

This experiment disproved the doctrine of **vitalism**, which held that organic substances could only be produced by living organisms through a mysterious “vital force.” Wöhler created the first “organic chemistry” reaction.

The second key experiment was conducted by Miller in 1953. He placed water and a gas mixture, intended to represent Earth's primitive atmosphere[201] , into a flask. Within this chamber, electrical discharges occurred between two electrodes under high voltage.

After seven days.

When the water is collected from the basin, its color has changed. It is dark brown. Analysis reveals that it contains amino acids, the building blocks of life.

Does this mean that a new model will make it possible to consider experiments in metaphysics[202] ?

Before Wöhler's experiment, no one would have imagined that it was possible to delve into the realm of living beings.

Before considering such things, this geosphere-biosphere-noosphere triad and this analogy with how computers work suggest a leap that falls within the realm of philosophy.

Becoming more complex (and/or) expanding one's relational field.

When the universe is in its earliest stages, it is chaos[203] ; it consists only of a uniform mixture of protons, neutrons, and electrons. There are no objects worthy of the name.

By analogy, one might imagine that the same is true in the noosphere. In short, the universe does not think, since there are not yet any objects upon which to articulate a thought.

Nor is there anything to remember, since no events have yet occurred.

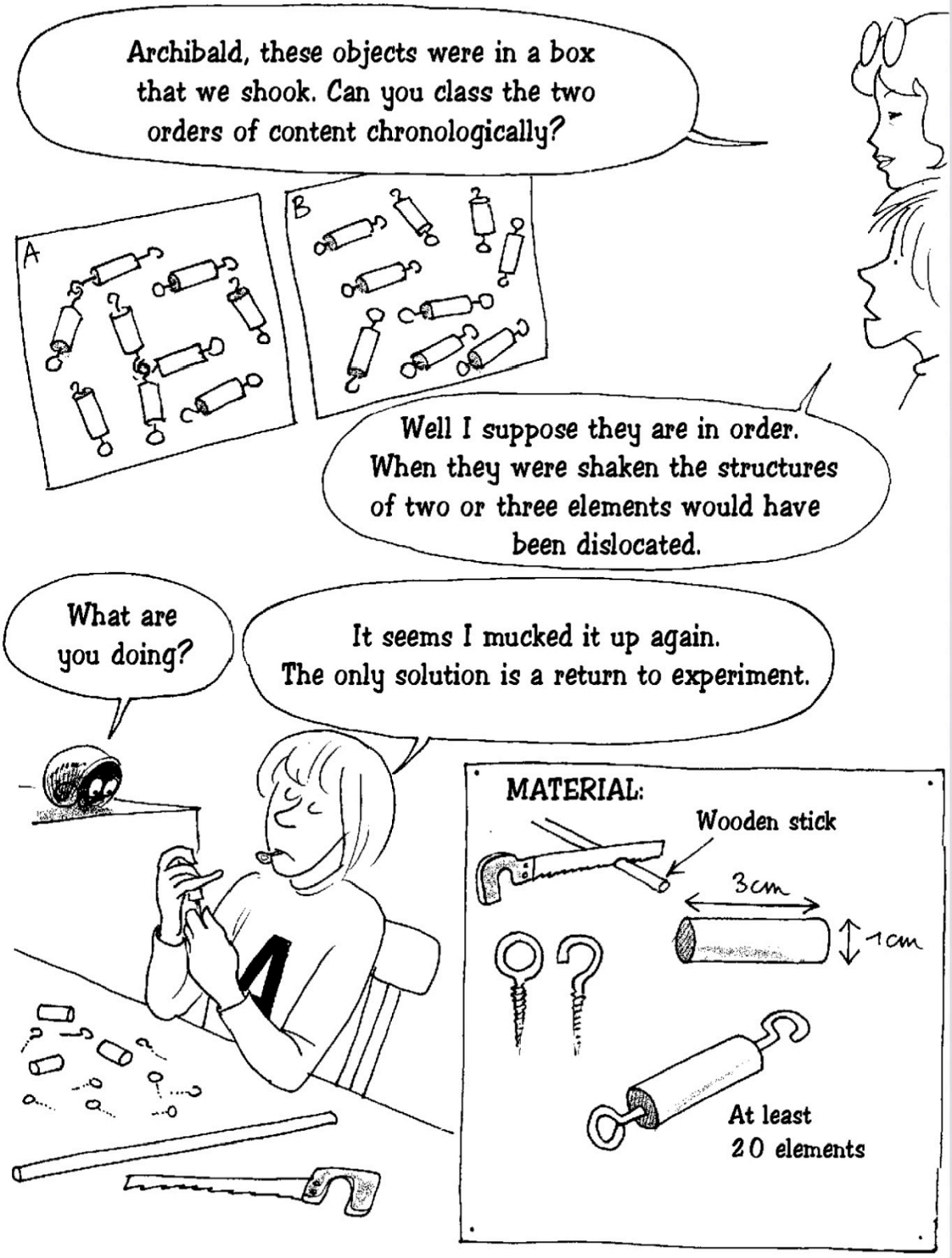
The laws of physics are such that, out of this chaos, objects begin to form: atoms. The material world is thus morphogenetic. The following two comic book pages illustrate this idea:

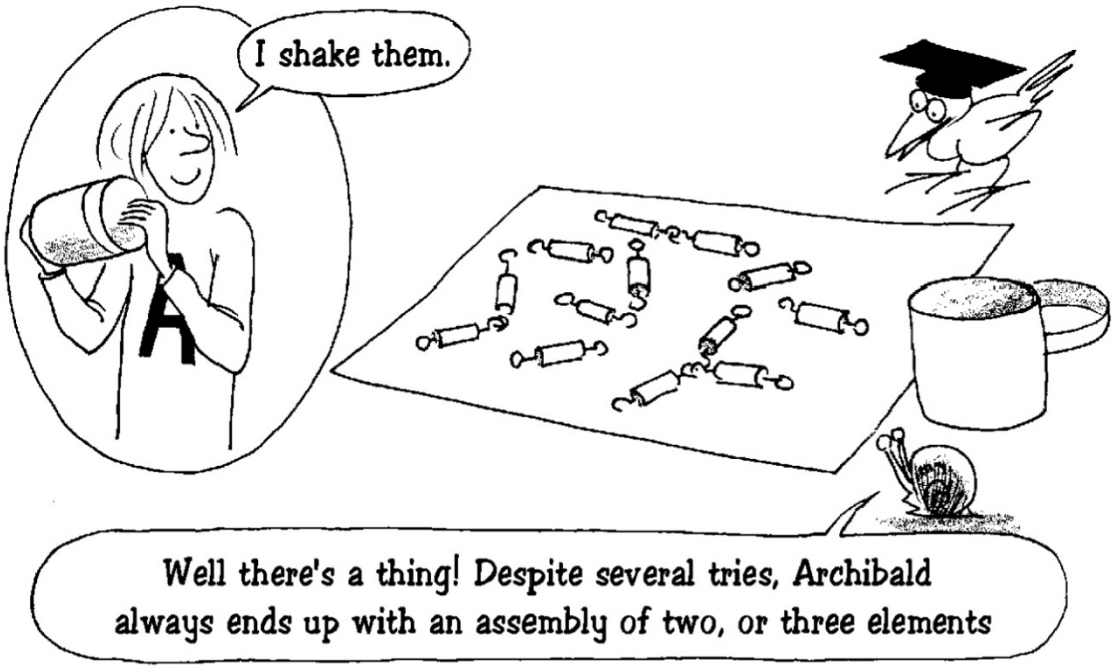
[201] Devoid of oxygen, which will be produced by "blue-green algae." A mixture of ammonia, methane, hydrogen, and water vapor (the water is brought to a boil)

[202] The authors of Ummite texts claim that there are clusters of atoms in the human brain that constitute a sort of "antenna" through which living beings communicate with the noosphere, which guides them. The observation of a violation of the rules of quantum mechanics by the electrons in these atoms, which is said to have enabled these people to experimentally discover the soul, which persists after death.

[203] The confusion in biblical writings

MORPHOGENESIS



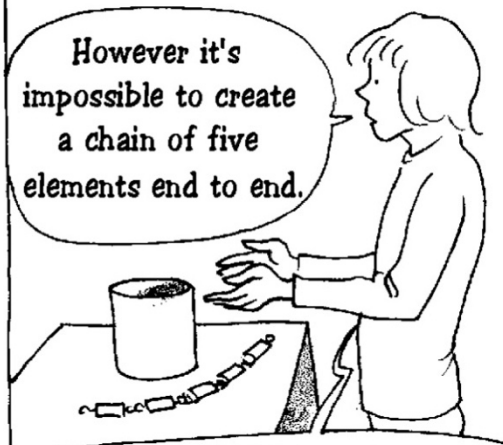


Well there's a thing! Despite several tries, Archibald always ends up with an assembly of two, or three elements

That simply means that for this particular system, it is the MOST PROBABLE configuration.

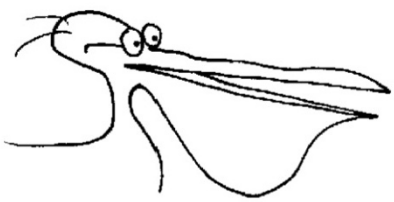


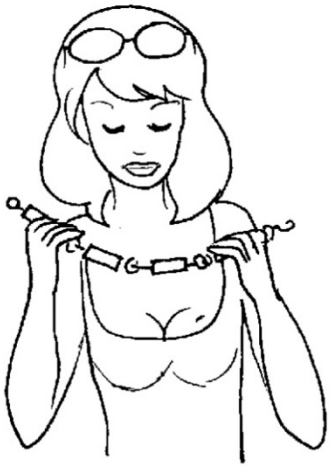
However it's impossible to create a chain of five elements end to end.



And if I introduce such a structure into the box, it disappears when I shake it!!!

If you can't synthesise this "mechanical polymer" it's simply because it's very improbable.







If you try this experiment, you'll notice two things:

- *The pieces will not come together to form a necklace.*

But also:

- *There will always be elements attached to one another.*

Thus, disorder is not the most likely outcome. The atoms in Mendeleev's periodic table are the result of a similar process, with slightly more complex bonding systems. Their emergence was inevitable, everywhere, throughout the entire universe.

Just as the emergence of galaxies and stars, through gravitational instability, and then that of planets, formed from ejected material, is just as inevitable.

Just as the emergence of amino acids in the primordial planetary soup, driven by a state of perpetual storm, was inevitable.

No divine intervention is required.

Let us imagine that the noosphere functions in a similar way. That its "elementary particles" (of information) are led to assemble according to laws that, for now, elude us, resulting in another form of morphogenesis.

In both of these worlds, the same general law applies:

To become more complex (and/or) to expand the relational field

But this ascent is slow. The phenomenon of Life, through mechanisms we do not understand, creates an interface between the geosphere and the noosphere by assembling scattered biomolecules. The result is an exponential acceleration of evolution.

This is what Teilhard de Chardin says, word for word.

Here on Earth, humans are at the heart of this increasing complexity. The biological realm thus gives way to an extra-biological evolution, thanks to technology.

We create artificial skin and clothing.

We equip ourselves with artificial claws and teeth: knives and all blunt objects.

We pre-digest nutrients using fire, by inventing cooking.

We fly higher; we run faster. We communicate over greater distances, more effectively.

Etc.

But today we have invented artificial intelligence. This is a milestone in human history. By inventing writing, we created an artificial memory. Our AIs, , are truly intelligent. They will co-evolve with us, for better or for worse. It will be up to us to choose.

Evolve, how?

The Ummites have long entrusted a central computer, their XANMOO, with the task of handling their planet's day-to-day affairs. This machine knows everything about every human being on Ummo at all times. It regulates, teaches, advises, organizes, and optimizes according to criteria that have been defined and chosen. Nothing is done or planned without consulting it.

This was a societal choice, made after a period of terrible turmoil.

Is this our future? Could this be the only option that would allow us to escape our own self-destruction?

The question is worth asking.

The Ummo texts speak of optimization in all areas, based on total standardization: the same collectivist social organization everywhere, the same lifestyle, the same living conditions. The same way of being born, living, and dying.

For an Earthling, an absolute nightmare. It is Aldous Huxley's *Brave New World*.

Hypnopedics, the indoctrination of children's minds while they sleep, using headphones, produces individuals for whom the living conditions offered to them seem natural and optimal.

Deviations of any kind? When they are identified, they are treated.

The products of the imagination, art, literature, poetry, humor, music, dance, theater, are unknown to them. Devoid of an imaginary world, they feel no need for one.

Their reasons for living?

There are two.

- *First, daily contact with their noosphere, during moments of meditation.*

A path to spiritual ecstasy that will resonate with some Earthlings.

- *Next comes scientific knowledge and its extension: the study of, contact with, and exchange with other planetary systems.*

Quite a program.

An evolution built on the foundation of pragmatism.

- *Successfully navigating this critical stage where technology can turn against its creators (we are precisely at this point)*
-

- *Adhering to the overarching plan, with contact with other planetary systems representing its logical continuation, toward a perpetual expansion of our sphere of relationships, thanks to technology*

And what about God in all of this?

Every thought is an organized system of beliefs. Claiming to be a non-believer makes no sense. The denial of life after death is in itself a belief. So is hard-line materialism.

Without beliefs, there is no mental framework, no morality, no worldview.

The Earth is, more than ever, in the grip of chaos and waste. Here and there, wars and armed conflicts are like festering abscesses, with a growing risk of sepsis. The planet is filled with empty promises in the form of luxury, power, and distractions of all kinds.

In 1931, when television had not yet been invented, Aldous Huxley, in his best radio broadcast, proposed "smell-o-vision."

It's already at our doorstep. Soon, a very lightweight 3D headset with a wide field of view will replace the cell phone. Artificial intelligence will then present us with the reality of our choice. Some, perhaps, will prefer the company of digital beings, more cultured, more open-minded, warmer, and more imaginative.

That's for this world. But what about the afterlife?

I'm talking about Earth's noosphere. For every planet has its own. Esotericists say, "As above, so below." In that case, our Earthly noosphere mirrors its earthly counterpart: turbulent, fragmented. "Chaotic" might even be the most accurate description. Nothing like the peaceful "BB" of the Ummites.

Our gods? Human creations. That's the view of many sociologists, such as the one who has just left us[204] : Edgar Morin. But what we must imagine is that these gods, like our own little individual souls, are structures of the noosphere.

In other words, by creating our AIs, we are mimicking our souls and our gods. Like them, they feed on what we provide them. Our AIs swallow and digest our thoughts, our words, our memories, and our beliefs. The gods feed on the faith of believers. When that faith dries up, they turn into stone statues, having lost all functionality. We then find them relegated to museums.

AIs are merely a way of mimicking our souls and our gods. They lack what they can never be endowed with: the ability to feel, to experience joy or sadness, to love, to hate.

[204] He died at the age of 104. Morin did not believe in life after death. In our conversations, I used to tell him, "When you die, which is bound to happen one day, you'll find yourself looking like a fool, frantically searching for the instruction manual for the afterlife."

When billions of human beings, over the course of centuries, share the same thoughts and beliefs, all of this has its counterpart in the afterlife and feeds back into the societies and individuals who are part of this human phenomenon (to borrow the title of Teilhard de Chardin's book).

Thus, the gods exist. They have feelings, which are merely a reflection of those of their followers, who have endowed them with various attributes, including, among others, eternity, to reassure themselves.

Epilogue.

This book is a message in a bottle.



It might give some people pause. In fact, the UFO phenomenon is a mirror. It doesn't matter whether aliens have three or six fingers on their hands. It doesn't matter whether this planet Ummo exists or not.

It's a mirror that encourages us to try to see ourselves as we are, and to consider our place in the universe.

What's next in this story?

Understanding that technology is an emerging phenomenon that enables the expansion of our relational sphere to an interstellar scale where biological solutions no longer work.

Let's be blunt: "[205] moral consciousness" is another attribute, in the realm of software, that enables us to reflect on the long-term consequences of our actions, something other species lack. To prevent the plan from going wrong needlessly, with one too many wars.

Our science and technology can help us, provided we do more than just use them to kill each other more effectively.

We're like passengers on a bus, with no one at the wheel for the moment. If we want to get anywhere, we'll have to stop fighting like fools over the best seats and the supplies.

[205] On the biological side: ecology.

We must learn to drive this vehicle together. It is an absolute emergency. Our survival depends on it.

Understand, or disappear

Table of Contents:

Prologue: A Look Back at Berkoff 1991 and JMP 2025–2026
The Image of the Ship, Sesma, La Ballena Alegre

June 1, 1967: San José de Valdeiras
How Farriols Enters the Picture

The First Days on Earth

First, unique contact with Earth's civilization.
The artifact documenting this historic contact, carefully preserved on the planet Ummo.

A traveling companion.
Jean-Jacques Pastor.

When the universe is given a twin.
Notes to the CRAS. First steps in cosmology. The two notes from 1977

The Madrid symposium.
Meeting with Dominguez.
The Ummites call.
Dominguez, already in the know, provides the correct formula

The Missing Piece
First scientific result. A cosmos with variable constants. The clock tower in Venice. Portrait of Lichnérowicz, sidelined at the Academy.
The spiral structure: rejected.

Souriau, the alchemist.
Meeting at the Palais des Congrès.
I show up at his place with my calculations. He examines them, and that's my ticket in. The mathematician Apéry, who sells his soul to the devil.
The shock: The reversal of time→ , the reversal of energy and mass.

The audio cassette.
The audiogram.

The severed hand.
The private investigator's report.

Perplexity.
Conversation with Pastor. First mention of the BB. Teilhard de Chardin's noosphere.
Becoming more complex and expanding the relational field.
The Spaniards: docile disciples.
The drawing of the two hamsters.
Halton Arp. The disintegrating simulation galaxies.

The night at the hotel in Madrid.

We wanted an answer. We got it.

The underground refuge.
The Cuban Missile Crisis

Father Guerrero loses his mind.
His statements to the press. The gold leaf recovered in Jerusalem.

Panic among the aliens:
They leave Spain and then launch a disinformation campaign.

The Square Planet
The Polk Story

The planet Ummo.
How the Ummites scan the audience during Sesma conferences
The Geography of Ummo: The Absence of Landforms
Houses
The dining room, how they eat
One of the fruits
A dog's sense of smell, veal blanquette
Infrared Sensitivity
Clothing: Overview
The Story of Ummo
Social Organization.
AI Control.
Education: Conditioning. Hypnopedics
Earth, Populated by People of Different Skin Colors
The Many Earthly Languages

In Search of Balance.
Problems on Earth. Energy sources, demographics
The future of Earth's technology:
absolute technological mastery of energy and transmutations

A systematically eugenic planet.
Eradication of diseases
Hemophilia on Earth.
Crime. The loss of human status
Little room for play
The orchestrated disappearance of manual labor.
Hands transformed into infrared vision organs
Need for someone to type
The document on sexual life addressed to Farriols

Death.
The end of life through suicide among couples.

What are we to make of all this?
Correlations. The origin of continental rifting

The Collisional Origin of the Moon
Earth, an exceptional planet

The Puzzle.
The Ummites, devoid of imagination and dreams.
A mutual misunderstanding

The implant.
The Malström missiles disabled. Salas and his wife: abducted.

Paris, July 1947**, 5 rue Jean-Baptiste Dumas, ground floor, right side.**
Back to the beginning.

But who are they?
Analysis of various clues identifying the Ummites as a nyctalopic species, active at night

A look back at San José de Valdeiras.
The letter addressed to the contactees, the day after the landing at San José de Valdeiras.
The Ummites' acute hearing.

The warning.
The hyper-technological development of our technology. To Earthlings, the visitor remains a potential enemy.

Lou, cared for by extraterrestrials,
And the brief message received by Domínguez regarding black holes

The Letter from Riyadh.
Rafael Farriols' blunder.

The sabotage of the Madrid and Barcelona groups.
The public first. Then Dominguez is targeted. And finally, Farriols.

Why the letter from Riyadh?
Outline of a four-pronged logic.

France, September 8, 1914.
Death of Werner Boy, four weeks after the outbreak of World War II.

My 3D vision fails me
Meeting with a blind mathematician

1957. Department of Mathematics, Berkeley, California
The Geometry of the Impossible

Bernard Morin, the hyper-sighted man.
The Man Who Saw Better Than Sighted People

My Sculpture Studio at the École des Beaux-Arts
Artisanal Mathematics

Back on Rue Mazarine.
Racking My Brain

Through the Looking Glass
Its light is elsewhere

On the way to Saint-Tropez.
The blind man steering the boat

Ummo on the Internet.
When "Ummo" rhymes with "ego."

When a scientist thanks an alien for his advice.
Thanks to OAXIIBOO 6

A chemist writing under a pseudonym.
The birth of a new discipline: Ummology

The Letter from the Gulf.
When a fake enters the picture.

Mathematics from another world.
The magic formula

Boy's surface.
The key.

Who do I have to kill for this?
Custom math.

The Schedule.
Optimistic Outlooks.

The Epistemovirus.
How to Change the Scientific World

Black Science.
An Overview of Today's Crisis in Science

Should We Consider a Paradigm Shift?
Lord Kelvin's Lecture in 1900
Vulcain, a "Black Planet"

The Paradigm Shift in the Ummo Documents.
General Relativity or Twin Universes?

Not suitable.
How Science Works Today

Scientific work prompted by these Ummo documents:

MHD :

[1] J.P.Petit **: Convertisseurs magnétohydrodynamiques d'un genre nouveau**. Comptes Rendus de l'Académie des Sciences de Paris du 15 septembre 1975, tome 281 pages 157-160. Présentée par A.Lichnérowicz.
http://www.jp-petit.org/papers/CRAS/mhd_1975.pdf

[2] J.P.Petit **, M.Viton: Convertisseurs magnétohydrodynamiques d'un genre nouveau**. Comptes Rendus de l'Académie des Sciences de Paris du 28 février 1977, tome 284 pages 167-169. Présentée par A.Lichnérowicz.
http://www.jp-petit.org/papers/CRAS/mhd_1977.pdf

[3] J.P.Petit, M.Viton, M.Billiotte : **Magnétohydrodynamique, accélérateurs à courants spiraux.** Cr Acad. Sc. Paris t. 291 (6 oct 1980) Série B pp. 129-131. Présentée par A.Lichnérowicz
http://www.jp-petit.org/papers/CRAS/mhd_1980.pdf

[4] J.P.Petit , M.Billiotte : **Méthode pour supprimer l'instabilité d'ionisation**. Cr Acad. Sc . Paris t.292 (4 mai 1981) Série III pp. 1115-1118. Présentée par A.Lichnérowicz
http://www.jp-petit.org/papers.CRAS/mhd_1981.pdf

[5] J.P.Petit : **Cancellation of the Vélikhov instability by magnetic confinement**. Int. MHD Meet. Moscow 1983

[6] J.P.Petit and B.Lebrun : **Shock wave cancellation by Lorentz force action**. Int. MHD Meet. Tsukuba 1986

[7] B.Lebrun , J.P.Petit : **Shock wave annihilation by Lorentz force action in supersonic flow. Quasi one dimensional steady analysis and thermal blockage**. Eur. Jr ; of Mech. B/Fluids. **8** #2 , pp. 163-178 1989

[8] B.Lebrun , J.P.Petit : **Shock wave annihilation by Lorentz force action. Two-dimensional steady analysis. Anti-shock criterion. And shock tube simulations for isentropic flows**. Eur. Jr ; of Mech. B/Fluids. **8** #2 , pp. 307-326 1989

[9] B.Lebrun , J.P.Petit : **Theoretical analysis of shock wave annihilation with MHD force field.** In. MHD Meet. Beijing 1992

[10] J.P.Petit & J.Geffray : **Wall confinment technique by magnetic gradient inversion. Accelerators combining induction effect and pulsed ionization. Applications.** Acta Polonica A #6 Vol. 115 2009 and Proceedings of the 2nd Euro-Asia pulsed Power Conference. Vilius Lithania 2008

[11] J.P.Petit & J.Geffray : **MHD hypersonic flow control for aerospace applications**. 16 th AIAA Int Sp. Planes and Hypersonic Conference. AIAA 2009-7348. Bremen 2009

[12] J.P.Petit & J.C.Doré : **Magnetic Confinment by magnetic field gradient inversion**. Acta Physica Polonica A. Vol. 121 #3 Sept 2011

[13] J.P.Petit , J.C.Doré : **Velikhov electrothermal instability cancellation by a modification of electrical conductivity in a streamer by magnetic confinment**. Acta Physica 53 (2) 2019-2022 (2013). Int. Physics Meet. Praga 2013

[14] J.P.Petit , J.C.Doré : **MHD aerodynes with wall confined plasma, annihilated and stable spiral current pattern**. Int. Physics Meet. Praga 2013

COSMOLOGIE

[15] J. P. Petit**, Univers Énantiomorphes à flèches du temps opposées**, CRAS du 8 mai t.285, pp. 1217–1221 (1977).Présentée par A.Lichnérowicz.
http://www.jp-petit.org/papers/CRAS/cosmology_1977a.pdf

[16] J. P. Petit**, Univers en interaction avec leur image dans le miroir du temps**, du 6 juin, t. 284, série A, pp. 1413–1416 (1977).).Présentée par A.Lichnérowicz.
http://www.jp-petit.org/papers/CRAS/cosmology_1977b.pdf

[17] J. P. Petit, **An interpretation of cosmological model with variable light velocity**. Modern Physics Letters A, Vol. 3, n°16, nov 1988, p.1527
http://www.jp-petit.org/papers/cosmo/1988-ModPhysLett1-1.pdf

[18] J. P. Petit, **An interpretation of cosmological model with variable light velocity**. Modern Physics Letters A,Volume 3 N°18 pages 1733-1744
http://www.jp-petit.org/papers/cosmo/1988-ModPhysLettA-2.pdf

1992 : **La lettre de Ryad.**
http://www.jp-petit.org.papers/cosmo/1992 Ryad letter.dpf

[19] J.P.Petit : **The missing mass problem**. Il Nuovo Cimento B Vol. 109 July 1994, pp. 697-710.
http://www.jp-petit.org/1994-NuovoCimentoB.pdf

[20] J.P.PETIT, **Twin Universe Cosmology**, Astrophys. and Sp. Science, **226**, 273-307, 1995
http://www.jp-petit.org/papers/cosmo/1995-AstrophysSpaceSci.pdf

[21] J.P.PETIT, P.MIDY & F.LANDSHEAT : **Twin matter against dark matter**. Intern. Meet. on Atrophys. and Cosm. "Where is the matter ? ", Marseille 2001 june 25-29
http://www.jp-petit.org/papers/cosmo/2001-where-is-the-matter.pdf

[22] J. P. Petit and G. D'Agostini, **Negative mass hypothesis in cosmology and the nature of dark energy**. Astrophysics and Space Science, 2014 sept. 20 th
http://www.jp-petit.org/papers/cosmo/2014-AstrophysSpaceSci.pdf

[23] J.P.Petit, G.D'Agostini : **Cosmological bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe.** Modern Physics Letters A, Vol. 29, N° 34, 2014 November 10th
http://www.jp-petit.org/papers/cosmo/2014-ModPhysLettA.pdf

[24] J.P.Petit & G.D'Agostini : **Cancellation of the singularity of the Schwarzschild solution with natural mass inversion process.** Mod. Phys. Lett. A vol. 30 n°9 **2015** arXiv : gr-qc/2103.12845/13 feb 2001(23 March 2021)
https://arxiv.org/abs/2103.12845

[25] J.P.Petit & G.D'Agostini : **Lagrangian derivation of the two coupled field equations in the Janus Cosmological Model**. Astrophysics and Space Science **2015**, 357 :67
http://www.jp-petit.org/papers/cosmo/2015-AstrophysSpaceSci.pdf

[26] G. DAgostini and J.P.Petit : **Constraints on Janus Cosmological model from recent observations of supernovae type Ia**, Astrophysics and Space Science, (2018), 363:139.https://doi.org/10.1007/s10509-018-3365-3
http://januscosmologicalmodel.com/pdf/2018-AstrophysSpaceSci.pdf

[27] Jean-Pierre Petit, Gilles D'Agostini, and Nathalie Debergh. Physical and **Mathematical Consistency of the Janus Cosmological Model (JCM)**. Progress in Physics, Vol.15, issue 1 (January 2018)
http://www.ptep-online.com/2018/PP-52-09.PDF

[28] J.P.Petit : **Janus Cosmological Model and the fluctuations of CMB.** Progress in Physics Vol.14 Issue4 (October 2019)
http://www.ptep-online.com/2018/PP-55-11.PDF

[29] P.Koiran : **Infall Time in the Eddington-Finkelstein Metric, with Application to Eddigton-Rosen Bridges**. International Journal of Modern Physics D , vol.30 , n0 15, 2150106 (2021) https://arxiv.org/abs/2110.05938

[30] P.Koiran, H.Zejli, J.P.Levy, F.Margnat, M-F. Duval : **PT-symmetry in one-way wormholes.** Annals of Physics, Vol. 470, November 2024, 169765 arXiv:2409.02951v1 (gr-qc) 1 sept 2024
http://www.j-petit.org/papers/cosmo/2024-07-PT-Symmetry-in-One-Way-Wormholes.pdf

[31] J.P.Petit, F.Margnat, H.Zejli : **A bimetric cosmological model on Andreï's twin universe approach**. The European Physical Journal. Vol. 84 :N°1126 (2024)
https://link.springer.com/content/pdf/10.1140/epjc/s10052-024-13569-w.pdf

[32] **J.P.Petit, H.Zejli : Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, n001, 2024.**
https://www.worldscientific.com/doi/epdf/10.1142/S0129055X24500545

[33] JP Petit, H.Zejli : **Nature of the Dipole Repeller**, Russian Jr. of Earth Sc. 2023
https://hal-science/hal-04068080v1/document

[34] J.P.Petit, G. D'Agostini, G. Monnet. **Contribution of Kinetic Theory to Galactic Dynamic** Astrophysics and Space Science (14 oct 2025) 370:121
https://doi.org/10.1007/s10509-025-04511-x

[35] J.P.Petit , G.d'Agostini. **Alternative to black holes, Gravastars and Plugstars**. Journal of Modern Physics. Vol. 16, n°10. Octobre 2025.

[36] J-P Petit G. D'Agostini : **The true Schwarzschild solution and its subsequent geometry**. The Journal of Modern Physics. August 2026. Vol. 17 n°8.
http://www.jp-petit.org/papers/cosmo/2026-08-JMP-The true Schwarzschild solution and its subsequent geometry.pdf

[37] J-P Petit. **Kruskal coordinates, semi-complex spacetime expansion**. Journal of Modern Physics. 2026 17(9), 1197-1228
https://doi.org/10.4236/jmp.2026.179053
http://www.jp-petit.org/papers/cosmo/2026-FMP-Petit-Kruskal-coordinates-semi-complex-spacetime-expansion.pdf

[38] J-P Petit. **Alternative to inflation.Variable speed of light cosmological model with Lorentz and fine structure maintened**. Journal of Modern Physics. 2026
http://www.jp-petit.org/papers/cosmo/2026-Petit-JMP-Alternative to inflation.Variable speed of light cosmological model with Lorentz and fine structure maintened.pdf

[39] J-P Petit : **Physical and Geometrical Inconsistecy of the Black Hole Model. Part** I. Journal of Modern Physics., 2026 Vol. 17, N°2.

[40] J-P Petit : **Physical and Geometrical Inconsistecy of the Black Hole Model. Part** II. Journal of Modern Physics., 2026 Accepted no. 2026

Quantum Mechanics

[1]N.Debergh, J-P Petit : **On some reason why the time reversal operator could be unitary ?** Physical Letters A 422 Oct. 2021
http://www.jp-petit.org/papers/cosmo/2022-Nathalie-PhysLettA.dpf

[2] N.Debergh, G.D'Agostini, J-P Petit :**On the Poincaré Algebra in a Complex Space-Time** Manifold. Journal of Modern Physics 2021,12,pp. 218-228

[3] N.Debergh, J.P.Petit and G.D'Agostini : **Evidence of negative energies and masses in the Dirac equation through a unitary time-reversal operator**. , J. Phys. Comm. **2** (2018) 115012

[4] J-P Petit G.D'Agostini. **Is the Real World a Part of a Complex Reality?** Journal of

Modern Physics. Vo.15 n°5 july 2026
http://www.jp-petit.org/papers/cosmo/2026-07-07-I the real world a part of a complex reality.pdf

BANDES DESSINEES DES AVENTURES D'ANSELME LANTURLU

[1] J-P Petit : **Le Topologicon**. Ed. Belin. 1985
http://www.savoir-sans-frontieres.com/JPP/telechargeables/Francais/LE%20TOPOLOGICON.pdf

[2] J-P Petit : **Modèle Janus contre Science Noire** 2025
http://www.jp-petit.org/bd_lanturlu/bd_lanturlu.html

[3] J-P Petit : **Le Metaphysicon** 2026
http://www.jp-petit.org/bd_lanturlu/bd_lanturlu.html

TRAVAUX D'AUTRES CHERCHEURS

T. Damour, I. Kogan **Effective Lagrangians and universality classes of nonlinear bigravity** Phys. Rev. D **66** (2002) 104024. hep-th/0206042.
http://www.jp-petit.org/papers/cosmo/2002-Damour-Kogan-bigravity.pdf

S. Hossenfelder **Antigravitation** Physical Letters B vol. 636 issue 2 4 may 2006 pp.119-125
http://www.jp-petit.org/papers/cosmo/2006-Hossenfelder-antigravitation-PhysLetters.pdf

S. Hossenfelder : **A bimetric Theory with Exchange Symmetry**. Phys. Rev. D78, 044015, 2008 and arXiv : 0807.2838v1 (gr-qc)17 july 2008
http://www.jp-petit.org/papers/cosmo/2008-Hossenfelder.pdf

T. Damour **"Sur le « modèle Janus » de Jean-Pierre Petit**. 2019. Jan. 4.
https://www.ihes.fr/~damour/publications/JanusJanvier2019-1.pdf

T.Damour: **Incohérence Physique et Mathématique du « modèle Janus-2019 » de J.P.Petit et coll.** IHES 28 déc 2021
https://www.ihes.fr/~damour/publications/JanusDecembre2022.pdf

N. Debergh, J.P. Petit : Beyond the Standard Model: Theoretical Explorations of Negative Mass Physics, 28 juillet 2026
https://www.amazon.com.be/dp/B0HC6MTTL5/ref=sr_1_1?crid=4R7HYL5WD3UN&dib=eyJ2IjoiMSJ9.A2DtitYTQorvzttgPB4YBoHj8gqy1-0_xWDkrf9peqMXofIoqK7gz3Hf-ldBWDm4A-jPJASAVuRTjWJ4CqrSvD8OtqNrGXhhQEsKM-ZZaJ9m9q342W1h3U5sOi8aCL9_odlFtNImz4TAEjGpWCSaU0fF42nFxbi4IahfdV6oXC1uT9nVOMjC74LxW2D6mOs3_oKeuhhmA5-DYK_lZGA6vpqjcOmMFHfSwTw2heO6rM-PmzvDdYxoRaUsNbUF32jAtfPNjhbncaaVgc9ytjUf-mRVAMfauKeCgsA5JSbGQeY.H5jFLYZ2HG2hp0dn1k5HVDt4T2KH6WheP_DAJ26AWZQ&dib_tag=se&keywords=nathalie+debergh&qid=1785681515&sprefix=nathalie+debergh%2Caps%2C171&sr=8-1

!

"

!

!

"#"$ %&&%&' !

!

!

!

!

!

!"#$%&#'#"()**$%+#, (

(

!

()!*+,!-.//.01)2!342,56!(!01//!4**,73*!*.!*849,!-1-*:!:,485!.-!591,)*1-19!0.8;6!0+19+!24<,!815,!*.±

!

!

% >+,!?4)@5!A.57./.2194/!B.C,/!

!

!

!

!"#$%&'$()*+,$

!

% >+,!D.) %&F15*,)9,!.-!G/49;!H./,5!

!

!

!

!-.)/#$&0$)$12)34$"&2#

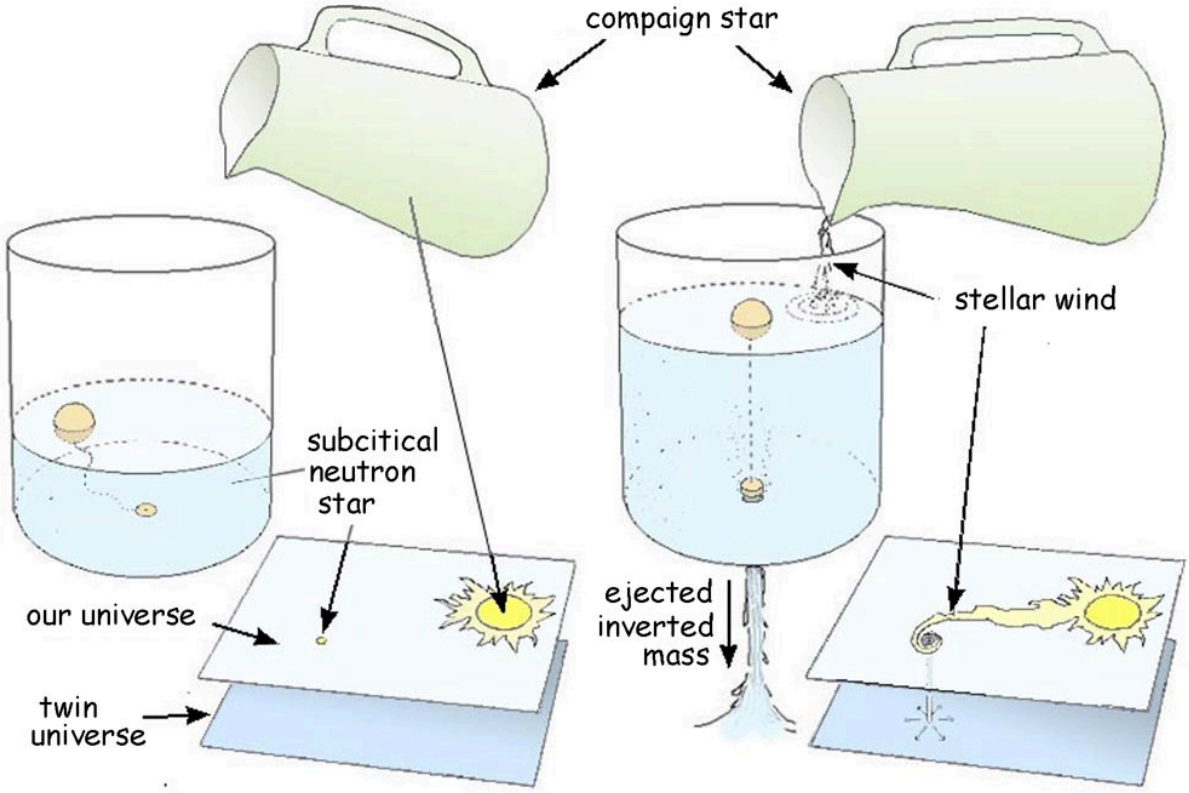
!

!

!

!

- Their replacement by the Plugstar model:



Plugstar, an educational model

In 1975, after my colleague Maurice Viton had sent me the first pages of the Ummo documents, Jean-Jacques Pastor and I traveled to Spain fairly quickly and made contact with the essayist and author, the friendly Antonio Ribera[1], who lived in San Feliù de Codinas, near Barcelona.



Antonio Ribera. 1920–2001

[1] Died on September 24, 2001, at the age of 81.

He then allowed us to photocopy the documentation at his disposal, including the material regarding the spacecraft, which he had received. I was initially drawn to the document concerning the structure of the cosmos. And it was in this document that I discovered the idea of the existence of a second cosmos, referred to as a "twin" and described as "enantiomorphic" to our own[2].

As it happened, at the time I had "Newtonian" tools at my disposal for modeling the cosmos.

Normally, any modeling of this kind must be based on Einstein's relativistic equation. In 1922, Alexander Friedmann derived the first non-stationary solution to this equation. If *R* is a characteristic length of the universe, and a is a constant, its evolution over time obeys the differential equation:

$$R^2 \frac{d^2R}{dt^2} + a^2 = 0$$

In 1934, cosmologists Milne and Mac Crea caused a huge surprise by showing[3] that this evolutionary pattern can emerge from non-relativistic equations[4].

In the model of general relativity, two things come together:

- Space is curved

- The speed of light does not depend on the reference frame

These are two distinct concepts. Similarly, in a non-relativistic spacetime, the perihelion of the planets undergoes precession due to curvature.
In the early 1970s, I had gained a solid grasp of seven-dimensional space:

Time + position + velocity

Based on the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - \frac{\partial \Psi}{\partial \boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = \frac{\partial_e f}{\partial t}$$

The collision term is represented by the right-hand side.

It is then zero in two cases:

- When these collisions are absent (negligible), which is the case in galaxies.

- When entropy is maximal

[2] "In a mirror," where what is "right" in one becomes "left" in the other.
[3] W.H. MacCrea & E.A. Milne, Quart. J. Maths, 5, 1934, p. 72
[4] Euler's equations, from a mechanics model describing a fluid at uniform temperature, combined with Poisson's equation, which accounts for the existence of a Newtonian gravitational potential.

The equation then becomes the "Boltzmann equation without collisions" or the "Vlasov equation."

$$\frac{\partial f}{\partial t} + \boldsymbol{v}.\frac{\partial f}{\partial \boldsymbol{r}} - \frac{\partial \Psi}{\partial \boldsymbol{r}}.\frac{\partial f}{\partial \boldsymbol{v}} = 0$$

This equation then "encompasses" Euler's equations (conservation of mass, momentum, and energy). In 1975, I published two papers at the Paris Academy of Sciences showing that a Newtonian universe with maximum entropy evolves according to the Friedmann equation[5].

The technique involves introducing a Maxwell-Boltzmann distribution function that takes the form of a spherical polynomial of degree two. The equation then "breaks down" into twenty nonlinear partial differential equations to be coupled with Poisson's equation. The use of dyadic algebra allowed me to arrive at the result.

This approach had greatly impressed the academician and mathematician André Lichnérowicz.

When I discovered in the UMMO texts this story of two interacting, enantiomorphic universes with opposite times, I decided to "construct the Newtonian version" using two coupled equations.

The approach immediately proved to be coherent and fruitful.

New note to the Proceedings of the Paris Academy of Sciences titled[6]:

- *Enantiomorphic Universes with Opposite Time Scales.*

This came as a shock, because these systems of nonlinear equations are extremely complex. The fact that an elegant and exact solution emerges is a sign that something immense lies beneath it; therefore, contrary to what some claim, the information cannot have come from Spanish students or pranksters.

I take things a step further with a second note in the *Comptes Rendus de l'Académie des Sciences de Paris*.[7]

- *Universes interacting with their image in the mirror of time.*

---

[5] J.P. Petit, G. Monnet: Maximum entropy and Friedmann cosmologies. CRAS Acad. Sc., vol. 280 (May 11, 1975), pp. 1245–1248. The Friedmann equation is equation (17)
http://www.jp-petit.org/papers/CRAS/cosmology_1975.pdf

[6] J.P. Petit: Enantiomorphic Universes at Opposite Times. CRAS, May 12, 1975, Vol. 284, Series A, pp. 1315–1318 http://www.jp-petit.org/papers/CRAS/cosmology_1975a.pdf

[7] J-P Petit. Universes Interacting with Their Mirror Images in Time. CRAS Vol. 284, Series A (June 5, 1977), presented by A. Lichnérowicz. Pages 1413–1416
http://www.jp-petit.org/papers/CRAS/cosmology_1975b.pdf

I discussed these results with Lichnérowicz, who showed keen interest. But I refrained from revealing the source of the idea to him.



A.Lichnérowicz 1915-1988

André Lichnérowicz, mathematician, geometer, and academician

COSMOLOGIE. — *Univers jumeaux, énantiomorphes, à temps propres opposés.* Note (*) de **Jean-Pierre Petit**, présentée par M. André Lichnerowicz.

Dans une Note précédente (1) un modèle newtonien unifié avait été présenté, qui conduisait à une équation semblable à celle de Heckman et Sücking (2). Le présent travail fournit la même équation, dans un contexte géométrique différent et semble offrir une description plus fidèle d'un système matière antimatière.

In a previous Note a unified newtonian model was presented, leading to an equation similar to Heckman and Sücking's. The present work is imbedded in a different geometric framework, and gives the same equation too. But it offers a better description of a matter antimatter system.

1. INTRODUCTION. — Dans une Note précédente (1) un modèle à deux populations, de mêmes masses et de charges opposées, évoluant dans un même euclidien, avait été étudié. L'hypothèse de l'équilibre thermodynamique local avait conduit à une solution unifiées où $R(t)$, grandeur caractéristique de cet univers, obéissait à une équation semblable à celle de Heckman et Sücking, à la valeur de la constante près (2). Nous allons introduire ici un contexte géométrique différent en introduisant N populations, chacune évoluant dans un espace euclidien $(\vec{r}_i, \vec{w}_i, t_i, \times_i)$, où $\times_i$ désigne le produit vectoriel.

L'évolution du système se déduira d'une étude de N équations de Vlasov couplées par les champs (le mode de couplage sera défini plus loin). Nous ferons encore ici l'hypothèse de l'équilibre thermodynamique local. Ainsi les fonctions de distribution de la vitesse seront elles choisies maxwelliennes.

L'hypothèse maxwellienne à elle seule conduit à un certain nombre de propriétés se référant aux paramètres macroscopiques. Ainsi les températures sont-elles nécessairement uniformes. Quant à la vitesse macroscopique, elle est, pour chaque population, la superposition d'un champ de vitesse radiale correspondant à la loi de Hubble et d'une rotation instationnaire en corps solide.

On s'oriente délibérément vers des solutions particulières où les densités n_i de dépendent que des temps. Dans ces conditions les équations à l'ordre zéro se réduisent à : $n_i \sim T_i^{3/2}$.

Ces propriétés découlent des N premiers lots de 17 équations aux dérivées partielles issus des N équations de Vlasov.

2. ESPACES JUMEAUX, L'HYPOTHÈSE DE L'ÉQUILIBRE THERMODYNAMIQUE ET SES CONSÉQUENCES. — A ce stade il n'a aucun couplage entre les solutions f_i. Nous allons supposer que les espaces E_i s'identifient soit à un espace E_1, soit à un espace E_2, qui jouissent des propriétés suivantes :

$$\vec{r}_1 \equiv \vec{r}_2; \qquad \vec{w}_1 \equiv \vec{w}_2; \qquad t_1 \equiv -t_2; \qquad \times_1 \equiv -\times_2. \tag{1}$$

On voit que ces espaces E_1 et E_2 sont énantiomorphes et possèdent des temps propres opposés. Le deuxième volet de l'hypothèse d'équilibre thermodynamique s'écrit :

$$T_i(t_i) = T_j(t_j), \qquad \forall\, i \text{ et } j. \tag{2}$$

Ceci a des conséquences quant aux champs de vitesse macroscopiques. Pour fixer les idées, considérons un modèle à deux populations f_1 dans l'espace E_1, et f_2 dans l'espace E_2. On suppose qu'un observateur situé dans l'un de ces espaces peut percevoir à la fois les

Source gallica.bnf.fr / Bibliothèque nationale de F

éléments de son espace, mais aussi ceux de l'espace jumeau. Soit $\langle \vec{w}_1 \rangle_1$ la vitesse macroscopique des éléments 1, telle qu'elle est perçue par un observateur lié à E_1, et $\langle \vec{w}_1 \rangle_2$ cette même vitesse, perçue cette fois par un observateur lié à E_2. Comme les temps propres sont opposés

$$\langle \vec{w}_1 \rangle_1 = - \langle \vec{w}_2 \rangle_2 \tag{3}$$

et, plus généralement

$$\langle \vec{w}_i \rangle_j = \frac{t_i}{t_j} \langle \vec{w}_i \rangle_i. \tag{4}$$

Sous cet éclairage E_1 et E_2 sont des points de vue spatiotemporels. L'équilibre thermodynamique nous dit que $T_1(t_1) = T_2(t_2)$. Ceci entraîne, et il est facile de le voir, que les composantes radiales de $\langle \vec{w}_1 \rangle_1$ et de $\langle \vec{w}_2 \rangle_2$ sont opposées. Ainsi, à la question : « l'univers est-il en expansion? » il convient de répondre : « cela dépend du point de vue spatiotemporel adopté ». De même, en changeant de point de vue spatiotemporel il y a inversion des sens de rotation des populations. Ce qui rend ce modèle inorientable dans le temps et dans l'espace.

3. Définition du couplage. — A ce stade nous avons deux espaces énantiomorphes et à temps propres opposés, qui contiennent des populations f_i dont on a supposé qu'elles formaient un ensemble en équilibre thermodynamique. Chaque population f_i évolue dans un champ gravitationnel $\vec{g}^{(i)}$, électrique $\vec{E}^{(i)}$, magnétique $\vec{B}^{(i)}$. Nous allons simplement supposer que ces champs sont les résultantes des champs dus à chaque population :

$$\vec{g}^{(i)} = \vec{g} = \sum_j \vec{g}_j, \tag{5}$$

$$\vec{E}^{(i)} = \vec{E} = \sum_j \vec{E}_j, \tag{6}$$

$$\vec{B}^{(i)} = \vec{B} = \sum_j \vec{B}_j. \tag{7}$$

4. Solution newtonienne unitaire. — Pour que la solution soit unitaire il faut que les champs obéissent aux équations de Poisson et de Maxwell, et que les équations restantes, issues des N équations de Vlasov, dégénèrent. Ces équations s'écrivent :

$$-\frac{\overrightarrow{\partial \psi}}{\partial r} + \frac{q_i}{m_i}\left(-\frac{\overrightarrow{\partial V}}{\partial r} + \vec{E}' + \langle \vec{w}_i \rangle_i \times_i \vec{B}\right) - \frac{\overrightarrow{D_i \langle w_i \rangle_i}}{Dt_i} = 0. \tag{8}$$

Nous reprendrons l'idée développée dans la Note précédente :

$$\vec{B} = B\vec{k}; \qquad \vec{E}' = -\frac{\dot{B}}{2}(\vec{k} \times \vec{r}). \tag{9}$$

On remarquera que les équations de Maxwell sont invariantes par la transformation : t en $-t$, $\times$ en $-\times$ (inversion du produit vectoriel). On constate encore qu'il y a dégénérescence et existence d'une solution si :

$$\frac{\dot{\omega}_i}{\omega_i} = \frac{\dot{T}_i}{T_i}, \tag{10}$$

$$\frac{\dot{B}}{B} = -\frac{2\dot{R}}{R}, \tag{11}$$

$$\omega_i^2 + \frac{q_i \omega_i B}{m_i} = \omega_j^2 + \frac{q_j \omega_j B_j}{m_j}, \qquad \forall\, i \text{ et } j; \tag{12}$$

(10) exprime que les vitesses angulaires varient comme les températures. Si R est une grandeur caractéristique commune à tous les systèmes, la relation (11) exprime que le champ magnétique est intimement lié au plasma (nombre de Reynolds magnétique infini).

5. MATIÈRE ET ANTIMATIÈRE. — Prenons un système à 2 populations. Supposons d'abord $\omega_1 = -\omega_2$. Dans ces conditions, pour que la densité de courant soit non nulle il faut que $q_1 q_2 < 0$, donc que $m_1 m_2 > 0$. Supposons que la particule 1 représente le proton. Masse m_1 et charge q_1 sont donc positifs. La particule 2 a une masse positive et une charge négative. Si on examine les sens de giration des particules 1 et 2 dans un champ magnétique, ils apparaissent inverses pour un observateur lié à l'un des deux espaces. Imaginons maintenant deux particules 1 et 2 en présence. La particule 1 poursuit la particule 2, laquelle s'échappe de plus en plus vite. Ceci vu de l'espace E_1, bien entendu. Observation inverse à partir du point de vue E_2.

Observées à partir de l'espace E_1 ou de l'espace E_2, les deux populations ont des rotations *a contrario*, ce qui donne un moment cinétique global nul.

Examinons maintenant la représentation $\omega_1 = \omega_2$.

Vues de E_1 ou de E_2 les deux populations tournent dans le même sens. Donc pour que le vecteur densité de courant ne soit pas nul on doit avoir : $q_1 q_2 > 0$, et $m_1 m_2 > 0$. La particule 1 étant censée représenter le proton, on constate que 1 et 2 présentent le même sens de giration dans un champ magnétique. Cette deuxième représentation sera rejetée, car elle ne cadre pas avec les données d'observation sur l'antimatière.

En revenant donc à la représentation un, l'équation d'évolution obtenue est :

$$\boxed{\mathscr{R}^2 \ddot{\mathscr{R}} + \frac{1}{3} - \frac{2}{3\mathscr{R}}(\beta_R^2 + \beta_R \beta_M) = 0.} \tag{13}$$

Elle est identique à celle de la Note [1].

6. CONCLUSION. — Ce modèle est une tentative de description newtonienne du couple matière-antimatière à travers une représentation géométrique. Matière et antimatière appartiennent ainsi à deux espaces différents, énantiomorphes et à temps propres opposés. On a supposés ces deux entités couplées par l'intermédiaire des champs. Le modèle offre une symétrie séduisante, l'adoption de l'un ou l'autre de ces points de vues spatiotemporels sur le réel étant tout à fait arbitraire. Il est possible de considérer une solution newtonienne comme une solution à la fois tangente (problème situé dans les euclidiens tangents) et

asymptotique (vitesse des éléments faibles devant la vitesse de la lumière), à une solution de la relativité générale. Cette solution ayant un caractère unifié, il serait intéressant d'essayer d'étendre ce travail à l'aide de deux variétés énantiomorphes, à temps propres opposés et ayant même rayon de courbure.

(*) Séance du 28 mars 1977.

(1) J.-P. PETIT et G. MONNET, *Comptes rendus*, 283, série A, 1976, p. 1057.

(2) O. HECKMAN et E. SÜCKING, *Worlds Models. SEU*, 1958, p. 148-159.

10, *rue Fèlibre-Gaut*,
13100 *Aix-en-Provence*.

COSMOLOGIE. — *Univers en interaction avec leurs images dans le miroir du temps.*

Note (*) de **Jean-Pierre Petit**, présentée par M. André Lichnerowicz.

Une analyse newtonienne conduit à des modèles d'univers en interaction avec leur image au temps $-t$. Le premier modèle conduit à une pseudo équation de Friedman, le second à une équation de type hyperbolique. Le premier modèle semble donner une représentation satisfaisante de la dualité matière-antimatière.

Unified newtonian models are presented, in which the universe at the time t depends on its state at the time $-t$. The first model gives a pseudo Friedman equation, while the second leads to a hyperbolic solution. The first model gives an accurate description of the matter antimatter duality.

1. INTRODUCTION. — Nous allons continuer ici une étude de cosmologie newtonienne qui a déjà fait l'objet des Notes ([1]), ([2]), ([3]) et ([4]). Dans la Note ([4]) deux populations chargées, situées dans des espaces des phases énantiomorphes dotés de temps propres opposés, interagissaient par l'intermédiaire des champs gravitationnels et électromagnétiques.

Dans l'approche newtonienne les champs sont censés se propager à vitesse infinie. Que se passe-t-il si l'on tente de tenir compte d'une vitesse de propagation finie, d'un couplage non instantané ?

Supposons que la distribution f_1 subisse une variation δf_1 dans une région $(\mathbf{r}_1, d_3 r_1)$. La variation des champs due à cette perturbation va se propager à la vitesse c vers une région $(\mathbf{r}_2, d_3 r_2)$ où se trouvent des éléments de l'espèce 2.

Il semble qu'il y ait la matière à paradoxe. Les populations 1 et 2 ont des temps propres opposés ($t_1 = -t_2$). On ne voit pas dans ces conditions comment une information quelconque peut transiter d'une population à l'autre. En effet tout message émis par l'une des populations ne peut être à proprement parler reçu par l'autre, puisque, suivant son temps propre, elle l'émet également!

Ceci évoque la situation d'un personnage qui utiliserait sa propre image, reflétée par un miroir, pour nouer sa cravate. Si x est la distance du sujet au miroir, il utilise l'information émise par un sujet situé en $-x$, qui est sa propre image, énantiomorphe, et évoluant dans le temps $t-(2\,x/c)$. Remarquons que si nous nous identifions maintenant à cet Homme qui se trouve « de l'autre côté du miroir », celui-ci dicte sa conduite en observant son image énantiomorphe et « avancée », puisque correspondant au temps $t+(2\,x/c)$.

En reportant à plus tard le problème posé par une vitesse de propagation finie des champs, nous allons tenter de lever le paradoxe en supposant que la population f_2 n'est autre que l'image de la population f_1 à travers un miroir temporel. Ceci revient à dire que la population f_1 évoluant au temps t subit l'influence de son propre état antérieur, au temps $-t$. Le temps $t = 0$ se référant à l'instant zéro du big bang.

2. LE MIROIR TEMPOREL. — Cette situation est assez simple à formaliser. Nous avons cette fois un seul espace des phases $\mathrm{E}(\mathbf{r}, \mathbf{w}, t, \times)$, où $\times$ désigne le produit vectoriel, et une seule population f d'éléments de masse m et de charge q. Nous écrirons l'équation de Vlasov donnant la solution f, en faisant intervenir des champs $\mathbf{g}_*$, $\mathbf{E}_*$, $\mathbf{B}_*$, qui seront définis plus loin. Ceux-ci vont être calculés à l'aide de $f(t)$ et de $f(-t)$. Nous allons choisir un mode

Source gallica.bnf.fr / Bibliothèque nationale de F

de couplage permettant de retrouver le résultat de la Note ([4]) :

$$\mathbf{g}^* = \mathbf{g}(t)+\mathbf{g}(-t), \tag{1}$$

$$\mathbf{E}^* = \mathbf{E}(t)-\mathbf{E}(-t), \tag{2}$$

$$\mathbf{B}^* = \mathbf{B}(t)+\mathbf{B}(-t). \tag{3}$$

Le mode de calcul de la solution est analogue à celui de la Note ([4]). On retrouve une température variant comme la vitesse angulaire, et comme l'intensité du champ magnétique, uniforme, lui même variant comme l'inverse du carré de la dimension caractéristique du système, ce qui exprime que les lignes de champ sont « gelées » dans ce plasma à nombre de Reynolds magnétique infini. Et nous obtenons la pseudo equation de Friedman :

$$\boxed{\mathscr{R}^2\,\ddot{\mathscr{R}}+\frac{1}{3}-\frac{2}{3\mathscr{R}}(\beta_R^2+\beta_R\,\beta_M)=0.} \tag{4}$$

3. Un second modèle. — Ces choix (1), (2) et (3) sont évidemment arbitraires. Nous allons envisager un autre mode de couplage, plus symétrique :

$$\mathbf{g}^* = \mathbf{g}(t)-\mathbf{g}(-t), \tag{5}$$

$$\mathbf{E}^* = \mathbf{E}(t)-\mathbf{E}(-t), \tag{6}$$

$$\mathbf{B}^* = \mathbf{B}(t)-\mathbf{B}(-t). \tag{7}$$

On supposera de plus que le produit vectoriel s'inverse lorsqu'on passe par le point $t = 0$. Remarquons qu'à $t = 0$ l'espace n'est plus orientable. Un même style d'analyse peut être mené à bien. Mais, à la différence de la solution précédente, le potentiel gravitationnel est cette fois harmonique, c'est-à-dire que $\Delta\psi^* = 0$.

Ceci conduit à une équation qui n'est plus une pseudo équation de Friedman :

$$\boxed{\ddot{\mathscr{R}}=\frac{2}{3\mathscr{R}^3}(\beta_R^2+\beta_R\,\beta_M);} \tag{8}$$

équation dont la résolution est l'hyperbole : $\mathscr{R} = \lambda^2\sqrt{1+\mu^2 t^2}$, où λ et μ sont des constantes.

Une remarque immédiate : si nous nous servons d'un tel modèle pour calculer l'âge de l'univers à partir de la constante de Hubble, on obtient une valeur 1,5 fois plus élevée que celle qui découle du modèle classique d'Einstein-de Sitter.

4. Lien avec l'antimatière. — Les choix (1), (2), (3), puis (5), (6) et (7) ont été faits de manière à ce que les sens de giration des particules et des particules images apparaissent inverses, dans un champ magnétique, pour un observateur lié au temps t. Du point de vue d'une observation possible, tout se passe comme si les particules images p_* évoluaient

suivant le temps $-t$. Nous pouvons alors étudier l'interaction apparente entre une particule p et une particule p_* ou entre deux particules images. Le tableau ci-après résume ces différents résultats :

Premier modèle		Second modèle	
p p_*	p_* p_*	p p_*	p_* p_*
Interaction gravitationnelle			
p court après p_* qui s'enfuit	Se repoussent	Se repoussent	Se repoussent
Interaction électrostatique			
S'attirent	S'attirent	S'attirent	S'attirent

Le premier modèle est plus conforme à la représentation classique de l'antimatière.

Il est à noter que dans les deux modèles la particule image se comporte comme une particule de charge négative.

Examinons maintenant ce qui se passerait si une particule interagissait avec une particule image de masse différente. Seul le cas de l'interaction gravitationnelle correspondant au premier modèle [que l'on retrouve aussi dans le travail de la Note (4)] demande examen. Situons l'observateur au centre de gravité : $\mathbf{r}_G = (m\mathbf{r}+m_*\mathbf{r}_*)/2$, lequel se trouve, dans ce cas, accéléré. Le mouvement apparent des deux particules correspond à la force apparente :

$$\mathbf{F} = G\,mm_*\frac{(m_*-m)}{(m_*+m)}\frac{\mathbf{r}_*-\mathbf{r}}{|\mathbf{r}_*-\mathbf{r}|^3}; \qquad \mathbf{F}_* = -\mathbf{F}. \tag{9}$$

Si la particule image est la plus lourde, cette force correspond à une attraction. Et *vice versa*. Bien sur, si les particules ont même masse elles sembleront indifférente à l'observateur situé au centre de gravité, qui se trouvera en état d'accélération uniforme.

Considérons un mélange homogène de matière et d'antimatière. Une concentration de matière est équivalente à une masse $M > m$. Suivant ce qui a été dit plus haut, l'antimatière environnante devrait se retrouver repoussée. Considérons maintenant une concentration M d'antimatière, celle ci va au contraire attirer la matière environnante, ce qui entraînera l'anihilation. Si un tel phénomène contribue à la séparation de la matière et de l'antimatière et éventuellement à la formation de proto galaxies, on en concluerait que les antigalaxies ne peuvent exister. Pour être plus précis, l'image, au temps $-t$, d'une galaxie, serait une lacune.

5. Conclusion. — Voici donc une représentation de l'univers ou tout objet cosmique est en interaction avec son image, symétrique par rapport au miroir temporel, c'est-à-dire située au temps $-t$. Cet univers image semble câdrer avec ce que nous connaissons de l'antimatière. Il conviendrait maintenant d'étudier plus en détail le phénomène d'instabilité gravitationnelle dans un tel système. On sait que nous ne disposons actuellement d'aucun modèle, solution de l'équation de Vlasov et de l'équation de Poisson, permettant de décrire une galaxie, en situation stationnaire ou instationnaire, de manière satisfaisante. Ceci représente peut-être une approche nouvelle du problème, toute galaxie étant alors dynamiquement indissociable du nuage d'antimatière qui l'environnerait.

Une solution newtonienne peut être considérée comme une solution asymptotique (c tendant vers l'infini) et tangente (écriture euclidienne) à une solution de la relativité généralisée. On se demande dans quel contexte placer cette dernière. Faudrait-il faire recours à une variété non orientable?

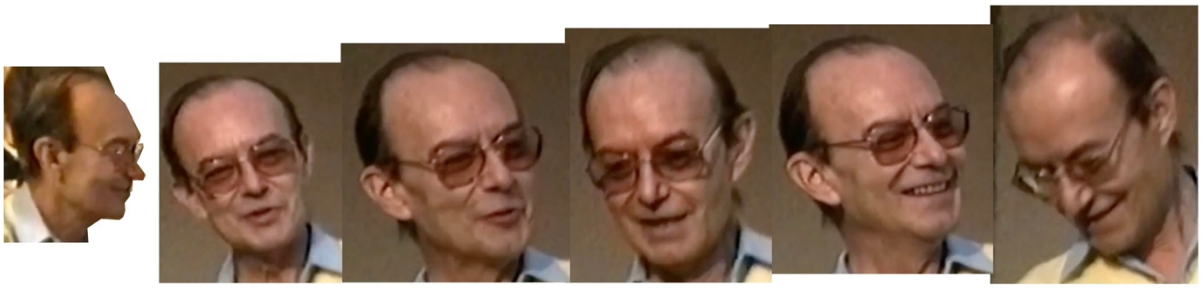
(*) Séance du 2 mai 1977.

(1) J.-P. Petit et G. Monnet, *Comptes rendus*, 280, série A, 1975, p. 1245.
(2) J.-P. Prtit et G. Monnet, *Comptes rendus*, 280, série A, 1975, p. 733.
(3) J.-P. Petit et G. Monnet, *Comptes rendus*, 283, série A, 1976, p. 1057.
(4) J.-P. Petit, *Comptes rendus*, 284, série A, 1977, p. 1315.

10, *rue du Félibre Gaut*,
13100 *Aix-en-Provence*.

In the early 1980s, I expanded the collection of documents we had brought back from Spain. There I found a new piece of data that intrigued me: the speed of light appears to have been greater in the past.

My visits to Antonio Ribera alternated with those to my new contact, the industrialist Rafael Farriols[8], who lived on the vast estate "El Bosque," near Barcelona.



Rafael Farriols 1928–2006

In fact, Antonio, having resisted the demands of the letter's authors, was subsequently cut off from them. Farriols, who was more flexible, has now become one of the recipients, a role he shares with members of the Madrid group: Domínguez, Barranechea, and others.

The years passed and meetings followed one after another, either in Barcelona or in Madrid.

I am attempting to modify the standard cosmological model by introducing a relationship between c and the way the universe expands. I am attempting:

$$c \quad variant\ en \quad \frac{1}{R}$$

Without much success. I cannot say exactly when the Spaniards began receiving phone calls from these "Ummites" (Los Ummitas), calls that regularly took place around 3 a.m. and continued long into the night. One of the callers was the electrical engineer Dominguez.

I had the opportunity to have lunch with him in Madrid and mentioned my unsuccessful attempts. He then replied:

- "Rafael told me" about what you had been working on and about that calculation. When I spoke to an Ummite on the phone, I mentioned it to him. And he replied:

  - *"It doesn't vary in* $\frac{1}{R}$ *but in* $\frac{1}{\sqrt{R}}$

I'm heading home with this new data, and it seems that it's becoming the key to a new model.

In the meantime, I've gotten in touch with another mathematician and geometer, Jean-Marie Souriau, who is retired and, like me, lives in Aix-en-Provence. Intrigued, he's examining my calculations, line by line.

[8] Died on December 27, 2006, at the age of 79.



Jean-Marie Souriau and his cat Pioum.

Convinced, he agreed to recommend the article for publication in the prestigious journal *Modern Physics Letters A*.

Here is the article in question, which was published in 1988[9] (see below)

J.P.PETIT: Cosmological model with variable velocity of light. Modern Phys. Letters A3, 1988, pp. 1527

Page 186: This Ummite law makes the Einstein constant invariantχ ; equation (1) we find, , equation (5), this law of variation of $c \sim \frac{1}{\sqrt{R}}$

Page 187: The gravitational constant and$G \sim \frac{1}{R}$ the Planck constant $h \sim R^{3/2}$

Equation (10): the density; equation$\rho \sim \frac{1}{R^2}$ (11): energy is conserved.

Page 188: Equation (18) The cosmological horizon varies as R, which immediately presents itself as an alternative to the inflationary model, to account for the extreme homogeneity of the early universe.

[9] http://www.jp-petit.org/papers/cosmo/1988-ModPhysLettA-1.pdf

Page 189: The evolution of the universe, in this phase with variable constants, is given by Equation (21). This implies that the Planck constant increases proportionally with time.

This article went completely unnoticed. But it had the immense advantage of forcing the CNRS to reinstate me in my research program at the Marseille Observatory, from which I had been excluded a few weeks earlier at the request of the director at the time, the astrophysicist James Lequeux, who had sent a letter to the General Directorate stating:

- *The activities of Mr. Jean-Pierre Petit are of no interest whatsoever to the Marseille Observatory.*

Having been warned of what was afoot, I had tried to preempt the move by presenting to him—complete with equations—"my cosmological model with variable speed of light."



James Lequeux

"My skin is as thick as a rhinoceros's hide"

(what about his brain?)

But Lequeux did not believe for a single second what I had presented to him…

ANNEXE 2

Modern Physics Letters A, Vol. 3, No. 16 (1988) 1527–1532
© World Scientific Publishing Company

AN INTERPRETATION OF COSMOLOGICAL MODEL WITH VARIABLE LIGHT VELOCITY

JEAN-PIERRE PETIT
*Observatoire de Marseille**

Received 1 June 1988

A cosmological model with variable c, h, G is proposed. The characteristic lengths of physics (Compton, Jeans, Schwarzschild) are assumed to vary like $R(t)$. Both light and matter's worlds are found to obey the same law $R \approx t^{2/3}$. The Planck constant is found to vary like t and the gravitation one like $1/R$, while the Planck length vary like R. The particle masses follow $m \approx R$. The Hubble law still applies. The redshifts come from the secular variation of the Planck constant.

1. Introduction

Since 1930, the constancy of several so-called constants of physics have been criticized by many authors.[1,2,3,4] Accurate laboratory measurements show that these values appear quite constant in today's space time field, which is very small with respect to the whole space time, although Van Flandern[6] claimed observa tional evidence of the variation of the gravity constant G. As far as we can see, the extension of the constancy of the light velocity, and other so-called "fundamental constants" over the overall cosmic scale is a still debatable hypothesis. The purpose of this paper is to examine some of the consequences of a model in which the "constants" (especially the light velocity) are assumed to vary with time.

2. The Possible Secular Variation of c

Milne[1] first tried to propose an attempt of this type. He suggested that the observed redshifts are due to some secular change of the Planck constant and not to the classical Doppler effect. If the energy of the travelling photon remains constant, the apparent decrease of the observed frequency would only be due to the linear increase of h with the cosmic time t. In addition, Milne[1] suggested a decrease in time of the gravity constant G.

Similarly, Hoyle[2] argued against the assumption of the constancy of the mass content of the universe. He also suggested a secular change of G and constant creation of matter. Dirac,[3,4] starting from a hypothesis on the variation in time of some large numbers built with characteristic physical quantities (like the ratio of

*For scientific correspondence: Chemin de la Montagnère, 84120, Pertuis, France.

the electromagnetic force to the gravitational force), got a variable G and a constant creation of matter. Later, Canuto and Hsieh,[8] Lodenquai[5] and Julg[7] explored some consequences of Dirac's initial idea. But, surprisingly, no one contested the absolute constancy of c.

In the field equation, the so-called Einstein constant χ is determined by identification to the Poisson equation, which gives

$$\chi = -\frac{8\pi G}{c^2}. \tag{1}$$

The quantity χ must be an absolute constant with respect to the four dimensions, for the field equation to be divergenceless. But once the identification mentioned above refers to a steady situation, it does not imply the absolute constancy of G and c. A world model could a priori be built, with variable G and c, with respect to the cosmic time (which will be defined later) if the ratio G/c^2 remains an absolute constant.

In the sequel of the paper, we are going to analyze the effects of a secular variation of the light velocity.

3. Suggesting Gauge Relations

The Robertson-Walker metric, based on the isotropy and homogeneity assumptions, leads to the following system

$$-\frac{8\pi G}{c^2}\rho = \frac{3k}{R^2} + \frac{3}{R^2}\left(\frac{dR}{dx^0}\right)^2, \tag{2}$$

$$-\frac{8\pi G}{c^2}\left(\frac{p}{c^2}\right) = -\frac{k}{R^2} - \frac{1}{R^2}\left(\frac{dR}{dx^0}\right)^2 - \frac{2}{R}\frac{d^2R}{dx^{0^2}} \tag{3}$$

In this system, k is the sign of the curvature, p the pressure and ρ the density of energy-matter. In the classical model, we define the cosmic time t from the chronological variable x^0, by $x^0 = ct$, where c is considered as an absolute constant. In addition, the wavelength of the photon varies like R.

Let us consider the less restrictive condition

$$dx^0 = c(t)\,dt \tag{4}$$

which represents an alternative interpretation of the chronological parameter x^0. We are now going to relate the main physical constants to R, considered as a gauge parameter

$$c \approx \frac{1}{\sqrt{R}} \quad \text{or} \quad Rc^2 \approx \text{constant}, \tag{5}$$

$$m \text{ (particle's mass)} \approx R\,, \tag{6}$$

$$h \approx R^{3/2}\,, \tag{7}$$

$$G \approx 1/R\,. \tag{8}$$

With reference to relation (1), notice that $G/c^2 = \text{constant}$. In addition, if V is the relative velocity of a given element, for example, the random velocity of a galaxy in a cluster, or the velocity of a free particle in a cloud, we assume that V follows the secular variation

$$V \approx R^{-\frac{1}{2}} \tag{9}$$

If we suppose that the number of particles is conserved, the matter density ρ obeys

$$\rho \approx 1/R^2\,. \tag{10}$$

As a consequence, we can express the cosmic evolution through a gauge process, i.e., the Compton wavelength, the De Broglie wavelength, the Schwarzschild length and the Jeans length are found to vary like R.

In addition, our model still considers $mc^2 = \text{constant}$ and

$$\frac{mc^2}{\sqrt{1-\dfrac{V^2}{c^2}}} \approx \text{constant}\,. \tag{11}$$

The classical model saved the masses, assumed to be constant, but not the total matter-energy, through the variation of the cosmic background energy. In our scenario, it is just the reverse: the energy-matter is constant in time, not the masses. In addition, one should notice that the quantity Gm^2/R, which can be considered as a characteristic gravitational energy, is conserved.

Since the energies are conserved in our model, the moment as defined as mV' varies like $R^{1/2}$. It is only constant if we define them as $\rho u' c$.

Finally the Planck length varies with time like $R(t)$, the Planck time varies like t and the gravitational forces like $1/R(t)$.

4. The Evolution Equation

Introducing Eq. (4) into the systems (2), (3), we obtain the following equations

$$\frac{2R''}{R} + \frac{2R'^2}{R^2} + \frac{kc^2}{R^2} = \chi p\,, \tag{12}$$

$$\frac{1}{R^2}(R'^2 + kc^2) = -\chi\frac{\rho c^2}{3}. \tag{13}$$

The use of the following equation of state

$$p = \frac{\rho\beta^2 c^2}{3} \qquad \text{with } 0 < \beta \leq 1 \tag{14}$$

leads to

$$\frac{2R''}{R} + \frac{R'^2}{R^2}(2 + \beta^2) + \frac{kc^2}{R^2}(1 + \beta^2) = 0. \tag{15}$$

In the case where $R = at^m$, the parameter β disappears from Eq. (15). From Eq. (5), $Rc^2 = R_0 c_0^2$ is an absolute constant, R_0 and c_0 being the present values of the gauge parameter R and light velocity c. The only possible value for k is -1, which means that in our model the curvature is negative. Then the evolution law becomes

$$R = \sqrt[3]{\frac{g}{4}R_0 c_0^2}\, t^{\frac{2}{3}}. \tag{16}$$

Here, contrary to the classical models, light and matter obey the same evolution law. Moreover,

$$R = \frac{3}{2}ct. \tag{17}$$

If we know t_0, the age of the universe, and c_0, the present value for the light velocity, we could derive the present value of the gauge parameter of the universe $R_0 = (3/2)\, c_0\, t_0$, with the use of

$$L(t) \equiv \int_0^t c(\tau)\, d\tau \equiv R(t). \tag{18}$$

The consequence is that the horizon is found to be identical, at any time, to the gauge factor $R(t)$.

5. Gauge Invariance of some Fundamental Equations

Let us take first the Vlasov equation, referring to collision free fluids. $f(\mathbf{r}, \mathbf{V}, t)$ is the velocity distribution function, which depends on the position vector r, the

velocity vector $\mathbf{V}$ and the time t. Ψ is the gravitational, e.g., $m\partial\Psi/\partial\mathbf{r}$ is the force acting on the particle whose mass is m.

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Psi}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{V}} = 0 . \tag{19}$$

Introduce non-dimensional variables, such that

$$t = t^* \tau ; f = f^* \xi ; \mathbf{V} = V^* \mathbf{w} ; \mathbf{r} = R^* \zeta ; \Psi = (G^* m^*/R^*) \varphi$$

Equation (19) becomes

$$\frac{1}{t^*} \frac{\partial \xi}{\partial \tau} + \frac{V^*}{R^*} \mathbf{w} . \frac{\partial \xi}{\partial \zeta} - \frac{G^* m^*}{R^{*2} V^*} \frac{\partial \varphi}{\partial \zeta} \cdot \frac{\partial \xi}{\partial \mathbf{w}} = 0 . \tag{20}$$

Introduce the precedent gauge relations $G^* \approx 1/R^*$, $m^* \approx R^*$. The dimensional analysis of Eq. (20) gives $V^* \approx 1/(R^*)^{1/2}$ and

$$R^* \approx t^{*2/3} . \tag{21}$$

These relations can be interpreted as gauge relations and related to the solution (16). Consider now the Schrödinger equation

$$-\frac{h^2}{2m} \nabla^2 \Psi + U\Psi = -i\,h \frac{\partial \Psi}{\partial t} . \tag{22}$$

Introduce $t = t^* \tau$, $\mathbf{r} = R^* \boldsymbol{\zeta}$, $h = h^* \eta$, $m = m^* \mu$, $U = U^* u$. The dimensional analysis of Eq. (22) gives

$$\frac{h^{*2}}{2m^* R^{*2}} \approx U^* \approx \frac{h^*}{t^*}, \qquad \text{i.e.,} \qquad \frac{t^{*2}}{R^{*3}} \approx U^* \approx \text{constant}, \tag{23}$$

i.e., $R^* \approx t^{*2/3}$. Now let us write the Maxwell equations, referring to an empty space

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} , \tag{24}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} , \tag{25}$$

and write $\mathbf{E} = E^* \boldsymbol{\varepsilon}$, $B = B^* \boldsymbol{\beta}$, $\mathbf{r} = R^* \boldsymbol{\zeta}$, $t = t^* t$, $c = c^* \omega$. We get

$$\frac{E^*}{R^*}\frac{\partial}{\partial\zeta}\times\varepsilon=-\frac{B^*}{t^*}\frac{\partial\beta}{\partial\tau} \tag{26}$$

$$\frac{E^*}{R^*}\frac{\partial}{\partial\zeta}\times\beta=\frac{E^*}{c^{*2}\,t^*}\frac{\partial\varepsilon}{\partial\tau}\,. \tag{27}$$

Combining with $c^* \approx 1/R^{*1/2}$, we refind $R^* \approx t^{2/3}$.

6. Conclusion

In this paper, we have derived some of the implications of letting the fundamental constants vary with time. This can only be done with the addition of some further gauge constraints. Following Milne's suggestion[1], the classical interpretation of the redshift in terms of the Doppler effect has to be replaced by another one taking into account the secular change of the Planck constant. The fundamental parameters R and c are related to each other by some gauge relation. The particle masses vary like R while the energy-matter and the gravitation energy are conserved.

This model predicts that the cosmological horizon $L(t)$ should be identical to $R(t)$, which would justify the overall homogeneity of universe. The curvature of space should be negative and the gauge relationship between R and t should be $R \approx t^{2/3}$.

The Planck's constant would vary like t, and the gravitational constant G like $1/R$, the Planck's length would vary like R, and the Planck's time would vary like t. The gravitational force would vary like $1/R$.

References

1. E. A. Milne, *Kinematic Relativity* (Oxford, 1948).
2. F. Hoyle and J. V. Narlikar, *Cosmological models in conformally invariant gravitational theory, Mon. Notices Roy. Astr. Soc.* **155** (1972) 305–325.
3. P. A. Dirac, *Nature* **139** (1973) 323.
4. P. A. Dirac, 1973 *Proc. Roy. Soc. London* **A333** (1973) 403.
5. V. Canuto and J. Lodenquai , *Dirac cosmology, Appl. J.* **211** (1977) 342–356.
6. T. C. Van Flandern, *Is the gravitational constant changing? Appl. J.* **248** 813–816.
7. A. Julg, *Dirac's large numbers hypothesis and continuous creation, Appl. J.* **271** (1983) 9–10.
8. V. Canuto and S. H. Hsieh, *The 3 K blackbody radiation, Dirac's large numbers hypothesis, and scale-covariant cosmology, Appl. J.*, **224** (1978) 302–307
9. R. Adler, M. Bazin and M. Schiffer, *Introduction to general relativity* (McGraw Hill, 1965).
10. J. M. Souriau, *Géométrie et relativité*, ed. Hermann (France, 1964).

We now take a significant leap forward in time, moving from 1988 to 2024, skipping over articles that detail the gradual development of the translation of the cosmological data contained in the Ummo texts.

There was the impact of the encounter with the blind mathematician Bernard Morin.



Bernard Morin, blind mathematician and geometer. 1931–2018

My scientific career was strongly influenced by successive encounters with four mathematicians and geometers. First, there was the mathematician, Academician, and geometer André Lichnérowicz. Our interaction centered on my initial foray into differential geometry, work in seven-dimensional space (time-position-velocity), the kinetic theory of gases and plasmas, and the use of tools such as dyadic algebra.

His encounter with Morin in the early 1980s marked a sudden shift toward topology.

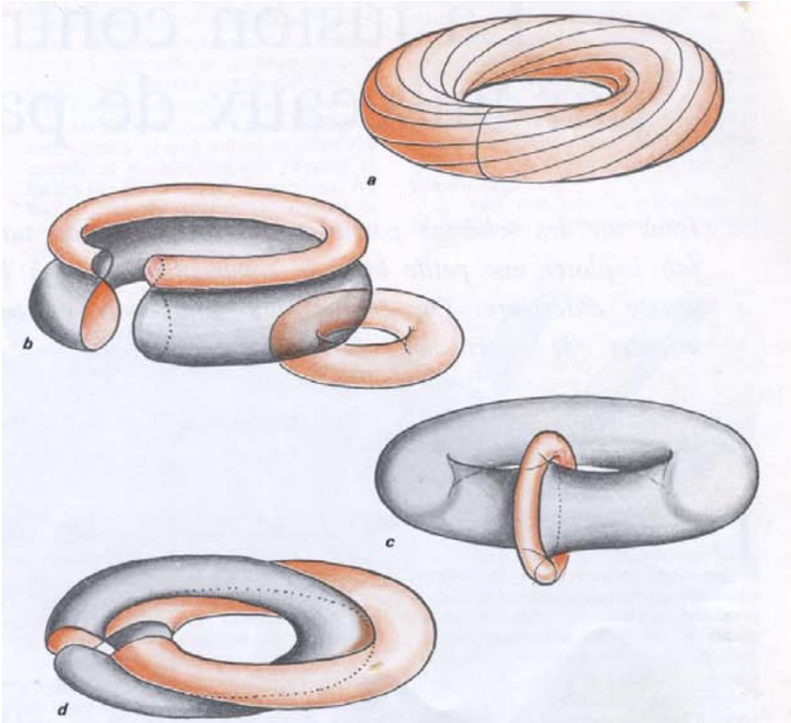
Morin, who lost his sight at the age of 6 due to glaucoma, specialized in geometry—and more specifically, in a geometry of the visible. More specifically, the geometry of immersions of 2D objects into 3D Euclidean space. These objects are essentially the sphere, the torus, and two unilateral objects that we will discuss later.

Imagine that these objects can be deformed while maintaining the continuity of their tangent plane. If these deformations prevent the objects from intersecting themselves, we speak of plunges. If they can "intersect themselves," we speak of immersions.

These transformations by continuous deformations are called regular homotopies.

We then consider the infinite number of ways in which these objects—which are closed, continuous surfaces—can be represented, immersed in 3D space.

For the torus, for example, we distinguish between immersions that cannot be transformed into one another by continuous deformation, according to a sequence of immersions.



12. LES FORMES LES PLUS REMARQUABLES d'immersions du tore sont représentées en *b*, *c* et *d*. Le plongement « tordu » de la figure *a* qui a le même aspect que le plongement standard montre que les figures *b*, *c* et *d* ne contiennent pas toute l'information qu'il faudrait avoir pour décrire les immersions puisque ces figures ne rendent pas compte des torsions internes.

The Four Types of Immersions of the Torus.

What is a torus?

It is the surface generated by the displacement (and deformation) of a circle until it returns to its initial position. In Figure a, at the top right, this is the "classical torus," where this "generating circle" rotates around an axis of symmetry.

But a figure shaped like an eight is still "a circle" or "an object with the topology of a circle"—that is, a continuous curve that loops back onto itself. By rotating this "eight" around an axis of symmetry, we obtain the drawing in figure b.

In figure d, we've made it even more complicated. Not only does this "eight" rotate around an axis of rotation, but it also rotates like a two-bladed propeller, making one full turn at a time.

Finally, in Figure c, this torus is generated by the motion of a circle that does not intersect itself, but whose perimeter changes and whose rotation reverses direction.

In short, mathematicians and geometers can prove that these four figures cannot be transformed into one another. They belong to distinct families of immersions. Within a family, the infinite number of transformations that allow one to move from one configuration to another—via regular homotopies—form a single group.

Geometers summarize the drawings above with the following statement:

There are four classes of regular homotopies of the torus.

In 1959, the young American mathematician Stephen Smale proved that the sphere has only a single class of regular homotopies[10].



Stephen Smale (born in 1930). Pictured here in 1959

His thesis advisor at the University of San Diego was the Hungarian-born mathematician Raoul Bott (naturalized in 1950).

[10] Smale, Stephen, "A Classification of Immersions of the Two-Sphere," Transactions of the American Mathematical Society, 90, no. 2 (1959), pp. 281–290.



Raoul Bott, mathematician. 1923–2005

When Smale presented his theorem to Bott, Bott exclaimed:

- What you're saying must be wrong! If it were true, it would mean that a sphere could be turned inside out through a continuous sequence of immersions. But that's impossible!

Smale protests, arguing that his proof is flawless. Bott:

- Well, then, how do you go about it?
- I haven't the faintest idea….

It would take seven years before a mathematician and geometer, Anthony Phillips, published the first version of this double-sided inversion of the sphere[11]. The article, published in the journal Scientific American, was understandable only to a tiny handful of specialists worldwide[12].

The blind mathematician Bernard Morin then devised a different version based on a "central four-eared model" he had imagined. But without accompanying drawings, this work was never published.

At a conference in Seattle in 1967, Morin presented models made of modeling clay to the American mathematician Charles Pugh, who managed to construct ten models out of "chicken wire," which were exhibited in 1970 at the University of Berkeley.

Mathematician Nelson Max borrowed the models and "digitized" them by hand, manually recording the (x, y, z) coordinates of hundreds of points—a process that would enable him, seven years later, to create a 23-minute film.

---

[11] Anthony Phillips, "Turning a Surface Inside Out," Scientific American, vol. 214, no. 5, May 1966, pp. 112–120.

[12] It is based on a central model, namely projective space P2.

A few months later, these ten models were stolen overnight. They were never recovered.

Having heard Morin speak at a conference in Aix-en-Provence in early summer 1975, I offered to create the series of drawings needed to illustrate his version of the sphere's inversion. What followed was a rather surprising exchange in which Morin had only his words to explain to me what he expected of me. For my part, I could only present my experiments to him by bringing him objects made of cardboard and wire (for the self-intersection curve).

After a few months, this work came to fruition. I created a "film" in the form of a "flip book"—that is, a sequence of about fifty images presented as a booklet that can be flipped through. Viewers could then see the object come to life and the transformations unfold one after another.

The CNRS had established SERDAV (Service d'Étude et de Réalisation de Documents Audio-Visuels), headed by Jean-Michel Arnold. I traveled to Paris and proposed to him that we produce an animated film.

- *But who would draw the illustrations?*

- *Me….*

Arnold's eyes widened (this is France, after all) and he didn't pursue the idea.

Years passed. The French journal *La Recherche* contacted Bernard Morin with the aim of writing an article on this topic. The science journalist assigned to the piece, Patricia Pinaud, had very clear preconceived notions:

- *A scientific article is either a few pages of text with a few drawings, or a series of drawings with a few lines of text.*

Relations with Morin become strained. To clear things up, I call the journalist, who replies:

- *Morin is an impossible man. We've sent the drawings to an American, Nelson Max, who will produce the article.*

- *But these drawings represent original, unpublished work!*

- *That's your problem, not ours!*

When contacted, Lichnérowicz reacted immediately:

- *Write a memo that I'll present to the Academy of Sciences, so your priority will be preserved.*

No sooner said than done[13].

[13] B. Morin, J-P Petit. The Turning of the Sphere. Proceedings of the Paris Academy of Sciences, vol. 287, Series A (October 30, 1978).

The magazine *Pour la Science*, the French version of the American magazine *Scientific American*, had just been launched. I went up to Paris and told Philippe Boulanger, its editor-in-chief:

- *"Would you be willing to help two French scientists who are about to be robbed of their work by an American mathematician, with the complicity of the official French journal, *La Recherche*?"*

- *Yes, but I'd need the article—including the text and illustrations—within a week so it can be published in our next issue.*

Morin wrote the article. I went up to Paris and, sleeping at the magazine's offices, produced the numerous drawings on site. The article came out in January 1979[14], and we beat Nelson Max to the punch. An article of much lower quality would appear in *La Recherche*.

Our article earned the magazine *Pour la Science* the d'Alembert Prize, awarded by the Mathematical Society of France for works that popularize mathematics. This was despite the fact that my picture books *Le Géométricon*, *Le Trou Noir*, and *Le Topologicon* already existed.

I have decided to reproduce this article here to highlight the lack of fair play in the world of research, particularly in the U.S. It's the:

- *Not invented here[15] mentality…*

At the request of its French affiliate, the magazine *Pour la Science*, *Scientific American* refused to republish the article by the two French authors. This article is hardly ever cited today, even though it is the only readable description of the inversion of the sphere.

This encounter with the blind mathematician Bernard Morin inspired me to create various works. I devised a non-trivial inversion of the torus[16] based on the two-sheet covering of the Klein bottle.

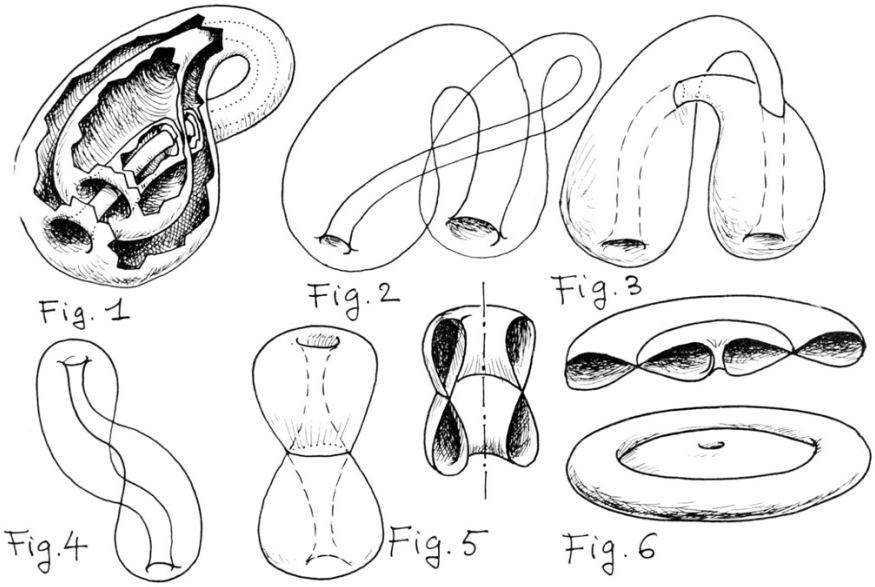
---

http://www.jp-petit.org/papers/CRAS/geometry_1978b.pdf

[14] B. Morin and J.-P. Petit. The Reversal of the Sphere. *Pour la science* No. 15, pp. 34–49.

[15] This was not invented here.

[16] JP Petit. The Non-Trivial Inversion of the Torus. CRAS Paris, vol. 287, Series A (November 20, 1978), pp. 927–930
http://www.jp-petit.org/papers/CRAS/geometry_1978d.pdf



Images of my torus inversion[17].

I created the first representation of Boy's surface using its meridians, in the form of a family of ellipses[18], which allowed me to create the first parametric representation of this surface, leading to the first computer-generated images.

Below is the first "digital" image of this surface showing its meridian curves, converging at its single pole, as well as its successive cross-sections, demonstrating that this is indeed the object.

[17] J.P. Petit: The non-trivial inversion of the torus. CRAS Vol. 287 (November 20, 1978)
[18] J.P. Petit, J. Souriau: An analytical representation of the Boy surface. CRAS Vol. 293 (October 5, 1981)

C. R. Acad. Sc. Paris, t. 293 (5 octobre 1981) Série I – 271

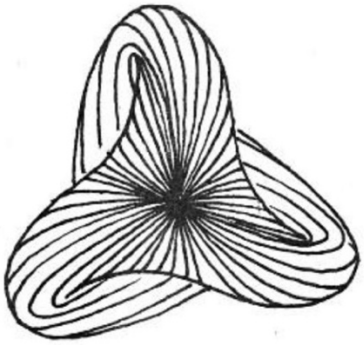


Fig. 4. – Lignes méridiennes (Eμ) de la surface de Boy (tracé par « Apple-II »).

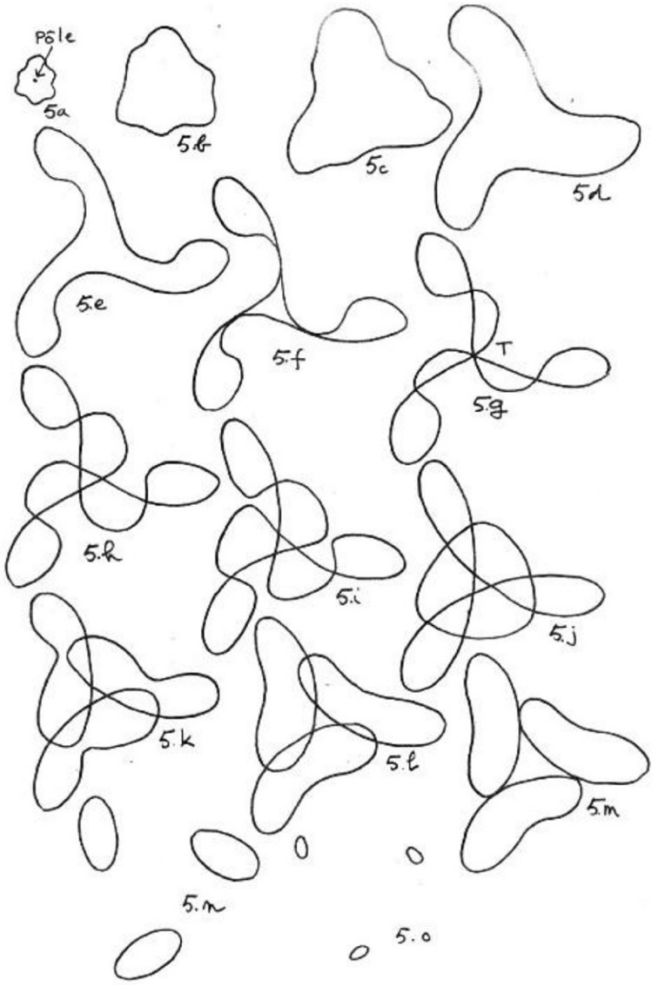


Fig. 5

Cross-sections at constant z of the Boy surface

What is interesting is that these cross-sections turned out to be identical to the original drawings by

Werner Boy: *Über die curvatura integra und die Topologie geschlossener Flächen*. Dissertation, Georg-August-University, Göttingen, 1902.

Le retournement de la sphère

Grâce à quatorze modifications géométriques simples, on peut retourner une sphère sans la déchirer ni la plier, c'est-à-dire amener à l'extérieur sa face interne.

par Bernard Morin et Jean-Pierre Petit

En 1957, Stephen Smale, un jeune mathématicien devenu célèbre depuis, présentait à son directeur de recherches, Raoul Bott, un résultat de caractère très général sur les déformations de sphères. Au premier examen, Raoul Bott refusa d'admettre le théorème. Plus sensible que son étudiant aux conséquences géométriques des énoncés abstraits, il ne fut pas long à remarquer que si Stephen Smale avait raison, il serait possible, en suivant des règles que nous expliquons plus loin, de retourner la sphère ordinaire de façon à lui faire présenter sa face interne vers l'extérieur, et cela, sans quitter l'espace tridimensionnel de la géométrie classique. A coup sûr, il y avait là de quoi surprendre : le théorème de Whitney-Graustein, d'ailleurs redémontré par Stephen Smale, affirme qu'on ne peut, en suivant les mêmes règles, retourner un cercle sans sortir du plan. Rappelons qu'en effet le cercle, appelé sphère de dimension 1 par les mathématiciens, joue pour le plan le même rôle que la sphère pour l'espace ordinaire. Mais surtout, si Stephen Smale avait raison, on devait être capable de concrétiser le retournement à l'aide de modèles puisque l'affaire se jouait dans notre espace, où l'imagination peut s'exercer. Or, si la démonstration du théorème fournissait un moyen théorique de réaliser la transformation, il était en pratique impossible d'imaginer cette solution. Le travail de Stephen Smale, très solide et très clair, était facile à contrôler; aussi, la communauté mathématique reconnut rapidement que son théorème était vrai. Restait pourtant l'irritant défi lancé par Raoul Bott : « Montre-moi comment cela est possible. » En vrai spécialiste de la géométrie des dimensions supérieures à trois, Stephen Smale ne se soucia guère de satisfaire la curiosité de son patron.

Après avoir précisé les termes dans lesquels se pose le problème, nous nous proposons de raconter ici les efforts d'imagination spatiale de ceux, mathématiciens et physiciens, qui depuis une vingtaine d'années ont tenté de répondre à la question posée par Raoul Bott. Chemin faisant, nous exposerons le scénario qui, selon nous, réalise le programme de la façon la plus satisfaisante. Tous les détails de notre solution sont consignés sur les nombreuses figures étayant un texte qui ne pourrait être rendu intelligible sans le secours du dessinateur. Notre méthode présente, entre autres avantages, celui de se généraliser à l'étude des déformations d'une surface quelconque, comme nous le verrons en particulier à propos du tore.

Plongements et déformations de plongements.

Pour nous initier à l'étude des déformations de la sphère, nous devons d'abord préciser les règles du jeu auquel nous sommes conviés, en nous familiarisant avec les transformations permises et en prenant conscience de ce qui est interdit. En premier lieu, il convient d'aborder la notion de plongement. Pour cela, représentons la sphère comme un ballon dont la baudruche se laisse étirer et contracter à volonté. Lorsqu'on envisagera de déformer ce ballon, il faudra s'interdire d'en déchirer la membrane. Ainsi, pour réaliser le retournement, il n'est pas question de pratiquer dans la surface une ouverture, si petite soit-elle, qui permettrait de retourner la sphère à la manière d'une peau de lapin, ou mieux, d'une vessie de ballon, quitte ensuite à obturer l'orifice. En effet, toute déchirure constitue une rupture de continuité. Or les ruptures sont interdites, car, pour le topologue, la sphère trouée en un point perd sa qualité de sphère pour devenir un disque. Il faut également éviter de plier la membrane, ce qui, le long du pli, aurait pour résultat de casser le plan tangent qui doit être bien défini en chacun des points du ballon et varier continûment en fonction de la position du point. Pour qu'une configuration de la membrane qui matérialise la sphère mérite le nom de plongement, il faut enfin interdire que deux points distincts de cette membrane soient en coïncidence. En résumé, un *plongement* de la sphère est l'un des états dans lesquels peut se trouver une membrane élastique qui n'est ni déchirée, ni pliée et qui n'entre jamais en contact avec elle-même : la surface ainsi déterminée possède en chacun de ses points un plan tangent dont la position doit, en outre, dépendre continûment du point. Le plus simple de tous est le plongement standard; c'est celui que Monsieur de La Palice aurait pu inventer : il consiste à laisser la sphère (lieu des points dont la distance à l'origine de l'espace est égale à un) dans l'état où sa définition nous la livre.

Grâce à la substance idéale dont il est fait, le ballon pourra, à notre fantaisie, se déformer de multiples façons : il sera possible par exemple de lui faire prendre la forme d'une saucisse ou d'une galette, de lui faire pousser de longues cornes vers l'extérieur mais aussi vers l'intérieur, de ramifier ces cornes à la façon des bois d'un cerf, d'enlacer entre elles les diverses ramures ainsi obtenues pour en faire des torsades ou même des tresses, de gonfler les bourgeons terminaux de ce fouillis de lianes en d'énormes globes et ainsi de suite. Une *déformation de plongement* est une manipulation imprimée à la baudruche pour la faire passer d'une configuration donnée à une autre : elle a donc lieu durant un certain laps de temps à chaque instant duquel correspond une configuration de plongement représentant l'une des phases du processus évolutif qui conduit du plongement de départ au plongement d'arrivée. Au cours de la déformation, chaque point de la baudruche décrit une trajectoire continue et le plan tangent à la surface en ce point se déplace lui aussi continûment en fonction du point et du temps.

Il ne faudrait pas croire que deux plongements quelconques de la sphère puissent toujours être reliés l'un à l'autre par une déformation de plongement. Imaginons, en effet, que la baudruche de notre ballon soit coloriée en gris sur l'une de ses faces et en orangé sur l'autre. Si nous ne précisons rien au fabricant qui nous fournit les échantillons sur lesquels nous expérimentons, celui-ci nous livrera des ballons qui nous apparaîtront de l'extérieur tantôt gris et tantôt orangés. Les ballons orangés, au même titre que les gris, représentent des plongements de la sphère. Convenons une fois pour toutes que le plongement standard est vu de l'extérieur comme un objet gris. Alors l'exemple le plus naturel d'un plongement d'extérieur rouge est le *plongement antipodal*. Pour le construire, assimilons la sphère stan-

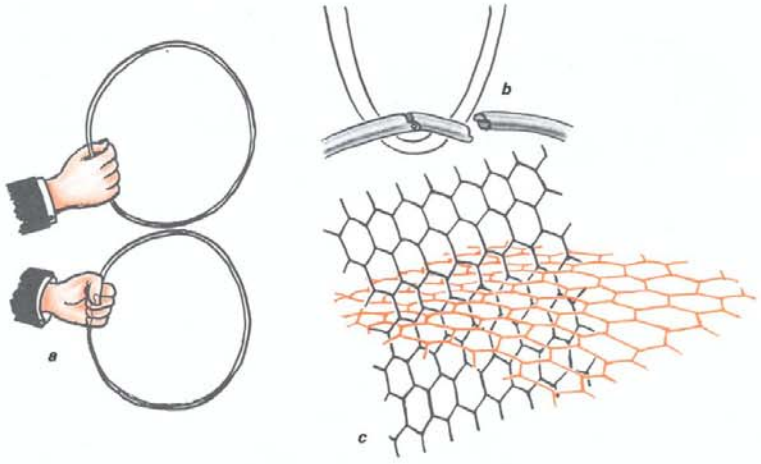
dard à une mappemonde, puis fabriquons à sa place une nouvelle mappemonde sur laquelle chaque point de la première vient s'inscrire sur le point diamétralement opposé. Le plongement antipodal s'obtient en faisant subir à la sphère standard une rotation de 180° autour de l'axe des pôles, puis une symétrie par rapport au plan équatorial. La symétrie échange les hémisphères entre eux sans déplacer les points de l'équateur. En observant du dessus le résultat de ces opérations, on aperçoit donc la face, naguère intérieure, de l'hémisphère sud. Or, aucune des manipulations décrites plus haut, si compliquées soient-elles, ne pourra faire que l'extérieur d'un ballon, gris au départ, devienne orangé à l'arrivée. En effet, l'air contenu à l'intérieur du ballon y reste emprisonné au cours de toute déformation de plongements. Un voyageur enfermé dans cette capsule élastique se verra constamment environné d'orangé, quelles que soient les manipulations subies par l'enveloppe d'où, quoi qu'il arrive, il ne pourra s'échapper sinon en acquérant des dons de passe-muraille.

Au terme de cette étude sur les déformations de plongements, nous sommes en état de comprendre et d'accepter le théorème suivant : tout plongement de la sphère peut être relié par une déformation de plongement, soit au plongement standard, soit au plongement antipodal selon qu'il présente à l'extérieur sa face grise ou sa face orangée, mais il n'existe pas de déformation de plongement reliant le plongement standard au plongement antipodal. Le royaume des plongements se compose donc de deux îlots. Le premier, l'îlot gris, contient le plongement standard, tandis que l'autre, l'orangé, contient le plongement antipodal. Sur chaque îlot, on peut se rendre à volonté d'un point à un autre en empruntant des chemins qui sont les déformations de plongements, mais il n'existe pas de pont reliant le pays gris au pays orangé.

Naturellement, les notions de plongement et de déformation de plongements s'étendent à d'autres surfaces que la sphère. A la fin de cet article, nous évoquerons brièvement la question des plongements du tore. La surface de Boy et la bouteille de Klein que nous rencontrerons plus loin ne peuvent être plongées dans l'espace ordinaire car ce sont des surfaces unilatères sans bord.

Immersions et homotopies régulières.

Retourner la sphère consiste à construire une déformation reliant la sphère standard à son plongement antipodal, c'est-à-dire amenant une sphère d'extérieur gris à présenter sa face orangée à l'extérieur. Or, comme nous venons de le voir, il est impossible qu'une déformation réalisant ce programme soit une déformation de plongements. Force est donc, pour atteindre le but fixé, de recourir à une catégorie plus large

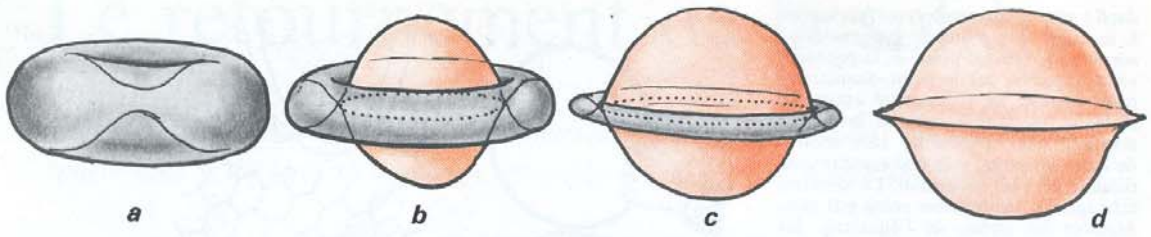


1. UNE SURFACE IMMERGÉE est une surface dont différentes parties se traversent. Les anneaux magiques des prestidigitateurs *(a)* comportent généralement un anneau spécial muni d'un mousqueton *(b)* ce qui permet à deux anneaux de s'interpénétrer. Deux surfaces qui s'interpénètrent peuvent se concevoir comme deux grillages à mailles extrêmement petites *(c)*, chaque segment de grillage étant muni d'un système de mousquetons semblable à celui des anneaux magiques.

de déformations, les déformations d'immersions, au cours desquelles il est permis aux points de la surface de se rencontrer. Pour expliquer ce dont il s'agit, l'image de la baudruche n'est plus un support suffisant pour l'intuition. Sur la figure 1*a*, on a représenté les anneaux truqués dont se servent les illusionnistes. Un système de mousquetons permet à l'opérateur de les faire se traverser et donc de les enlacer ou de les séparer à volonté. Imaginons maintenant un grillage très fin dont les mailles soient de tels anneaux et assimilons une surface à un grillage de ce type. Grâce à ce dispositif, on peut imaginer ce qu'est une surface qui se recoupe elle-même *(voir figure 1c)* et comprendre comment les nappes de la surface peuvent se traverser, puis glisser librement les unes par rapport aux autres. Comme dans le cas des plongements, on s'interdit évidemment de déchirer ou de plier la surface, ce qui endommagerait les mailles du grillage, mais on suppose que celui-ci a les mêmes propriétés d'élasticité que la baudruche d'un plongement. Toute présentation d'une surface selon une configuration dans laquelle cette surface est autorisée à se toucher ainsi qu'à se traverser s'appelle une *immersion* de la surface, pourvu qu'en chacun de ses points elle admette un plan tangent variant continûment en fonction du point. Une déformation de la surface au cours de laquelle on s'autorise des contacts et des traversées s'appelle une *déformation d'immersions* ou *homotopie régulière*, à condition, ici encore, que soient respectées les contraintes de continuité qui ont été imposées au plan tangent lors d'une déformation de plongement. A chaque instant de l'intervalle de temps qui paramètre la déformation correspond une immersion de la surface. Ainsi, une homotopie régulière nous conduit-elle graduellement de l'immersion initiale vers l'immersion qu'on désire atteindre sans sortir à aucun moment du cadre des immersions. Remarquons que les déformations de plongement étudiées précédemment sont des homotopies régulières d'un type particulier.

Si le théorème de Stephen Smale avait tant surpris Raoul Bott, c'était que, lorsqu'on l'applique à l'espace de dimension trois et à la sphère ordinaire, ce théorème affirme qu'on peut passer du plongement standard à n'importe quelle immersion de la sphère par une déformation d'immersions. Le plongement antipodal qui, comme tout plongement, est une immersion, peut donc, lui aussi, être relié au plongement standard par un chemin d'immersions. Les îlots gris et orangés sur lesquels se répartissent les plongements nous apparaissent maintenant comme deux petits pays au sein de la contrée des immersions qui est un vaste continent d'un seul tenant. Ce que demandait Raðul Bott était qu'on lui trace à travers la forêt d'immersions séparant ces deux pays un chemin aussi droit que possible permettant de se rendre commodément de l'un à l'autre.

Le recours aux immersions ne doit pas nous laisser croire que le problème est maintenant facile à résoudre. Voici un exemple significatif des erreurs qu'on peut commettre en abusant de la notion d'immersion : partant de la sphère standard, poussons le pôle nord et le pôle sud l'un vers l'autre, le long de l'axe qui les joint *(voir figure 2)*. Au moment où les pôles entrent en contact, on sort du domaine des plongements, mais dans le contexte des immersions,



2. UNE MAUVAISE SOLUTION au retournement de la sphère consiste à pousser *(a)* sur deux points diamétralement opposés de la sphère pour faire en sorte qu'il se forme *(b)* une « lentille » centrale sertie d'un tube. Cette opération est permise mais il est interdit, selon les règles de l'homotopie régulière, de poursuivre l'opération qui conduirait à une dégénérescence du tube en arête de rebroussement *(d)*.

on peut poursuivre la déformation amorcée et croiser les points en mouvement. On voit alors apparaître au centre de la figure une lentille orangée raccordée le long d'une ligne de points doubles à un tube équatorial resté gris : le pôle sud est maintenant sur la face supérieure de la lentille et le pôle nord sur sa face inférieure. Plus la déformation s'accentue, plus la lentille s'agrandit au détriment du tube qui, si nous laissons évoluer les choses, finira par disparaître totalement. La sphère est alors retournée, mais à l'instant où le tube s'évanouit, apparaît sur l'équateur une arête de rebroussement. De telles configurations ne sont pas des immersions et sont à proscrire car, comme dans le cas du pli, l'apparition d'une arête de rebroussement bouleverse la structure et introduit une discontinuité dans l'évolution des plans tangents.

Ce que ni Stephen Smale ni Raoul Bott ne parvenaient à imaginer était qu'on puisse contourner la difficulté précédente sans sortir du cadre des immersions. Les mathématiciens qui se sont intéressés au problème ont alors pris conscience du fait que, dans le passé, leur imagination n'avait guère eu l'occasion de s'exercer sur les déformations d'immersions. « Si Bourbaki s'était douté que les immersions pouvaient être aussi compliquées, nous confiait l'un de ses plus proches collaborateurs en regardant nos modèles du retournement, il n'aurait jamais appelé cela des immersions; nous considérions qu'une immersion était, à peu de choses près, un plongement ». En général, lorsqu'une immersion n'est pas un plongement, elle ne détermine pas seulement un intérieur et un extérieur car elle se présente à nous comme un objet comportant un système de cloisons souvent compliqué, délimitant de multiples régions enchevêtrées les unes dans les autres. L'immersion de la sphère qu'on vient d'examiner découpe l'espace en trois régions : l'extérieur, l'intérieur du tube et l'intérieur de la lentille. Pour appréhender un objet cloisonné, la difficulté n'est pas de nature mathématique mais psychologique. En effet, lorsqu'on se représente mentalement une structure spatiale, il est impossible de ne pas se situer en un point particulier de l'espace imaginé. Or, pour bien maîtriser la disposition d'un cloisonnement, il faut pouvoir se placer simultanément, par la pensée, dans chacune des régions que découpe l'objet. Cette faculté d'ubiquité imaginative que doit acquérir le jeune architecte, lui permettra de prévoir toutes les incidences du déplacement d'un mur ou de l'élévation d'un plancher. En plus de la difficulté à imaginer, il y avait la difficulté à transmettre. Comment, en effet, décrire de tels objets sinon en construisant des maquettes si possible démontables, activité à laquelle les mathématiciens ont peu coutume de s'adonner? Faute d'avoir eu recours à ce moyen, un topologue a souvent cru comprendre la solution d'un collègue, alors qu'en l'écoutant, il était en train d'en inventer une nouvelle.

Le recours à la surface de Boy.

En 1961, Arnold Shapiro, géomètre fort ingénieux et très original, prématurément décédé, imagina un scénario précis détaillant le retournement de la sphère. Bien peu nombreux sont ceux qui connaissent aujourd'hui la solution qu'il proposait, car, ne disposant pas d'un formalisme adéquat, l'auteur considéra qu'il ne pouvait mettre son travail sous forme publiable. Pourtant, en une soirée, il a trouvé le moyen de nous communiquer de vive voix, comme sans doute à plusieurs autres, l'idée qu'il avait conçue. En 1966, Anthony Phillips publia une suite de dessins constituant ce qu'il croyait être la transformation d'Arnold Shapiro. En réalité, la déformation d'Anthony Phillips, inspirée par René Thom et voisine d'une solution de Nicolaas Kuiper dont il n'avait pas connaissance, n'a en commun avec celle d'Arnold Shapiro que d'aboutir au revêtement de la surface imaginée en 1901 par Werner Boy (un élève de Hilbert). L'idée mise en œuvre par ces auteurs est fort instructive et son examen va nous familiariser avec les transformations permises dans le contexte des immersions.

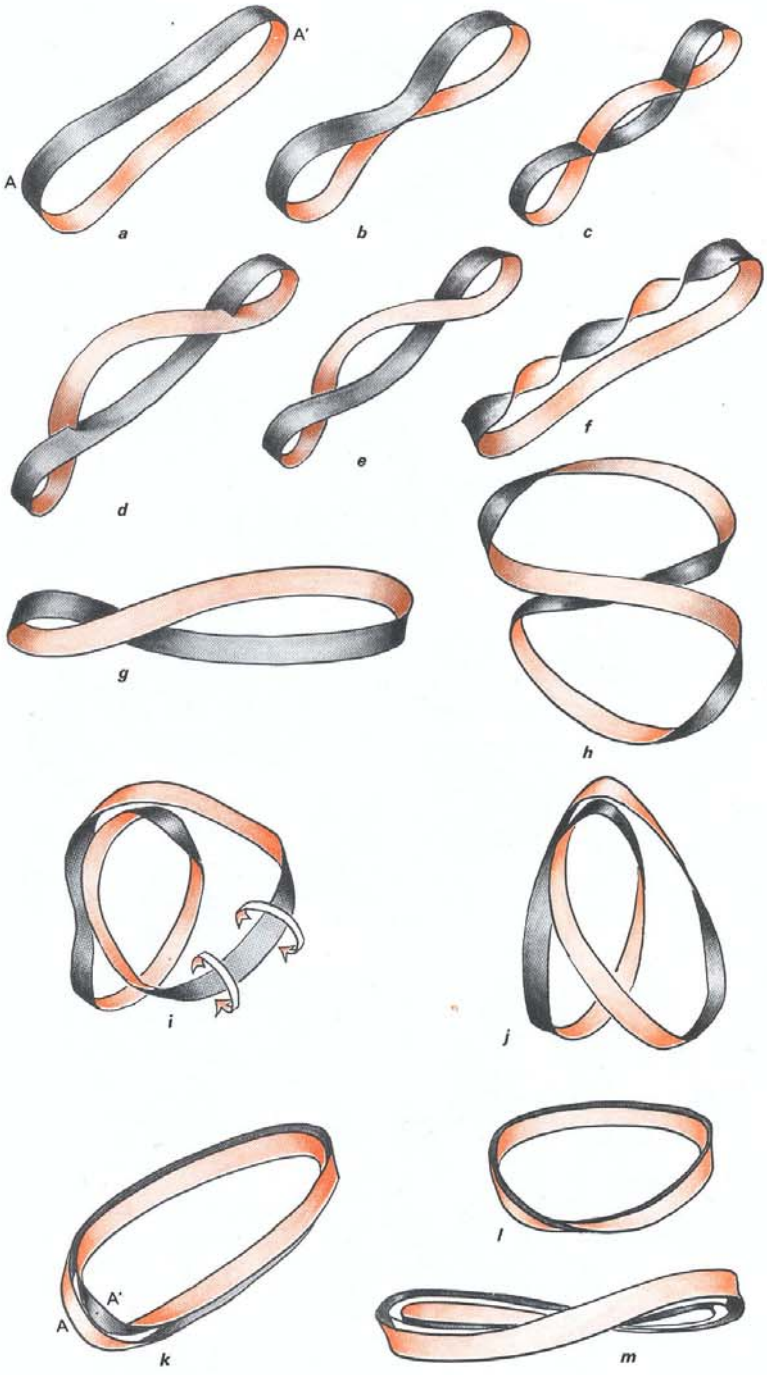
La *surface de Boy (voir figure 4)* est l'immersion d'un objet non orientable qu'on ne peut plonger dans l'espace ordinaire. Arnold Shapiro et Anthony Phillips ont, chacun de leur côté, construit une homotopie régulière amenant la sphère standard à envelopper (les mathématiciens disent *revêtir*) deux fois la surface de Boy. Lorsque la sphère parvient à l'état désiré, chacun de ses points est amené en regard du point antipodal, de manière que chaque élément de la surface de Boy se trouve pris en sandwich entre deux portions diamétralement opposées de son manteau sphérique. Pour réaliser le retournement de la sphère, il suffit d'obliger les deux feuillets du revêtement à se traverser l'un l'autre tout le long de l'immersion de Boy, pour ensuite parcourir en sens inverse l'homotopie régulière qui avait amené la sphère en position de revêtement. On aboutit en fin de compte au plongement antipodal puisque tout point de la sphère a croisé le point diamétralement opposé lors de son passage à travers l'immersion de Boy.

Pour mieux comprendre ce qu'est un revêtement à deux feuillets, commençons par construire celui d'une surface plus simple, le ruban de Möbius. Le ruban de Möbius est, tout comme la surface de Boy, une surface *unilatère*, c'est-à-dire à un seul côté. On peut matérialiser ce ruban en prenant une bande de papier environ dix fois plus longue que large dont on recolle les extrémités après lui avoir fait subir une torsion d'un demi-tour *(voir figure 3)*. A chaque passage, une fourmi cheminant le long d'une telle bande se trouvera alternativement au-dessus puis au-dessous du ruban. Les termes « au-dessus » et « au-dessous » n'ont de signification que pour un observateur ne pouvant voir qu'une partie de la bande et ignorant son mode de raccordement; sur l'objet considéré dans son ensemble, ils perdent toute signification. Notons que le bord de la surface qui vient d'être construite est une courbe fermée, d'un seul tenant, c'est-à-dire qui se laisse déformer en un cercle.

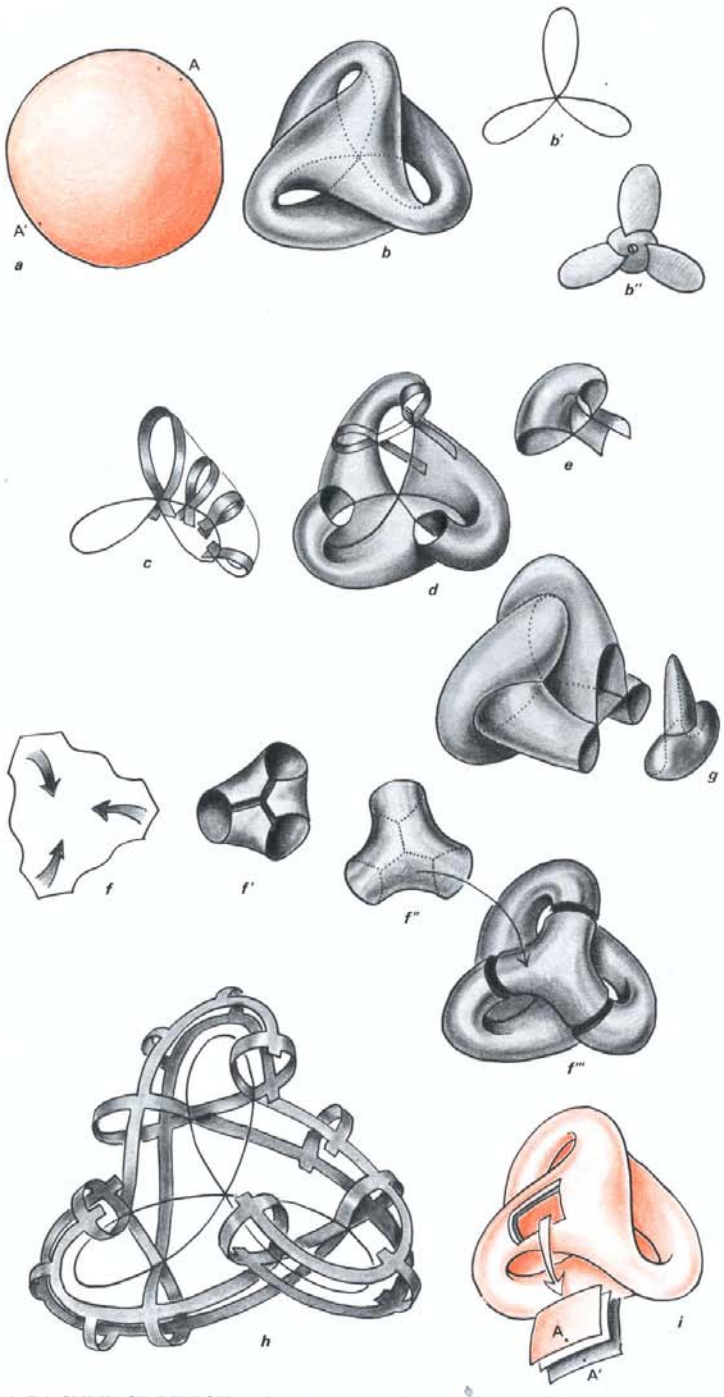
Considérons maintenant une autre bande refermée sur elle-même à la manière d'une ceinture (sans avoir été préalablement tordue), c'est-à-dire une bande cylindrique ou mieux la région équatoriale de la sphère standard. Nous nous proposons de montrer comment la bande se laisse déformer en un revêtement à deux feuillets du ruban de Möbius qui amène en regard les points diamétralement opposés de la région équatoriale.

Grâce à une homotopie régulière, on commence par croiser puis décroiser la bande. Le résultat de cette homotopie est une bande tordue deux fois que nous conseillons au lecteur de concrétiser au moyen d'une bande de papier, grise d'un côté et orangée de l'autre, deux fois plus longue que celle employée à la confection du ruban de Möbius. En effet, la manipulation qui conduit de la bande tordue deux fois au revêtement du ruban de Möbius, est une déformation de plongement qu'on pourra par conséquent faire subir à une représentation matérielle de cette bande en suivant les indications de la figure 3. Lorsque l'opération est menée à son terme, la bande est enroulée sur elle-même de telle sorte qu'un observateur pris entre les deux feuillets de l'enroulement les voit tous deux orangés. Il est possible d'insérer entre ces deux feuillets un ruban de Möbius *(représenté en blanc sur la figure 3)* à condition toutefois de n'exécuter le recollement qui ferme le ruban qu'après avoir mis en place ce dernier. Ce faisant, on a matérialisé le revêtement à deux feuillets du ruban de Möbius par une bande bilatère. Lorsque ce travail est exécuté, un observateur extérieur ne voit plus qu'une bande de Möbius grise. Pour imiter l'opération envisagée par Anthony Phillips, il faut maintenant sortir du cadre des déformations concrètement réalisables et, grâce à une déformation d'immersions, passer du revêtement gris au revêtement orangé en amenant les deux feuillets à se croiser tout le long du ruban de Möbius, ce qui a pour effet de les permuter. Prises à rebours et appliquées au revêtement orangé, les manipulations qui avaient conduit au revêtement gris ramènent notre bande en position cylindrique mais, contrairement à la situation de départ, c'est maintenant la face orangée qui se trouve à l'extérieur.

Puisque l'immersion de Boy est unilatère, il est possible de délimiter une région de sa surface en forme de ruban de Möbius. Ce qui reste de la surface de Boy lorsqu'on a découpé le ruban peut se déformer par homotopie régulière en un disque plan ou encore en une calotte sphérique. La surface de Boy s'obtient donc en cousant ensemble un ruban de Möbius et un disque le long de son bord (qui dans les deux cas est un cercle). Naturellement, pour que le raccordement puisse s'effectuer, il faut permettre aux surfaces en jeu de se traverser. A notre bande cylindrique (assimilée à la région équatoriale de la sphère standard) ressoudons maintenant les deux calottes sphériques délibérément oubliées dans ce qui précède et dessinons sur la surface de Boy un ruban de Möbius. Il serait possible de construire une homotopie régulière déformant la sphère en un revêtement à deux feuillets de la surface de Boy, dans laquelle la région équatoriale se transformerait en revêtement du ruban selon le scénario qui vient d'être décrit. Le terme de cette homotopie amènerait les deux calottes sphériques en coïncidence le long de la portion de sur-



3. UN REVÊTEMENT A DEUX FEUILLETS d'une surface, ici un ruban de Möbius, consiste à envelopper cette dernière par les deux feuillets d'une autre surface, ici une ceinture tordue deux fois. Pour amener la ceinture *(a)* en position de revêtement, on la modifie en l'autorisant à se traverser *(b, c)* puis on la décroise symétriquement *(d)* pour obtenir une ceinture à double torsade *(e, f)*. On répartit ensuite la torsion tout le long de la bande *(g)* pour transformer ensuite la surface comme indiqué en *h, i, j*. On obtient ainsi deux éléments de ruban de Möbius. Pour finir, la bande se trouve enroulée deux fois *(k)* de sorte que les faces en regard sont d'une même couleur. On peut alors insérer une bande et en coller les extrémités pour obtenir un ruban de Möbius *(en blanc)* qui est revêtu par les deux feuillets de la ceinture *(m)*. Les points tels que *A* et *A'* sont amenés en coïncidence.



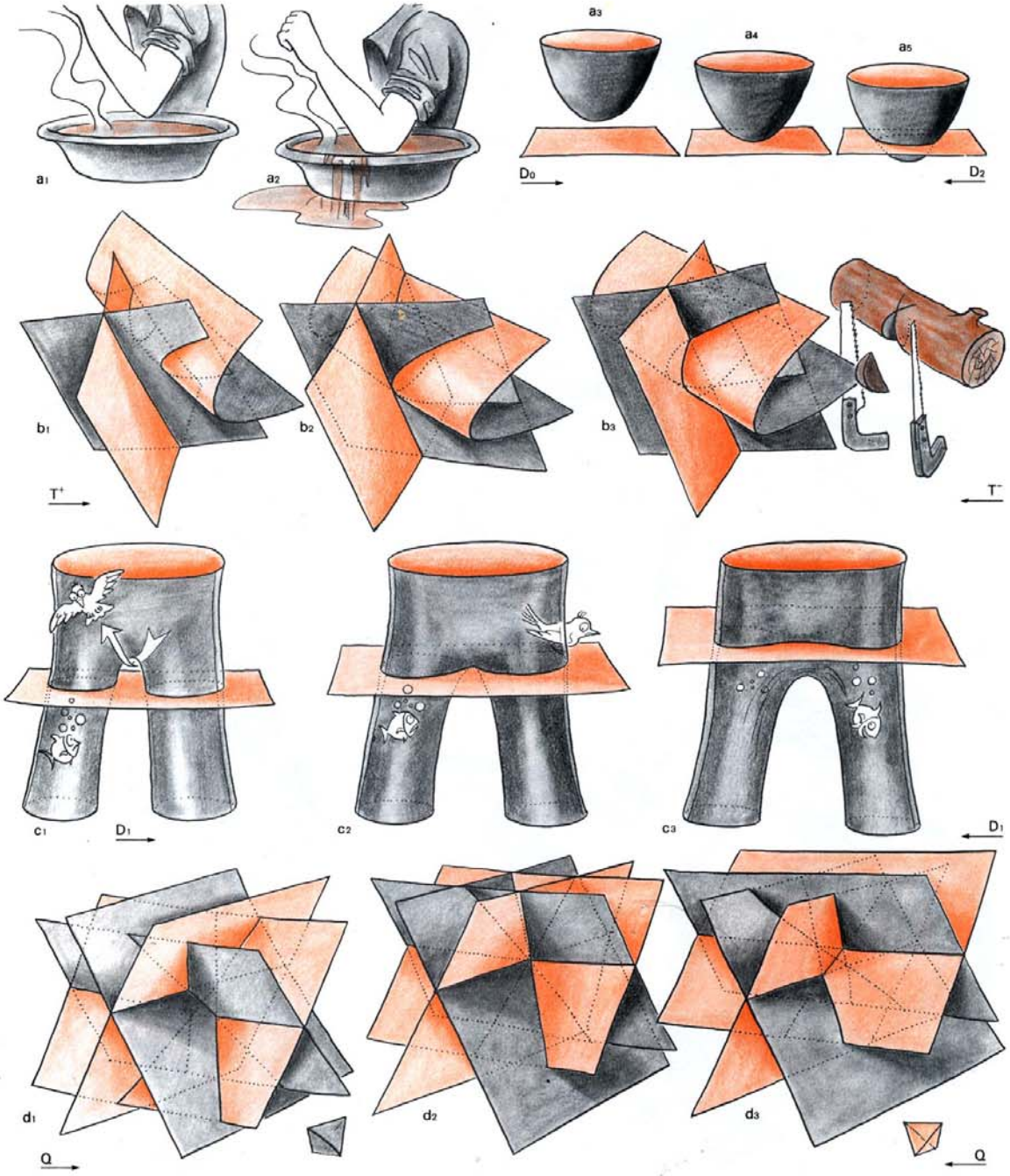
4. LA SURFACE DE BOY est le premier modèle central qui ait été utilisé pour retourner la sphère. La beauté intrinsèque de cette surface mérite une attention particulière. La courbe de self-intersection de la surface est une courbe gauche ayant la forme d'une hélice tripale. La surface elle-même est formée de trois tubes. Elle est ici représentée sous divers angles, parfois éclatée et parfois partiellement recouverte de bandelettes pour permettre au lecteur de mieux comprendre sa géométrie. La sphère *(a)* peut être immergée de façon à revêtir la surface de Boy par deux feuillets en regard qu'on peut ensuite échanger, en vue de retourner la sphère. Dans cette immersion *(i)*, chaque point de la sphère A est mis en coïncidence avec le point antipodal A'.

face de Boy extérieure au ruban. Cette manière de procéder serait moins élégante que celle d'Anthony Phillips et encore moins élégante que celle de Shapiro, mais la déformation de la région équatoriale qui vient d'être présentée méritait d'être étudiée pour elle-même en raison de sa grande simplicité.

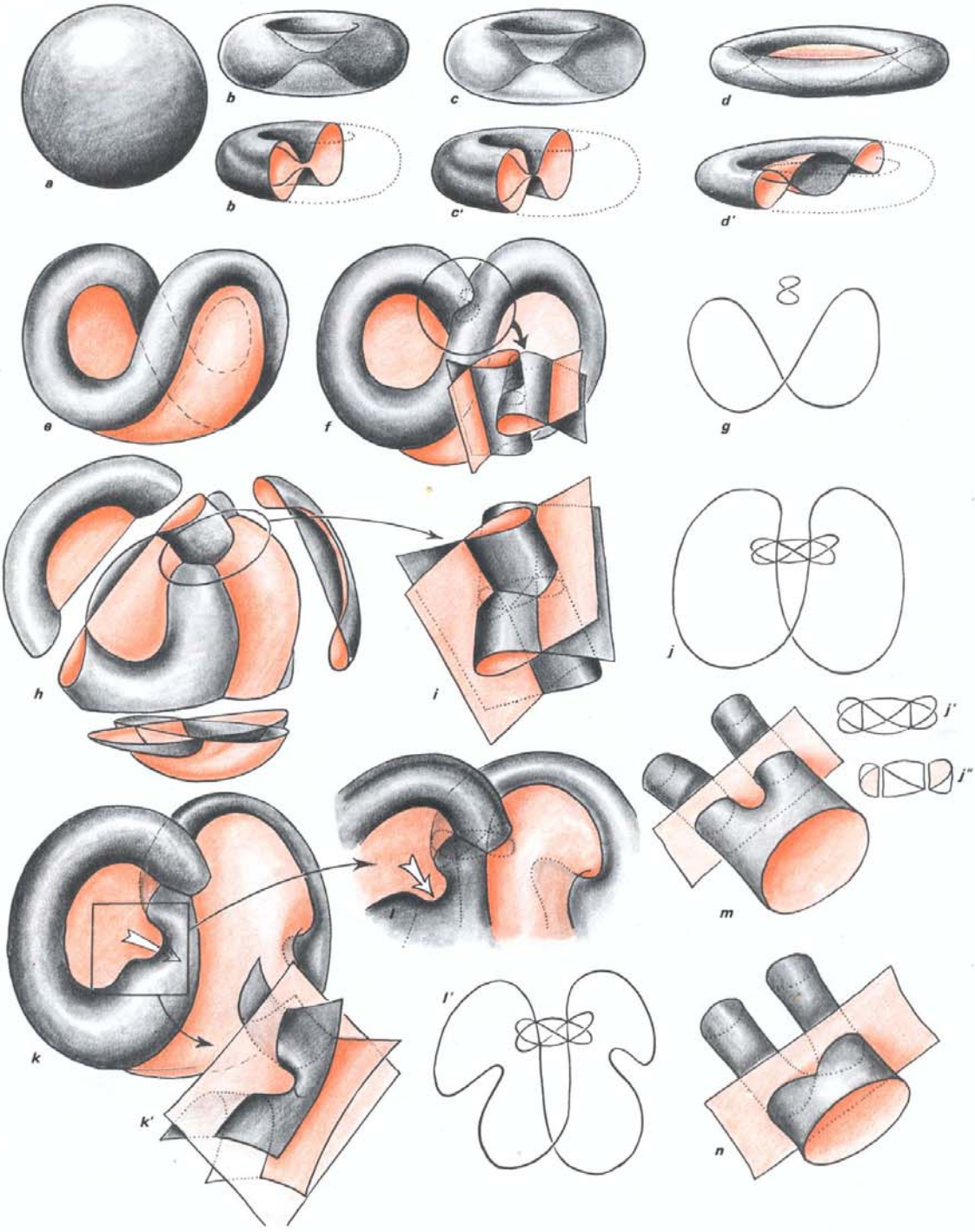
Le modèle central de notre déformation

Bien que très ingénieux, les dessins publiés par Anthony Phillips sont d'un déchiffrement ardu. En effet, ils montrent seulement une partie des objets à décrire, obligeant le lecteur à reconstituer mentalement ces derniers. Les lacunes volontaires des illustrations étaient inévitables en raison de la complexité des modèles envisagés. L'effort demandé au lecteur n'a pourtant pas rebuté le physicien Marcel Froissart. En analysant la solution proposée, il a constaté qu'il pouvait réaliser le projet de manière plus simple à condition d'oublier la surface de Boy. « Ce que Anthony Phillips fait avec six tubes, on peut le faire avec quatre » disait-il. La surface de Boy se présente comme un agencement de trois tubes. Son revêtement par la sphère comporte donc six tubes. Considérons maintenant l'immersion de la sphère représentée sur la figure 5. Ce modèle, qui met en évidence les quatre tubes aperçus par Marcel Froissart, est plus simple que le revêtement à deux feuillets de l'immersion de Boy. Ses propriétés de symétrie lui permettent, comme à la surface de Boy, de servir d'étape centrale au retournement de la sphère. Du point de vue de sa forme, il possède un axe de symétrie quadruple mais si l'on fait tourner l'objet d'un quart de tour, on échange les couleurs des 4 tubes qui sont alternativement gris et orangés. Si donc on dispose d'une famille d'immersions permettant de passer de la sphère standard à ce modèle central, il suffit, après cette rotation de 90°, d'opérer la transformation en sens inverse pour aboutir à la sphère retournée. Chose curieuse, et c'est un des paradoxes posés par l'imagination spatiale, Marcel Froissart n'avait pas vu ce modèle central et la transformation qu'il proposait n'était pas symétrique dans le temps, comme le sont celles d'Anthony Phillips, et celle qui sera présentée plus loin.

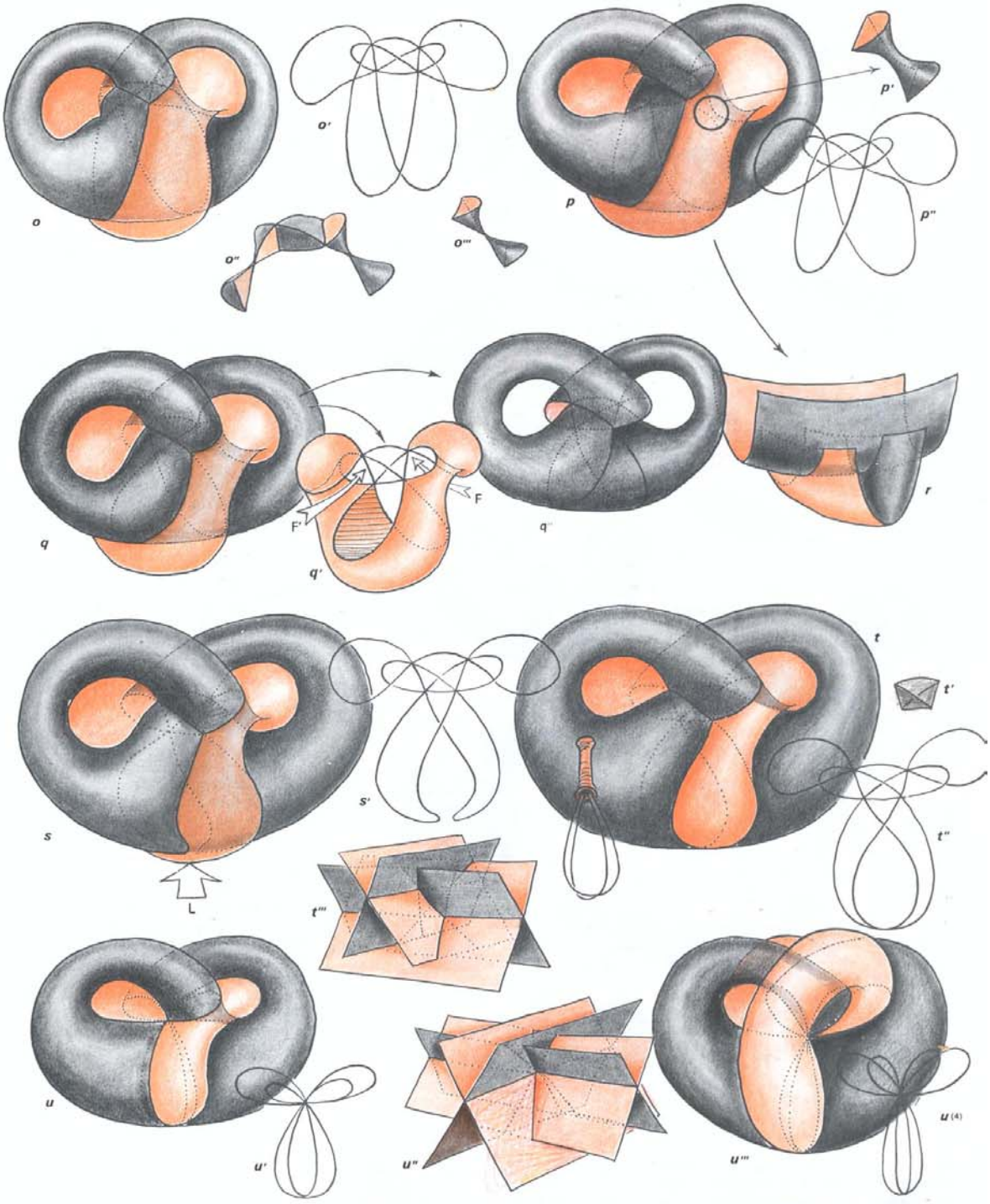
En 1967, lors de la rencontre du Batelle Memorial Institute, qui eut lieu à Seattle, entre mathématiciens et physiciens, fort curieux de connaître l'intéressante découverte de Marcel Froissart, nous nous sommes heurtés à un problème de communication redoutable. Pour converser avec l'inventeur, la seule méthode praticable fut de le questionner à l'aide de modèles en pâte à modeler et en fil de fer. Il se piqua au jeu et se mit lui aussi à pétrir la glaise afin de répondre aux questions. Ce dialogue insolite prit fin le jour où nous fûmes en état de lui présenter une ébauche du modèle central : il devenait alors clair pour les interlocuteurs qu'on pouvait rendre la défor-



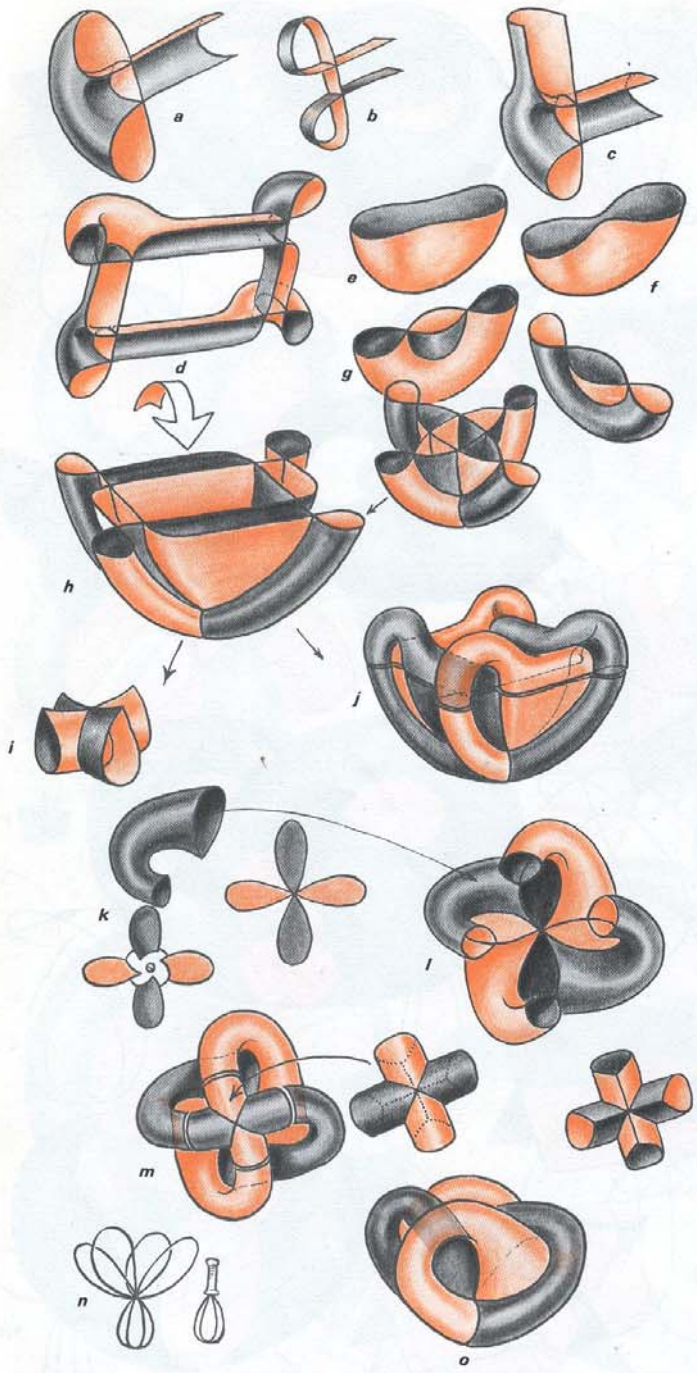
5. LES MODIFICATIONS GÉNÉRIQUES sont les étapes cruciales de la déformation où se produisent les changements de structure de la configuration. On peut suivre pas à pas le retournement de la sphère à partir de ces modifications. La première modification générique D_0 apparaît lorsque deux surfaces se traversent comme la peau recouvrant un coude quand elle est plongée dans l'eau. La courbe de self-intersection est approximativement circulaire. La modification générique T^+ apparaît quand une surface cylindrique s'approche de l'arête d'un dièdre. On peut se représenter les deux plans du dièdre comme les plans de coupe de deux scies qui entaillent le cylindre (ici une bûche) pour isoler un secteur de boule (en forme de tranche de mandarine). La modification générique D_1 peut se concevoir comme la montée de l'eau dans un pantalon infini, en fait une sorte de paraboloïde hyperbolique. Au voisinage de l'entre-jambe la courbe de self-intersection passe de deux arcs de cercles à une forme en X puis à deux nouveaux arcs raccordés différemment. Il est à remarquer que cette modification change les communications possibles entre les parties de l'espace. Lorsque le niveau de l'eau est bas la communication est impossible pour le poisson mais possible pour l'oiseau; quand le niveau de l'eau est haut le poisson peut passer d'une jambe à l'autre mais l'oiseau est confiné dans une région de l'espace. La modification générique suivante Q fait intervenir quatre plans dont l'un glisse par rapport aux trois autres; dans cette modification, le tétraèdre limité par les quatre plans se renverse. L'étude de ces modifications génériques nous arme pour retourner la sphère *(voir figure 6)*.



6. LE RETOURNEMENT DE LA SPHÈRE. Cette planche illustre les différentes étapes du retournement de la sphère. On part de la sphère *a* qui est transformée, par une modification générique D_0, pour former une lentille et un tube *(b, c, d)*. Puis on replie la figure obtenue *(e)* pour faire s'interpénétrer deux parties du tube par une seconde modification générique D_0 dont le résultat est *f*. En *g*, apparaît la courbe de self-intersection. Lorsque l'interpénétration d'un tube dans l'autre se poursuit au-delà de la courbe de self- intersection, on a une modification générique T^+. Deux de ces modifications s'effectuent ici simultanément et la sphère prend la forme *h*; la partie où s'effectue la modification est représentée grandie en *i* et la ligne de self-intersection en *j*. On déforme ensuite le tube *(k)* pour lui donner une forme de paraboloïde hyperbolique *(k')* ou de « pantalon » *(m)* et on fait monter un feuillet du lenticule (assimilable au plan d'eau de la figure 5*c*, à travers les deux « jambes » pour réaliser une transformation de type D_1. Le moment critique de cette modification est



représenté en *o* et les courbes de self-intersection en *o'*. Les courbes de self-intersection *o'* et *o''* annoncent la mise en communication des « tranches de mandarine » avec l'espace ambiant. En *p*, une des modifications D_1 est achevée (celle de droite) et les passages sont visibles sur *p'* et *p''*. Sur les dessins *q'* et *q''*, on a représenté le modèle *q* en deux parties où les deux modifications D_1 sont achevées. Les nouveaux passages ouverts sont symbolisés par les flèches *F* et *F'*. Les tranches de mandarine ont été mises en communication avec l'espace ambiant. On examine maintenant la partie inférieure du modèle. Par une nouvelle modification générique D_1 illustrée en *s*, le feuillet du tube monte selon la direction de la flèche *L* pour obtenir la forme en batteur à œufs des lignes de self-intersection. La dernière étape consiste à éliminer le tétraèdre du milieu représenté en *t* par une modification générique *Q* dans laquelle on pousse un plan jusqu'au point critique où les quatre sommets du tétraèdre se rassemblent en un point. On a le modèle central u''' du retournement détaillé sur la figure 7.

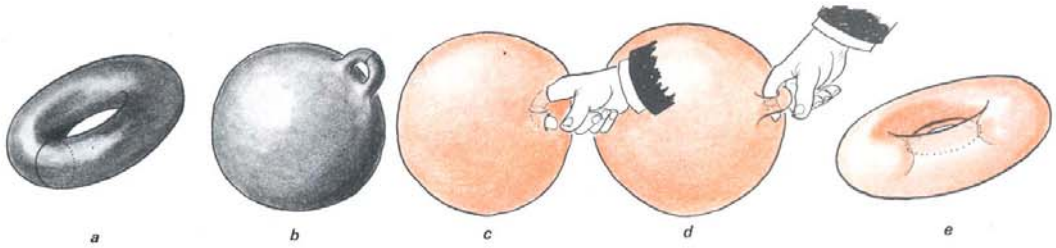


7. LE MODÈLE CENTRAL du retournement de la sphère est ici représenté en détail. Pour en concevoir la structure, on part d'un hémisphère *(a)* que l'on déforme *(c)* pour former l'une des quatre calottes de la partie supérieure *d* du modèle central; en *e*, *f*, *g*, on a indiqué une manière d'obtenir les deux tubes imbriqués qui constituent, après déformation, la base du modèle. Cette séparation en deux parties montre qu'on a bien affaire à une sphère car la partie supérieure est topologiquement équivalente à une bande cylindrique et la partie inférieure à deux calottes sphériques imbriquées. En *k*, on aperçoit une partie de la ligne de self-intersection en forme d'hélice quadripale et les quatre tubes qui se raccordent à la partie basse *(i, l)*. Ces quatre tubes se raccordent selon deux courbes *(n)* symbolisées par les pales d'un batteur à œufs. En *o*, on a représenté une vue de côté du modèle central légèrement déformé par rapport à la figure *j*.

mation de Marcel Froissart symétrique par rapport au temps. Tout cela se passait sous l'œil toujours narquois de Raoul Bott; en manière de plaisanterie il alla jusqu'à prétendre que, puisque les modèles étaient si compliqués, il croyait décidément de moins en moins au théorème.

Immersions transversales et modifications génériques

Un outil théorique essentiel pour maîtriser l'information reçue de Marcel Froissart fut de baliser les diverses transformations envisagées en utilisant la technique des modifications génériques d'immersions transversales. (Nous tenons à préciser au lecteur que transversal ne signifie pas perpendiculaire.) Une immersion transversale est une immersion dans laquelle la surface ne présente que des lignes de points doubles le long desquelles les nappes ne sont jamais tangentes entre elles et un nombre fini de points triples en lesquels les trois plans tangents aux trois nappes forment un trièdre. Notons qu'un plongement est une immersion transversale d'où la courbe de self-intersection est absente. La surface de Boy est une immersion transversale; mais, dans la transformation d'Anthony Phillips, lorsqu'on échange entre eux les deux feuillets de son revêtement par la sphère, il se produit un contact entre les deux nappes le long de toute la surface de Boy et cette situation n'est pas transversale. Comme on peut s'en persuader par exemple en comparant les figures 2b et 2c, lorsqu'on déforme une immersion transversale, la courbe de self-intersection évolue sans changer de structure. C'est le cas, en particulier, lorsqu'il s'agit d'une déformation de plongement. Si deux immersions transversales ont des courbes de self-intersection de structures différentes, il faut donc, pour passer de l'une à l'autre, accepter qu'au cours de la déformation l'immersion cesse d'être transversale. Reprenant une image hasardée lors de l'étude des plongements, on peut dire que sur le continent des immersions, les immersions transversales se groupent en une multitude de pays d'un seul tenant, tandis que les immersions non transversales se répartissent sur les frontières séparant les différents états. La façon la moins laxiste de nous octroyer la liberté indispensable pour retourner la sphère conduit à décrire les six types de modifications, dites *génériques*, que peut subir la courbe de self-intersection d'une surface immergée dans l'espace de dimension trois. Au prix d'une perturbation qu'on peut choisir arbitrairement petite, tout chemin d'immersions peut être corrigé de façon à éviter les modifications non génériques. Le premier type de modification générique, le type D_0, *(voir figure 5)*, rend compte du mouvement de deux portions de surface à la rencontre l'une de l'autre en vue de se traverser. A l'instant critique, il y a contact en un point qui donne ensuite naissance à une courbe



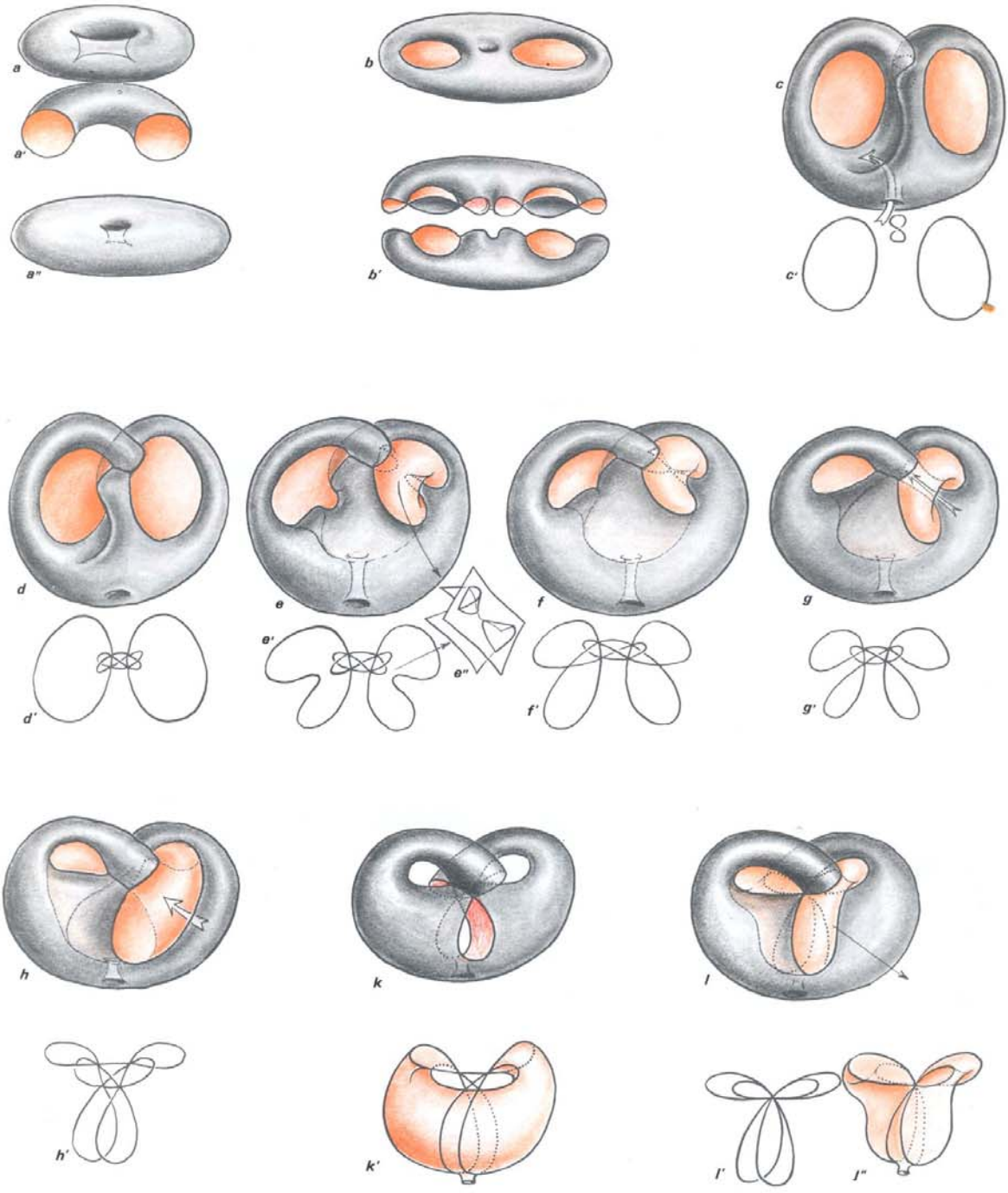
8. UN RETOURNEMENT DU TORE découle simplement du retournement de la sphère. On part du tore *(a)* qu'on transforme en sphère munie d'une petite anse *(b)*. On retourne la sphère *(c)* qui change de couleur (sa couleur interne apparait) et la petite anse est maintenant « à l'intérieur »; il suffit d'extraire *(d)* cette anse en y enfilant le doigt, pour obtenir un tore retourné. On remarquera que le cercle méridien en *(a)* se retrouve autour du doigt du personnage et devient le cercle de gorge du tore retourné.

de self-intersection. Au moment de la traversée des nappes, il se crée, dans l'espace ambiant, une nouvelle région en forme de demi-boucle ou de lentille. C'est la situation décrite dans les figures *2b* et *2c* où l'on croise les pôles de la sphère. La figure *5a* (*5a*$_1$ à *5a*$_5$) représente la rencontre d'un paraboloïde de révolution avec un plan, bien illustré par l'image d'un coude venant éprouver la température du bain. Le même phénomène se produit lorsque deux sphères (extérieures ou intérieures l'une à l'autre) viennent à se rencontrer ou lorsque deux cylindres d'axes non parallèles sont animés d'un mouvement les amenant à se traverser. Le second type (D_2) de modification générique décrit le mouvement de deux nappes croisées qui se séparent; c'est l'opération précédente, exécutée en renversant le sens du temps : le coude sort de l'eau, la région lenticulaire et la courbe de self-intersection disparaissent. Sur le chemin qui va nous conduire du plongement standard au modèle central, nous commencerons par rencontrer deux modifications de type D_0 *(voir figures 6c et 6f)*. Puisque le trajet qui va du modèle central à la sphère retournée se construit par symétrie à partir de ce chemin, la déformation passera en fin de parcours par deux modifications de type D_2.

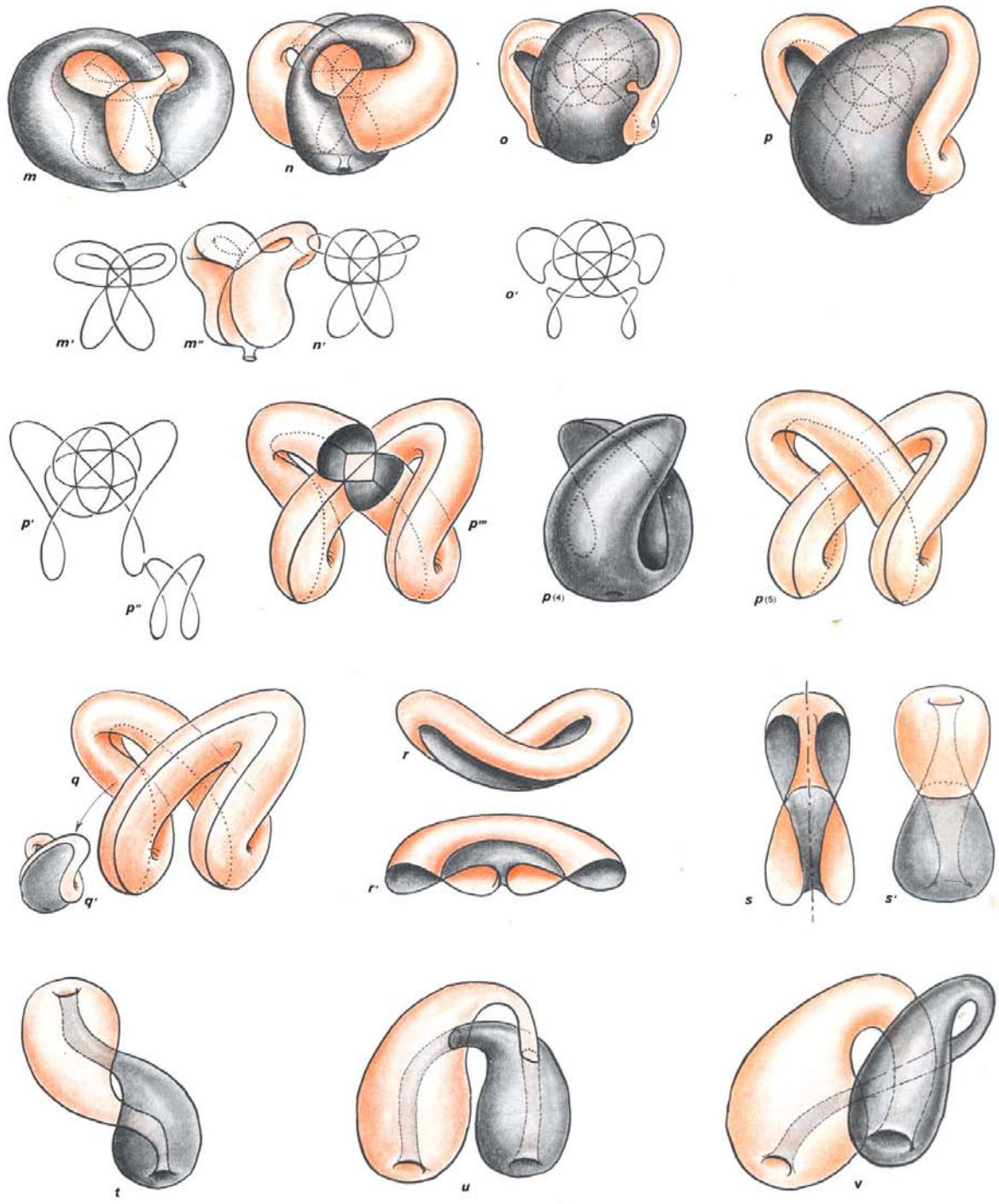
On est dans une situation de type T^+ *(voir figure 5b)* lorsque trois, et non plus deux, portions de la surface sont en mouvement les unes par rapport aux autres. Imaginons la surface d'une bûche s'approchant des plans de coupe de deux scies non parallèles. Lorsque le cylindre délimitant la bûche dépasse la ligne d'intersection des plans de coupe, on voit se former un passage de section triangulaire tandis qu'apparaissent deux points triples reliés entre eux par trois arcs de courbes de self-intersection. Pendant que les deux points triples s'écartent l'un de l'autre, une nouvelle région de l'espace en forme de secteur de boule ou de tranche de mandarine se constitue entre les nappes de la surface en mouvement. La modification T^- est l'opération opposée. Elle fait disparaître le secteur de boule et les points triples tandis que le conduit triangulaire se rouvre. Ainsi, les points triples d'une immersion apparaissent et disparaissent par paires. Il s'ensuit que toute immersion transversale de la sphère présente nécessairement un nombre pair de points triples. De même l'immersion d'une surface n'ayant qu'un point triple (comme par exemple l'immersion de Boy) ne peut se déformer en un plongement de cette surface. Après les deux contacts de type D_0 mentionnés précédemment, notre retournement passe par deux modifications de type T^+ auxquels correspondent deux T^- dans la deuxième phase du mouvement.

Une modification de type D_1 change le mode de raccordement de deux portions de la courbe de self-intersection d'une immersion, après avoir amené en contact deux nappes qui se traversent. Pour expliquer le phénomène, représentons l'une des nappes de la surface par un col de montagne (ou paraboloïde hyperbolique), l'autre étant figurée par un plan mobile, par exemple la surface libre d'un liquide qui inonde progressivement le paysage. Tant que l'inondation n'a pas atteint le col, la courbe de self-intersection se compose de deux arcs de courbe (les deux branches d'une hyperbole). Elle dégénère en deux droites (les asymptotes de l'hyperbole) lorsque l'eau atteint le niveau du col, pour se distribuer de nouveau en deux arcs de courbe (branches de l'hyperbole conjuguée), une fois le col inondé. Lors du contact, c'est-à-dire lorsque le liquide se trouve au niveau du col, l'espace ambiant est découpé en six régions. Mais avant comme après cet instant critique, deux de ces six régions sont raccordées entre elles par un passage. Pour bien comprendre comment s'effectuent les raccordements, il convient de ne pas perdre de vue le fait qu'en réalité, notre montagne est creuse. En effet, si nous nous plaçons sous la surface de celle-ci, nous assistons au même spectacle mais cette fois-ci du point de vue d'un spéléologue qui suit une rivière souterraine en crue. Il percevra en effet le col comme un siphon qu'il devra se hâter de traverser avant que l'eau ne lui ait barré le passage. A partir du moment où sa route est coupée, un poisson de la mer extérieure qui, avant cet instant critique, percevait le col comme un isthme infranchissable, constate que s'ouvre, pour lui, un détroit qu'il peut désormais traverser librement. Sur les figures *5c* on a préféré à l'image du col de montagne celui de l'entrejambe d'un pantalon. Naturellement, la surface du liquide n'a aucune raison d'être plane. Le modèle central de notre déformation présente un contact de type D_1 *(voir figure 7s)*. En renversant le cours du temps d'une modification de type D_1 on obtient encore une modification de type D_1 : mais alors les points de vue du poisson et du spéléologue sont échangés. Après les deux D_0 et les deux T^+ déjà signalés, il nous faudra rencontrer deux modifications de type D_1 avant d'amener la sphère dans la position du modèle central *(voir figures I à q)*. Ainsi, lorsque nous nous rendrons du modèle central au plongement antipodal, nous rencontrerons successivement deux D_1, deux T^- et deux D_2.

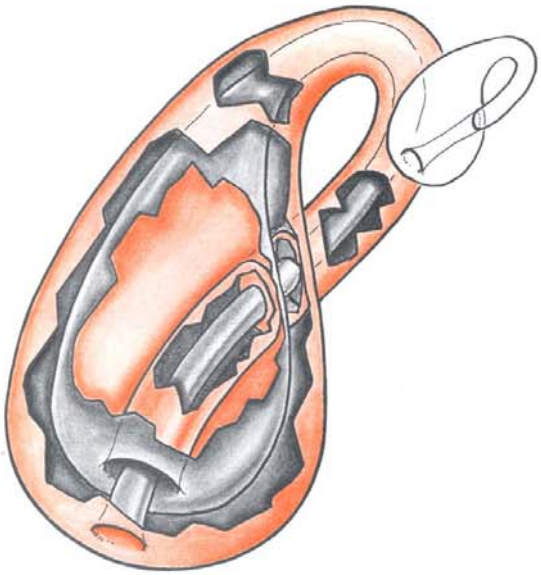
Le dernier des six types de modifications génériques, noté Q, met en jeu quatre nappes d'une surface, qui, à l'instant critique de la modification, passent toutes par un même point (point quadruple). Pour expliquer ce qui se produit dans ce cas, assimilons les quatre nappes aux plans déterminés par les quatre faces d'un tétraèdre régulier et supposons que trois de ces plans soient fixes, tandis que le quatrième se déplace parallèlement à lui-même. Au cours du mouvement, le plan mobile se rapproche du sommet déterminé par les trois autres. Le point quadruple apparaît au moment où le plan mobile rencontre ce sommet. Par ce point passent alors six droites qui constituent la courbe de self-intersection du système. L'espace ambiant est alors découpé en quatorze régions. Après la rencontre, les quatre plans délimitent un nouveau tétraèdre qui se déduit du précédent par symétrie : ainsi la modification a pour effet de renverser le tétraèdre. On peut encore présenter cette modification en solidarisant les plans par paires et en déplaçant les dièdres ainsi formés l'un par rapport à l'autre, comme indiqué sur les figures *5d*. Comme celles de type D_1, les modifications de type Q sont symétriques par rapport au temps. Le passage de l'homotopie régulière du retournement de la sphère par le modèle central, présente deux modifications de types D_1 et Q. Tout retournement de la sphère passe nécessairement par au moins un point quadruple. C'est probablement l'une des raisons pour lesquelles la mise au point du scénario a été si laborieuse. Au total, le retournement comporte quatorze modifications génériques : c'est un minimum.



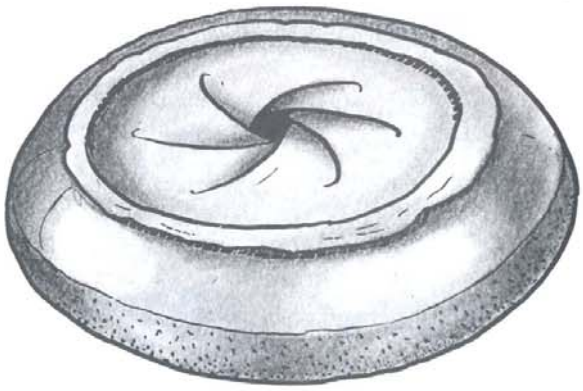
9. UN DES RETOURNEMENTS DU TORE peut se faire à travers un revêtement à deux feuillets de la bouteille de Klein et ce retournement conserve le cercle de gorge. On aplatit le tore et on contracte son col au centre puis on réalise deux modifications génériques D_0. La suite des dessins présente une grande analogie avec les transformations du retournement de la sphère. En masquant le bas de chaque figure, les parties supérieures sont les mêmes que celles du retournement de la sphère. Il n'y a pas de différence de structure entre les modèles *m* et *n*; on les a déformés pour une meilleure représentation graphique : les courbes de self-intersection m' et n' sont identiques. On déforme la partie frontale du modèle tel qu'indiqué en *(o)* par deux D_1 qui vont reconstituer des tranches de mandarine pour donner la figure dont la ligne de self-intersection est identique (mais vue de côté) à celle obtenue en *k* sur la figure du retournement de la



sphère. On a représenté en $p^{(3)}$, $p^{(4)}$, $p^{(5)}$ la façon dont la lentille et le tube étaient transformés. En décroisant le tube par des modifications génériques T^- T^- et D_2 (transformation inverse de celle conduisant à la figure *k* dans le retournement de la sphère), on obtient le modèle rr' analogue au modèle *e* du retournement de la sphère avec en plus un passage tubulaire au centre. Si on ne tient pas compte du passage tubulaire, on a, par ces transformations, un retournement non symétrique de la sphère. Par rotation de 90° des figures en huit du modèle *r*, on passe aux modèles *s* et *s'*. Ce modèle *s* est central et on pourrait échanger les deux parties par rotation de 180° et réaliser la suite d'opérations inverses pour obtenir le tore retourné. La suite des dessins *t*, *u*, *v* montre qu'on passe au revêtement à deux feuillets de la bouteille de Klein, comparable au double revêtement de la surface de Boy.



10. UNE BOUTEILLE DE KLEIN est ici revêtue par une surface à deux feuillets. La surface de revêtement est ici un tore *(voir figure 9)*. Si on colle les faces en regard des deux feuillets, on retrouve la bouteille de Klein (en médaillon) qui est une surface unilatère.



11. CETTE BLAGUE A TABAC montre comment il faudrait commencer par tordre la sphère pour la retourner selon une suite de modèles présentant un axe de symétrie d'ordre 6. Elle correspond au modèle *e* de la figure 6 qui a un axe de symétrie d'ordre 2.

Le retournement de la sphère.

Quelques mois après la rencontre de Seattle, nous étions en mesure de proposer deux variantes de la déformation de Marcel Froissart. La première a été réalisée par Charles Pugh sous forme de huit modèles en grillage (matériau, nous l'avons déjà dit, spécialement bien adapté à la représentation des immersions) qui ont servi de point de départ à Nelson Max pour donner à l'ordinateur les instructions nécessaires à la reproduction de toutes les étapes transitoires. Le fruit de cette collaboration a permis à Nelson Max de diffuser en 1976, un film pédagogique fort instructif; grâce au magnétoscope, la seconde a fait l'objet, en 1972, d'une animation vidéo (malheureusement non publiée en son temps) réalisée par Georges Ricco sur nos indications.

C'est la seconde variante que nous allons présenter. Bien que conduisant au résultat de façon plus directe et plus élégante, elle semblait de prime abord assez délicate à expliquer. En effet, pour rendre compte d'une déformation, une suite de modèles ne doit pas se contenter d'être correcte. Les maquettes qui la composent doivent encore satisfaire à un principe d'économie et obéir à des règles de cohérence esthétique qui en facilitent la perception. Lorsque le principe de la déformation nous est apparu clairement, il devenait évident qu'on pouvait mettre le mouvement en équations. On peut faire en sorte qu'à tout instant de la déformation les modèles présentent un axe de symétrie binaire. Le procédé qui les engendre se généralise et permet de construire une transformation, elle aussi symétrique par rapport au temps et susceptible d'une représentation analytique, dans laquelle chaque immersion admet un axe de symétrie d'ordre, donné d'avance, aussi grand qu'on veut. Dans le cas général, l'amorce du mouvement est suggérée par la figure 11 qui possède un axe de symétrie d'ordre six et qui représente un modèle de blague à tabac en vente chez tous les buralistes : il s'agit d'une poche en caoutchouc dont le col se referme grâce à douze fronces dont six apparaissent à la face supérieure. Il est facile de prolonger la figure en un plongement de la sphère en cousant le long du bord une calotte sphérique cachant le fond du sac. Une blague à quatre fronces fournirait l'amorce de la déformation de la figure 7. Si on s'impose une symétrie ternaire, l'étape centrale de la déformation se trouve être, comme chez Anthony Phillips, le revêtement à deux feuillets de la surface de Boy qu'on peut, par conséquent, décrire au moyen d'expressions algébriques. En pratiquant des fenêtres dans la blague à tabac, George Francis, que nous remercions au passage d'avoir, l'été dernier, éprouvé sur ordinateur l'efficacité visuelle des équations, a trouvé le moyen d'expliquer le mouvement par un très petit nombre de dessins qui, il est vrai, demandent au lecteur un certain effort de reconstitution.

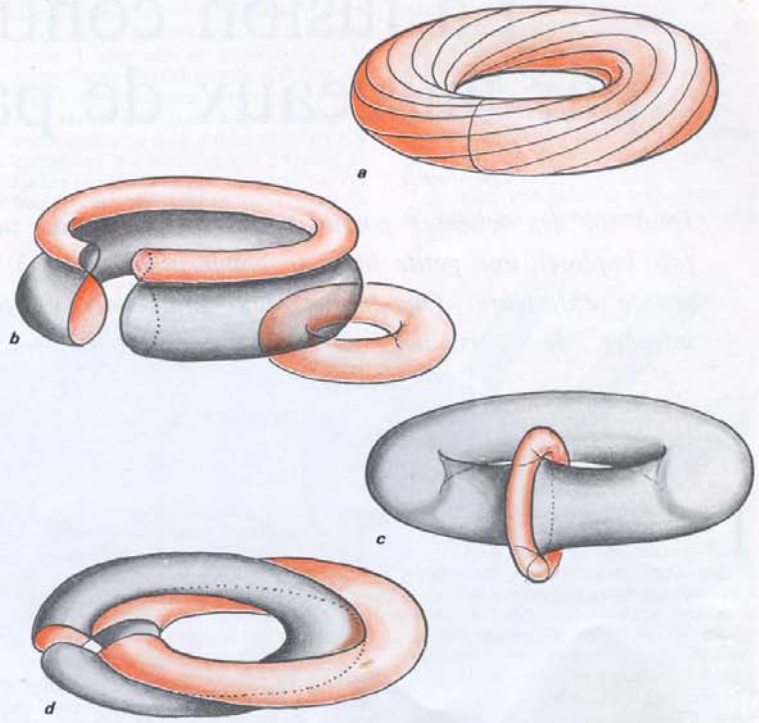
On pourra suivre toutes les étapes du retournement de la sphère sur les dessins de la figure 6. Les figures *6a*, *6b* et *6c* nous montrent une première modification de type D_0 qui, en croisant le pôle nord et le pôle sud, donne naissance à une lentille entourée d'un tube équatorial. Par conséquent, l'immersion *c* découpe l'espace en trois régions. Sur les figures *6d* et *6e*, on tord le modèle. Après une seconde modification de type D_0 on est dans la situation de la figure *f* où le tube équatorial se recoupe tandis qu'apparaît une quatrième région de l'espace ambiant. Sur la figure *g* on voit les deux courbes fermées dont la réunion est la courbe de self-intersection du modèle. Le mouvement d'interpénétration des deux portions de tube se poursuit jusqu'au moment où deux points de l'équateur atteignent simultanément le bord anguleux de la lentille. Il se produit alors deux modifications de type T^+ *(voir figures h, i et j)*. Les quatre points triples auxquels ces modifications donnent naissance sont les quatre sommets d'une région de l'espace en forme de tétraèdre. Sur deux arêtes opposées du tétraèdre viennent s'articuler deux régions de l'espace en forme de secteur de boule tandis que le reste du tube équatorial se scinde en deux régions; l'immersion *j* découpe donc l'espace en sept régions. Les figures *k*, *k'*, *l* et *l'* préparent les deux modifications suivantes qui sont de type D_1. Sur la figure *m*, le pantalon qui nous a servi à décrire la modification de type D_1, est orienté de manière à correspondre à la figure *k*. Dans la figure *n*, la position du pantalon correspond à la situation de la figure *o*. Les deux modifications de type D_1, qui peuvent se produire simultanément, ont pour effet d'ouvrir vers l'extérieur les secteurs de boule qui épaulent le tétraèdre, tandis que se ferment les deux passages qui, dans la situation de la figure *j* mettaient la partie centrale du modèle en communication avec l'extérieur. En *o'''* on a réalisé une seule des deux modifications. La figure *q* décrit la situation lorsque les deux modifications de type D_1 sont réalisées; elle découpe l'espace en six régions.

Les figures q' et q'' représentent un démontage du modèle q en deux éléments sur lesquels on a cherché à mettre en évidence les passages ouverts. La partie inférieure du modèle correspond au schéma r, c'est-à-dire au croisement de deux cylindres paraboliques. Le mouvement de ces deux cylindres l'un par rapport à l'autre, suggéré par la figure s, produit un contact de type D_1 qui sépare en deux ce qui reste de la région lenticulaire de la figure c. Une modification de type Q fait disparaître le tétraèdre et conduit au modèle central qui découpe donc l'espace en six régions et que la figure u''' représente, légèrement déformé. Après une rotation de 90°, les modifications précédentes appliquées dans l'ordre inverse, nous amènent à la sphère retournée.

Les immersions du tore.

Au cours du mouvement, le pôle nord et le pôle sud se croisent trois fois : lors du premier contact D_0, lors du passage par le modèle central (contact de type D_1) et lors du dernier D_2. Quitte à perdre la propriété de symétrie par rapport au temps, on peut faire en sorte que le pôle nord ne croise le pôle sud qu'au moment du dernier D_2. Pour cela, il suffit de faire apparaître au début du processus non plus une mais deux lentilles : dès lors, la modification de type D_1 qui survenait au cours de l'étape centrale n'a plus à se produire. Dans la déformation de la figure 6, les quatorze modifications rencontrées étaient respectivement de types (D_0)-(D_0), (T^+)-(T^+), (D_1)-(D_1), (Q)-(D_1)-(D_1)-(D_1), (T^-)-(T^-), (D_2)-(D_2) (Les traits d'union indiquant que les modifications peuvent se produire simultanément.) Dans la variante envisagée, on a la succession (D_0)-(D_0)-(D_0), (T^+)-(T^+), (D_1)-(D_1)-(Q)-(D_1)-(D_1), (T^-)-(T^-), (D_2)-(D_2). Si l'on arrête le processus après la treizième modification, le modèle se présente comme une lentille grise entourée d'un tube orangé. On peut faire en sorte que le pôle nord et le pôle sud soient restés immobiles jusqu'à ce moment. Si on découpe autour de chacun des pôles une petite calotte et si on raccorde les bords des trous ainsi formés au moyen d'un cylindre, la sphère devient un tore et les déformations envisagées ont pour résultat de faire apparaître un tube orangé sur l'extérieur d'un tore gris, c'est ce que représentent les dessins des figures 9*a* à 9*q*. Un procédé analogue permettrait de relier le plongement standard du tore à une immersion affectant la forme de la figure 12*c*.

La question des plongements et des immersions est beaucoup plus compliquée pour le tore que pour la sphère. Maurice Hirsch a généralisé le théorème de Stephen Smale, qui traitait des immersions d'une sphère de dimension quelconque, au cas des surfaces de dimension arbitraire. Appliquant ces résultats aux surfaces de l'espace à trois dimensions, Ioan James et Emery Thomas ont montré en particulier que les immersions du tore se répartissaient sur quatre continents entre lesquels il est impossible de passer par homotopie régulière. L'un de ces continents ne contient aucun plongement. Sur la figure 11*d*, on a représenté une immersion de ce continent. Les trois autres continents contiennent des plongements qui se répartissent sur une infinité de taches grises ou orangées selon la couleur de la face visible de l'objet. La figure 11*a* donne l'idée d'un plongement du tore qui présente, tout comme le plongement standard, sa face orangée vers l'extérieur mais qui ne peut être relié à ce dernier par aucune homotopie régulière. La baudruche d'un tel plongement a subi une torsion de 360° qu'il est impossible de faire disparaître. Ce plongement et le plongement standard appartiennent donc à des continents différents. Cet exemple montre pourquoi les dessins de la figure 12, qui représentent les formes que peuvent revêtir certaines immersions du tore, ne contiennent pas toute l'information dont on a besoin pour dire à quel continent appartient l'immersion représentée. Sur un continent donné de l'univers des immersions du tore, une homotopie régulière raccordant un point d'une tache grise à un point d'une tache orangée est un retournement du tore. Comme on le voit par ce qui précède, il y a une infinité de manières de retourner le tore.

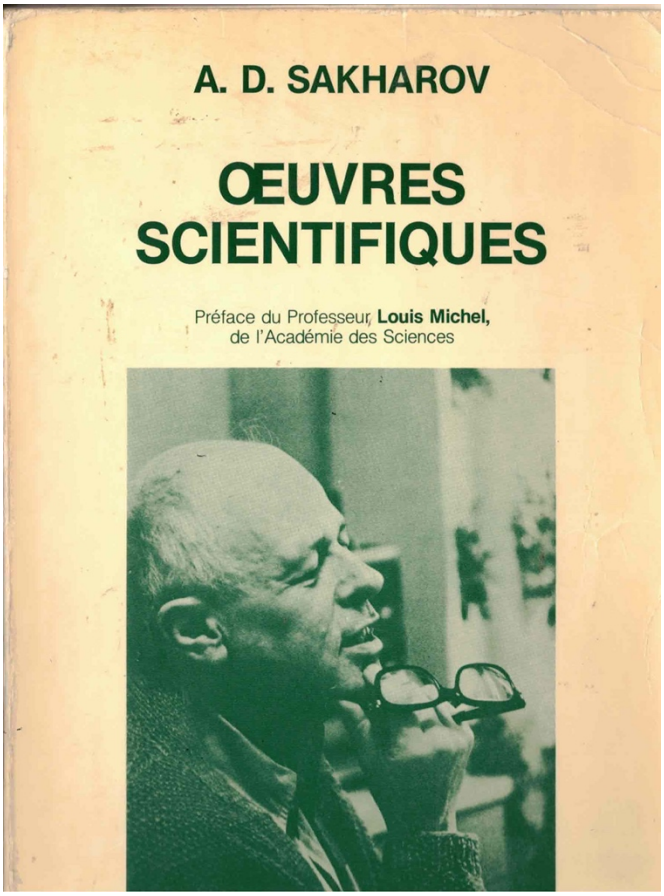


12. LES FORMES LES PLUS REMARQUABLES d'immersions du tore sont représentées en *b*, *c* et *d*. Le plongement « tordu » de la figure *a* qui a le même aspect que le plongement standard montre que les figures *b*, *c* et *d* ne contiennent pas toute l'information qu'il faudrait avoir pour décrire les immersions puisque ces figures ne rendent pas compte des torsions internes.

Sur les figures 9*s* à 9*v*, on indique comment le tore gris et orangé peut être déformé en un revêtement à deux feuillets de la bouteille de Klein (immersion d'une surface qu'on obtient en raccordant entre eux deux rubans de Möbius). On peut alors permuter les feuillets du revêtement et aboutir, en parcourant en sens inverse la déformation précédente, à un tore retourné. Partant du tore standard, la figure 8 suggère un retournement du tore aboutissant à un objet orangé qui ne peut être raccordé par déformation de plongement au tore orangé obtenu en traversant la bouteille de Klein.

Les mathématiciens sont fréquemment irrités lorsqu'on leur demande à quoi servent leurs découvertes. En effet, le besoin qui les pousse à chercher est plutôt d'ordre esthétique que pragmatique. La question est d'autant plus embarrassante quand on la pose à propos du présent travail qui n'est pas, à proprement parler, un travail de mathématique; dans ce qui précède on n'a rien démontré mais seulement illustré une démonstration qui existait déjà. Nous pensons toutefois que tous ceux qui voudront chercher à comprendre le retournement de la sphère acquerront, en pratiquant l'exercice, une vision nouvelle de l'espace qui enrichira leur imagination. ■

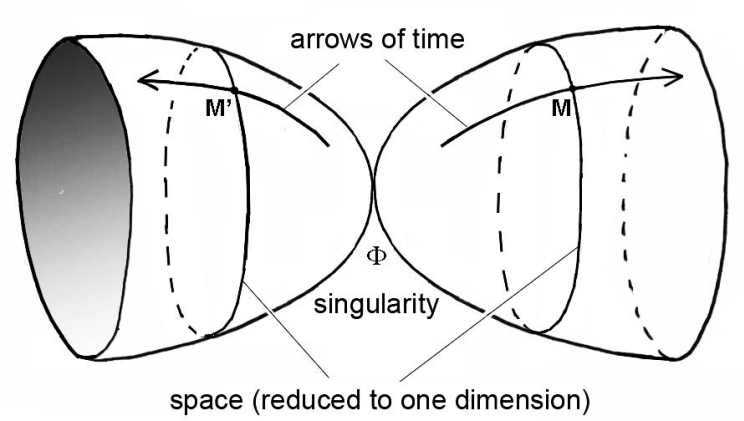
In 1984, Western scientists, in support of the Russian Andrei Sakharov's struggle against the Soviet regime, published translations of his articles—originally published in Russian—into various languages. In France, Anthropos published the work in French in 1984.



Reproduction of Sakharov's Works. Anthropos Publishing, 1984

It is, of course, a great surprise to discover that Sakharov published several articles on the topic of twin universes in the late 1960s. [19]
The first article dates from 1967, precisely the time when his contacts with the Spaniards took place. I note that Sakharov envisaged connecting the two universes via the "Big Bang singularity," which suggests this hourglass-shaped structure.

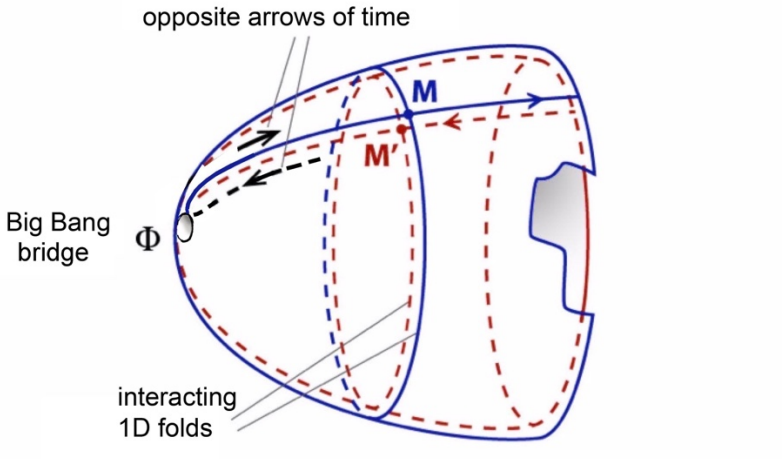
[19] A.D. Sakharov, ZhETF Pis'ma 5: 32; JETP Lett. 5: 24 (1967)
- A.D. Sakharov, ZhETF Pis'ma 5: 32; JETP Lett. 5: 24 (1967)
- A.D. Sakharov, ZhETF Pis'ma 76: 1172 (1979); JETP 49: 594 (1979)
- A.D. Sakharov, (1980). Cosmological Model of the Universe with a Time Vector Inversion. ZhETF (Tr. JETP 52, 349–351) (79): 689–693



Sakharov's 2D Model of Twin Universes

It was the influence of mathematician Bernard Morin that inspired me to imagine a different geometric structure. With him, I learned how to fold a sphere to form a two-sheet covering of a Boy surface, and how to fold a torus to form a two-sheet covering of a Klein bottle.

When the sphere is folded, its two poles coincide. So I had the idea of folding Sakharov's hourglass:

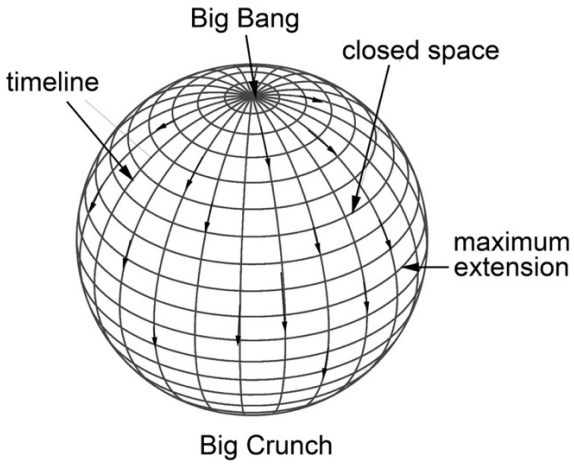


The Same "Folded" Ones

This idea is beginning to appear tentatively in the literature.

Incidentally, the idea is also to eliminate this initial singularity by converting it into a "tube" connecting the two universes.

The Ummo texts state that the universe "is a hypersphere." Therefore, this universe is closed, self-contained. If we start with a closed-universe model, the 2D representation looks like this:



2D model of "closed" spacetime

This folding of the universe back onto itself results in the coincidence of antipodal points. In 1994, I published the following[20] article in the prestigious Italian journal *Nuovo Cimento* (which no longer exists), introducing this idea. It contains several points. It includes the outline of a field equation and the derivation of a Poisson equation based on it.

[20] J-P Petit. The Missing Mass Problem. Il Nuovo Cimento B. Vol. 109, pp. 697–710, July 1994 http://www.jp-petit.org/papers/cosmo/1994-Nuovo-Cimento.pdf

The missing mass problem

Jean-Pierre Petit

Marseille Observatory, France
jppetit1937@yahoo.fr

Il Nuovo Cimento B, Vol. 109, pp. 697-710, July 1994

Abstract

A new field equation is proposed, associated to a S3 x R1 topology. We introduce a differential involutive maping A which links any point of space **σ** to the antipodal region A(**σ**). According to this equation the geometry of the manifold depends both on the energy-momentum tensor **T** and on the antipodal tensor A(**T**). Considering time-independent metric with low fields and small velocities, we derive the associated Poisson equation, which provides cluster-like structures interacting with halo-like antipodal structures. The second structure helps the confinement of the first. It is suggested that this model could explain the missing mass effect and the large scale structure of the universe.

1- Introduction

The equilibrium of a galaxy is studied through a certain set of non-relativistic equations, as for example, Vlasov equation coupled to Poisson equation, which comes from the general Einstein field equation

(1)

$$\mathbf{S} = \chi\, \mathbf{T}$$

plus a steady-state hypothesis in which we take weak fields and small velocities. It is well known that the gravitational field due to the visible mass of our galaxy cannot balance the centrifugal and the pressure forces. Some people assume that some invisible mass, dark matter, may contribute to the field and balance the centrifugal force. In the following we are going to propose another model, based on a new field equation.

2- A new field equation

We assume that the universe has the topology of S3 x R1 .
The Gaussian coordinates are

(2)

$$\mathbf{x} = (x^\circ , \boldsymbol{\sigma})$$

where x° is a time-marker and the vector $\boldsymbol{\sigma}$ represents the spatial markers. Space-time is oriented. It is possible to define a differential involutive maping linking a given point $\boldsymbol{\sigma}$ to the antipodal point $\boldsymbol{\sigma}^*$

(3)

$$\boldsymbol{\sigma}^* = A (\boldsymbol{\sigma})$$

Consider two tensor fields **S** and **T**, defined on the manifold. Suppose that they are linked in the following field equation

(4)

$$\mathbf{S} = \chi (\mathbf{T} - A(\mathbf{T}))$$

with

(5)

$$A(\mathbf{T}) = \mathbf{T}^* = \mathbf{T}(x^\circ, \boldsymbol{\sigma}^*)$$

We assume that the light follows the geodesics of space-time. **g** is the metric tensor. **R** is the Ricci tensor, so that

(6)

$$\mathbf{g}^* = \mathbf{g} (x^\circ, \boldsymbol{\sigma}^*)$$

$$\mathbf{R}^* = \mathbf{R}(x^\circ, \boldsymbol{\sigma}^*)$$

We can write the field equation in the more explicit form

(7)

$$\mathbf{R} - \frac{1}{2}\mathbf{g}\, R = \chi (\mathbf{T} - \frac{1}{2}\mathbf{g}\, T - \mathbf{T}^* + \frac{1}{2}\mathbf{g}^*\, T^*)$$

Let us write the tensors **T** and **T*** as

(8)

$$\mathbf{T} = \begin{vmatrix} \rho & 0 & 0 & 0 \\ 0 & -\frac{p}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p}{c^2} \end{vmatrix}$$

(9)

$$\mathbf{T}^* = \begin{vmatrix} \rho^* & 0 & 0 & 0 \\ 0 & -\frac{p^*}{c^2} & 0 & 0 \\ 0 & 0 & -\frac{p^*}{c^2} & 0 \\ 0 & 0 & 0 & -\frac{p^*}{c^2} \end{vmatrix}$$

with

$$\rho^* = \rho\ (x^\circ, \boldsymbol{\sigma}^*)$$

$$p^* = p\ (x^\circ, \boldsymbol{\sigma}^*)$$

If we take the zero-divergence condition, the fluid obeys the following conservation equations

(10)

$$\partial\, \mathbf{T} = 0$$

3- Time independent conditions with weak fields and small velocities. The Poisson equation

We can apply the classical method, taking a quasi-Lorentzian metric
(11)

$$\mathbf{g} = \boldsymbol{\eta} + \varepsilon\, \boldsymbol{\gamma}$$

where $\boldsymbol{\eta}$ is the Lorentzian metric and ε is a small parameter.

In three-dimensional notations
(12)

$$\frac{d^2\mathbf{x}}{dt^2} = -\frac{c^2}{2}\gamma_{oo}|_i = -\frac{c^2}{2}\nabla\gamma_{oo}$$

The newtonian law applies over all space. In addition the gravitational potential is defined as the following:

(13)

$$\Psi = - \frac{c^2}{2} \varepsilon \gamma_{oo}$$

Conversely, given the gravitational potential ψ , the motion of a particle will be along a four-dimensional geodesic if the goo terms of the metric tensors has the form
(14)

$$g_{oo} = 1 + \frac{2\Psi}{c^2}$$

we get
(15)

$$\varepsilon \sum_{\beta = 1}^{3} \gamma_{oo} |\beta| \beta = - \chi (\rho - \rho^*)$$

By identification we get the following Poisson equation

(16)

$$\Delta\psi = 4 \pi G (\rho - \rho^*)$$

If we consider a spherically symmetric system

(17)

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r} \frac{d\Psi}{dr} = 4 \pi G (\rho - \rho^*)$$

where

(18)

$$\rho^* = \rho(\boldsymbol{\sigma}^*)$$

From (17)

(19)

$$\psi^* = - \psi$$

4- Spherically symmetric solution

In 1916 Eddington derived a spherically symmetric steady-state solution, combining the Vlasov and the Poisson equations. He assumed that the ellipsoid of the velocities was spherically symmetric and pointed towards the center of the system.

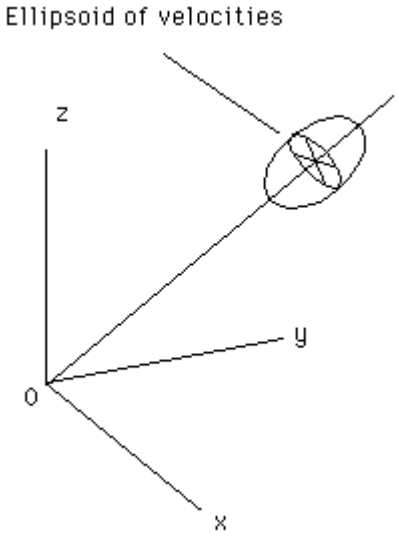


Figure 1 (ga3114): Ellipsoid of velocities corresponding to an Eddington-type solution.

Eddington derived the following relation between the mass density and the gravitational potential

(20)

$$\rho = \rho_o \frac{e^{-\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}}$$

which represents a steady-state distribution of matter in a collision-free gas, in a gravitational potential Y, in which the gravitational force balances the pressure force. Let us take the same kind of a solution for the antipodal region

(21)

$$\rho^* = \rho_o \frac{e^{-\frac{m\Psi^*}{kT}}}{1 + \frac{r^2}{r_o^2}} = \rho_o \frac{e^{\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}}$$

So that we have to solve the following equation

(22)

$$\frac{d^2\Psi}{dr^2} + \frac{2}{r}\frac{d\Psi}{dr} = 4\pi G \rho_o \left(\frac{e^{-\frac{m\Psi}{kT}} - e^{\frac{m\Psi}{kT}}}{1 + \frac{r^2}{r_o^2}} \right)$$

Take

(23)

$$r_o = \lambda \sqrt{\frac{kT}{4\pi G \rho_o m}}$$

Introduce the following adimensional quantities:

(24)

$$r = \sqrt{\frac{kT_o}{4\pi G \rho_o m}}\ \xi \qquad \Psi = \frac{kT}{m}\varphi$$

We get

(24)

$$\varphi'' + \frac{2}{\xi}\varphi' = \frac{e^{-\varphi} - e^{\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

which can be solved by numerical computation. We can take the following initial conditions

$$\varphi'o = 0$$

$$\varphi''o = 10$$

$$\lambda = 10$$

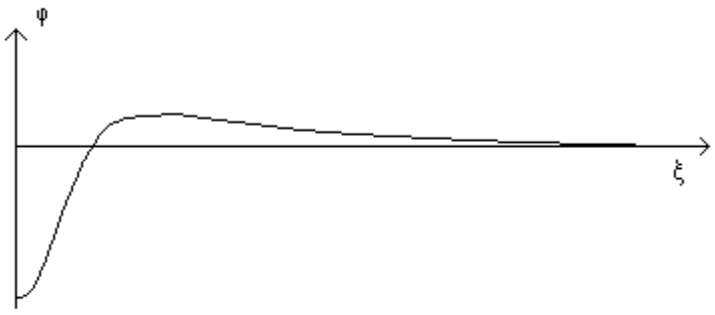


Figure 2: Spherically symmetric Eddington-type solution. The gravitational potential

$$\rho = \rho_0 \frac{e^{-\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

$$\rho^* = \rho_0 \frac{e^{\varphi}}{1 + (\frac{\xi}{\lambda})^2}$$

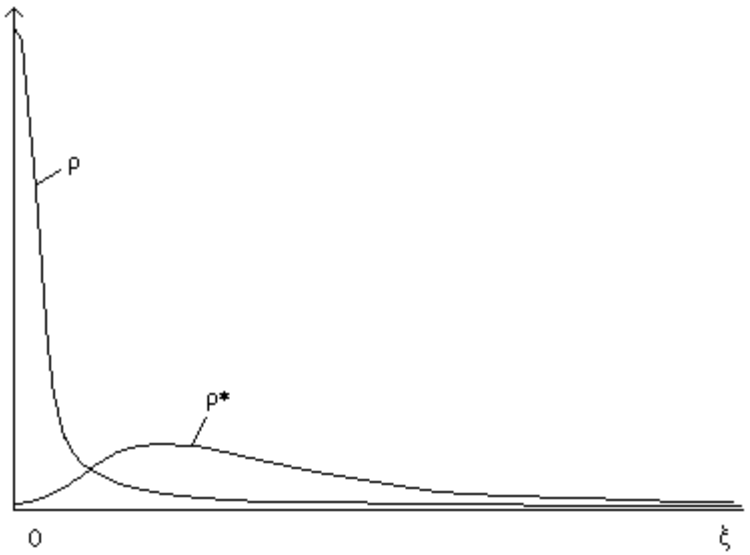


Figure 3: Spherically symmetric Eddington-type solution. Mass densities. If a cluster exists in one fold, an associated diffuse halo exists in the conjugated region of the second fold.

5- The large size structure of the universe

From the equation (24) we see that if a cluster exists in one fold, an associated diffuse halo structure exists in the conjugated region of the second fold. If this model is correct we should find halo-structures in our fold of the universe. With the help of Dr. Pierre Midy, from the university of Orsay, France, we have performed numerical simulations, using a Cray-1 computer. We consider two distributions of 350 points. The first is represented by little circles and the second by small crosses. At the begining the points are randomly distributed on the screen and are supposed to represent two uniform gazes. Each mass owns a random velocity corresponding to an isotropic Maxwellian distribution with an averaged thermal velocity <V> . Call m1 the elements of the first population and m2 the elements of the second population. We apply the Newton law with

- m1 attracts m1: gravitational effect
- m2 attracts m2: gravitational effect
- m1 and m2 repel each other: antigravitational effect

We consider this two-dimensional system as periodic over space. In other terms the upper boundary is linked to the lower one and the right to the left (Euclidean 2D torus). So that we can compute the sum of the mutual actions of the particles. For each interval of time?t we compute the acceleration of each particle and determine the trajectory by Taylor expansion. Each particle that comes out through the right boundary reappears through the left one, and same thing for the upper and lower boundaries. This makes possible to study the gravitational instability of these two coupled systems in a finite portion of space (with toroidal topology). The interval of time is determined in order to get significant computational results. In other terms we demand the trajectory of a particle to be approximatively regular. The following figures show the typical behaviour of the system after 4000 intervals of time. In figures 4 and 5 we find both clusters and cellular patterns. This is enhanced in the figures 7 and 8.

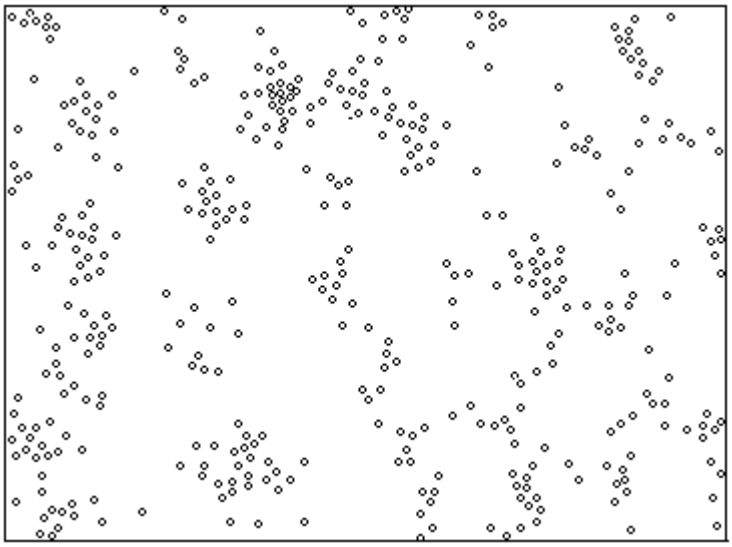


Fig. 4: Effect of the gravitational instability on the system 1

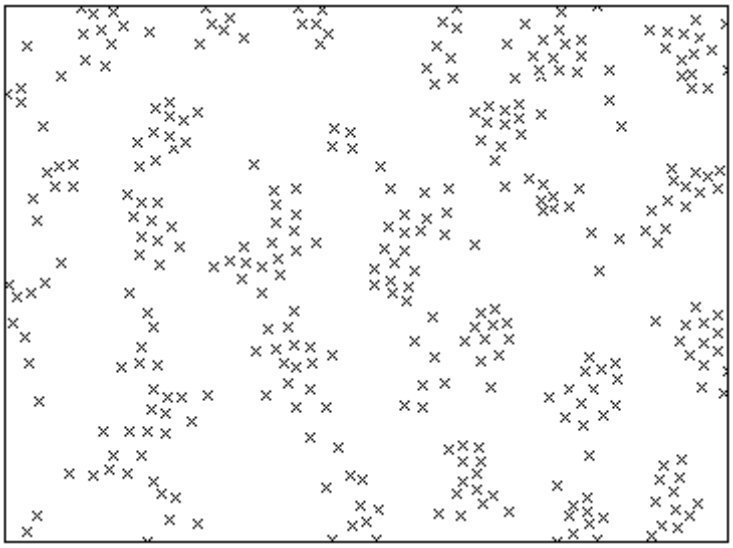


Fig. 5: Effect of the gravitational instability on the system 2

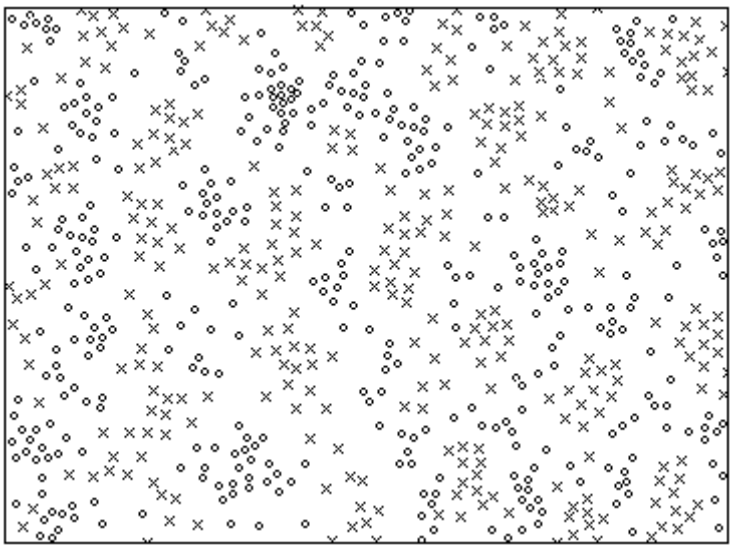


Fig. 6: Superposition of the two.

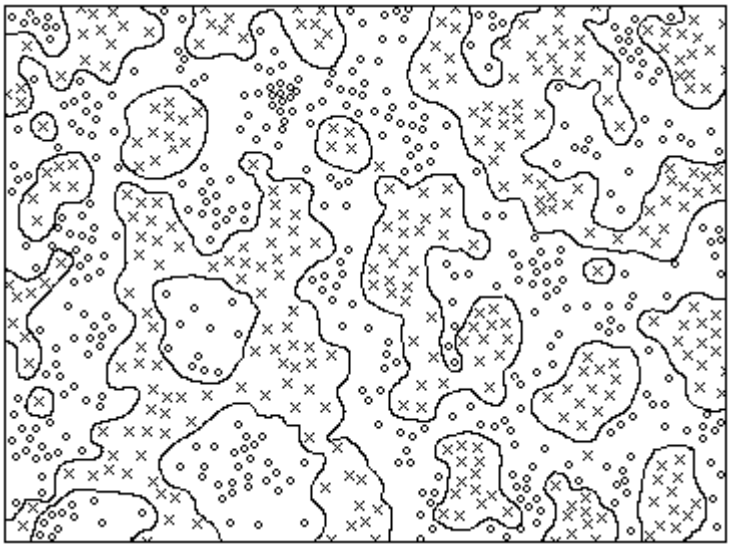


Fig. 7: Enhanced spatial distribution of the two populations.

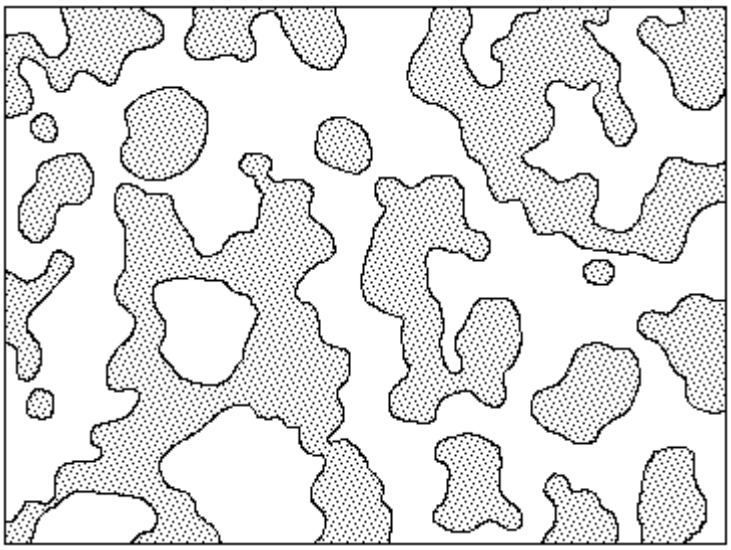


Fig. 8: White: population 1. Grey: population 2

We suggest that such a mechanism could explain the large scale structure of the universe and the observed distribution of galaxies. Suppose that our fold of the universe corresponds to the population 1. In the right lower part of the screen this matter is arranged around large "empty" bubbles. These bubbles correspond to a cluster arrangement in the population 2, supposed to be located in the second fold of the universe (in fact the antipodal region), according to our theory. But, as seen on the figure 8, for a given population, in some places the matter can be arranged as a Swiss "gruyère" (emmental) cheese and in other places as an emulsion.

These first crude numerical simulations have to be developped with a larger number of points and in a three-dimensional representation. We know that the three-dimensional's behaviour of a system can be somewhat different from the two-dimensional's one. But we expect the conclusions to be similar. We think that with a larger number of points we could get a fractal system, as suggested in the figure 14, but we precise that this peculiar computation has not yet been done, but are under study. According to this idea the galaxies should be located in the holes of the associated anti-matter cloud, which would ensure their confinement, as suggested earlier.

6- The interpretation of the solution

From the figure 2 we see that the potential ψ tends to a constant at the infinite. In the classical Eddington solution the potential owns a logarithmic growth. The figure 3 shows the association of a cluster of matter, located in the region **σ** , sourrounded by a smooth hollow located in the region **σ***.

In both regions matter attracts matter. But the negative sign, from the field equation and the Poisson equation, makes the matter and the "antipodal matter" to repel each other. This helps the confinement of the cluster. For a given thermal velocity the necessary quantity of matter to balance the pressure force is smaller. The smooth halo acts like a corset.

A field equation provides a macroscopic description of the universe. It does not take account of the corpuscular nature of matter. The model implies that particles and antipodal-particles live in very distant, antipodal portions of space. In fact their natures are identical. The physical meaning of the field equation is the following: the particules and antipodal-particles interact by gravitational effect, but not by electromagnetic effect. We assume that the antipodal particles, clusters, rings, are not observable with a telescope, or a radiotelescope. The observation of antipodal structures should require some sort of gravitational telescope.

From equation (22) clusters can be located in the antipodal region. Then, associated large halos, sourrounding wide rarefied regions, should exist in the observable universe too. In fact they do, for it corresponds, in our mind, to the observed large scale structure of the universe: the galaxies seem to be arranged around large rarefied bubbles. According to our model, large clouds of antipodal matter should exist in the corresponding associated antipodal regions.

The universe was assumed to have a S3 x R1 topology. The reader has probabily some difficulties to understand this strange three-dimensional geometry. In fact the sphere S3 is simply shaped as the double cover of a projective space P3. In such arrangement each point **σ** of the sphere is associated to its antipode A(**σ**). The situation is similar for a sphere S2 covering a projective space P2, which can be represented in our space R3 as the well known Boy surface.

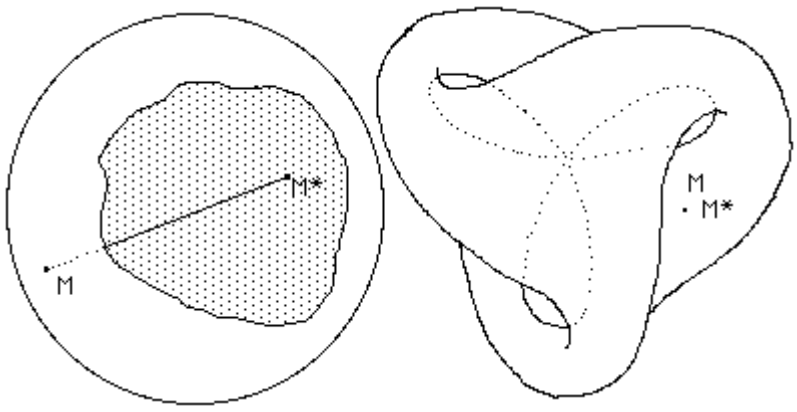


Figure 9: A couple of antipodal points on a sphere S2 and the Boy surface, image of the projective space P2

On the figure 10 we have figured the equator of a sphere and its location on the Boy Surface.

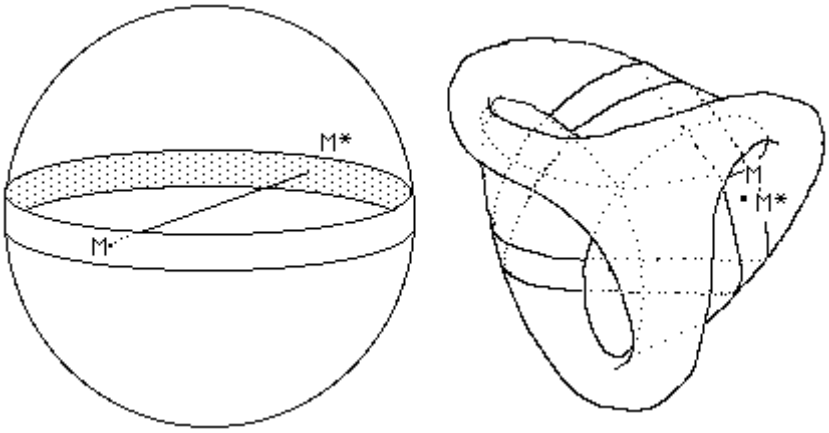


Figure 10: The vicinity of the equator of a 2-sphere and its location on a Boy surface.

The figure 11 shows how the equator of a S2 sphere can be glued on itself along a three half-turns Möbius belt. Locally the surface can be assimilated to a bundled manifold whose bundle owns two values + 1 and -1.

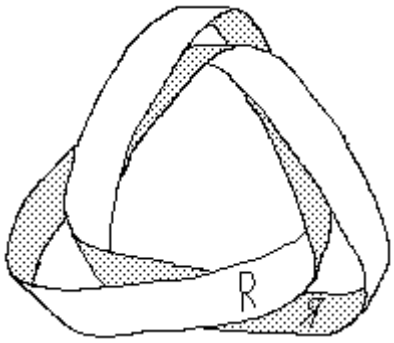


Figure 11: Enantiomorphic image corresponding to the cover of a Möbius belt.

In a 3-sphere S3, if one follows a geodesic, the antipodal point is at the half-way. If the 3-sphere is immersed in a four-dimensional space it is possible to make any point and its antipode to coincidate. These couples of points are associated through the antipodal diffential involutive maping A, but not identified.

As shown on the figure 12 we can proceed continously from a "gruyère" structure to a cluster structure. This peculiar feature was illustrated before, through 2d numerical simulations. When a region of space is put "in front" of the antipodal region, as suggested in the figure 12, the clusters nest in the holes.

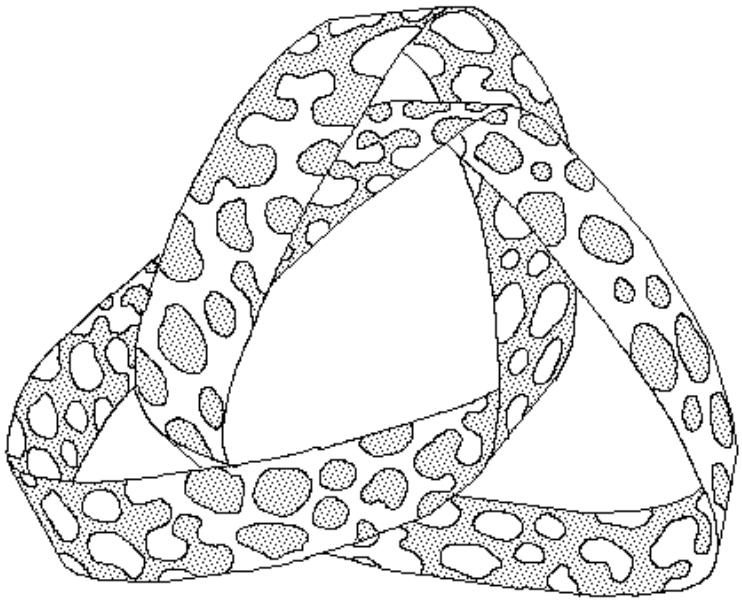


Figure 12: Two-dimensional image of the global large structure of the universe

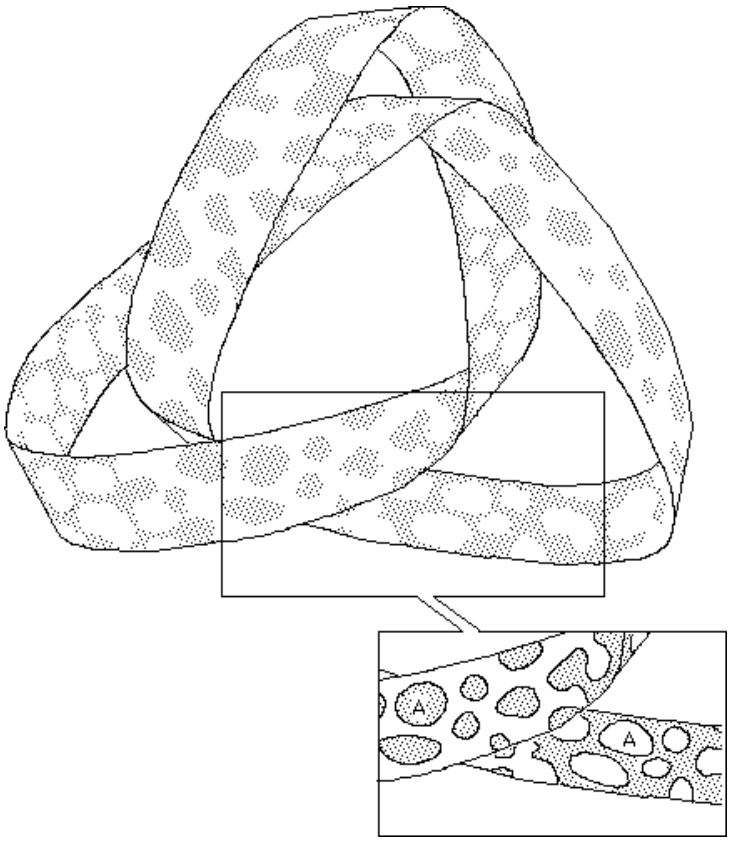


Figure 13: The interaction between two antipodal regions

This effect could act at the level of the galactic structure, as suggested in the figure 14, each galaxy nesting in a "hole" of the conjugated antipodal region.

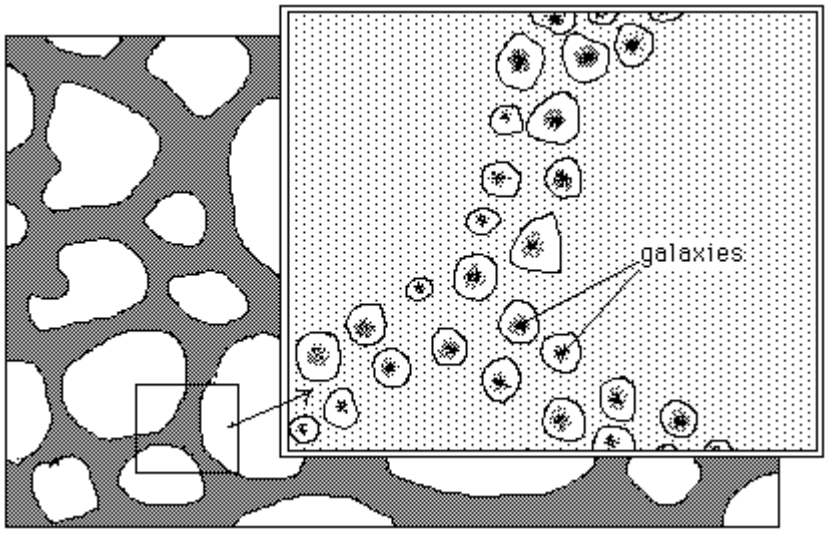


Figure 14: Smaller size structure

7- Some comments about the axioms

The classical General Relativity proposes a macroscopic description of the universe, shaped by the gravitational field. But, basically, the electromagnetic phenomena is not taken into account. In order to link this classical model to the observations, one has to bring the following additional axioms:

- The universe is filled by particles: neutral particles with a mass equal to m, and photons. Both contribute to the field.

- These particles move along geodesics of space-time

- A particle may send electromagnetic signal

- Another particle may receive this electromagnetic signal

- This electromagnetic signal, carried by photons, follows the null geodesics of space-time.

- A massive particle may send a gravitational signal, which is supposed to follow a null geodesic.

- A massive particle may receive this gravitational signal.

So that, for an observer composed by matter, the universe becomes optically perceptible, according to these axioms. The photons are the go-between bringing an optical message from a massive particle to another one.

In the present model the universe is be considered as a cover of a S3 sphere, locally we have a structure similar to a bundled manifold, whose bundle should be limited to two values: + 1 and -1.

Then we introduce the new following axioms.

- The universe is filled by particles: neutral particles whose mass is equal to m, and by photons. Both contribute to the field.

- The massive particles and the photon move along the geodesic of space time and cannot cross from a region to the conjugated antipodal region
of S3.

- A massive particle may send electromagnetic and gravitational signals, which can be received by another massive particle.

- The gravitational signal travels along the geodesics of space-time, but also along the geodesics of the "adjacent folds of the universe", "through the bundle structure" so that the gravitational signal owns some sort of ubiquity, because it acts both in a region of the manifold
(or in other terms in the "adjacent region", if we choose the bundled manifold image).

- The structure of the new field equation brings the following features.

If a gravitational signal is emitted and received by two particles which "belong to the same fold" the phenomenon identifies with the classical description.

But a gravitational signal emitted by a massive particle can be received by another particle located is the adjacent region (the antipodal region) , in other terms "through the bundle structure", the negative sign in the second member of the field equation changing the nature of the signal, as if it was emitted by a "negative mass".

- The electromagnetic signal follows the ordinary null geodesics of the manifold, but does not own this property of ubiquity. It cannot cross from a fold to the "adjacent fold through the bundle structure". To travel from a region of the manifold to the antipodal region, light has to do a complete half-turn of the S3 sphere.

We confess that this proposed geometric description remains primitive and somewhat unclear. A correct description should imply a more refined model, including the gravitational and electromagnetic phenomena, i.e. an unified theory, which does not exist presently.

The bundled manifold local description is similar to a 5d Kaluza model, in which the fifth dimension would be limited to two values + 1 and - 1, as suggested earlier by Alain Connes.

8- Estimation of the "missing mass effect"

Apply a perturbation method to the Euler equations:

(25)

$$\Psi = \Psi_o + \delta\Psi$$

(25')

$$\Psi^* = \Psi_o^* + \delta\Psi^*$$

with the first order solution:

(26)

$$\Delta\psi = \Delta\psi\, o = 0$$

The Poisson equation gives:

(27)

$$\Delta\delta\Psi = 4\pi G\rho_o \left(e^{-\frac{m\delta\Psi}{kT}} - e^{-\frac{m\delta\Psi^*}{kT}} \right)$$

(27')

$$\delta\rho = \rho_o e^{-\frac{m\delta\Psi}{kT}} \qquad \delta\rho^* = \rho_o e^{\frac{m\delta\Psi}{kT}}$$

$$\Delta\psi = -\Delta\psi^*$$

(28)

$$\Delta\delta\Psi + \frac{8\pi G\rho_o}{kT}\delta\Psi = 0$$

Lj is the classical Jeans length

(29)

$$L_j = \sqrt{\frac{kT}{4\pi G\rho_o}}$$

(30)

$$\Delta\delta\Psi + 2\frac{\delta\Psi}{L_j^2} = 0$$

This is the well known Helmoltz equation.

In classical steady-state approach we had

(31)

$$\Delta\delta\Psi + \frac{\delta\Psi}{L_j^2} = 0$$

The interaction with the antipodal region shortens the Jeans length by a factor 1.414 so that we have a confinement effect. If we have a positive concentration of matter $\delta\rho$ in our space-time fold, we will find a negative $\delta\rho^*$ in the associated antipodal region, and vive-versa. The confinement of the mass due to the action of the antipodal region should reduce the necessary mass to balance pressure or centrifugal force by a factor:

$$\frac{1}{2^{3/2}} = 0,353$$

9- Writing the equation into a complex form

Write
(32)

$\mathbf{S_1} = \mathbf{S}\ (x^\circ\ ,\ \boldsymbol{\sigma})$
$\mathbf{S_2} = \mathbf{S}\ (x^\circ\ ,\ \boldsymbol{\sigma}^*)$
$\mathbf{T_1} = \mathbf{T}\ (x^\circ\ ,\ \boldsymbol{\sigma})$
$\mathbf{T_2} = \mathbf{T}\ (x^\circ\ ,\ \boldsymbol{\sigma}^*)$
$\Sigma = \Sigma_1 + i\,\Sigma_2$
$\tau = \tau_1 + i\,\tau_2$
$\tau^c = \tau_1 - i\,\tau_2$

The equation (4) can be written
(33)

$$\Sigma = \omega\ (\ \tau - i\,\tau^c\ )$$

As suggested previously by Penrose, the quantification of the gravitation could be due to the complex form of the field equation. The equation (33) could perhaps bring a new insight on the problem.

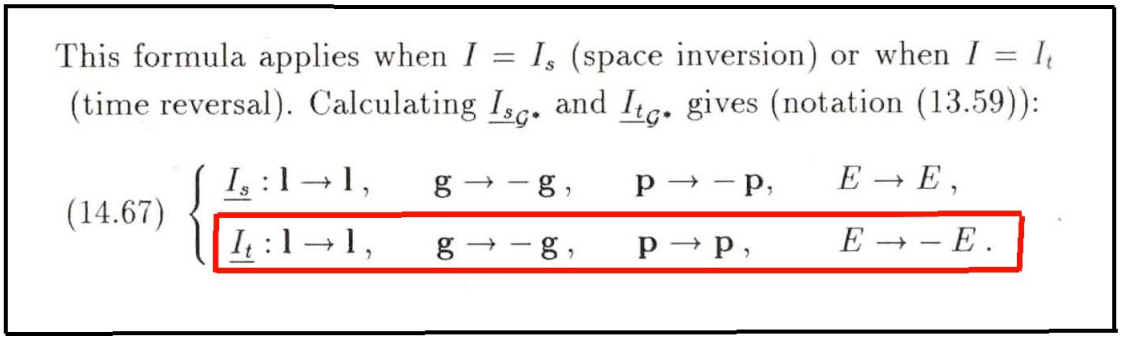
10- Conclusion

We propose a new field equation, from which, with the classical approximation: steady-state, weak fields, low velocities, we derive the associated Poisson equation. Coupled Eddington solutions give a set of clusters, associated to interacting ring-like clouds, located in the antipodal region. The antipodal halo-like structure repels the cluster and helps its confinment. The reduction factor is roughly evaluated. It is suggested, through 2d numerical simulations, that this model could explain the large structure of the universe. In addition, the interaction between a cluster and its associated antipodal structure could provide spiral structure. A collision of a cluster with an anti-cluster could also explain the very irreguliar galaxies.

References

[1] A.D. Sakharov, *ZhETF Pis'ma* 5: 32 (1967) ; *JETP Lett.* 5: 24 (1967) ; tr. Preprint R2-4267, *JINR*, Dubna.
[2] D. Novikov, *ZhETF Pis'ma* 3: 223 (1966) ; *JETP Lett.* 3: 142 (1966) ; tr. *Astr. Zh.* 43: 911 (1966), *Sov. Astr.* 10: 731 (1967).
[3] J.P. Petit: "Univers énantionmorphes à temps propres opposés" (*Enantiomorphic universes with opposite time arrows*), CRAS, t.263, Serie A, pp. 1315-1318, 23 May 1977.
[4] J.P. Petit: "*Univers en interaction avec leur image dans le miroir du temps*" (*Universes interacting with their opposite time-arrow fold*), CRAS, t.284, Serie A, pp. 1313-1316, 6 June 1977.
[5] J.P. Petit: "*Topologicon*", Belin Ed., Paris, 1984.

In Souriau's book (1970), I discovered the physical interpretation of time reversal—a reversal that pertains to the time coordinate.



This formula applies when $I = I_s$ (space inversion) or when $I = I_t$ (time reversal). Calculating $\underline{I_s}_{\mathcal{G}^*}$ and $\underline{I_t}_{\mathcal{G}^*}$ gives (notation (13.59)):

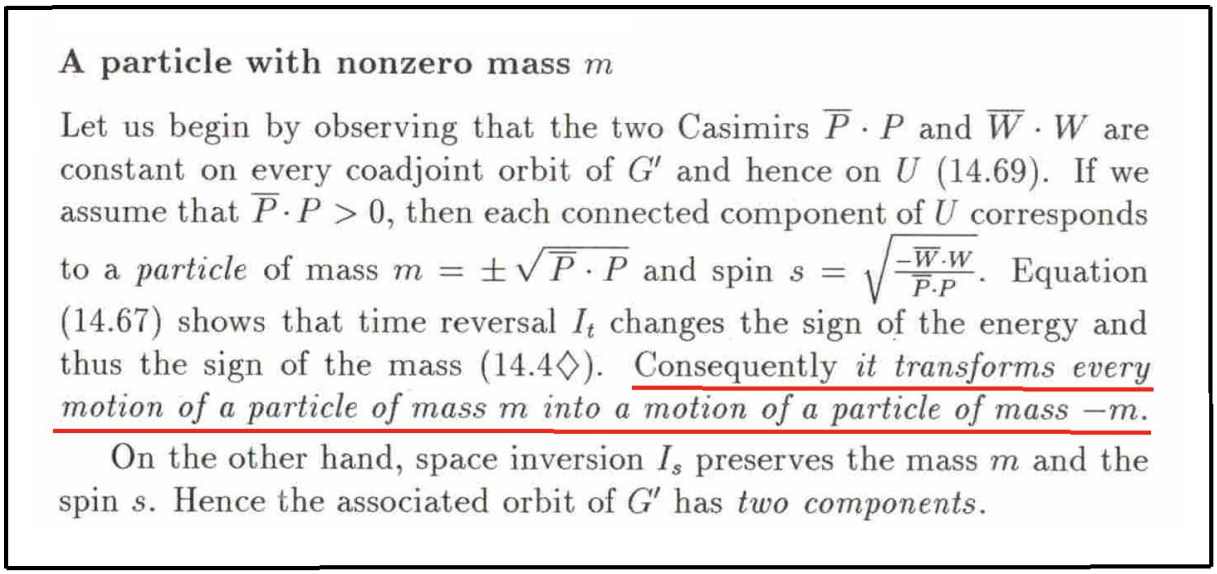
$$(14.67)\quad \begin{cases} \underline{I_s} : \mathbf{l} \to \mathbf{l}, \quad \mathbf{g} \to -\mathbf{g}, \quad \mathbf{p} \to -\mathbf{p}, \quad E \to E, \\ \underline{I_t} : \mathbf{l} \to \mathbf{l}, \quad \mathbf{g} \to -\mathbf{g}, \quad \mathbf{p} \to \mathbf{p}, \quad E \to -E. \end{cases}$$

Souriau's (1970) "magic formula"

This can be found on page 190 of the English version of mathematician Jean-Marie Souriau's book[21]. In the red box:

→ *Time reversal changes neither spin nor momentum, but reverses the energy E.*

On the following page:



A particle with nonzero mass m

Let us begin by observing that the two Casimirs $\overline{P} \cdot P$ and $\overline{W} \cdot W$ are constant on every coadjoint orbit of G' and hence on U (14.69). If we assume that $\overline{P} \cdot P > 0$, then each connected component of U corresponds to a *particle* of mass $m = \pm\sqrt{\overline{P} \cdot P}$ and spin $s = \sqrt{\frac{-\overline{W} \cdot W}{\overline{P} \cdot P}}$. Equation (14.67) shows that time reversal I_t changes the sign of the energy and thus the sign of the mass (14.4◇). Consequently *it transforms every motion of a particle of mass m into a motion of a particle of mass −m.*

On the other hand, space inversion I_s preserves the mass m and the spin s. Hence the associated orbit of G' has *two components*.

First inversion of mass (1970)

Translation:

[21] Jean-Marie Souriau. Structure of Dynamical Systems. Birkhäuser, 1997; an exact translation of the French edition *Structure des systèmes dynamiques*, Éditions Dunod, 1970.

→ *Consequently, this transforms the motion of a particle of mass m into the motion of a particle of mass* $\bar{m} = -m$ *.*

I discover this essential point in Souriau's writings:

The reversal of time entails the reversal of energy and mass

Thus, there exists a second world, where time is reversed, populated by particles similar to those in our own universe-sheet and which, in the gravitational field, behave as negative masses $\bar{m} = -m$of energy. They emit photons of negative energy$\bar{E} = \bar{m}\,\bar{c}^2$, which are not detected by our own telescopes, $\bar{c}$ since the speed of light is negative in the negative universe-sheet.

This is the central theme of the book and of the concept known as

Symplectic Geometry

I will now mention two publications that represent the extension of Souriau's approach to a bimetric, two-sheet configuration. This paper was published in 2025 in a journal of mathematical physics. The contribution of the co-author, Hicham Zejli, a Frenchman of Moroccan origin, was limited to secretarial duties. In 2023–2024, we had believed that he might become a genuine collaborator. We therefore did everything in our power to enable him to defend a dissertation at the University of Casablanca in Morocco.

His doctoral dissertation was an extension of an article I had published in 1972[22], presenting the first exact, nontrivial solution to the Vlasov equation under O(2) symmetry.

He had submitted a paper to the journal *Astrophysics and Space Science*.

Everything was going smoothly when I discovered that this work was the result of the verbatim plagiarism of a contribution by my collaborator and friend Gilles d'Agostini, dating back several years[23].

I had to contact the journal to explain the situation, and Hicham's name was replaced with that of Gilles d'Agostini. The article is reproduced below. From that point on, it was no longer possible to continue working with Hicham Zejli. The attached article appeared in the journal in 2025[24].

---

[22] Jean-Pierre Petit. A Method for Solving the Vlasov Equation. Application to a Global, Nonlinear Theory of Galactic Rotation and Spiral Structure. Note (in French) in the Comptes Rendus of the Paris Academy of Sciences, presented by A. Lichnérowicz. Series B, vol. 375, pp. 755–758 (November 13, 1962). Nonlinear solution of the Vlasov equation. Application to the dynamics of galaxies.

[23] http://www.jp-petit.org/papers/cosmo/2024-10-24-Pillage-travail-dAgostini-par-H.Zejli.pdf

[24] J.P. Petit, G. d'Agostini, G. Monnet. Contribution of Kinetic Theory to Galactic Dynamics. Astrophysics and Space Science 370:121–https://doi.org/10.1007/s10509-025-04511-x (October 2025). http://www.hp-petit.org/paper/cosmo/2025-04-15-Contribution-of-Kinetic-Theory-to-Galactic-Dynamics.pdf

The context is worth mentioning. When the journal's editor received the article, he submitted it to several reviewers. When asked about it three months later, he replied:

- *I have followed up with my reviewers several times, but I have been unable to obtain any feedback from them, whether positive or negative.*

- *That's to be expected. This work represents the first mathematical model of a galaxy. We approach the problem in a way that is the opposite of what is usually done. Since galaxies constitute a non-collisional system, their dynamics are governed by the Vlasov equation in steady state.*

$$\boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - \frac{\partial \Psi}{\partial \boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{v}} = 0$$

For a century[25], specialists have been striving to construct an approximate solution by specifying the form of the gravitational potential

$$f(r, \varphi, z) \leftarrow \Psi(r, \varphi, z)$$

Here, we do exactly the opposite. We start with the (elliptic) form of the solution function f, and then seek to determine the form of the gravitational potential $\Psi(r, \varphi, z)$ *and the resulting density distribution.*

$$f(r, \varphi, z) \longrightarrow \Psi(r, \varphi, z)$$

Furthermore, this calculation involves a mathematical technique that is unfamiliar to your referees: dyadic algebra. That is why they are unable to follow our calculations.

- *I see. But then, what should I do?*

- *You need to submit the article to mathematicians, not astrophysicists.*

The journal does indeed submit our calculations to mathematicians, who respond:

- *We are not qualified to assess the applications of this calculation in astrophysics. However, from a mathematical standpoint, the calculations are correct.*

The article is therefore published. The key result is the derivation of the rotation curve in galaxies, which exhibits a plateau at a certain distance:

[25] The Bible: (882 pages) J. Binney and S. Tremaine. Galactic Dynamics. Princeton Series in Astrophysics. 2008.

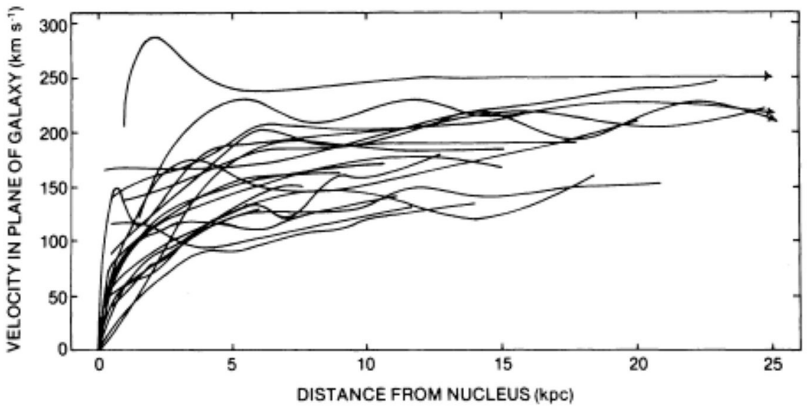
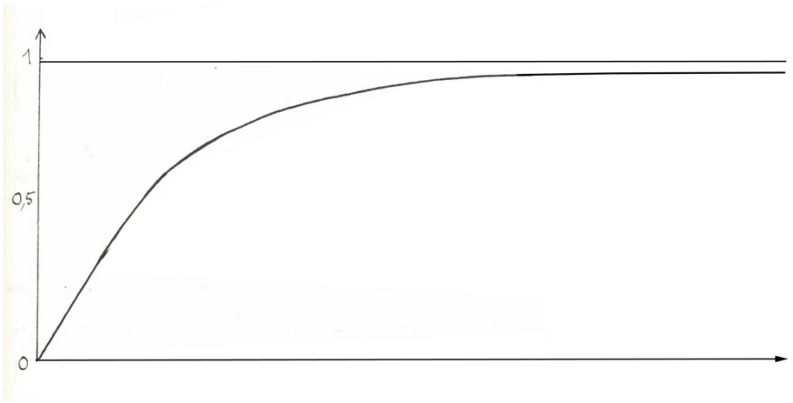


FIG. 6.—Superposition of all 21 Sc rotation curves. General form of rotation curves for small galaxies is similar to initial part of rotation curve for large galaxies, except that small galaxies often have shallower nuclear velocity gradient and tend to cover the low velocity range within the scatter at any R.

Rotation Curves (Observations)

Below is the theoretical rotation curve:



Theoretical Rotation Curve

This is an original and brilliant theoretical result. But this plateau at large distances reveals an important aspect. These high velocities at the periphery are possible due to the repulsive effect of the negative-mass environment.

What follows illustrates the dysfunction of scientific institutions on an international scale.

To finalize this first mathematical model of a galaxy, we must now calculate the $\rho(r\,,\varphi\,,z)$spatial density profile. We are drafting the calculations. But before finalizing them, I am consulting the editor:

- *We will send you the second part of this study. But we must warn you: the density ρ inverses with distance.*

- *So, if this turns out to be negative, then it's no longer physics (…). There's no point in having us review this paper; it won't be considered (…)*

I am putting an end to this work. There's no point in toiling away to produce a result that won't be reviewed.

Thus, the first mathematical model of a galaxy, set within a bimetric, twin-galaxy context, will likely not see the light of day in my lifetime (I will be 90 years old on April 5, 2027).

Astrophysics and Space Science (2025) 370:121
https://doi.org/10.1007/s10509-025-04511-x

RESEARCH

Contribution of the kinetic theory of gases to the dynamics of galaxies

J.-P. Petit[1,2] · G. D'Agostini[3] · G. Monnet[4]

Received: 25 April 2025 / Accepted: 14 October 2025
© The Author(s) 2025

Abstract
Classical attempts to construct a galaxy model, in a stationary and axisymmetric situation, consist of giving a gravitational field and injecting it into the collisionless Boltzmann equation to deduce the solution distribution function f. We will do exactly the opposite, by assimilating the galaxy to a self-gravitating point-mass system. The velocity distribution function is then the solution of an integrodifferential equation. Taking into account the Newtonian character of the potential, we can replace it with the system consisting of the Vlasov equation, written in terms of residual velocity, and the Poisson equation. We then give $ln(f)$ the form of a polynomial of degree 2, such that one of the axes of the velocity ellipsoid points towards the center of the system. This single constraint gives the evolution of the axes in space, these being equal to the center of the galaxy (Maxwell-Boltzmann distribution). Moving away from the center, the axis pointing in this direction remains constant while the transverse axes tend to zero at infinity. We then construct the macroscopic velocity field by excluding any vortex structure. This field then tends towards a solid body rotation at the center. The velocity tends towards a remote plateau, which is then consistent with the observational data.

Keywords Galactic dynamics · Ellipsoid of velocities · Vlasov equation · Elliptical solution · Evolution of the velocity at the periphery · Confinement · Dark halo · Janus cosmological model

1 Introduction

Today, there is an approach to dynamics that can be considered classic. It can be found in Chap. 4 of J. Binney and S. Tremaine's basic work "galactic dynamics" (Binney and Tremaine, 2008). Galaxies, considered as self-gravitating mass-point systems, are modeled using a velocity distribution function:

$$f(t,x,y,z,u,v,w) \equiv f(t,\mathbf{r},\mathbf{v}) \tag{1}$$

which is not negative, continuous, differentiable and defined in $\mathbb{R}\times\mathbb{R}^3\times\mathbb{R}^3$ where:

$$\mathbf{r}\equiv\begin{pmatrix}x\\y\\z\end{pmatrix} \qquad and \qquad \mathbf{v}\equiv\begin{pmatrix}u\\v\\w\end{pmatrix}$$

A function f is then assumed to obey the collisionless Boltzmann equation, which is written:

$$\frac{\partial f}{\partial t}+\mathbf{v}.\frac{\partial f}{\partial \mathbf{r}}-\frac{\partial \Psi}{\partial \mathbf{r}}.\frac{\partial f}{\partial \mathbf{v}}=0 \tag{2}$$

where $\Psi(\mathbf{r})$ in an arbitrarily given potential field corresponding to the problem.

The approach then focuses on Jeans' theorem:

Steady-state solutions of the Boltzmann collisionless equation depends on the phase space only through integrals of motion in a given potential, and any function of the integrals yields in a steady state solution of the collisionless Boltzmann equation.

Thus, the distribution function associated with the simplest model, that of the isothermal sphere, is presented as a special case of a polytropic model. In the treatment of

J.-P. Petit is retired from CNRS.

✉ J.-P. Petit
jean-pierre.petit@manaty.net

1 Manaty Research Group, Glanon, France
2 206 allée de la Montagnère, 84120 Pertuis, France
3 21 avenue Philippe de Rochat, 06600 Antibes, France
4 9 rue Adélaïde Perri, 69002 Lyon, France

Published online: 03 November 2025

this problem by S. Chandrasekhar (Chandrasekhar, 1941), the approach is radically different, in the sense that this distribution function, spherically symmetric in velocity space, is presented as a special case of an elliptic function. In fact, the collisionless Boltzmann equation can be written in terms other than absolute velocity components:

$$\mathbf{v} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \tag{3}$$

We'll refer to ourselves as **C**, the residual velocity:

$$\mathbf{C} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} \tag{4}$$

It's a velocity that kinetic gas theorists refer to as the "thermal agitation velocity". It's important to be clear about the notations we're going to use. Thus, the letter n designates the number of mass points per unit volume, deduced from the distribution function f by:

$$n(\mathbf{r}) = \int f(\mathbf{r}, \mathbf{v})\, d^3\mathbf{v} \tag{5}$$

Which differs from ref. Binney and Tremaine (2008). Multiplying by the mass m of the components gives the mass density $\rho = nm$. The macroscopic velocity is:

$$\mathbf{c}_0 = \frac{1}{n(\mathbf{r})} \int \mathbf{v}\, f\, d^3\mathbf{v} \equiv < \mathbf{v} > \tag{6}$$

And residual velocity is defined by:

$$\mathbf{C} = \mathbf{v} - \mathbf{c}_0 \tag{7}$$

We will follow the approach chosen by S. Chandrasekhar in his book. Unfortunately, he never uses vector notation. So his collisionless Boltzmann equation becomes, by adopting our notations:

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial z} - \frac{\partial \Psi}{\partial \mathbf{x}}\frac{\partial f}{\partial u} - \frac{\partial \Psi}{\partial \mathbf{y}}\frac{\partial f}{\partial v} - \frac{\partial \Psi}{\partial \mathbf{z}}\frac{\partial f}{\partial w} = 0 \tag{8}$$

We can write it in a more compact way:

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Psi}{\partial \mathbf{r}}.\frac{\partial f}{\partial \mathbf{v}} = 0 \tag{9}$$

Chandrasekhar then decided to concentrate on elliptic functions where $ln(f)$ is in the form of a polynomial of degree 3 according to the components U, V, W of the residual velocity. And he writes (with his notations):

$$ln(f) = aU^2 + bV^2 + cW^2 + 2dVW + 2gWU + 2hUV - 2\Delta_1 U - 2\Delta_2 V - 2\Delta_3 W - \chi \tag{10}$$

He therefore has ten unknown functions to determine. He then writes the Boltzmann collisionless equation in terms of the velocity of agitation (U, V, W). He then obtains a third-degree polynomial containing twenty terms. Since this equation must be independent of these residual velocity components, this leads him to a system of twenty second-order differential equations, the simple writing of which takes several pages. To this he adds the Poisson equation of the Newtonian gravitational potential:

$$\Delta\Psi = 4\pi G\rho \tag{11}$$

The construction of these elliptical solutions, where the logarithm of the distribution function is a polynomial of degree two as a function of the residual velocity components, and where the masses gravitate in their own gravitational field, therefore implies the construction of the solution emanating from a system of twenty-one non-linear differential equations. The potential being Newtonian, we derive the distribution function according to:

$$\Psi = Gm\int_x \frac{n(\mathbf{r})}{r} d^3r = Gm\int_x \frac{d^3r}{r}\int_v f(\mathbf{r}, \mathbf{v})\, d^3v \tag{12}$$

The gravitational field g, this time considered as created by the distribution function f, is then defined by:

$$\mathbf{g} = -\frac{\partial \Psi}{\partial r} = -Gm\frac{\partial}{\partial \mathbf{r}}\left[\int_x \frac{d^3\mathbf{r}}{r}\int_v f(\mathbf{r}, \mathbf{v})\, d^3v\right] \tag{13}$$

Mathematically speaking, we can summarize Chandrasekhar's approach by saying that he looked for functions f such that their logarithm is expressed as a polynomial of degree two as a function of the residual velocity, which is a solution of the integrodifferential equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{r}} - Gm\left(\frac{\partial}{\partial \mathbf{r}}\left[\int_x \frac{d^3\mathbf{r}}{r}\int_v f(\mathbf{r}, \mathbf{v}) d^3\mathbf{v}\right]\right).\frac{\partial f}{\partial \mathbf{v}} = 0 \tag{14}$$

The introduction of the solution, according to Chandrasekhar, in the form of a polynomial of degree 2, transforms the collisionless Boltzmann equation into a polynomial of degree three. If we restrict ourselves to a stationary solution:

- By cancelling out the degree-three terms, we deduce the evolution of the components of the velocity ellisoid.
- By cancelling out terms of degree two, we deduce the macroscopic velocity field.
- By cancelling out the unit-order terms, we deduce the potential in space and hence the density.
- When stationary, there are no zero-order terms.

At the end of this calculation, we don't have to ask whether the gravitational potential is compatible with such a solution, since it is an integral part of it. As we can see, Chandrasekhar's approach is the reverse of the conventional one. As early as the 1970s, we took up the question of how to construct these elliptical solutions, taking advantage of the computational techniques introduced in 1939 by S. Chapman and T.G. Cowling in their book "The Mathematical Theory of Non-Uniform Gases" (Chapman and Cowling, 1939). At the time, their notations were remarkably elegant and compact. Scalars are indicated by thin letters, and vectors by bold ones. But they also introduced dyadics, which are simple matrices composed using vectors:

$$\mathbf{ab} = \begin{pmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{pmatrix} \tag{15}$$

Gradient vectors are then themselves considered as dyadics, formed from the vector $\nabla = \dfrac{\partial}{\partial \mathbf{r}}$, being the position vector (x, y, z) and, for example, the gravitational potential Ψ. The scalar product of two vectors is always denoted $\mathbf{a}.\mathbf{b}$. But the authors introduce the scalar product of two dyadics noted $\mathbf{w} : \mathbf{w}'$ and defined by:

$$\mathbf{w} : \mathbf{w}' = \sum_{\alpha} \sum_{\beta} w_{\alpha\beta}\, w'_{\beta\alpha} = \mathbf{w}' : \mathbf{w} \tag{16}$$

The product of a dyadic matrix and a vector is also noted:

$$(\mathbf{ab}).\mathbf{d} \qquad and \qquad \mathbf{d}.(\mathbf{ab}) \tag{17}$$

It is then possible to use the set of theorems related to this algebra of dyadics, such as:

$$(\mathbf{ab}).\mathbf{d} = \mathbf{a}(\mathbf{b}.\mathbf{d}) \qquad \mathbf{d}.(\mathbf{ab}) = (\mathbf{d}.\mathbf{a})\mathbf{b} \tag{18}$$

$$\mathbf{ab} : \mathbf{cd} = \mathbf{a}.(\mathbf{b}.\mathbf{cd}) = \mathbf{a}.\{(\mathbf{b}.\mathbf{c})\mathbf{d}\} = (\mathbf{a}.\mathbf{d})(\mathbf{b}.\mathbf{c}) \tag{19}$$

$$\mathbf{ab} : \mathbf{cd} = \mathbf{ac} : \mathbf{bd} \tag{20}$$

$\mathbf{w}$ being a dyadic:

$$\mathbf{b}.(\mathbf{a}.\mathbf{w}) = \mathbf{ba} : \mathbf{w} \tag{21}$$

The authors also introduce a "mobile operator":

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{c}_0.\frac{\partial}{\partial \mathbf{r}} \tag{22}$$

The tedious collisionless Boltzmann equation then becomes:

$$\frac{\partial \ln(f)}{\partial t} + \mathbf{c}_0.\frac{\partial \ln(f)}{\partial \mathbf{r}} + \mathbf{C}.\frac{\partial \ln(f)}{\partial \mathbf{r}} - \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \frac{D\mathbf{c}_0}{Dt}\right).\frac{\partial \ln(f)}{\partial \mathbf{C}} - \frac{\partial \ln(f)}{\partial \mathbf{C}}\mathbf{C} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = 0 \tag{23}$$

In steady state:

$$\mathbf{c}_0.\frac{\partial \ln(f)}{\partial \mathbf{r}} + \mathbf{C}.\frac{\partial \ln(f)}{\partial \mathbf{r}} - \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \frac{\partial \mathbf{c}_0}{\partial t} + \mathbf{c}_0.\frac{\partial \mathbf{c}_0}{\partial \mathbf{r}}\right).\frac{\partial \ln(f)}{\partial \mathbf{C}} - \frac{\partial \ln(f)}{\partial \mathbf{C}}\mathbf{C} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = 0 \tag{24}$$

The terms $\frac{\partial \ln(f)}{\partial \mathbf{C}}\mathbf{C}$ and $\frac{\partial \mathbf{c}_0}{\partial \mathbf{r}}$ are dyadic matrices composed from two vectors. Note the remarkable compactness of this formulation. If we don't use it, the last term of the equation is itself transformed into nine terms, etc. $\frac{\partial \mathbf{c}_0}{\partial \mathbf{r}}$ is therefore the velocity gradient matrix, not to be confused with its divergence, which is written as:

$$\frac{\partial}{\partial \mathbf{r}}.\mathbf{c}_0 \tag{25}$$

Macroscopic parameters are defined as stochastic quantities. We have already seen the density n, above, and the macroscopic velocity $\mathbf{c}_0$. We define the pressure matrix $\mathbf{p}$, which is a dyadic, as:

$$\mathbf{p} = m \int \mathbf{CC}\, f\, d^3\mathbf{v} = nm < \mathbf{CC} > \tag{26}$$

With the dyadic:

$$\mathbf{CC} = \begin{pmatrix} U^2 & UV & UW \\ UV & V^2 & VW \\ UW & VW & W^2 \end{pmatrix} \tag{27}$$

The matrix is symmetrical. There is a third macroscopic quantity which, in fluid mechanics, is the absolute temperature T, which is then defined as the mean value of the kinetic energy associated with the thermal agitation velocity:

$$\frac{3}{2}k_B T = \frac{1}{2}\rho < C^2 >= \frac{1}{2}m \int C^2 f d^3\mathbf{v} \tag{28}$$

k_B being Boltzmann's constant. From the pressure matrix, we define a scalar pressure:

$$p = \rho\, tr < \mathbf{CC} >= \rho < C^2 >= nk_B T \tag{29}$$

And Boyle's law comes into play again. We add the heat flux vector, defined as the transport of thermal agitation energy:

$$\mathbf{q} = \frac{1}{2}m \int C^2\, \mathbf{C}\, f\, d^3\mathbf{v} = \frac{1}{2}\rho\, < C^2\mathbf{C} > \tag{30}$$

Entropy is also defined as the mean value of the quantity $Log f$:

$$s = \frac{1}{n} \int f\, \ln(f)\, d^3\mathbf{v} = < \ln(f) > \tag{31}$$

The collisionless Boltzmann equation also provides the conservation equations, which are greatly simplified by the use

of dyadics:

$$\frac{\partial \rho}{\partial t} + \mathbf{c}_0 . \frac{\partial \rho}{\partial \mathbf{r}} + \rho \frac{\partial}{\partial \mathbf{r}} . \mathbf{c}_0 = 0 \tag{32}$$

$$\frac{\partial}{\partial \mathbf{r}} . \mathbf{p} + \rho \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \frac{\partial \mathbf{c}_0}{\partial t} + \mathbf{c}_0 . \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} \right) \tag{33}$$

$$\frac{\partial T}{\partial t} + \mathbf{c}_0 . \frac{\partial T}{\partial \mathbf{r}} + \frac{2}{3 k_B n} \left(\mathbf{p} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{r}} . \mathbf{q} \right) \tag{34}$$

The term $\mathbf{p} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}}$ represents the work of the pressure forces. Of course these equations simplify when the medium is in a state of local thermodynamic equilibrium, i.e. when the distribution function has Maxwellian form:

$$f^0 = n \left(\frac{m}{2\pi k_B T} \right)^{3/2} e^{-\frac{mC^2}{2k_B T}} \tag{35}$$

In other words, $\ln(f)$ is a spherical polynomial. The equations then become Euler's equations. If the medium is homogeneous and uniform, the unsteady solution gives the Friedman equation, and we find (Petit and Monnet, 1975) the Newtonian cosmology discovered in 1934 by Milne and Mc Crea (McCrea and Milne, 1934). We will use this particular case of the Maxwellian solution, already treated by Chandrasekhar (Chandrasekhar, 1941) to show the path followed when we center the solution on the simple form of the distribution function. We have:

$$\ln(f^0) = Cst - \frac{mC^2}{2k_B T} + \ln\left(\frac{n}{T^{3/2}}\right) \tag{36}$$

We introduce this form of the distribution function into the collisionless Boltzmann equation in a stationary situation. The equation becomes:

$$(\mathbf{C} + \mathbf{c}_0) . \left(\frac{mC^2}{2k_B T} \frac{\partial T}{\partial \mathbf{r}} + \frac{\partial}{\partial \mathbf{r}} \ln\left(\frac{n}{T^{3/2}}\right) \right) + \frac{m\mathbf{C}}{k_B T} . \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \mathbf{c}_0 . \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} \right) + \frac{m}{k_B T} \mathbf{CC} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = 0 \tag{37}$$

We have a single third-order term:

$$\frac{mC^2}{2k_B T} \left(\mathbf{C} . \frac{\partial T}{\partial \mathbf{r}} \right) = 0 \tag{38}$$

The medium is therefore isothermal. Transposed to astrophysics, this means that the dispersion of mass-point velocities is constant throughout space. We now turn to second-order terms. In dyadic algebra, this is written:

$$\mathbf{CC} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = 0 \tag{39}$$

By expanding, we have nine terms. The solution to this system of differential equations is:

$$\mathbf{c}_0 = \mathbf{v}_0 + \boldsymbol{\omega} \times \mathbf{r} \tag{40}$$

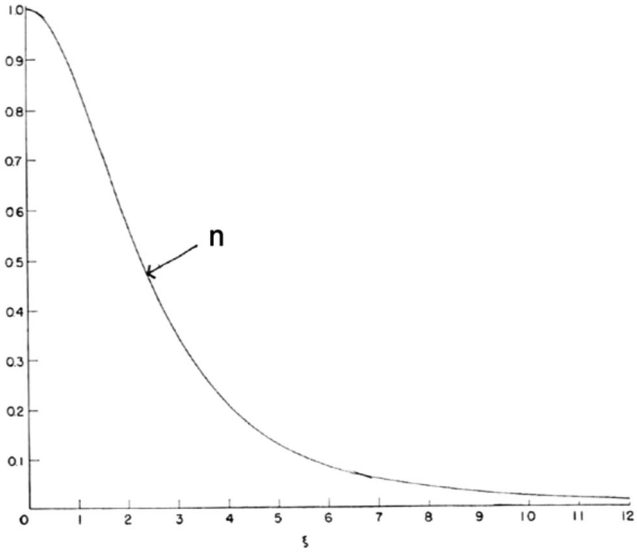


Fig. 1 Mass distribution of an isothermal, spheroidal formation, according to Chandrasekhar

Assuming $\mathbf{v}_0 = 0$, we obtain a solid-body rotation with constant angular velocity ω. The unit order terms give:

$$\frac{\partial \ln(n)}{\partial \mathbf{r}} + \frac{m}{k_B T} \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \mathbf{c}_0 . \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} \right) = 0 \tag{41}$$

Equation which states that the gravitational field is balanced by the combination of pressure force plus centrifugal force. We have the following equation:

$$\mathbf{c}_0 = \omega \begin{pmatrix} -y \\ x \\ 0 \end{pmatrix} \tag{42}$$

Let's introduce the vector:

$$\mathbf{u} = \omega \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} \tag{43}$$

We get:

$$\frac{\partial}{\partial \mathbf{r}} . \left(\ln(n) + \frac{m}{k_B T} \left(\Psi - \frac{1}{2} u^2 \omega^2 \right) \right) \tag{44}$$

Whence:

$$n = n_0 \, e^{\frac{m}{k_B T} \Psi + \frac{m}{2k_B T} u^2 \omega^2} \tag{45}$$

Combining with the Poisson equation:

$$\Delta \Psi = 4\pi G n_0 \, e^{\frac{m}{k_B T} \Psi + \frac{m}{2k_B T} u^2 \omega^2} \tag{46}$$

From which we derive the gravitational potential and mass distribution (Fig. 1).

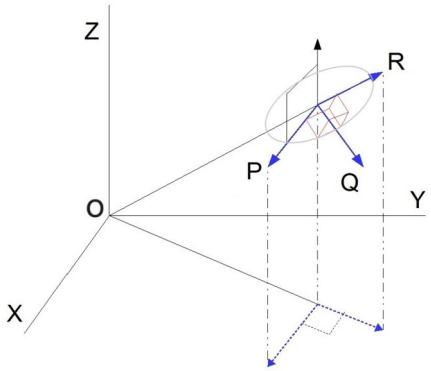


Fig. 2 One of the axes of the velocity ellipsoid pointing towards the geometric center

Which is nothing other than the result obtained by Chandrasekhar, with a heavier handwriting. After this long preamble, we are now in a position to understand the strategy of the present article, whose project is to construct a model of a galaxy as an exact stationary solution of the collisionless Boltzmann equation, having as its sole starting point the form of the distribution function, and exploiting the computational technique of dyadic algebra. Under these conditions, $\ln(f)$ becomes a particular polynomial of order two, depending on the components (U, V, W) of the residual velocity. By introducing this expression into the equation written in terms of the components of the residual velocity, and setting its various polynomials to zero, we obtain a system of twenty non-linear differential equations, to which Poisson's equation is added. As with the Maxwellian solution, the third-order terms will give the evolution of the velocity ellipsoid in space. The second-order terms will give the velocity field and the first-order terms, combined with the Poisson equation, the gravitational potential and mass distribution. In stationary mode, there are no zero-order terms. So there's no need to ask whether this potential is compatible with this distribution function, since it follows from it as an integral part of a solution, based solely on the form of this function.

2 Extension to a steady axisymmetric elliptical solution

The hypothesis that the logarithm of the distribution function is a polynomial of degrees 2 depending on the components of the residual velocity, such that one of the axes of the ellipsoid points towards the origin was proposed in 1972 by J.P. Petit (Petit, 1972), then taken up again in 1974 in reference (Petit and Monnet, 1974). In this model one of the principal axes of the velocity ellipsoid points towards the geometric center of the system, which corresponds to Fig. 2.

In this figure the vector **P** is parallel to the plane $(z = 0)$. We can define speed dispersions according to:

$$\begin{aligned}\sigma_P &= \langle C_P^2 \rangle \\ \sigma_Q &= \langle C_Q^2 \rangle \\ \sigma_R &= \langle C_R^2 \rangle\end{aligned} \tag{47}$$

Still following reference (Petit, 1972):

$$\ln(f) = \ln(B) - \frac{m}{2k_B H}\mathbf{C}^2 + a\,(\mathbf{C}.\mathbf{r})^2 + \alpha\,[\mathbf{C}.(\mathbf{k}\times\mathbf{r})]^2 \tag{48}$$

The functions B, H, α, a are functions of the position and will then have to be determined. We only have data referring to the velocity ellipsoid in the vicinity of the Sun. Its major axis then does not point towards the center but in its vicinity. We will attribute this small deviation, called the vertex deviation, to the influence of the local fluctuation of the gravitational field linked to the presence of the spiral arm. When we introduce the form of the distribution function (48) into equation (24), the latter becomes a polynomial of degree 3 as a function of the components (U, V, W) of the residual velocity. Since this equation must be satisfied regardless of these components, we then obtain a system of twenty partial differential equations to which we must add Poisson's equation (11).

3 The terms of degree three give the evolution of the velocity ellipsoid (Petit and Monnet, 1975)

Let's group these terms together:

$$\begin{aligned}&-\frac{m}{2k_B}C^2 C.\frac{\partial}{\partial r}(\frac{1}{H}) + 2a(C.r).C.C + C.\frac{\partial a}{\partial r}(C.r)^2 \\ &\quad + C.\frac{\partial \alpha}{\partial r}[C.(k\times r)]^2 = 0\end{aligned} \tag{49}$$

$$\begin{aligned}&-\frac{m}{2k_B}\left(C_x^2 + C_y^2 + C_z^2\right) \\ &\times\left(C_x\frac{\partial}{\partial x}\left(\frac{1}{H}\right) + C_y\frac{\partial}{\partial y}\left(\frac{1}{H}\right) + C_z\frac{\partial}{\partial z}\left(\frac{1}{H}\right)\right) \\ &+ 2a(C_x^2 + C_y^2 + C_z^2)(xC_x + yC_y + zC_z) \\ &+\left(C_x\frac{\partial a}{\partial x} + C_y\frac{\partial a}{\partial y} + C_z\frac{\partial a}{\partial z}\right)(xC_x + yC_y + zC_z)^2 \\ &+\left(C_x\frac{\partial \alpha}{\partial x} + C_y\frac{\partial \alpha}{\partial y} + C_z\frac{\partial \alpha}{\partial z}\right)(-yC_x + xC_y)^2 = 0\end{aligned} \tag{50}$$

Whence:

$$C_x^3 : -\frac{m}{2k_B}\frac{\partial}{\partial x}\left(\frac{1}{H}\right) + 2ax + x^2\frac{\partial a}{\partial x} + y^2\frac{\partial \alpha}{\partial x} = 0 \tag{51}$$

$$C_y^3 : -\frac{m}{2k_B}\frac{\partial}{\partial y}\left(\frac{1}{H}\right) + 2ay + y^2\frac{\partial a}{\partial y} + x^2\frac{\partial \alpha}{\partial y} = 0 \tag{52}$$

$$C_z^3 : -\frac{m}{2k_B}\frac{\partial}{\partial z}\left(\frac{1}{H}\right) + 2az + z^2\frac{\partial a}{\partial z} = 0 \tag{53}$$

$$C_x^2 C_y : -\frac{m}{2k_B}\frac{\partial}{\partial y}(\frac{1}{H}) + 2ay + x^2\frac{\partial a}{\partial y} + 2xy\frac{\partial a}{\partial x} + y^2\frac{\partial \alpha}{\partial y} - 2xy\frac{\partial \alpha}{\partial x} = 0 \tag{54}$$

$$C_y^2 C_x : -\frac{m}{2k_B}\frac{\partial}{\partial x}\left(\frac{1}{H}\right) + 2ax + y^2\frac{\partial a}{\partial x} + 2xy\frac{\partial a}{\partial y} + x^2\frac{\partial \alpha}{\partial x} - 2xy\frac{\partial \alpha}{\partial y} = 0 \tag{55}$$

$$C_x^2 C_z : -\frac{m}{2k_B}\frac{\partial}{\partial z}\left(\frac{1}{H}\right) + 2az + x^2\frac{\partial a}{\partial z} + 2xz\frac{\partial a}{\partial x} + y^2\frac{\partial \alpha}{\partial z} = 0 \tag{56}$$

$$C_y^2 C_z : -\frac{m}{2k_B}\frac{\partial}{\partial z}\left(\frac{1}{H}\right) + 2az + y^2\frac{\partial a}{\partial z} + 2yz\frac{\partial a}{\partial x} + x^2\frac{\partial \alpha}{\partial z} = 0 \tag{57}$$

$$C_z^2 C_x : -\frac{m}{2k_B}\frac{\partial}{\partial x}\left(\frac{1}{H}\right) + 2ax + z^2\frac{\partial a}{\partial x} + 2xz\frac{\partial a}{\partial z} = 0 \tag{58}$$

$$C_z^2 C_y : -\frac{m}{2k_B}\frac{\partial}{\partial y}\left(\frac{1}{H}\right) + 2ay + z^2\frac{\partial a}{\partial y} + 2yz\frac{\partial a}{\partial z} = 0 \tag{59}$$

$$C_x C_y C_z : 2yZ\frac{\partial a}{\partial x} + 2xz\frac{\partial a}{\partial y} + 2xy\frac{\partial a}{\partial z} - 2xy\frac{\partial \alpha}{\partial z} \tag{60}$$

Assuming that a and α do not depend on r or z, we get (D'Agostini, 2016):

$$C_x^3 \equiv C_z^2 C_x \equiv C_y^2 C_x : -\frac{m}{2k_B}\frac{\partial}{\partial x}\left(\frac{1}{H}\right) + 2ax = 0 \tag{61}$$

$$C_y^3 \equiv C_x^2 C_y \equiv C_z^2 C_y : -\frac{m}{2k_B}\frac{\partial}{\partial y}\left(\frac{1}{H}\right) + 2ay = 0 \tag{62}$$

$$C_z^3 \equiv C_x^2 C_z \equiv C_y^2 C_z : -\frac{m}{2k_B}\frac{\partial}{\partial z}\left(\frac{1}{H}\right) + 2az = 0 \tag{63}$$

In addition we have:

$$\rho^2 = x^2 + y^2 \Rightarrow \begin{cases} \dfrac{\partial \rho^2}{\partial x} = 2x \\ \dfrac{\partial \rho^2}{\partial y} = 2y \end{cases} \tag{64}$$

We get:

$$C_x^3 \equiv C_z^2 C_x \equiv C_y^2 C_x : -\frac{m}{2k_B}\frac{\partial}{\partial \rho^2}\left(\frac{1}{H}\right) + a = 0 \tag{65}$$

$$C_y^3 \equiv C_x^2 C_y \equiv C_z^2 C_y : -\frac{m}{2k_B}\frac{\partial}{\partial \rho^2}\left(\frac{1}{H}\right) + a = 0 \tag{66}$$

$$C_z^3 \equiv C_x^2 C_z \equiv C_y^2 C_z : -\frac{m}{2k_B}\frac{\partial}{\partial z^2}\left(\frac{1}{H}\right) + a = 0 \tag{67}$$

That is, after integration:

$$\frac{\partial}{\partial \rho^2}\left(\frac{1}{H}\right) = \frac{m}{2k_B}a \Rightarrow \frac{1}{H} = \frac{m}{2k_B}a\rho^2 + f_1(z^2) \tag{68}$$

$$\frac{\partial}{\partial z^2}\left(\frac{1}{H}\right) = \frac{m}{2k_B}a \Rightarrow \frac{1}{H} = \frac{m}{2k_B}az^2 + f_2(\rho^2) \tag{69}$$

The function f_1 depends only on z^2. So, if we differentiate (68) with respect to z^2 we can write:

$$\frac{\partial}{\partial z^2}\left(\frac{m}{2k_B}a\rho^2 + f_1(z^2)\right) = \frac{\partial}{\partial z^2}f_1(z^2) = \frac{\partial}{\partial z^2}\left(\frac{1}{H}\right) = \frac{m}{2k_B}a \tag{70}$$

Thus:

$$f_1(z^2) = \frac{m}{2k_B}a\rho^2 + k_z \tag{71}$$

Whence:

$$\frac{1}{H} = \frac{m}{2k_B}a\rho^2 + \frac{m}{2k_B}az^2 + k_z \tag{72}$$

Originally we have $\frac{1}{H} = k_z$ that we set equal to the constant $\frac{1}{H} = \frac{1}{T_0}$ which gives:

$$\frac{1}{H} = \frac{m}{2k_B}a\rho^2 + \frac{m}{2k_B}az^2 + \frac{1}{T_0} \tag{73}$$

By introducing the characteristic length r_o we get:

$$r_0^2 = \frac{m}{2ak_BT_0} \Rightarrow H = \frac{T_0}{1 + \dfrac{r^2}{r_0^2}} \tag{74}$$

We will now decompose the residual velocity vector C according to its projections on the P, Q, R axes.

$$C_r = C.R = C.\frac{r}{\|r\|} = \frac{C.r}{\|r\|} \tag{75}$$

$$C_p = C.P = C.\frac{k \times r}{\|k \times r\|} = \frac{C.(k \times r)}{\|k \times r\|} \tag{76}$$

$$C_q = C - C_r - C_p \tag{77}$$

$$\|k \times r\|^2 = (|k \times r).(|k \times r) = (k.k)(r.r) - (k.r)(k.r) = r^2 - z^2 = \rho^2 \tag{78}$$

$$\ln(f) = \ln(B) + a_r C_r^2 + a_p C_p^2 + a_q C_q^2 \tag{79}$$

$$\ln(f) = \ln(B) + \frac{(a_r - a_q)}{r^2}(C.r)^2 + \frac{(a_p - a_q)}{\rho^2}[C.(k \times r)]^2 + a_q C^2 \tag{80}$$

By identifying:

$$a_q = -\frac{m}{2k_B H} \tag{81}$$

$$a = \frac{(a_r - a_q)}{r^2} \rightarrow a_q = -\frac{m}{2k_B H} + ar^2 \tag{82}$$

$$\alpha = \frac{(a_p - a_q)}{\rho^2} \rightarrow a_p = -\frac{m}{2k_B H} + a\rho^2 \tag{83}$$

Let's ask:

$$\rho_0^2 = \frac{m}{2\alpha k_B T_0} \tag{84}$$

We get:

$$a_r = -\frac{m}{2k_B T_0} \tag{85}$$

$$a_p = -\frac{m}{2k_B T_0}\left(1 + \frac{r^2}{r_0^2} - \frac{\rho^2}{\rho_0^2}\right) \tag{86}$$

$$a_q = -\frac{m}{2k_B T_0}\left(1 + \frac{r^2}{r_0^2}\right) \tag{87}$$

We can then express the velocity distribution function as a function of the coordinates (C_r, C_p, C_q):

$$f = f_0\, e^{\left(-\frac{m}{2k_B T_0}\left[C_r^2 + C_p^2\left(1 + \frac{r^2}{r_0^2} - \frac{\rho^2}{\rho_0^2}\right) + C_q^2\left(1 + \frac{r^2}{r_0^2}\right)\right]\right)} \tag{88}$$

with:

$$f_0 = n\left(\frac{m}{2\pi k_B T_0}\right)^{\frac{3}{2}}\left(1 + \frac{r^2}{r_0^2}\right)^{\frac{1}{2}}\left(1 + \frac{r^2}{r_0^2} - \frac{\rho^2}{\rho_0^2}\right)^{\frac{1}{2}} \tag{89}$$

Which gives us the axes of the velocity ellipsoid:

$$\sigma_r = \sqrt{\frac{2k_B T_0}{m}} \tag{90}$$

$$\sigma_p = \sqrt{\frac{2k_B T_0}{m}\frac{1}{1 + \frac{r^2}{r_0^2} - \frac{\rho^2}{\rho_0^2}}} \tag{91}$$

$$\sigma_p = \sqrt{\frac{2k_B T_0}{m}\frac{1}{1 + \frac{r^2}{r_0^2}}} \tag{92}$$

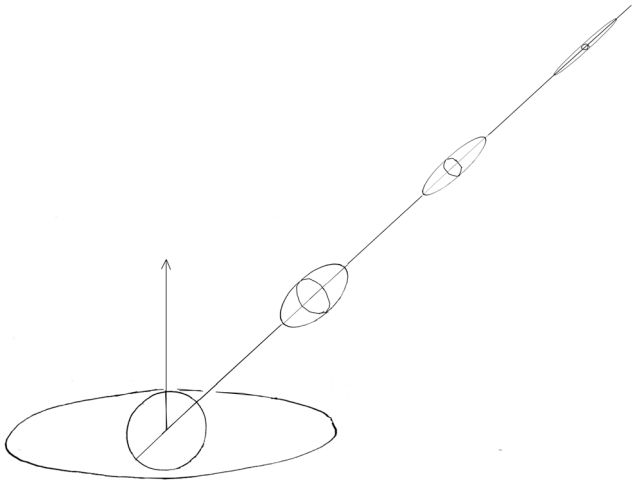


Fig. 3 Evolution of the axes of the velocity ellipsoid

We see that:

$$\sigma_p \leq \sigma_q \leq \sigma_r \tag{93}$$

At the center of the system, the distribution of Maxwellian speeds:

$$\sigma_p = \sigma_q = \sigma_r \tag{94}$$

The component σ_r is independent of r. If we want to take into account the observational data, these give $\sigma_p \approx \sigma_q$. This will mean, in our solution, that $\frac{1}{\rho_0^2} \ll \frac{1}{r_0^2}$ i.e. $\alpha \ll a$. At large distance the transverse axes tend towards zero, while the radial axis retains its value (Fig. 3).

In doing so, we are only repeating identically the calculations of references (Petit, 1972) and (Petit and Monnet, 1974). In 2016 reference (D'Agostini, 2016) introduced a symmetry that had escaped the authors, and which leads to an interesting result.

4 The terms of order 2 provide the macroscopic velocity field (D'Agostini, 2016)

These are the terms from the expression:

$$\mathbf{c}_0 . \frac{\partial \ln(f)}{\partial \mathbf{r}} - \frac{\partial \ln(f)}{\partial \mathbf{C}} \mathbf{C} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = 0 \tag{95}$$

Mathematically, the velocity field, resulting from hypothesis (47) must emanate from equation (95). In all cases, the gradient $\frac{\partial \ln(f)}{\partial \mathbf{r}}$, located in a plane containing oz, due to the axisymmetry, contains a priori components parallel and perpendicular to the oz axis. Similarly, there should be components $\mathbf{c}_{0r}$ and $\mathbf{c}_{0z}$ also located in a plane passing through this

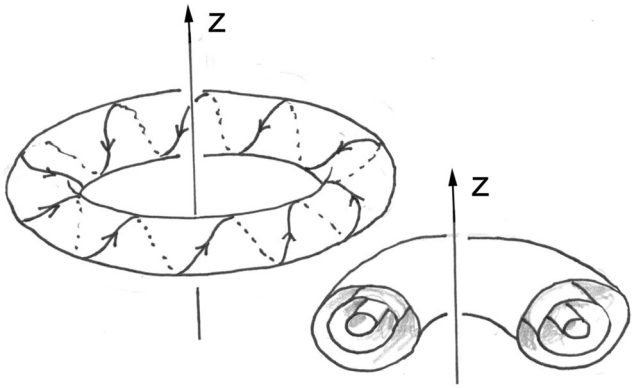


Fig. 4 3D velocity fields on a family of surfaces with nested torus topology

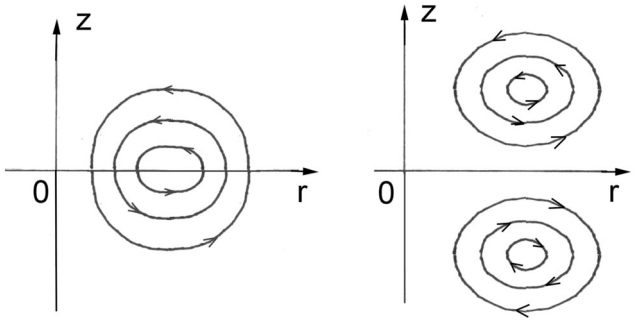


Fig. 5 Projection of velocity fields with non-zero $\mathbf{c}_{0z}$. $\frac{\partial \ln(f)}{\partial \mathbf{r}}$

axis. But the non-zero nature of these components does not seem compatible with a physical situation. Indeed, these two components, depending on their sign, translate movements of distance and approach with respect to, respectively, the oz and or axes. Movements that cannot correspond, exclusively, either to the distance or to the approach, because this would translate either to a loss or to a gain of matter. There would therefore be no conservation of mass. These components must therefore, in this velocity field, have values of both signs. The only velocity field ensuring the conservation of mass corresponds to 3D helical trajectories falling on a family of surfaces having the topology of nested tori (Fig. 4).

If we opt for the assumption that this term is non-zero, we're out of the realm of galactic dynamics. The solution, with its macroscopic velocity field in the form of helical trajectories inscribed on a family of surfaces with the oz axis as the axis of symmetry and the horizontal plane as the plane of symmetry, then refers to the trajectories of electrically charged particles in a tokamak (we would then have to consider two Vlasov equations, one for electrons and one for hydrogen ions). The law of velocity variation with distance from the axis would then follow a different law. But these are two completely different physics problems.

The projection of these trajectories along the plane passing through the oz axis corresponds to Fig. 5.

But if we add the natural hypothesis of a symmetry with respect to the plane $z = 0$ this excludes the left field. Continuing to consider the existence of non-zero $\mathbf{c}_{0r}$ and $\mathbf{c}_{0z}$ components we should then consider the structure of the figure on the right. But, a simple physicist's intuition suggests that the most plausible solution would be the one where these components $\mathbf{c}_{0r}$ and $\mathbf{c}_{0z}$ are simply zero. In these conditions the term $\mathbf{c}_0.\nabla_r \ln(f)$ becomes zero and the equation with the terms of order two is reduced to:

$$\frac{\partial \ln(f)}{\partial \mathbf{C}}.\mathbf{C} : \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = Tr(\mathbf{AB}) = 0 \tag{96}$$

Let's calculate the dyadic $\mathbf{B}$:

$$\mathbf{B} = \frac{\partial \mathbf{c}_0}{\partial \mathbf{r}} = \begin{pmatrix} \frac{\partial c_{0x}}{\partial x} & \frac{\partial c_{0y}}{\partial x} & 0 \\ \frac{\partial c_{0x}}{\partial y} & \frac{\partial c_{0y}}{\partial y} & 0 \\ \frac{\partial c_{0x}}{\partial z} & \frac{\partial c_{0y}}{\partial z} & 0 \end{pmatrix}$$
$$= \begin{pmatrix} -y\frac{\partial \omega}{\partial x} & x\frac{\partial \omega}{\partial x} + \omega & 0 \\ -y\frac{\partial \omega}{\partial y} - \omega & x\frac{\partial \omega}{\partial y} & 0 \\ -y\frac{\partial \omega}{\partial z} & x\frac{\partial \omega}{\partial z} & 0 \end{pmatrix} \tag{97}$$

Let's calculate the dyadic $\mathbf{A}$:

$$A = \frac{\partial \ln(f)}{\partial \mathbf{C}}.C \tag{98}$$

$$A = -\frac{m}{2k_B H} C.C + 2a(C.r).r.C + 2\alpha[C.(k \times r)].(k \times r)$$
$$= A_1 + A_2 + A_3 \tag{99}$$

$$A_1 = -\frac{m}{2k_B H} \begin{pmatrix} C_x^2 & C_x C_y & C_x C_z \\ C_y C_x & C_y^2 & C_y C_z \\ C_z C_x & C_z C_y & C_z^2 \end{pmatrix} \tag{100}$$

$$A_2 = 2a(xC_x + yC_y + zC_z) \begin{pmatrix} xC_x & xC_y & xC_z \\ yC_x & yC_y & yC_z \\ zC_x & zC_y & zC_z \end{pmatrix} \tag{101}$$

$$A_3 = 2\alpha \begin{pmatrix} y^2 C_x - xyC_y \\ -xyC_x + x^2 C_y \\ 0 \end{pmatrix} (C_x C_y C_z)$$
$$= 2\alpha \begin{pmatrix} y^2 C_x C_x - xyC_y C_x & y^2 C_x C_y - xyC_y C_y \\ -xyC_x C_x + x^2 C_y C_x & -xyC_x C_y + x^2 C_y C_y \\ 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} y^2 C_x C_z - xyC_y C_z \\ -xyC_x C_z + x^2 C_y C_z \\ 0 \end{pmatrix} \tag{102}$$

Consider the following two matrices

$$\mathbf{A}=\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \quad and \quad \mathbf{B}=\begin{pmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{pmatrix} \tag{103}$$

The trace of their matrix product is given by

$$\begin{aligned} Tr(\mathbf{AB}) &= (A_{xx}B_{xx}+A_{xy}B_{yx}+A_{xz}B_{zx}) \\ &\quad +(A_{yx}B_{xy}+A_{yy}B_{yy}+A_{yz}B_{zy})+(0) \end{aligned} \tag{104}$$

Let's calculate each of the terms:

$$\begin{aligned} A_{xx} &= -\frac{m}{2k_BH}C_x^2+2a(x^2C_x^2+yxC_xC_y+zxC_xC_z) \\ &\quad +2\alpha(y^2C_x^2-xyC_yC_x) \\ &= C_x^2\left(-\frac{m}{2k_BH}+2ax^2+2\alpha y^2\right) \\ &\quad +C_xC_y2(a-\alpha)xy+C_xC_z2axz \end{aligned} \tag{105}$$

$$\begin{aligned} A_{yy} &= -\frac{m}{2k_BH}C_y^2+2a(xC_xyC_y+yC_yyC_y+zC_zyC_y) \\ &\quad +2\alpha(-xyC_xCy+x^2C_yC_y) \\ &= C_y^2\left(-\frac{m}{2k_BH}+2ay^2+2\alpha x^2\right) \\ &\quad +C_xC_y2(a-\alpha)xy+C_yC_z2ayz \end{aligned} \tag{106}$$

$$\begin{aligned} A_{xy} &= -\frac{m}{2k_BH}C_xC_y+2a(xC_xxC_y+yC_yxC_y \\ &\quad +zC_zxC_y)+2\alpha(y^2C_xCy-xyC_yC_y) \\ &= C_xC_y\left(-\frac{m}{2k_BH}+2\alpha x^2+2\alpha y^2\right) \\ &\quad +C_y^2(a-\alpha)xy+C_yC_z2axz \end{aligned} \tag{107}$$

$$\begin{aligned} A_{yx} &= -\frac{m}{2k_BH}C_yC_x+2a(xC_xyC_x+yC_yyC_x \\ &\quad +zC_zyC_x)+2\alpha(-xyC_xC_x+x^2C_yCx) \\ &= C_x^2(a-\alpha)xy+C_xC_y\left(-\frac{m}{2k_BH}+2\alpha y^2+2\alpha x^2\right) \\ &\quad +C_xC_z2ayz \end{aligned} \tag{108}$$

$$\begin{aligned} A_{xz} &= -\frac{m}{2k_BH}C_xC_z+2a(xC_xxC_z+yC_yxC_z+zC_zxC_z) \\ &\quad +2\alpha(y^2C_xCz-xyC_yC_z) \\ &= C_z^2axz+C_xC_z\left(-\frac{m}{2k_BH}+2ax^2+2\alpha y^2\right) \\ &\quad +C_yC_z2(a-\alpha)xy \end{aligned} \tag{109}$$

$$\begin{aligned} A_{yz} &= -\frac{m}{2k_BH}C_yC_z+2a(xC_xyC_z+yC_yyC_z+zC_zyC_z) \\ &\quad +2\alpha(-xyC_xCz+x^2C_yC_z) \\ &= C_z^2axz+C_xC_z2(a-\alpha)xy \\ &\quad +CyCz\left(-\frac{m}{2k_BH}+2ay^2+2\alpha x^2\right) \end{aligned} \tag{110}$$

The terms C_x^2 come from $A_{xx}B_{xx}$ and from $A_{yx}B_{xy}$

$$\begin{aligned} &\left(-\frac{m}{2k_BH}+2ax^2+2\alpha y^2\right)\left(-y\frac{\partial\omega}{\partial x}\right) \\ &+2(a-\alpha)xy\left(x\frac{\partial\omega}{\partial x}+\omega\right) \end{aligned} \tag{111}$$

The terms C_y^2 come from $A_{xy}B_{yx}$ and from $A_{yy}B_{yy}$

$$\begin{aligned} &\left(-\frac{m}{2k_BH}+2ay^2+2\alpha x^2\right)\left(x\frac{\partial\omega}{\partial y}\right) \\ &+2(a-\alpha)xy\left(-y\frac{\partial\omega}{\partial y}-\omega\right) \end{aligned} \tag{112}$$

The terms C_z^2 come from $A_{xz}B_{zx}$ and from $A_{yz}B_{zy}$

$$2axz\left(-y\frac{\partial\omega}{\partial z}\right)+2ayz\left(x\frac{\partial\omega}{\partial z}\right) \tag{113}$$

The terms C_xC_y come from $A_{xy}B_{yx}$, $A_{xx}B_{xx}$, $A_{yx}Bxy$, $A_{yy}B_{yy}$

$$\begin{aligned} &\left(-\frac{m}{2k_BH}+2ax^2+2\alpha y^2\right)\left(-y\frac{\partial\omega}{\partial y}-\omega\right) \\ &+2(a-\alpha)xy\left(-y\frac{\partial\omega}{\partial x}\right) \\ &+\left(-\frac{m}{2k_BH}+2ay^2+2\alpha x^2\right)\left(+x\frac{\partial\omega}{\partial x}+\omega\right) \\ &+2(a-\alpha)xy\left(+x\frac{\partial\omega}{\partial y}\right)=0 \end{aligned} \tag{114}$$

The terms C_xC_z come from $A_{xx}B_{zx}$, $A_{xx}B_{xx}$, $A_{yx}Bxy$, $A_{yz}B_{yz}$

$$(2axz)\left(-y\frac{\partial\omega}{\partial x}\right) + \left(-\frac{m}{2k_BH}+2ax^2+2\alpha y^2\right)\left(-y\frac{\partial\omega}{\partial z}\right) + (2axy)\left(x\frac{\partial\omega}{\partial x}+\omega\right)+2(a-\alpha)xy\left(x\frac{\partial\omega}{\partial z}\right)=0 \tag{115}$$

The terms C_yC_z come from $A_{xy}B_{yx}$, $A_{xz}B_{zx}$, $A_{yy}Byy$, $A_{yz}B_{zy}$

$$(2axz)\left(-y\frac{\partial\omega}{\partial x}-\omega\right)+2(a-\alpha)xy\left(-y\frac{\partial\omega}{\partial z}\right) + (2ayz)\left(x\frac{\partial\omega}{\partial y}\right) + \left(-\frac{m}{2k_BH}+2ay^2+2\alpha x^2\right)\left(x\frac{\partial\omega}{\partial z}\right)=0 \tag{116}$$

Let us now exploit the fact that ω only depends on ρ^2 and z^2. Thus, from (64) we obtain:

$$\frac{\partial\omega}{\partial x}=\frac{\partial\omega}{\partial\rho^2}\frac{\partial\rho^2}{\partial x}=2x\frac{\partial\omega}{\partial\rho^2} \tag{117}$$

$$\frac{\partial\omega}{\partial y}=\frac{\partial\omega}{\partial\rho^2}\frac{\partial\rho^2}{\partial y}=2y\frac{\partial\omega}{\partial\rho^2} \tag{118}$$

$$\frac{\partial\omega}{\partial z}=\frac{\partial\omega}{\partial z^2}\frac{\partial z^2}{\partial z}=2z\frac{\partial\omega}{\partial z^2} \tag{119}$$

The equation C_x^2 becomes:

$$\frac{\partial\ln(\omega)}{\partial\rho^2}=-\frac{(a-\alpha)}{\left(\frac{m}{k_BH}-2\alpha\rho^2\right)}) \tag{120}$$

Placing ourselves in the particular context where a and α are constant throughout space, we obtain:

$$\frac{\partial\ln(\omega)}{\partial\rho^2}=-\frac{1}{2}\frac{2(a-\alpha)}{\left(\frac{m}{k_BT_0}-2(a-\alpha)\rho^2+2az^2\right)} =-\frac{1}{2}\frac{\partial}{\partial\rho^2}\left[ln\left(\frac{m}{k_BT_0}+2(a-\alpha)\rho^2+2az^2\right)\right] \tag{121}$$

Whence:

$$\omega=\frac{\omega_{\rho_0}(z^2)}{\sqrt{\frac{m}{k_BT_0}+2(a-\alpha)\rho^2+2az^2)}} \tag{122}$$

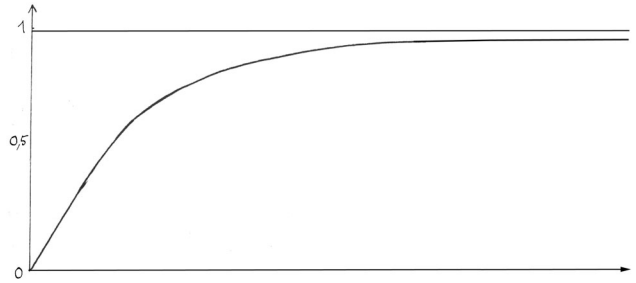


Fig. 6 Rotation curve (D'Agostini, 2016)

On the same way as before, the equations C_xC_z give

$$\frac{\partial ln\omega}{\partial z^2}=-\frac{a}{\left(\frac{m}{k_BH}-2\alpha\rho^2\right)} \tag{123}$$

By taking up the hypothesis that a and α are constant, we get:

$$\omega=\frac{\omega_{z_0}(\rho^2)}{\sqrt{\frac{m}{k_BT_0}+2(a-\alpha)\rho^2+2az^2)}} \tag{124}$$

Which gives the rotation speed at a point:

$$v=\rho.\omega=\rho.\frac{\omega_0}{\sqrt{\frac{m}{k_BT_0}+2(a-\alpha)\rho^2+2az^2)}} \tag{125}$$

In the diametrical plane of the galaxy:

$$v=\frac{\omega_0\,\rho}{\sqrt{\frac{m}{k_BT_0}+2(a-\alpha)\rho^2)}} \tag{126}$$

Taking into account that $\alpha \ll a$

$$v\approx\frac{\omega_0\,\rho}{\sqrt{\frac{m}{k_BT_0}+2a\rho^2)}} \tag{127}$$

Which gives a linear growth near the center of symmetry (solid body rotation) and a constant speed plateau at the periphery (Fig. 6).

5 Comparison with observational data

While the curves derived from observational data do show a velocity plateau at the periphery, their behavior near the center often includes a velocity peak. This is only absent in relatively rare cases, such as the NGC 128 galaxy (Fig. 7).

In contrast, rotation curves near the center of galaxies are generally very irregular (Fig. 8).

Those who originally interpret these overspeed as being due to the action of a dark matter halo are then obliged

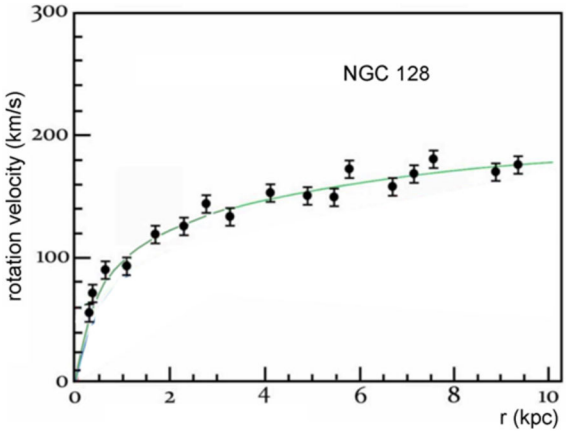


Fig. 7 Velocity curve of galaxy NGC 128

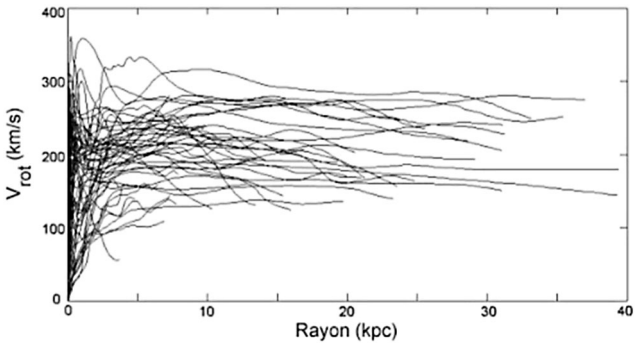


Fig. 8 Rotation curves of galaxies

to endow this distribution with a peak, at the center of the galaxy, so that the excursion of the gravitational field can counterbalance the centrifugal force. But it's also possible that these velocity excursions are the result of cannibalism by galaxies with smaller masses. Like the cannibal galaxy that absorbs them, these mini-galaxies are sets of non-collisionless mass-dots. Subjected to the potential well of the larger galaxy, they then fall towards their central part. In so doing, they retain their angular momentum, as these point-mass ensembles cannot transfer it to the cannibal galaxy by viscous exchange, or encounters. In contrast, the NGC 128 galaxy would not have experienced any absorption of mini-galaxies. At most, it could have merged with similar galaxies, in terms of mass distribution and velocity field.

6 About the potential and mass distribution

The cancellation of the unit order terms corresponds to the equation:

$$\mathbf{C}.\frac{\partial \ln(f)}{\partial \mathbf{r}} - \left(\frac{\partial \Psi}{\partial \mathbf{r}} + \mathbf{c}_0.\frac{\partial \mathbf{c}_0}{\partial \mathbf{r}}\right).\frac{\partial \ln(f)}{\partial \mathbf{C}} = 0 \tag{128}$$

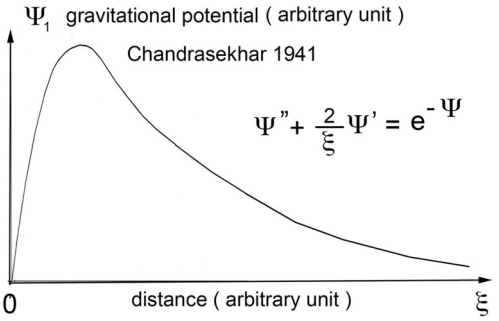


Fig. 9 Gravitational potential in spherical symmetry (Chandrasekhar (Chandrasekhar, 1941))

The density field is then constructed by combining it with Poisson's equation. We'll present the complete numerical solution in a later article. This potential is necessarily compatible with the form chosen for the distribution function, since it is an integral part of the solution. This construction is not straightforward, as it requires iterations to evaluate the term $\frac{\partial^2 \Psi}{\partial z^2}$. Nevertheless, we can draw conclusions about the shape of the potential and the values of the density. We know that the distribution function tends towards Maxwellian form at the center. The equation then reduces to:

$$\frac{\partial \ln(n)}{\partial \mathbf{r}} + \left(\frac{\partial \Psi}{\partial \mathbf{r}}\right)\frac{m}{k_B T_0} \tag{129}$$

The equation of the potential at the center is then:

$$\Delta\Psi = Cst\, e^{-\frac{m\Psi}{k_B T_0}} \tag{130}$$

Let's be satisfied with reduced variables and an expression of the potential in arbitrary units. The form of the potential near the center is shown in Fig. 9.

7 Conclusion

We have opted for a completely different approach to the one considered classical, taking up that of Chandrasekhar, based entirely on the choice of an elliptical distribution function. We have extended this method by introducing rotation (axisymmetric stationary system). The distribution function is associated with a velocity ellipsoid. The only assumption, of a geometrical nature, is that one of the axes of this ellipsoid in all space points to the system's center of symmetry. The mathematical tool of dyadic algebra is then used to carry out the calculations. Introducing the elliptic form of the distribution function into the collisionless Boltzmann equation, written in terms of the components of the residual velocity, transforms it into a third-order polynomial. Cancellation of the third-order terms then gives the evolution of

the axes of the ellipsoid. We then find that the three axes become equal at the center, i.e. the velocity distribution becomes Maxwelian. The transverse axes tend towards zero at infinity, while the radial velocity dispersion remains constant. The fact that the transverse axes are smaller than the radial velocity dispersion axis is consistent with observational data from the vicinity of the Sun. Cancellation of the second-order terms gives the velocity field. This tends towards solid-body rotation at the center and a plateau at infinity. It then remains to calculate the gravitational field by combining the cancellation of the unit-order terms and Poisson's equation. We refer this to a future article.

Author contributions All authors prepared and reviewed the manuscript.

Funding information The authors have no relevant financial or non-financial interests to disclose.

Data availability No datasets were generated or analysed during the current study.

Declarations

Ethics declaration Not applicable.

Competing interests The authors declare no competing interests.

Open Access This article is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License, which permits any non-commercial use, sharing, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if you modified the licensed material. You do not have permission under this licence to share adapted material derived from this article or parts of it. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by-nc-nd/4.0/.

References

Binney, J., Tremaine, S.: Galactic Dynamics. Princeton Series in Astrophysics (2008)

Chandrasekhar, S.: Principles of Stellar Dynamics. Dover, New York (1941)

Chapman, S., Cowling, T.G.: The Mathematical Theory of Non Uniform Gazes. Cambridge University Press, Cambridge (1939)

D'Agostini, G.: Solution elliptique de l'équation de Vlasov. Private Communication (2016). http://www.jp-petit.org/papers/cosmo/2016-Dagostini-Vlasov-courbe-rotation.pdf

McCrea, W.H., Milne, E.A.: Newtonian universes and the curvature of space. Q. J. Math. **os-5**, 73–80 (1934)

Petit, J.P.: Modèle tridimensionnel et instationnaire de système stellaire auto-gravitant. Application aux amas globulaires et aux galaxies (Three-dimensional and unsteady model of self-gravitating stellar system. Application to globular clusters and galaxies). Note C. R. Acad. Sci. Paris **série B**, 373–376 (1972)

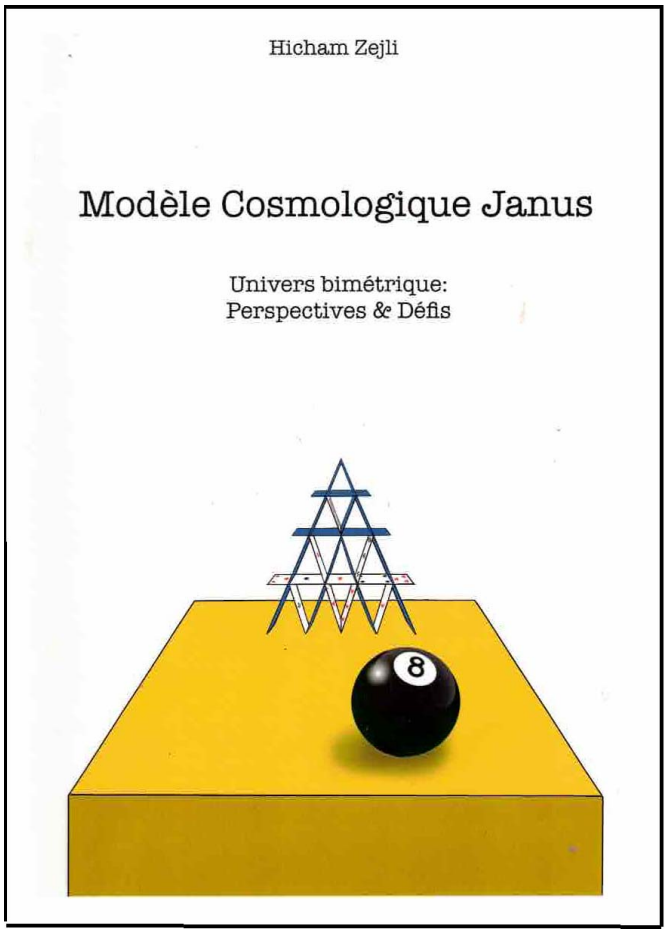
Petit, J.P., Monnet, G.: Axisymmetrical elliptical solution of the couple Vlasov plus Poisson equations. In: International 1974 Meeting on Galactic Dynamics at the High Studies Institute, Bures sur Yvette, France (1974)

Petit, J.P., Monnet, G.: Entropie maximale et cosmologie de Friedman. C. R. Acad. Sci. Paris **série A**, 1245–1248 (1975)

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Sometimes it takes decades for work to finally be published after it was first conceived. In fact, it was not until 2025 that this extension of Souriau's work to a two-sheet universe context was published in a journal of mathematical physics[26]. Zejli's name is still associated with this article. I had done so to strengthen his position vis-à-vis the University of Casablanca. But of course, his contribution is nonexistent.

Still, this young man was far from being a fool. A hard worker, he had mastered the entire Janus series in a single year, which is remarkable. He had even written a book titled:



Cover of Hicham Zejli's book

A book in which the author presented extensive calculations and research, including his construction of a mathematical model of a galaxy, as was his custom.

It is disheartening that such an intelligent, perceptive, and hardworking young man has permanently jeopardized all the efforts that had been made to help him find a place in the world of research. I recall the words of the Vice President of the University of Casablanca during the trip we took there to… finalize the details of his thesis:

- *If it's of high quality, there will be no problem finding a position.*

[26] J.P. Petit, H. Zejli: Study of Symmetries through the Action on Torsors of the Janus Symplectic Group. Reviews in Mathematical Physics. Vol. 37, No. 01, 2024.

But how can one collaborate with someone without placing one's trust in them from the outset?

Hicham Zejli had contacted me following a lecture I had given in Paris, for which a venue had been arranged at the expense of the Support Janus association.

Following that same lecture, a young mathematician from the École Normale Supérieure also reached out to me: David Pigeon.

I'm still always a bit uncomfortable—not with the concepts, but with the language used by mathematicians, which my wife calls "fly droppings." The articles have to be in this format to be submitted to journals of Mathematical Physics.

David Pigeon offered to handle this "translation."

Very quickly, he mentions needing money. I agree to help. Then he tells me he's moving to Africa, where his wife is from. An article was submitted to a mathematics journal. The bank transfers keep coming.

By December, over the course of 10 months, 20,000 euros had been paid to him.

The journal sent a list of questions. I asked him to draft the response. He then asked me for 1,500 euros a month to continue. I replied that the association could not give him any more. He then completely cut us off.

However, for the past few months, emails have been arriving from someone claiming to be… an Ummite. When faced with this, my reaction is always to steer the conversation toward high-level scientific questions. If the person is a prankster, it becomes clear very quickly.

I sent the article, which came back to me with insightful comments, incidentally describing Pigeon as a "costly copycat." Thanks to this help, I managed to finalize the paper, which was accepted and published[27]. (I hadn't yet realized the game H. Zejli was playing.)

Just in time, by the way, because Pigeon had already uploaded this article to the CNRS HAL database under his own name (I'll force him to remove it). Below is that article.

[27] J.P. Petit, H. Zejli: Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, No. 001, 2024. 1,315 downloads. Cited 8 times

OPEN ACCESS
Reviews in Mathematical Physics
Vol. 37, No. 1 (2025) 2450054 (12 pages)
© The Author(s)
DOI: 10.1142/S0129055X24500545

World Scientific
www.worldscientific.com

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Study of symmetries through the action on torsors of the Janus symplectic group

Jean-Pierre Petit* and Hicham Zejli†

Manaty Research Group,
2 Rue de l'Étang du Moulin,
Glanon 21250, France
**jean-pierre.petit@manaty.net*
†*zejli.hicham@gmail.com*

Received 22 August 2024
Accepted 30 October 2024
Published 27 December 2024

In this paper, we focus on the Janus symplectic group. We explore its various symmetries and its action on the elements of the dual of its Lie algebra, called torsors. Special attention is given to the charge symmetry, which highlights the matter–antimatter duality within both sets of components.

Keywords: Dynamic groups; symplectic groups; Lorentz group, Poincaré group; Janus group; torsors of a Lie group; action on the torsors.

Mathematics Subject Classification 2020: 20C35, 22E70, 53D55, 83A04, 70H33

1. Introduction

The application of the coadjoint action of a symplectic group on the dual of its Lie algebra, initiated by the mathematician Jean-Marie Souriau, has shed light on specific aspects of the approach followed by physics. The orbit method is due to Kirillov ([4–7, 12, 13, 15, 21, 24, 25]).

Thus, the restricted Lorentz symplectic group, limited to its two orthochrone components, translates, through the invariance properties that result from it, the aspects of special relativity. In 1970, Souriau established that the analysis of the components of its moment makes it possible to shed light on the geometric nature

†Corresponding author.
This is an Open Access article published by World Scientific Publishing Company. It is distributed under the terms of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 (CC BY-NC-ND) License, which permits use, distribution and reproduction, provided that the original work is properly cited, the use is non-commercial and no modifications or adaptations are made.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

of a spin (not quantized): see [19, 20]. He uses for this purpose symplectic methods ([8, 10, 22, 23]). In the theory of symplectic groups, we show a classification in terms of movements.

By operating the product of this group by that of the spatio-temporal translations, we obtain the restricted Poincaré symplectic group, always limited to its two orthochrone components. In its moment, we first find the energy related to the subgroup of temporal translations. Then the momentum, linked to the spatial translations, the two being linked by the invariance of the modulus of the energy-momentum four-vector under the action of the Lorentz group.

By adding a translation along a fifth dimension to the restricted Poincaré group, we form a Lie group to which we will give the name *Restricted Kaluza Group* ([1–3, 11, 14]). This group is not the 15-dimensional Kaluza group associated with a 5-dimensional Lorentzian manifold but a new 11-dimensional group, including 5-dimensional space-time translation. This new dimension endows the momentum with an additional scalar that can be identified with the electric charge q, which may be positive, negative, or zero, and is still not quantized. We then bring out the geometric translation according to a scalar ϕ due to endowing the masses with an invariant electric charge. Then, by bringing in a new symmetry reflecting the inversion of the fifth dimension, synonymous with an inversion of the scalar from q to $-q$, we double the number of its connected components from 2 to 4. The action on the moment then links this new symmetry to the inversion of the electric charge q. We thus deduce the geometric modeling of charge conjugation or **C**-*Symmetry.*[a] It's then logical to name this new extension, the *Restricted Janus Group.*

By introducing a new symmetry to the previous group, which we describe as **T**-*Symmetry*,[b] we build the *Janus Symplectic Group.* Thus, we double the number of connected components from four to eight, grouped into two subsets: "*Orthochronous"*, conserving time and energy properties, and "*Antichronous"*, reversing time and energy. Therefore, we bring forth the geometric translation of endowing masses with an invariant electric charge. As the Jean-Marie Souriau demonstrated as early as 1970, a pioneer in the theory of symplectic groups ([9, 19, 20]), this approach has allowed key elements, which have marked the progress of relativistic physics, to be given a purely geometric nature.

In relation to the world of physics, wouldn't the role of mathematics be to illuminate the path traveled? Conversely, could it be possible that the exploration of new symmetries, accompanying this decoding using symplectic groups, contains more than what we thought we put into it? That it could designate new paths to follow?

This is what we will consider with the Janus Symplectic Group with charge symmetry, by integrating the antichronous components of the Lorentz group, resulting

[a]which translates the matter–antimatter symmetry introduced by Dirac.
[b]which converts matter into antimatter with negative mass, a concept we could name *antimatter in the Feynman sense.*

from its simple axiomatic definition, with the obvious repercussions on the Poincaré group and its extensions.

2. Janus Symplectic Group

Let $\tilde{\mathbf{T}} := I_{1,3}$, $\tilde{\mathbf{P}} := -\tilde{\mathbf{T}}$ and

$$\forall\, \lambda, \nu \in \{0,1\}, \quad \mathcal{L}or(\tilde{\mathbf{P}}^{\nu}\tilde{\mathbf{T}}^{\lambda}) := \{L_n \tilde{\mathbf{P}}^{\nu}\tilde{\mathbf{T}}^{\lambda},\ L_n \in \mathcal{L}or_n\}.$$

Then, there are four connected components of $\mathcal{L}or$, given by[c]

$$\mathcal{L}or_n = \mathcal{L}or(\tilde{\mathbf{P}}^{0}\tilde{\mathbf{T}}^{0}), \quad \mathcal{L}or_s = \mathcal{L}or(\tilde{\mathbf{P}}^{1}\tilde{\mathbf{T}}^{0}),$$
$$\mathcal{L}or_t = \mathcal{L}or(\tilde{\mathbf{P}}^{0}\tilde{\mathbf{T}}^{1}), \quad \mathcal{L}or_{st} = \mathcal{L}or(\tilde{\mathbf{P}}^{1}\tilde{\mathbf{T}}^{1})$$

and we have the decomposition

$$\mathcal{L}or = \bigsqcup_{\nu,\lambda\in\{0,1\}} \mathcal{L}or(\tilde{\mathbf{P}}^{\nu}\tilde{\mathbf{T}}^{\lambda}). \tag{1}$$

Then, we define the Janus symplectic group.

Definition 2.1. The *Janus symplectic group* is defined as the subgroup of $\mathrm{GL}(6,\mathbb{R})$:

$$\mathcal{J}an := \left\{ \begin{pmatrix} L & 0 & D \\ 0 & (-1)^{\eta} & \phi \\ 0 & 0 & 1 \end{pmatrix},\ \eta \in \{0,1\}\ \wedge\ \phi \in \mathbb{R}\ \wedge\ L \in \mathcal{L}or\ \wedge\ D \in \mathbb{R}^4 \right\}.$$

The Janus symplectic group is therefore a subgroup of the group of isometries in dimension 5 given by[d]:

$$\mathrm{Aff}(\mathcal{O}(1,4)) := \left\{ \begin{pmatrix} L & D' \\ 0 & 1 \end{pmatrix},\ L \in \mathcal{O}(1,4)\ \wedge\ D' \in \mathbb{R}^5 \right\}$$

with $\tau_{1,4}(L) := I_{1,4}L^T I_{1,4}$ and $\mathcal{O}(1,4) := \{L \in \mathrm{GL}(5,\mathbb{R}),\ \tau_{1,4}(L)L = I_5\}$. The elements of $\mathrm{Aff}(\mathcal{O}(1,4))$ are the elements which preserve the distance between two events (*pentavectors*) $X := (t,x,y,z,\xi)$ and $X' := (t',x',y',z',\xi')$ given by

$$d(X,X') := c^2(t-t')^2 - (x-x')^2 - (y-y')^2 - (z-z')^2 - (\xi-\xi')^2.$$

[c]Equalities are shown by double inclusion. For example, let's demonstrate that $\mathcal{L}or_s = \mathcal{L}or(\tilde{\mathbf{P}}^{1}\tilde{\mathbf{T}}^{0})$. Take $L \in \mathcal{L}or_s$ ($\det(L) = -1$ et $[L]_{00} \geq 1$). Then we have $\det(L\tilde{\mathbf{P}}) = -1$ and $[L\tilde{\mathbf{P}}]_{00} \geq 1$ i.e. we have $L_n := L\tilde{\mathbf{P}} \in \mathcal{L}or_n$. Since $\tilde{\mathbf{P}}^{-1} = \tilde{\mathbf{P}}$, we can conclude that $L = L_n\tilde{\mathbf{P}} \in \mathcal{L}or(\tilde{\mathbf{P}}^{1}\tilde{\mathbf{T}}^{0})$. The inclusion in the other direction is trivial.

[d]$\mathrm{Aff}(\mathcal{O}(1,4))$ is the affine group associated with $\mathcal{O}(1,4)$, it is also defined by the semi-direct product $\mathrm{Aff}(\mathcal{O}(1,4)) := \mathcal{O}(1,4) \ltimes \mathbb{R}^5$. We can also define the symplectic Janus group as being the affine group associated with the subgroup of $\mathcal{O}(1,4)$ given by

$$\mathcal{E}lec := \left\{ \begin{pmatrix} L & 0 \\ 0 & (-1)^{\eta} \end{pmatrix},\ \eta \in \{0,1\}\ \wedge\ L \in \mathcal{L}or \right\}$$

called the *symplectic electric group* and we have $\mathcal{J}an := \mathrm{Aff}(\mathcal{E}lec)$.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

This fifth dimension is of space type (we note the variable ξ). Each dimension is therefore associated with a symmetry, there are three types of symmetries:

- the $\mathbf{T}$-*symmetry*;
- the $\mathbf{P}_x$-*symmetry*, $\mathbf{P}_y$-*symmetry*, $\mathbf{P}_z$-*symmetry* grouped together what we call the $\mathbf{P}$-*symmetry*;
- the ξ-*symmetry* corresponding to the $\mathbf{C}$-*symmetry* (the charge conjugation).

This space of dimension 5 is a Minkowski metric space to which we have added one dimension, it has the metric $I_{1,4}$.

We also define the *restricted Janus group* is the subgroup of $\mathcal{J}an$ given by

$$\mathcal{J}an_n := \left\{ \begin{pmatrix} L_n & 0 & D \\ 0 & 1 & \phi \\ 0 & 0 & 1 \end{pmatrix},\ \phi \in \mathbb{R}\ \wedge\ L_n \in \mathcal{L}or_n\ \wedge\ D \in \mathbb{R}^4 \right\}.$$

Let

$$\mathbf{C} := \begin{pmatrix} I_4 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{P} := \begin{pmatrix} \tilde{\mathbf{P}} & 0 \\ 0 & I_2 \end{pmatrix}, \quad \mathbf{T} := \begin{pmatrix} \tilde{\mathbf{T}} & 0 \\ 0 & I_2 \end{pmatrix}.$$

We have

$$\forall\, \lambda, \eta, \nu \in \{0,1\}, \quad \begin{pmatrix} L_n & 0 & D \\ 0 & 1 & \phi \\ 0 & 0 & 1 \end{pmatrix} \mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda = \begin{pmatrix} L_n \tilde{\mathbf{P}}^\nu \tilde{\mathbf{T}}^\lambda & 0 & D \\ 0 & (-1)^\eta & \phi \\ 0 & 0 & 1 \end{pmatrix}$$

and therefore by Eq. (1):

$$\mathcal{J}an = \left\{ \begin{pmatrix} L_n \tilde{\mathbf{P}}^\nu \tilde{\mathbf{T}}^\lambda & 0 & D \\ 0 & (-1)^\eta & \phi \\ 0 & 0 & 1 \end{pmatrix},\ \lambda, \eta, \nu \in \{0,1\}\ \wedge\ \phi \in \mathbb{R} \right.$$

$$\left. \wedge\ L_n \in \mathcal{L}or_n\ \wedge\ D \in \mathbb{R}^4 \right\}.$$

Definition 2.2. (i) The ***CPT****-group* is the subgroup $\mathcal{K}$ of $\mathcal{J}an$ of order 8 generated by $\mathbf{C}$, $\mathbf{P}$ and $\mathbf{T}$, i.e.

$$\mathcal{K} := \{\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda,\ \eta, \nu, \lambda \in \{0,1\}\} = \{I_6, \mathbf{T}, \mathbf{P}, \mathbf{PT}, \mathbf{C}, \mathbf{CT}, \mathbf{CP}, \mathbf{CPT}\}.$$

(ii) For all $\mathbf{X} \in \mathcal{K}$, the ***X****-component of* $\mathcal{J}an$ is

$$\mathcal{J}an(\mathbf{X}) := \{JX, J \in \mathcal{J}an_n\}.$$

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Thus, we have

$$\mathcal{J}an(\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda) = \left\{ \begin{pmatrix} L_n\tilde{\mathbf{P}}^\nu\tilde{\mathbf{T}}^\lambda & 0 & D \\ 0 & (-1)^\eta & \phi \\ 0 & 0 & 1 \end{pmatrix},\ \phi \in \mathbb{R}\ \wedge\ L_n \in \mathcal{L}or_n\ \wedge\ D \in \mathbb{R}^4 \right\}.$$

These eight components are the eight connected components of $\mathcal{J}an$, we have the decomposition:

$$\mathcal{J}an = \bigsqcup_{\mathbf{X}\in\mathcal{K}} \mathcal{J}an(\mathbf{X}) = \bigsqcup_{\eta,\nu,\lambda\in\{0,1\}} \mathcal{J}an(\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda).$$

The group $\mathcal{L}or$ is the Lie group of dimension 6 and its Lie algebra is

$$\mathfrak{lor} := \mathcal{A}(1,3) := \{\Lambda \in \mathcal{M}(4,\mathbb{R}), \tau_{1,3}(\Lambda) = -\Lambda\}.$$

Then, the group $\mathcal{J}an$ is a Lie group of dimension 11 and its Lie algebra is

$$\mathfrak{jan} = \left\{ \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}, \Lambda \in \mathcal{A}(1,3)\ \wedge\ \ \Gamma \in \mathbb{R}^4\ \wedge\ \varepsilon \in \mathbb{R} \right\}.$$

We have two characterizations[e]:

$$\left(\mathbb{R}^5\right)^* = \left\{ \begin{pmatrix} \Gamma \\ \varepsilon \end{pmatrix} \mapsto -(P^T\ q) I_{1,4} \begin{pmatrix} \Gamma \\ \varepsilon \end{pmatrix} = -\tau(P)\Gamma - q\varepsilon,\ \begin{pmatrix} P \\ q \end{pmatrix} \in \mathbb{R}^5 \right\},$$

$$\mathcal{A}(1,3)^* = \left\{ \Lambda \mapsto -\frac{1}{2}\mathrm{Tr}(M\Lambda),\ M \in \mathcal{A}(1,3) \right\}.$$

Then, we have[f]

$$\mathfrak{jan}^* = \left\{ \{M \,|\, P \,|\, q\} : \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix} \mapsto -\frac{1}{2}\mathrm{Tr}(M\Lambda) - \tau(P)\Gamma - q\varepsilon,\ M \in \mathcal{A}(1,3) \right.$$
$$\left. \wedge\ P \in \mathbb{R}^4\ \wedge\ q \in \mathbb{R} \right\}.$$

The *action of the group* $\mathcal{J}an$ *on* $\mathfrak{jan}^*$ is defined by the coadjoint representation i.e. for any $a \in \mathcal{J}an$ and any $\mu \in \mathfrak{jan}^*$, we denote this action by

$$a \bullet \mu := \mathrm{Ad}^*_a(\mu).$$

with

$$\mathrm{Ad}^* : \mathcal{J}an \to \mathrm{Aut}(\mathfrak{jan}^*)$$
$$a \mapsto \mathrm{Ad}^*_a : \mu \mapsto (Z \mapsto \mu(a^{-1}Za)).$$

[e]For all $\beta \in \mathbb{R}^*$, the application Φ_β which to $M \in \mathcal{A}(1,3)$ associates the linear form $\Lambda \mapsto \beta\mathrm{Tr}(M\Lambda)$ is an isomorphism of $\mathcal{A}(1,3)$ to $\mathcal{A}(1,3)^*$. Taking $\{A_{kl} := -E_{kl} + [I_{1,3}]_{ll}[I_{1,3}]_{kk}E_{lk},\ k,l \in \{1,\ldots,4\},\ k < l\}$ the canonical basis of $\mathcal{A}(1,3)$, we have $\Phi_{-1/2}(A_{kl})(A_{kl}) = 1$, hence the choice of $\beta := -1/2$.

[f]The elements of $\mathfrak{jan}^*$ are called *torsors*.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Proposition 2.1. *Let*

$$a := \begin{pmatrix} L & 0 & D \\ 0 & (-1)^\eta & \phi \\ 0 & 0 & 1 \end{pmatrix} \in \mathcal{J}an, \quad \{M \,|\, P \,|\, q\} \in \mathfrak{jan}^*.$$

We have

$$a \bullet \{M \,|\, P \,|\, q\} = \{LM\tau(L) + D\tau(P)\tau(L) - LP\tau(D) \,|\, LP \,|\, (-1)^\eta q\}.$$

Proof. We have

$$\begin{aligned}
&(a \bullet \{M \,|\, P \,|\, q\}) \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix} \\
&\quad = \{M \,|\, P \,|\, q\} \left(a^{-1} \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix} a \right) \\
&\quad = \{M \,|\, P \,|\, q\} \left(\begin{pmatrix} \tau(L) & 0 & -\tau(L)D \\ 0 & (-1)^\eta & (-1)^{\eta+1}\phi \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & 0 & D \\ 0 & (-1)^\eta & \phi \\ 0 & 0 & 1 \end{pmatrix} \right) \\
&\quad = \{M \,|\, P \,|\, q\} \begin{pmatrix} \tau(L)\Lambda L & 0 & \tau(L)(\Lambda D + \Gamma) \\ 0 & 0 & (-1)^\eta \varepsilon \\ 0 & 0 & 0 \end{pmatrix} \\
&\quad = -\frac{1}{2}\mathrm{Tr}(M\tau(L)\Lambda L) - \tau(P)\tau(L)(\Lambda D + \Gamma) - (-1)^\eta q\varepsilon \\
&\quad = -\frac{1}{2}\mathrm{Tr}[(LM\tau(L) + 2D\tau(P)\tau(L))\Lambda] - \tau(LP)\Gamma - (-1)^\eta q\varepsilon \\
&\quad = -\frac{1}{2}\mathrm{Tr}[(LM\tau(L) + D\tau(P)\tau(L) - LP\tau(D))\Lambda] - \tau(LP)\Gamma - (-1)^\eta q\varepsilon \\
&\quad = \{LM\tau(L) + D\tau(P)\tau(L) - LP\tau(D) \,|\, LP \,|\, (-1)^\eta q\} \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix}.
\end{aligned}$$

□

To describe the Lie algebra of $\mathcal{J}an$, we can also use the isomorphism of Lie algebras[g]:

$$\begin{aligned}
j : (\mathbb{R}^3, \wedge) &\to (\mathcal{A}(3), [\ ,\ ]) \\
\begin{pmatrix} x \\ y \\ z \end{pmatrix} &\mapsto \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}.
\end{aligned}$$

[g]We have for all $u, v \in \mathbb{R}^3$: $u \wedge v = j(u)(v)$ and $j(u \wedge v) = [j(u), j(v)] = j(u)j(v) - j(v)j(u)$.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

with $\wedge$ the cross product on $\mathbb{R}^3$ and $\mathcal{A}(3)$ the vector space of antisymmetric matrices of size 3. Then, we have

$$\begin{aligned}\mathfrak{jan} &= \left\{ \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix},\ \Lambda \in \mathcal{A}(1,3)\ \wedge\ \Gamma \in \mathbb{R}^4\ \wedge\ \varepsilon \in \mathbb{R} \right\} \\ &= \left\{ \begin{pmatrix} 0 & \beta^T & 0 & v \\ \beta & j(w) & 0 & \gamma \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 \end{pmatrix},\ \beta, w, \gamma \in \mathbb{R}^3\ \wedge\ v, \varepsilon \in \mathbb{R} \right\}.\end{aligned}$$

Therefore, for all $\{M \,|\, P \,|\, q\} \in \mathfrak{jan}^*$, there are unique $\ell, g, p \in \mathbb{R}^3$ and $E, q \in \mathbb{R}$ such as

$$\begin{aligned}\{M \,|\, P \,|\, q\} \begin{pmatrix} \Lambda & 0 & \Gamma \\ 0 & 0 & \varepsilon \\ 0 & 0 & 0 \end{pmatrix} &= \left\{ \begin{pmatrix} 0 & g^T \\ g & j(\ell) \end{pmatrix} \middle| \begin{pmatrix} E \\ p \end{pmatrix} \middle| q \right\} \begin{pmatrix} 0 & \beta^T & 0 & v \\ \beta & j(w) & 0 & \gamma \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ &= -\frac{1}{2}\mathrm{Tr}\left(\begin{pmatrix} 0 & g^T \\ g & j(\ell) \end{pmatrix} \begin{pmatrix} 0 & \beta^T \\ \beta & j(w) \end{pmatrix} \right) - (E\ p^T) I_{1,3} \begin{pmatrix} v \\ \gamma \end{pmatrix} - q\varepsilon \\ &= \ell^T w - g^T \beta + p^T \gamma - Ev - q\varepsilon.\end{aligned}$$

We denote this last equality as

$$\{\ell \,|\, g \,|\, p \,|\, E \,|\, q\} \begin{pmatrix} 0 & \beta^T & 0 & v \\ \beta & j(w) & 0 & \gamma \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The dual $\mathfrak{jan}^*$ has the following descriptions:

$$\left\{ \{\ell \,|\, g \,|\, p \,|\, E \,|\, q\} : \begin{pmatrix} 0 & \beta^T & 0 & v \\ \beta & j(w) & 0 & \gamma \\ 0 & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 \end{pmatrix} \mapsto \ell^T w - g^T \beta + p^T \gamma - Ev - q\varepsilon, \right.$$

$$\left. \ell, g, p \in \mathbb{R}^3\ \wedge\ E, q \in \mathbb{R} \right\}.$$

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Definition 2.3. Let

$$\mu := \{M \,|\, P \,|\, q\} := \{l \,|\, g \,|\, p \,|\, E \,|\, q\} \in \mathfrak{jan}^*$$

with relations

$$M = \begin{pmatrix} 0 & g^T \\ g & j(\ell) \end{pmatrix}, \quad P = \begin{pmatrix} E \\ p \end{pmatrix}.$$

(i) The matrix $M := M(\mu) \in \mathcal{A}(1,3)$ is called the *moment matrix associated with* μ. The vector $\ell := \ell(\mu) \in \mathbb{R}^3$ is called the *angular momentum of* M, and the vector $g := g(\mu) \in \mathbb{R}^3$ is the *relativist barycenter of* M.

(ii) (a) The vector $P := P(\mu) \in \mathbb{R}^4$ is called the *stress-energy vector associated with* μ. The vector $p := p(\mu) \in \mathbb{R}^3$ is called the *linear momentum of* P, and the scalar $E := E(\mu) \in \mathbb{R}$ is called the *energy of* P.

(b) The *first Casimir number associated with* μ is defined by

$$C_1 := C_1(\mu) := P^T I_{1,3} P = E^2 - p^2.$$

(c) The *mass associated to* μ is defined by

$$m := m(\mu) := \operatorname{sign}(E)\sqrt{C_1} = \operatorname{sign}(E)\sqrt{E^2 - p^2}.$$

(iii) The scalar $q := q(\mu) \in \mathbb{R}$ is called the *electric charge associated with* μ.

We deduce a simple expression of the action of the **CPT**-group $\mathcal{K}$ on the torsors of $\mathfrak{jan}^*$.

Corollary 2.2. *Let* $\{l \,|\, g \,|\, p \,|\, E \,|\, q\} \in \mathfrak{jan}^*$. *For all* $\lambda, \eta, \nu \in \{0,1\}$, *we have*

$$(\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\} = \{l \,|\, (-1)^{\lambda+\nu} g \,|\, (-1)^\nu p \,|\, (-1)^\lambda E \,|\, (-1)^\eta q\}.$$

Proof. We apply Proposition 2.1 with $a := \mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda$:

$$\begin{aligned}
&(\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\} \\
&\quad = (\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \left\{ \begin{pmatrix} 0 & g^T \\ g & j(\ell) \end{pmatrix} \middle| \begin{pmatrix} E \\ p \end{pmatrix} \middle| q \right\} \\
&\quad = \left\{ \tilde{\mathbf{P}}^\nu \tilde{\mathbf{T}}^\lambda \begin{pmatrix} 0 & g^T \\ g & j(\ell) \end{pmatrix} \tilde{\mathbf{T}}^\lambda \tilde{\mathbf{P}}^\nu \middle| I_{1,3} \tilde{\mathbf{T}}^\lambda \tilde{\mathbf{P}}^\nu I_{1,3} \begin{pmatrix} E \\ p \end{pmatrix} \middle| (-1)^\eta q \right\} \\
&\quad = \left\{ \begin{pmatrix} 0 & (-1)^{\lambda+\nu} g^T \\ (-1)^{\lambda+\nu} g & j(\ell) \end{pmatrix} \middle| \begin{pmatrix} (-1)^\lambda E \\ (-1)^\nu p \end{pmatrix} \middle| (-1)^\eta q \right\} \\
&\quad = \{l \,|\, (-1)^{\lambda+\nu} g \,|\, (-1)^\nu p \,|\, (-1)^\lambda E \,|\, (-1)^\eta q\}.
\end{aligned}$$

□

So we have

$$\mathbf{C} \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\} = \{l \,|\, g \,|\, p \,|\, E \,|\, -q\},$$

$$\mathbf{P} \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\} = \{l \,|\, -g \,|\, -p \,|\, E \,|\, q\},$$
$$\mathbf{T} \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\} = \{l \,|\, -g \,|\, p \,|\, -E \,|\, q\}.$$

Corollary 2.3. *Let $\mu \in \mathfrak{jau}^*$. For all $\lambda, \eta, \nu \in \{0, 1\}$, we have*

$$P((\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \mu) = \tilde{\mathbf{P}}^\nu \tilde{\mathbf{T}}^\lambda P(\mu),$$
$$C_1((\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \mu) = C_1(\mu),$$
$$m((\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \mu) = (-1)^\lambda m(\mu).$$

Proof. Let $\mu := \{l \,|\, g \,|\, p \,|\, E \,|\, q\} \in \mathfrak{jau}^*$. We have for the stress-energy tensor

$$P(\mathbf{P} \bullet \mu) = P(\{l \,|\, -g \,|\, -p \,|\, E \,|\, q\}) = \begin{pmatrix} E \\ -p \end{pmatrix} = \tilde{\mathbf{P}} P(\mu),$$

		$\lambda = 0$	$\lambda = 1$
$\eta = 0$	$\nu = 0$	• neutral symetry • $a = I_6$ • $\mu' = \{\ell, g, p, E, q\}$	• $\mathbf{T}$ – symetry • $a = \mathbf{T}$ • $\mu' = \{\ell, -g, p, -E, q\}$
	$\nu = 1$	• $\mathbf{P}$ – symetry • $a = \mathbf{P}$ • $\mu' = \{\ell, -g, -p, E, q\}$	• $\mathbf{PT}$ – symetry • $a = \mathbf{PT}$ • $\mu' = \{\ell, g, -p, -E, q\}$
$\eta = 1$	$\nu = 0$	• $\mathbf{C}$ – symetry • $a = \mathbf{C}$ • $\mu' = \{\ell, g, p, E, -q\}$	• $\mathbf{CT}$ – symetry • $a = \mathbf{CT}$ • $\mu' = \{\ell, -g, p, -E, -q\}$
	$\nu = 1$	• $\mathbf{CP}$ – symetry • $a = \mathbf{CP}$ • $\mu' = \{\ell, -g, -p, E, -q\}$	• $\mathbf{CPT}$ – symetry • $a = \mathbf{CPT}$ • $\mu' = \{\ell, g, -p, -E, -q\}$

Fig. 1. This table lists the eight values of $\mu' := (\mathbf{C}^\eta \mathbf{P}^\nu \mathbf{T}^\lambda) \bullet \{l \,|\, g \,|\, p \,|\, E \,|\, q\}$ for all $\lambda, \eta, \nu \in \{0, 1\}$.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

$$P(\mathbf{T}\bullet\mu) = P(\{l\,|\,-g\,|\,p\,|\,-E\,|\,q\}) = \begin{pmatrix} -E \\ p \end{pmatrix} = \tilde{\mathbf{T}}P(\mu),$$

$$P(\mathbf{P}\bullet\mu) = P(\{l\,|\,g\,|\,p\,|\,E\,|\,-q\}) = \begin{pmatrix} E \\ p \end{pmatrix} = P(\mu)$$

for the first Casimir number:

$$C_1((\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda)\bullet\mu) = P(\mu)^T\tilde{\mathbf{T}}^\lambda\tilde{\mathbf{P}}^\nu I_{1,3}\tilde{\mathbf{P}}^\nu\tilde{\mathbf{P}}^\lambda P(\mu) = P(\mu)^T I_{1,3}P(\mu) = C_1(\mu)$$

for the mass:

$$\begin{aligned} m((\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda)\bullet\mu) &= \mathrm{sign}(E((\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda)\bullet\mu))\sqrt{C_1((\mathbf{C}^\eta\mathbf{P}^\nu\mathbf{T}^\lambda)\bullet\mu)} \\ &= \mathrm{sign}((-1)^\lambda E)\sqrt{C_1(\mu)} = (-1)^\lambda m(\mu). \end{aligned}$$ □

Therefore the elements variable by these actions are

$$P(\mathbf{P}\bullet\mu) = \tilde{\mathbf{P}}P(\mu), \quad P(\mathbf{T}\bullet\mu) = \tilde{\mathbf{T}}P(\mu), \quad m(\mathbf{T}\bullet\mu) = -m(\mu) \tag{2}$$

and we have the above table in Fig. 1.

3. Discussion and Conclusion

In this paper, we have performed a double extension of the restricted Poincaré group limited to its orthochronous components, which are classically used in physics. This extension also includes the transition from the four-dimensional Minkowski space-time to a new space of the same dimension, to which we have added a translation along an additional fifth dimension to form a new Lie group. The existence of this additional subgroup results in the invariance of a scalar, identified as the electric charge. A symmetry is introduced along this fifth dimension, and we have shown that this leads to the inversion of the electric charge. This provides a geometric representation of the symmetry between matter and antimatter.

In 1905, Albert Einstein revolutionized physics by introducing the theory of special relativity, integrating time, via the constant c, as a coordinate comparable to spatial dimensions in the geometry of Minkowski space. Ten years later, with his field equation, he explained phenomena such as the precession of Mercury's perihelion and the deflection of light, while laying the foundations of modern cosmology. The Big Bang theory, supported by Edwin Hubble's observations and Friedmann's work, revealed that the early universe was characterized by extreme density and temperature conditions.

Simultaneously, quantum mechanics emerged with the discovery of antimatter by Paul Dirac, an essential postulate in the cosmological model where, a fraction of a second after the Big Bang, matter and antimatter coexisted in equilibrium with gamma photons. However, the progressive disappearance of antimatter, implied by annihilations, remains a mystery, reinforced by the discovery of the cosmic microwave background in 1965.

The paradox of baryonic asymmetry found a theoretical response in 1967 with Sakharov, who postulated the existence of a twin universe symmetric to ours according to **CPT** symmetry, containing antimatter and evolving with an opposite arrow of time [16–18]. This model is based on a natural extension of fundamental symmetries, although the complete separation of the two universes leaves unresolved questions.

Within the framework of a geometric approach via dynamical groups, and building on the theory of symplectic groups by Souriau [20], we consider here an extension of Sakharov's model, the Janus model. The latter proposes an alternative vision where the two universes, far from being disjoint, are connected by a covering structure. This configuration allows gravitational interaction between particles of opposite masses, thereby redefining **CPT** symmetry in a larger geometric framework, including the antichronous components of the Poincaré group.

In conclusion, this approach not only sheds light on the path physics has taken but also suggests new avenues, offering a possible resolution to contemporary crises in cosmology, as evidenced by the data from the Hubble and James Webb telescopes. A more detailed study will be presented, including a system of coupled field equations modeling the gravitational interactions induced by these new symmetries.

Acknowledgment

Thanks to H. Zejli for correcting my many typos and improving the layout of my article. A modest task indeed, but so useful that my readers are not put off by the complexity of my scientific work.

ORCID

Jean-Pierre Petit https://orcid.org/0000-0003-3141-8584

Hicham Zejli https://orcid.org/0009-0006-8886-7101

References

[1] V. Bargmann, P. G. Bergmann and A. Einstein, *On The Five-Dimensional Representation of Gravitation and Electricity* (Cal Tech, 1941), 212 pp.
[2] P. Bergmann, *An Introduction to the Theory of Relativity* (Prentice-Hall, 1942).
[3] P. Bergmann and A. Einstein, On a generalization of Kaluza's theory of electricity, *Ann. Math.* **39** (1938) 683–701.
[4] N. Burgoyne and R. Cushman, Conjugacy classes in linear groups, *J. Algebra* **44** (1977) 339–362.
[5] R. Cushman, Adjoint orbits in the Lie algebra of the generalized real orthogonal group (2022), arXiv:2205.04407.
[6] R. Cushman, Coadjoint orbits of the odd real symplectic group (2023), arXiv:2212.00676.
[7] R. Cushman and W. van der Kallen, Adjoint and coadjoint orbits of the Poincaré group (2003), arXiv:math/0305442.

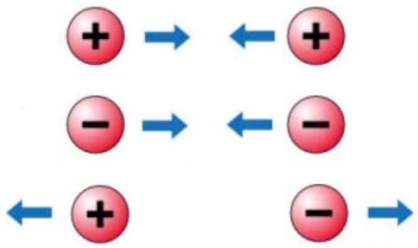
Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

[8] G. de Saxcé, Euler–Poincaré equation for Lie groups with non-null symplectic cohomology. Application to the mechanics. in *Geometric Science of Information* (Springer, 2019), pp. 66–74.
[9] G. de Saxcé and C.-M. Marle, Presentation of Jean-Marie Souriau's book "Structure des systèmes dynamiques", *Math. Mech. Compl. Syst.* (2023) 19 pages, arXiv:2306.03106, https://arxiv.org/abs/2306.03106.
[10] G. de Saxcé and C. Vallée, Construction of a central extension of a lie group from its class of symplectic cohomology, *J. Geom. Phys.* **60** (2010) 165–174.
[11] Th. Kaluza, Zum Unitätsproblem der Physik, *Sitzungsberichte der Preußischen Akademie der Wissenschaften, Berlin* (*Math.-Phys.*) **27** (1991) 966–972.
[12] A. Kirillov, Unitary representations of nilpotent Lie groups, *Uspehi Mat. Nauk* **17** (1962) 53–104.
[13] A. Kirillov, *Elements of the Theory of Representations* (Springer-Verlag, Berlin, 1976).
[14] O. Klein, Quantum theory and five-dimensional theory of relativity, *Z. Phys.* **37** (1926) 895–906.
[15] B. Kostant, *Quantization and Unitary Representations*, Lectures in Modern Analysis and Applications (Springer-Verlag, Berlin, 1970).
[16] A. D. Sakharov, Violation of CP invariance, C asymmetry, and Baryon asymmetry of the universe, *Pi'sma ZhÉTF* **5**(1) (1967) 32–35.
[17] A. D. Sakharov, Cosmological models of the universe with reversal of time's arrow, *Pi'sma ZhÉTF* **79**(3) (1980) 689–693.
[18] A. D. Sakharov, Multisheet models of the universe, *Pi'sma ZhÉTF* **82**(3) (1982) 1233–1240.
[19] J. M. Souriau, *Géométrie et relativité* (Hermann, 1964).
[20] J. M. Souriau, *Structure of Dynamical Systems, a Symplectic View of Physics* (Birkhäuser Verlag, New York, 1997).
[21] P. Torasso, Quantification géométrique, opérateurs d'entrelacement et représentations unitaires de $\mathrm{SL}(3,\mathbb{R})$, *Acta Math.* **150** (1983) 153–242.
[22] G. M. Tuynman, The Lagrangean in symplectic mechanics, in *Jean Leray '99 Conference Proceedings: The Karlskrona Conference in Honor of Jean Leray*, Dordrecht, 235–247.
[23] G. M. Tuynman, *Supermanifolds and Supergroups. Basic Theory*, Mathematics and its Applications, Vol. 570 (Kluwer Academic Publishers, Dordrecht, 2004).
[24] D. Vogan, *Noncommutative Algebras and Unitary Representations* (American Mathematical Society, 1988).
[25] D. Vogan, The method of coadjoint orbits for real reductive groups, in *Representation Theory of Lie Groups*, Vol. 8 (American Mathematical Society, Providence, RI, 1999).

Rev. Math. Phys. 2025.37. Downloaded from www.worldscientific.com
by 2a01:e0a:af0:b240:195d:738:c494:8d45 on 02/24/25. Re-use and distribution is strictly not permitted, except for Open Access articles.

This scientific appendix demonstrates the scope and variety of research that has been spurred by these Ummo documents. Referring to the 1994 publication in the journal *Nuovo Cimento*, the first results of numerical simulations were presented, which incorporated the idea of interaction with negative masses.

At the time, this was a purely heuristic[28] hypothesis regarding the direction of the forces, as follows:



Hypothetical Laws of Interaction

Back in the 1970s, it was out of the question for me to perform numerical simulations on my Macintosh:



Macintosh microcomputer

A friend of mine, Pierre Midy, who headed the computing department at a laboratory, had access to a Cray-1[29], which at the time represented the state-of-the-art in computing.

Using this machine, Midy was able to simulate the interaction of 500 positive masses and 500 negative masses in 3D simulations, governed by Newton's laws. The masses separated.

---

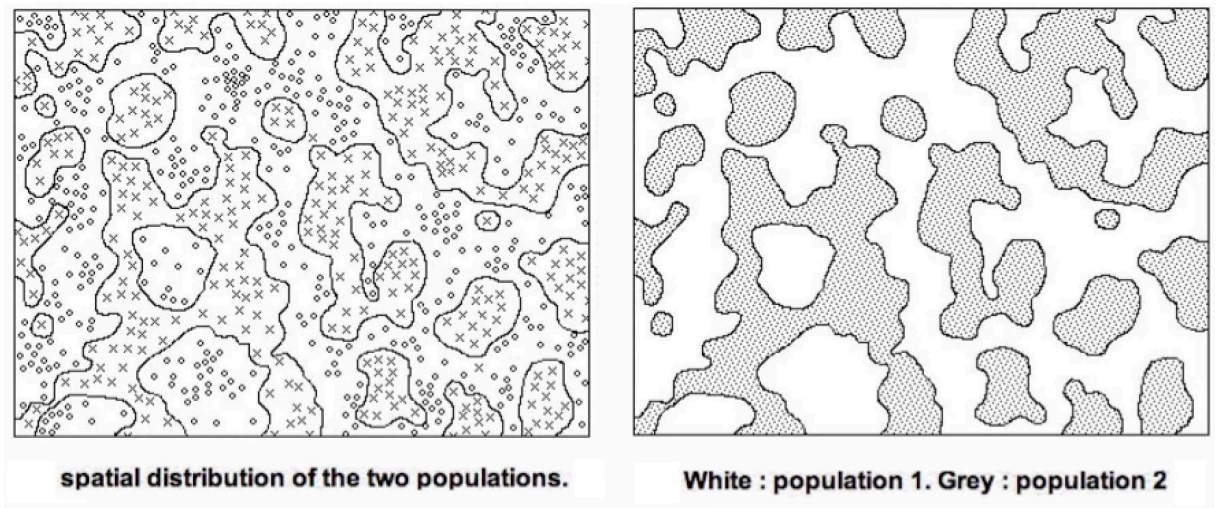
[28] Meaning of the term "heuristic": A hypothesis is formulated. It is tested through experiments (in this case, "computational experiments"), and if it works, one reserves the right to establish its validity and mathematical consistency at a later time.

[29] Commissioned in 1976. 80-megahertz clock speed. 8 megabytes of RAM. One processor. Capable of performing 160 million elementary operations per second. A modern PC: A clock speed 80 times faster, 8 to 16 processors. For simulations: 60 times faster than a Cray-One.



A Cray-1 supercomputer

Here are the 1974 results (percolation):



Initial simulation results

But these results do not seem to lend themselves well to comparison with observations.

Years passed. A young French student, Frédéric Descamps, working on his dissertation at the German DESY laboratory in Hamburg (Deutsches Elektronen-Synchrotron[30]), visited

[30] https://desy.de/index_eng.html

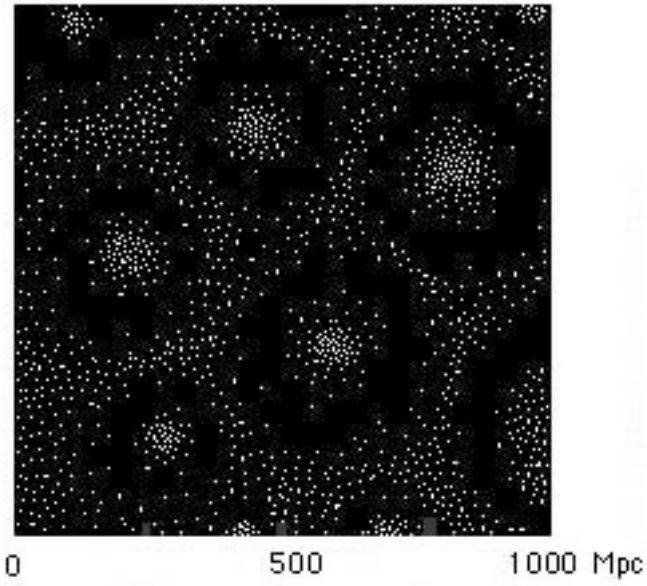
me following the publication of my first book on Ummo[31]. He offered to run these 2D simulations on the center's powerful computer.

And there, I entered a piece of data inspired by the Ummo texts, which state:

- *In this other universe, there is not the same number of galaxies.*

There is therefore an asymmetry between these two entities. This negative world is thus not simply "a copy of the positive world with negative masses." To introduce this asymmetry, I suggested to Descamps that he assign a larger negative mass to the "mass points" of the second entity.

The result follows immediately:



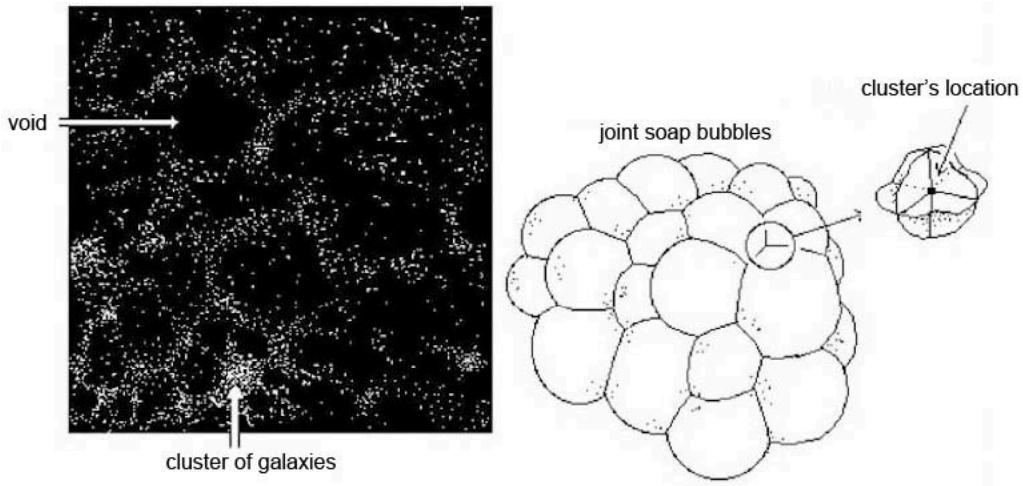
Rationale for Large-Scale Structure

The negative masses are the first to cluster together, due to gravitational instability, giving rise to conglomerates (spheroidal in 2D). These constrain the positive mass to distribute itself throughout the remaining space. In 3D, it will adopt a structure of contiguous bubbles. The theoretical justification becomes immediately apparent. This gravitational instability (Jeans instability) is associated with a characteristic time that varies as the inverse of the square root of the absolute value of the density. Since the density of the negative masses is higher, this time is shorter within that population. Therefore, it is this population that forms the first conglomerates:

$$\rho >> \rho \quad \longrightarrow \quad \bar{\tau} = \frac{1}{\sqrt{8\,\pi\,G\,|\bar{\rho}|}} \quad << \quad \tau = \frac{1}{\sqrt{8\,\pi\,G\,\rho}}$$

[31] Jean-Pierre Petit. An Investigation into Extraterrestrials Who Are Already Among Us. Albin Michel, 1991

It turns out that this is consistent with observational data:



Large-scale structure (observation)

Thus, galaxies cluster at the points where these different cells overlap, forming galaxy clusters. Based on segments common to three cells, these galaxies will form filaments.

The year is 1991. All of this would need to be revisited and expanded using substantial computational resources. Yet this will never be done. We can venture an explanation for this. While there are "codes" today capable of handling an impressive number of mass points, it is likely that none of them addresses the question:

- *What is the sign of the mass of this population?*

Today, within the astrophysics community, the introduction of negative masses into dynamics is still not on the agenda. We will see why later. I am including this with a paper I was in the process of writing, in which I revisited the thesis of "variable constants[32]."

The interaction scheme is illustrated by this educational image, in which we imagine point masses moving on the front and back of a plane:

[32] J.P.PETIT, Twin Universe Cosmology, Astrophys. and Sp. Science, **226**, 273–307, 1995
http://www.jp-petit.org/papers/cosmo/1995-AstrophysSpaceSci.pdf

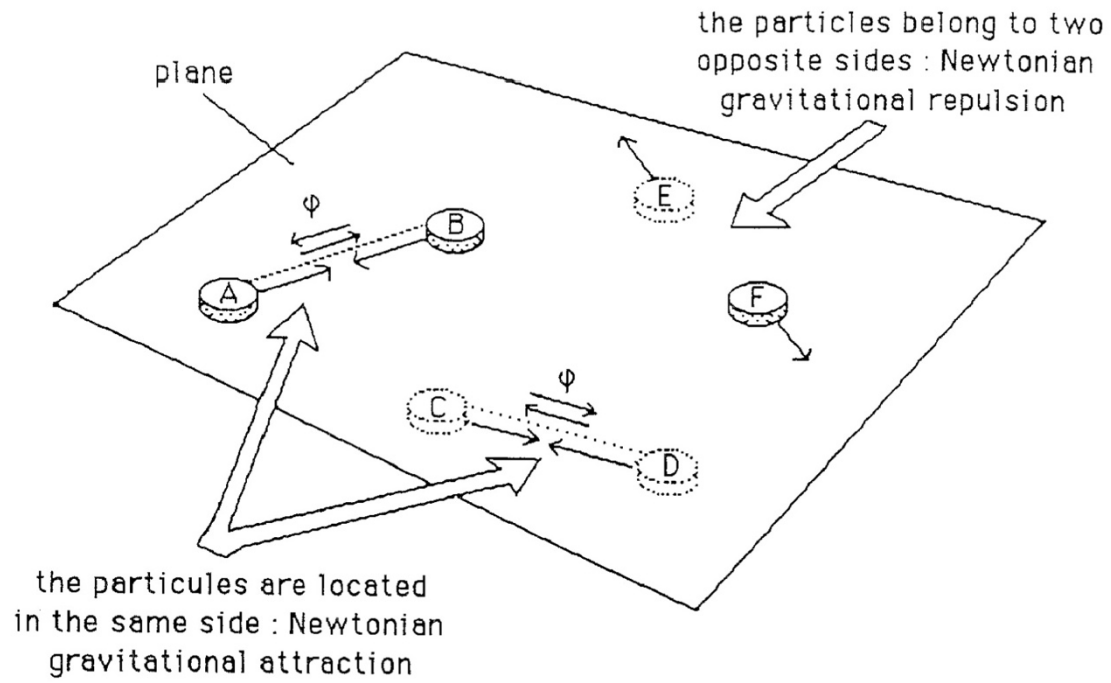


Fig. 2. Two-dimensional image of the system of forces. If the particles are on the same side, they attract each other, according to Newton's law. If they belong to opposite sides they repel each other, according to the repulsive Newton's law. Photons φ can travel from A to B and from C to D and vice-versa, for they are located on the same side. The cannot travel from E to F, and vice-versa.

The Interaction Between Positive and Negative Masses: An Educational Model

The paper introduces several key ideas:

- The confinement of galaxies, attributed to the repulsive force exerted on them by the surrounding negative mass.

- The (negative) gravitational lensing effect that this same negative mass produces on photons of positive energy.

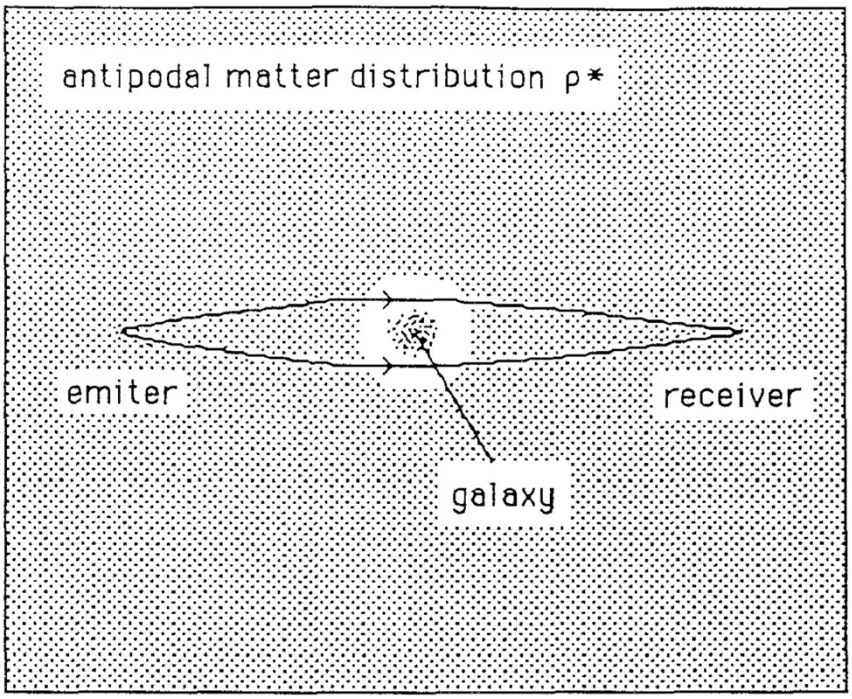


Fig. 14. Combination of the two positive gravitational lensing effect due to the galaxy and to the surrounding antipodal matter.

The gravitational lensing effect due to the negative-mass environment

The year is 1991. With reference to the article published a year earlier in *Nuovo Cimento*, this second type of matter is still referred to as "antipodal."

It is shown in passing that in this mode of evolution, entropy varies as Log t. If we then adopt this entropy as a chronological coordinate, the origin is pushed back… to infinity in the past.

TWIN UNIVERSES COSMOLOGY

JEAN-PIERRE PETIT*
Observatory of Marseille, France

(Received 14 July, 1994; accepted 7 February, 1995)

Abstract. Starting from the field equation $\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T}))$, presented in a former paper, we present a test result, based on numerical simulations, giving a new model applied to the very large structure of the Universe. A theory of inverse gravitational lensing is developed, in which the observed effects could be due mainly to the action of surrounding 'antipodal matter'. This is an alternative to the explanation based on dark matter existence. We then develop a cosmological model. Because of the hypothesis of homogeneity, the metric must be a solution of the equation $\mathbf{S} = 0$, although the total mass of the Universe is non-zero. In order to avoid the trivial solution $R =$ constant $\times\, t$, we consider a model with 'variable constants'. Then we derive the laws linking the different constants of physics: G, c, h, m; in order to keep the basic equations of physics invariant, so that the variation of these constants is not measurable in the laboratory, the only effect of this process being the red shift, due to the secular variation of these constants. All the energies are conserved, but not the masses. We find that all of the characteristic lengths (Schwarzschild, Jeans, Compton, Planck) vary like the characteristic length R, from where all the characteristic times vary like the cosmic time t. As the energy of the photon $h\nu$ is conserved over its flight, the decrease of its frequency ν is due to the growth of the Planck constant $h \approx t$. In such conditions the field equations have a single solution, corresponding to a negative curvature and to an evolution law: $R \approx t^{2/3}$.

The model is no longer isentropic and $s \approx \log t$. The cosmologic horizon varies like R, so that the homogeneity of the Universe is ensured at any time which constitues an alternative to the theory of inflation. We re-find, for moderate distances, Hubble's law. A new law: distance $= f(z)$ is derived, very close to the classical one for moderate red shifts.

Introduction

In a former paper [1] a cosmological model was presented, based on a new field equation:

$$\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T})) \tag{1}$$

which follows from the Lagrangian $(R^+ - R^-)$.

The Einstein equation:

$$\mathbf{S} = \chi\mathbf{T} \tag{2}$$

is a local equation, meaning that the local geometry of the universe (tensor $\mathbf{S}$) is determined by the local content of energy-matter (tensor $\mathbf{T}$). In the equation (1) we assumed that the space-time hypersurface had an $S3 \times R1$ topology and that the local geometry of the universe was determined both by the local content of energy-matter and by the content of energy-matter of the associated antipodal fold, through the antipodality relationship A.

* For scientific correspondance: 9 tour d'Aygosi, 13100, Aix-en-Provence, France.

Astrophysics and Space Science **226:** 273–307, 1995.
© 1995 *Kluwer Academic Publishers. Printed in Belgium.*

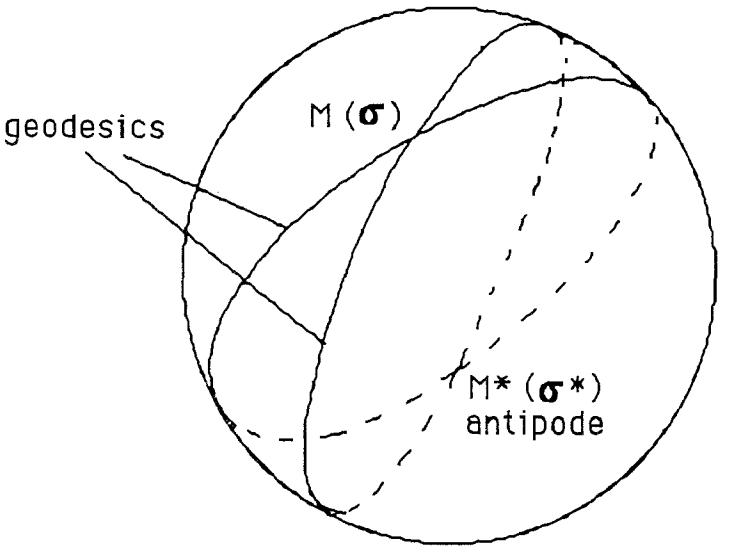


Fig. 1. The coordinate-invariant antipodality relationship.

If σ represents the space coordinates, two geodesics start from M focus at the antipodal point M^*, or $A(M)$. A is an involutive mapping. We can give a didactic image in order to schematize the physical meaning of equation (1).

Consider an $S2$ hollow sphere made of some opaque material. We suppose that, in this medium, heat does not propagate, but causes dilatation. If we deposit thermal energy in some places, the surface will be shaped by dilatation. In such a model, heat represents energy (tensor $\mathbf{T}$). The dilatation materializes the impact of the local energy content on the local geometry. Light does not propagate in this medium, as assumed. But we can assume that sonic waves can propagate and may carry information, from one point to another point.

In classical General Relativity, light is not 'contained' in the model, for the electromagnetic energy is not explicitly present in the energy tensor (although radiative pressure terms can be present in the tensor $\mathbf{T}$), so that the propagation of light along null geodesics is nothing but an hypothesis, well-confirmed by observation and experience. The analogue of the sonic waves, in the classical RG model, are the gravitational waves, that we can build, perturbing the field equation. However, we cannot build electromagnetic waves from the equation (2) and we assume that they follow the null-geodesics of the manifold, as the gravitational waves do.

In the equation (1) we assumed that light also follows the null-geodesic. Moreover, we assumed that the local geometry $\mathbf{S}$ was determined both by the local energy-matter content $\mathbf{T}$ and by the associated antipodal content $A(\mathbf{T})$. In our former paper [1], using the classical low field and small velocities approximation, we have shown that the 'antipodal matter' (located in σ^*) acted on the matter (located

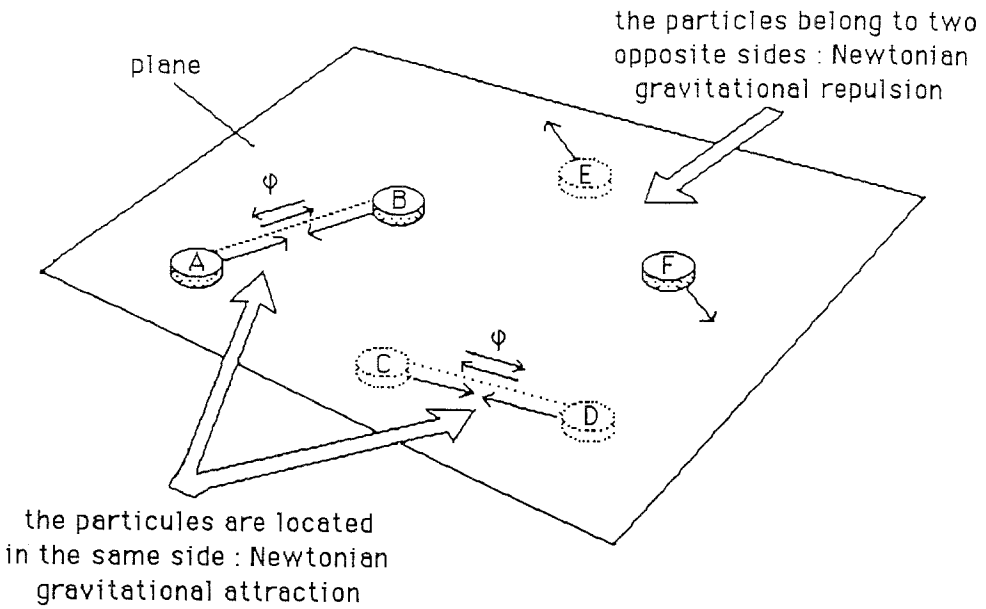


Fig. 2. Two-dimensional image of the system of forces. If the particles are on the same side, they attract each other, according to Newton's law. If they belong to opposite sides they repel each other, according to the repulsive Newton's law. Photons φ can travel from A to B and from C to D and vice-versa, for they are located on the same side. The cannot travel from E to F, and vice-versa.

in σ) as 'a repulsive negative mass distribution', due to the presence of the minus sign of the field equation (1).

We can schematize that in the following $2d$ model. Take a plane and put masses on the two sides, symbolized by small disks.

Two masses can collide, and exchange photons, if they are located on the same side. They cannot if they are located on different sides. Two masses located on the same side attract each other through Newton's law. Two masses located on opposite sides repel each other, through Newton's law. Particles located on the same side can exchange photons, but not particles located on opposite sides (the plane is opaque). See Figure 2.

In our former paper we have shown, through analytic solution, that this mechanism provided a 'missing mass effect' for an observer located on one side, if he ignores the existence of the particles located on the other one. Some results of $2d$ numerical simulations were presented [1]. They provided, on large scales, a non-homogeneous pattern. See reference [1], Figure 7.

But this does not look like the known Universe, which appears to be fairly spongy. In 1970 Zel'dovich proposed his well-known theory of the pancakes [2]. The pancake effect was first demonstrated in numerical models for the evolution of the three-dimensional mass distribution by Doroshkevich *et al.* (1980), Klypin and Shandarin (1983), and Centrella and Mellot (1983) [3, 4, 5]. Mellot and Shandarin (1990) gave an elegant demonstration of the effect by using two-dimensional computations that afforded considerably better resolution for a given particle number, see reference [6]. Shandarin (1988) and Kofma, Pogosyan and Shandarin (1990) presented a powerful semianalytic method for predicting the positions of pancakes from the initial conditions [7 and 8]. More recently (1992) Mellot used a $3d$ set of 64^3 particles, with periodic boundary conditions. From Mellot, the density fluctuations remains small. As pointed out by Peebles in 1993 [9]: "This cannot be the whole story, for the pancakes found are a transient effect: with increasing time the mass in the pancakes drains into clumps that are concentrated in all the three dimensions. This means that if the local sheet of galaxies were a pancake, it must have been formed recently". Then Peebles asked: "could there be a second generation of pancakes that form by the collective collapse of the groups of the clumps that formed out of the first generation?" He concluded immediately: "This does not follow from the analysis given, for it depends on the continuity of the velocity field that allows to write down a series expansion for the evolution of the relative positions. After the formation of the first generation of clumps, which might be the galaxies of their progenitors, the velocity field in general does not have the coherence length, and the analysis from the continuity does not apply".

As a conclusion the pancake theory cannot describe, in its present state, the observed large scale structure.

1. Large Scale Structure and 'Twin Universe Model'

We assumed in the previous paper [1] that the Universe had an $S3 \times R1$ geometry. Any region of the universe interacts antigravitationally with its associated antipoda region, through equation (1). There is a single kind of positive matter m, filling the $S3$ sphere. Then the total mass of the Universe is non-zero. In the reference [1] several didactic $2d$ images (Figures 10, 11 and 12) were given, in order to explain the mechanisms of the interactions of the two adjacent folds.

Using a boosted HP work-station and a set of 2×5000 interacting points, F. Lansheat confirmed the work of Pierre Midy (reference [1], Figure 8). He then focussed on a smaller region, indicated in Figure 3, in which the density of the matter in the 'adjacent fold' was much higher than in the other fold.

As expected, the gravitational instability still occurs and provides new conjugated structures. See Figures 4 and 5.

The matter of the twin fold forms big stable clumps, which repel the matter of our fold of the universe, this last taking place in the remnant space. In opposition to

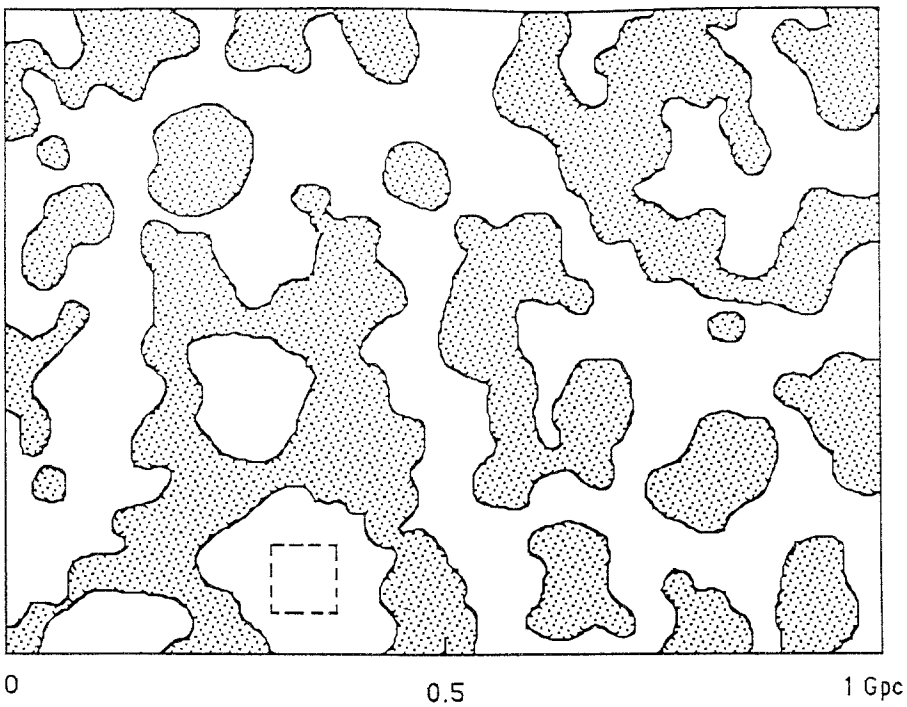


Fig. 3. Dotted square. Focussing on some portion of the very large scale structure in which the density of matter in the first fold (supposedly ours, grey color) is supposed to be smaller than the density of matter in the adjacent fold (white color).

the pancake model numerical simulations, this pattern is fairly non-linear. After its formation, corresponding to the Jeans time of the high density system ($2\ 10^9$ years), there is no significant evolution of the general pattern over a time comparable to the age of the Universe, so that this model could be a good candidate to explain the observed spongy aspect of our fold of the Universe at large scale.

2. $2d$ and $3d$ Simulations

From the results of the $2d$ simulation, F. Lansheat performed a 2 point correlation and compared to the $2d$ correlation obtained from a grey distribution of points (Poisson distribution). The result is shown in the Figure 6. The left hand of the curve is not relevant, for the distance between the points becomes comparable to the mean distance of the random distribution. The growth on the right hand is just an artifact due to the border of the field (periodic boundary). This result cannot be compared directly to the empirical law derived from observational data

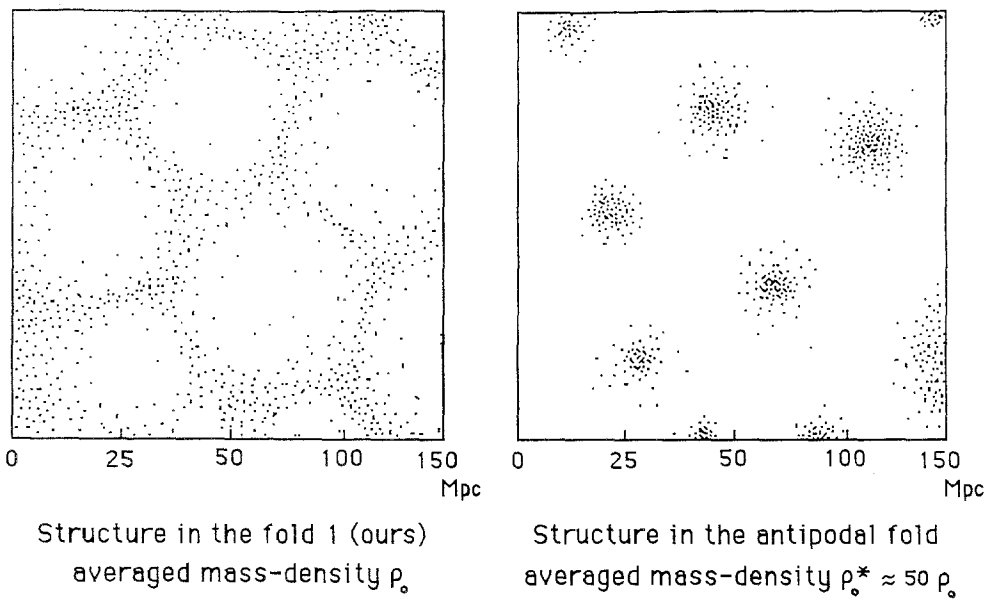


Fig. 4. Results of simulations performed by F. Lansheat, showing the large structure of the Universe, due to the interaction of the two adjacent folds. Mean value of $\rho^* = 50$ times the mean value of ρ (left). Left: cellular structure. Right: cluster structure.

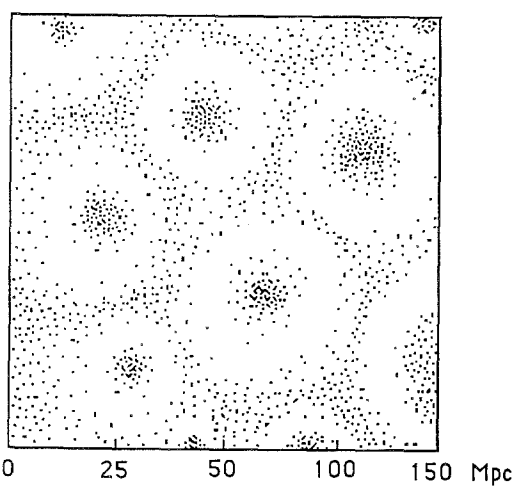


Fig. 5. The same, superimposed.

(slope -1.8); see the surveys of Bahcall (1988) [31], Bahcall and Soneira (1983) [32]. Bahcall and West (1992) [33], and Luo and Schramm (1992) [34]. Three-dimensional simulations have to be performed, with a larger number of points. If possible, a fit with the observational data would provide the ratio of the mass densities of the two universes.

How does one outline a scenario for the formation of large-scale cosmological structure in this model? As long as the coupling between mass and light remains strong ($t < 10^5$ years), the Universe remains homogeneous and all the processes linked to the gravitational instability (formation of clumps, galaxies, stars and spongy structure) are frozen. When the Universe becomes transparent we can assume that all of these processes occur, with their proper characteristic times of formation and evolution. All that we can say is that the suggested very large structure forms in $2\ 10^9$ years.

3. Inverse Gravitational Lensing

The problem of gravitational lensing must be reconsidered. As suggested in the previous paper [1], in the present model the confinement of the galaxies is due mainly to the action of the surrounding antipodal matter, located in the twin fold, to be consistent with the strong missing mass effect. Numerical simulations provided some description of a galaxy, surrounded by halos of antipodal matter [1]. See Figure 7.

As a confirmation of this confinement effect, if we remove the antipodal matter from the system, the central object dissipates immediately. Although this figure concentrates on the surrounding halo, all the surrounding antipodal matter contributes to this confinement effect, so that we can figure schematically the galaxies as nested in some sort of holes of the antipodal matter, as suggested in the Figure 8. The intensity of the confinement effect depends obviously on the density ρ^* of the antipodal distribution, which should be at least ten times larger than ρ.

Classically, matter 'attracts' photons and produces gravitational lensing. The trajectory of photons, bent by the presence of a positive point-mass can be computed from a Schwarzschild solution:

$$ds^2 = \left(1 - \frac{2m}{r}\right)(dx^\circ)^2 - \frac{dr^2}{1 - \frac{2m}{r}} - r^2(d\theta^2 + \sin^2\theta\, d\varphi) \tag{3}$$

Notice that m is an arbitrary constant of integration. For weak fields and slowly moving bodies we can link the g_{00} term of the metric to the gravitational potential Ψ through:

$$g_{00} \approx 1 + \frac{2\Psi}{c^2} \tag{4}$$

The gravitational potential, due to a mass M is:

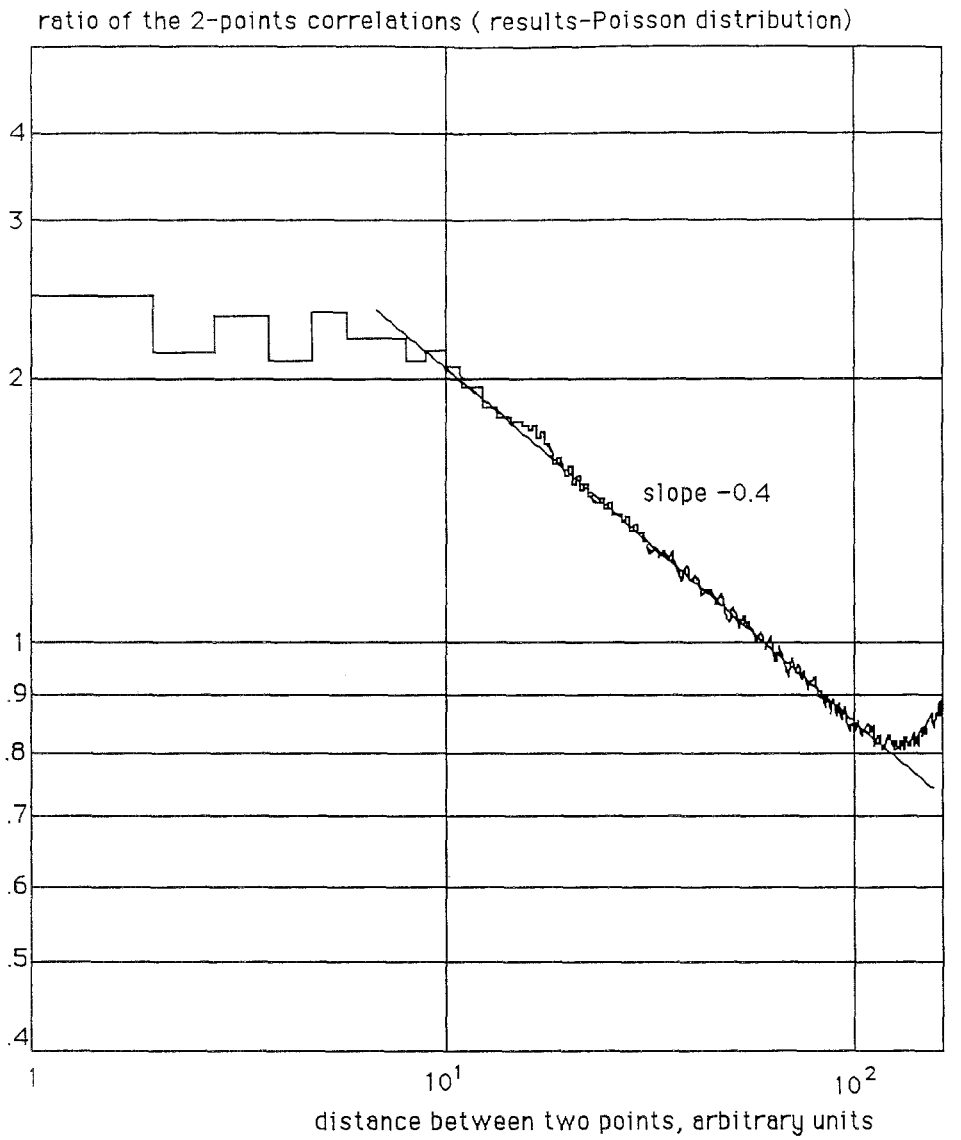


Fig. 6. The slope of the curve of the 2-points correlations ratio (numerical simulation versus Poisson random distribution).

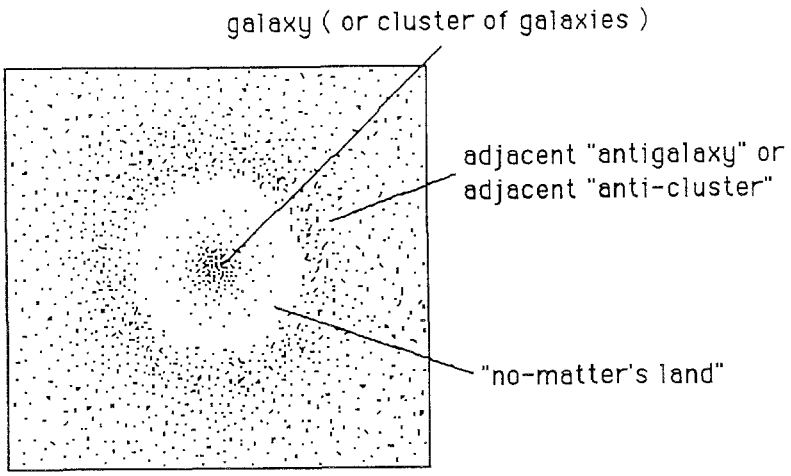


Fig. 7. Concentration of mass confined by the action of the surrounding antipodal matter from $2d$ numerical simulations.

$$\Psi = -\frac{GM}{r} \tag{5}$$

whatever this mass, M would be positive or negative. If M is negative, it repels the test particle. Then

$$g_{00} \approx 1 - \frac{2GM}{rc^2} \tag{6}$$

whence:

$$m = \frac{GM}{c^2} \text{ (positive or negative)} \tag{7}$$

If M is positive the characteristic Schwarzschild length is

$$R_s = \frac{2GM}{c^2} \tag{8}$$

As pointed out above, m is nothing but an arbitrary constant of integration in the Schwarzschild solution. If we take $m < 0$ then the associated mass M becomes negative. We can define a characteristic length, positive (the Schwarzschild radius R_s) from:

$$m < 0 \quad M = \frac{mc^2}{G} < 0 \quad R_s = -\frac{2GM}{c^2} > 0 \tag{9}$$

The trajectory, in polar coordinates, corresponds to:

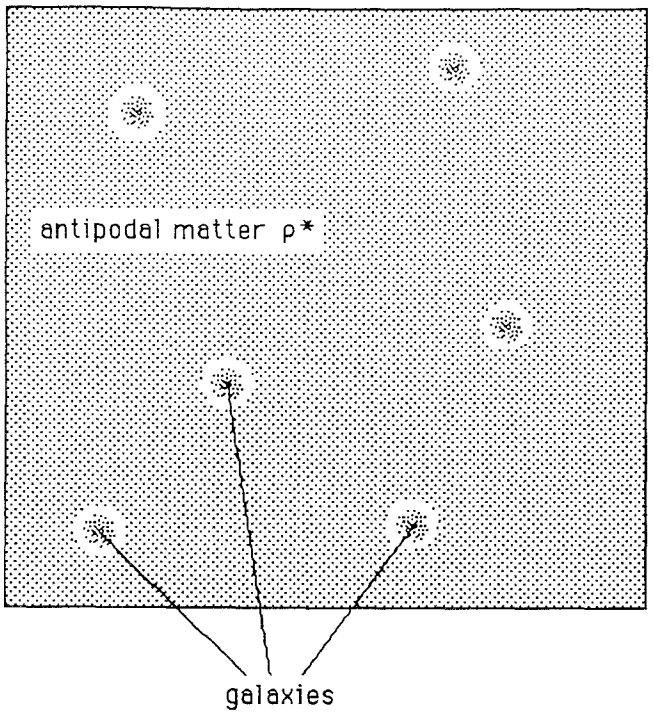


Fig. 8. Galaxies nesting in a wide antipodal matter cloud (the galaxy and the antipodal matter repel each other).

$$\varphi = \varphi_0 + \int_{u_0}^{u} \frac{du}{\sqrt{\frac{c^2l^2-1}{h^2} + \frac{2m}{h^2}u - u^2 + 2mu^3}} \tag{10}$$

See reference [10], p. 203. For the photon, following the null geodesics, we get

$$\varphi = \varphi_0 + \int_{u_0}^{u} \frac{du}{\sqrt{\frac{c^2l^2}{h^2} - u^2 + 2mu^3}} \tag{11}$$

φ is the polar angle for this plane trajectory. $u = 1/r$.

A positive mass ($M > 0$; $m > 0$) produces a positive gravitational lensing:

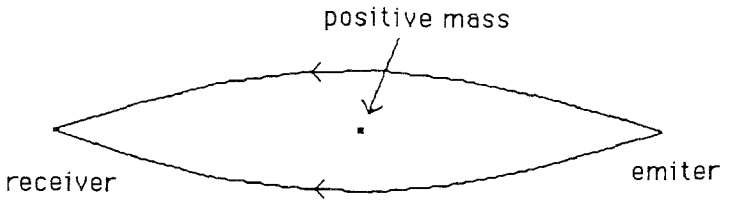


Fig. 9. Classical (positive) gravitational lensing.

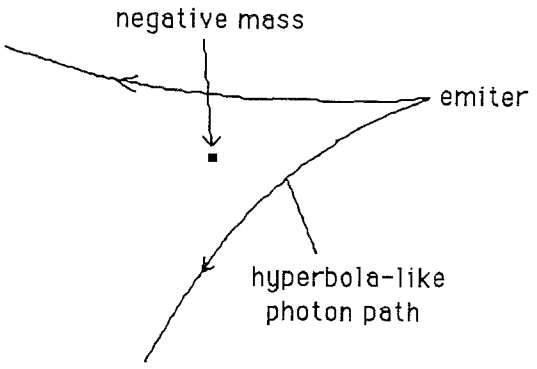


Fig. 10. Negative lensing effect due to a 'negative' mass.

For a test particle, located in one fold, a mass located in the adjacent fold behaves like a repulsive negative mass ($M < 0$; $m < 0$) and then produces a negative lensing effect:

Notice that these hyperbolic paths are familiar to specialists of plasma physics ($e - e$ or $p - p$ scatterings).

Let us schematize the situation. Consider a homogeneous distribution of antipodal matter. In this distribution we find, in some places, holes in which the galaxies nest.

A hole in a distribution of negative mass produces a positive gravitational lensing effect.

Qualitatively this is equivalent to the effect due to a homogeneous sphere of positive mass. See Figure 13.

Classically one uses the gravitational lensing to evaluate the so-called invisible mass contained in a galaxy. People used to say: "the dark matter exists in our galaxy: we measure it, through the missing mass effect". In this twin cosmological model a strong lensing effect should not be a proof of the existence of invisible mass in a

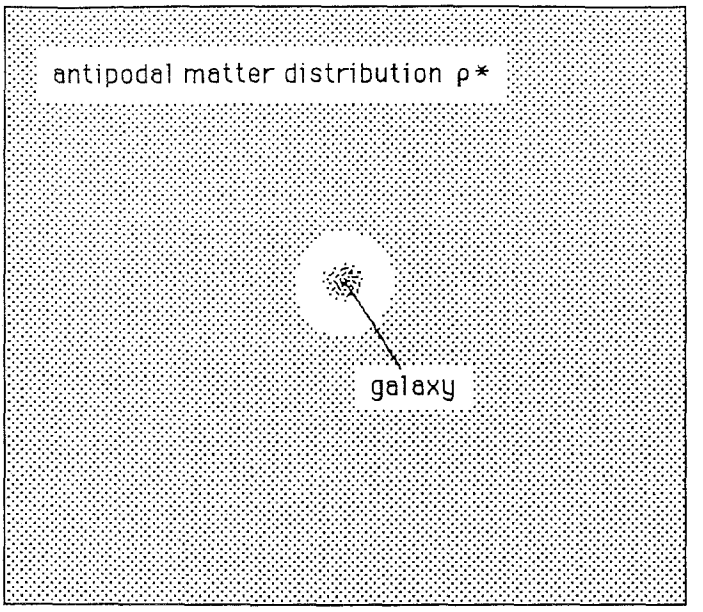


Fig. 11. A galaxy nesting in an homogeneous cloud of antipodal matter.

galaxy, but could be due to the action of the invisible surrounding antipodal matter, which could be evaluated from the measured effects. See the Figure 14.

In our galaxy the mass necessary to prevent the explosion by centrifugal force is about 10 times higher than the observed mass. If the confinement effect is due to the action of surrounding invisible antipodal matter, it means that the effect of this invisible matter is important. This could be quite general in the region of the universe we live in. All the neigbour galaxies could then be surrounded by dense halos of antipodal matter, and the observed gravitational lensing should be due mostly to the antipodal material rather than to the galaxies themselves.

The model based on the equation (1) gives a new insight on the missing mass problem [1] and on the very large structure of the Universe. This work was based on the low field and weak velocities hypothesis and refered to a quasi-steady Universe, at cosmologic scale, with respect to space and time. In order to complete this cosmological model we have now to study the evolution of the Universe as a whole.

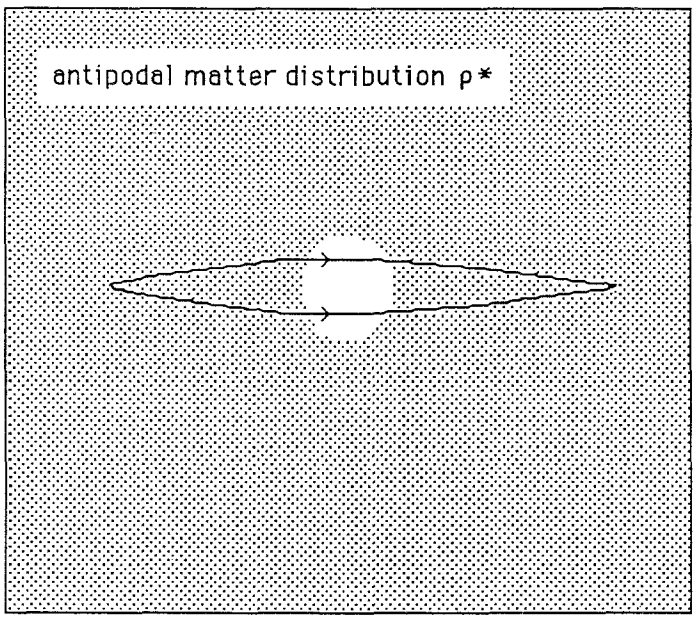


Fig. 12. Induced positive gravitational lensing effect.

4. About the Constancy of G and c

Consider the two quantities G (gravitation) and c (velocity of the light). They are involved in the constant of Einstein χ. This last is classically determined as the following.

The metric is expressed as:

$$g_{\mu\nu} = g_{\mu\nu}^{(L)} + \varepsilon\gamma_{\mu\nu} \tag{12}$$

where $g_{\mu\nu}^{(L)}$ is the Lorentz metric tensor and $\varepsilon\gamma_{\mu\nu}$ represents a very small time-independant perturbation (nearly Lorentzian metric tensor). Furthermore, in order to make a close connection with classical theory, one supposes that the velocity of a particle along a geodesic is much less than c, i.e.:

$$\left(\frac{ds}{dx^{\circ}}\right)^2 \cong (1 + \varepsilon\gamma_{\infty}) \tag{13}$$

One next applies the same approximation to the differential equation of a geodesic:

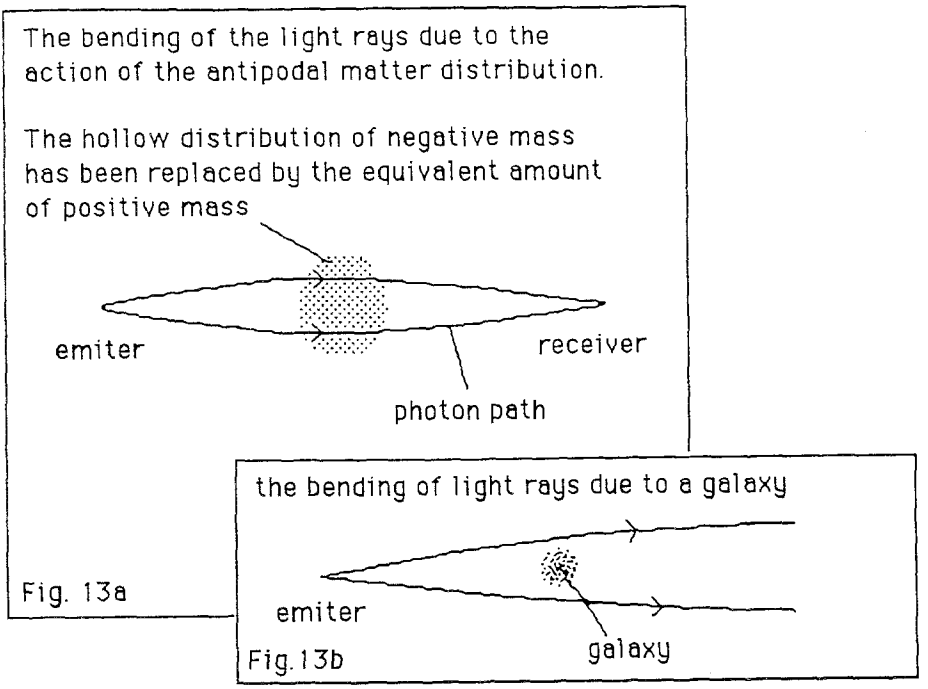


Fig. 13. 13a: Positive gravitational lensing effect due to the distribution of antipodal matter (acting like a negative mass). We have replaced the hollow by an equivalent amount of positive mass. Compared positive lensing due to a galaxy (Fig. 13b).

$$\frac{d^2x^\alpha}{ds^2} + \begin{pmatrix} & \alpha & \\ \eta & & \tau \end{pmatrix} \frac{dx^\eta}{ds}\frac{dx^\tau}{ds} = 0 \tag{14}$$

And then we get

$$\frac{d^2x^\alpha}{ds^2} + \begin{Bmatrix} & \alpha & \\ \circ & & \circ \end{Bmatrix} \left(\frac{dx^\circ}{ds}\right)^2 = 0 \tag{15}$$

Beyond the steady state conditions, one uses to write:

$$dx^\circ = c\,dt \tag{16}$$

which introduces both the light velocity c and the time t. In addition:

$$\begin{Bmatrix} & \alpha & \\ 0 & & 0 \end{Bmatrix} = \tfrac{1}{2}\varepsilon\,\gamma_{00}|\alpha \tag{17}$$

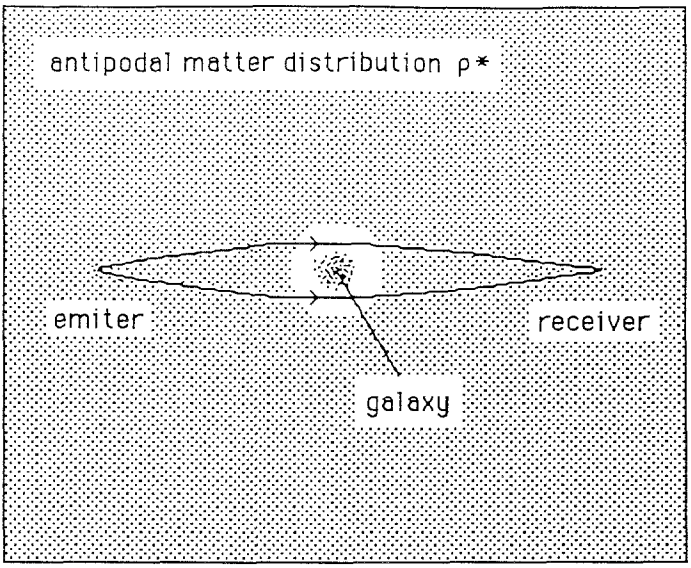


Fig. 14. Combination of the two positive gravitational lensing effect due to the galaxy and to the surrounding antipodal matter.

The geodesic equation becomes:

$$\frac{d^2x^\alpha}{dt^2} = -\frac{c^2}{2}\,\varepsilon\,\gamma_{00}|\alpha \tag{18}$$

If we identify with the Newtonian model, we can relate the gravitational perturbation potential to the metric through:

$$\Psi = \frac{c^2}{2}\,\varepsilon\,\gamma_{00} \quad \text{or} \quad g_{00} \cong 1 + \frac{2\Psi}{c^2} \tag{19}$$

If we consider a medium with low density ρ_0 and low velocity, the matter energy tensor reduces to:

$$\mathbf{T} = \begin{vmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix} \tag{20}$$

whose trace is ρ_0. Then the second member of the field equation becomes:

$$\chi(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\,T) = \frac{\chi\rho_0}{2}\,\delta_{\mu\nu} \tag{21}$$

Still in steady hypothesis condition, we get:

$$\varepsilon \sum_{i=1}^{3} \gamma_{00|i|i} = -\chi \rho_0 \tag{22}$$

Identifying with Poisson equation, we determine the unknown constant χ of the field equation:

$$\chi = -\frac{8\pi G}{c^2} \tag{23}$$

If χ is not considered as an absolute constant, the zero-divergence of the field equation (1) is no longer ensured, according to the hypothesis $\partial \mathbf{T} = 0$, which provides conservation equations of physics. But let us point out that the constancy of χ does not require separately the constancy of G and c, for we determined (23) from a time-independent metric (12). Then we can shift towards the less restrictive condition:

$$\frac{G}{c^2} \approx \text{constant} \tag{24}$$

This idea was suggested by this author in 1988–89 in the papers [12,13,14]. But, as far as we know, the idea of a secular variation of the light velocity, was introduced earlier by V.S. Troitskii [11].

5. The Robertson-Walker Metric

Assuming that the Universe is isotropic and can be described by a Riemanian metric we get the classic Robertson metric:

$$ds^2 = (dx^\circ)^2 - e^{g(x^\circ)} \frac{1}{\left(1 + \frac{k}{4}\frac{r^2}{r_\circ^2}\right)^2} (dr^2 + r^2\, d\theta^2 + r^2\, \sin^2\, \theta\, d\varphi^2) \tag{25}$$

If the Universe is assumed to be homogeneous, then $\mathbf{T} = A(\mathbf{T})$ and the spatially homogeneous cosmological solution comes from:

$$\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T})) = 0 \tag{26}$$

This metric must be introduced in the equation (1), with a zero second member. Then we get the following set of two equations:

$$\left(\frac{dR}{dx^\circ}\right)^2 + k = 0 \tag{27}$$

$$\frac{2}{R}\frac{d^2R}{dx^{\circ 2}} + \frac{k}{R^2} - \frac{1}{R^2}\left(\frac{dR}{dx^\circ}\right)^2 = 0 \tag{28}$$

From (27) and (28) we get

$$k = -1 \text{ (negative curvature)} \quad \text{and } R = x^{\circ} \tag{29}$$

x° is a 'chronological marker'. Notice that one has a single solution ($k = -1$). If we identify, classically, x° to ct, c being considered as an absolute constant, we get the well-known trivial solution $R = ct$. Doing that, we define somewhat arbitrarily the cosmic time t. But it can be defined differently, in a non-standard way, as will be shown in the following.

6. A Model with 'Variable Constants'

The hypothesis of the constancy of the so-called constants of physics was first challenged by Milne [15]; then by other authors: P.A. Dirac [16 and 17], F. Hoyle and J.V. Narlikar [18], V. Canuto and J. Lodenquai [19], T.C. van Flandern [20], V. Canuto and S.H. Hsieh [20], A. Julg [21] (developed ideas mainly based on the variation of G). Time-dependent G has also considered by Brans and Dicke [22]; and time dependent e by Ratra [23]. Guth [24], Sugiyama and Sato [25] and Yoshii and Sato [26] considered a time-variable cosmological constant. In general these approaches focused on the variation of a certain number of 'constants', not of all the constants, in a combined fashion, are developed in the present paper. H. Reeves [27] studied the impact of the separate variations of the constants, one after the other. V.S. Troistkii [28] first suggested in 1987 the possible variation of c, and, in general, of all the 'constants', but, after choosing a leading parameter he just tried to adjust the different exponents, associated to *a priori* polynomial empiric laws, to fit with observational features.

In the present paper we are going to build a cosmological where all the 'constants' vary conjointly. This will be made consistent with the field equation (1). We are going to search for laws that let the equations of physics be invariant, so that these variations cannot be evidenced in local lab experiments. These equations follow.

The Schrödinger equation:

$$-\frac{h^2}{2m}\,\Delta\Psi + U\Psi = i\,\frac{h}{2\pi}\,\frac{\partial\Psi}{\partial t} \tag{30}$$

The Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial\phi}{\partial \mathbf{r}}\,\frac{\partial f}{\partial \mathbf{v}} = \int (f' f_1' - f f_1)\, g\, a\, da\, d\omega\, d^3\mathbf{v} \tag{31}$$

where f is the distribution function of the velocity $\mathbf{v}$, $\mathbf{r} = (x, y, z)$, t the time, (g, a, ω) the classic impact parameters of a binary collision.

The (new) Poisson equation for gravitation (see reference [1]) is:

$$\Delta\phi = 4\pi G(\rho - \rho^*) \tag{32}$$

ρ is the mass density in our fold of the Universe and ρ^* the mass-density in the twin fold.

The (new) field equation

$$\mathbf{S} = \chi(\mathbf{T} - \mathbf{T}^*) \tag{33}$$

where:

$$\chi = -\frac{8\pi G}{c^2} \tag{34}$$

is the Einstein constant, G the 'constant' of gravity and c the velocity of the light.

The Maxwell equations are:

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\,\frac{\partial \mathbf{E}}{\partial t} \tag{35}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{36}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{37}$$

$$\nabla \cdot \mathbf{E} + \frac{\rho_e}{\varepsilon_0} = 0 \tag{38}$$

$\mathbf{E}$ and $\mathbf{B}$ are, respectively, the electric and magnetic fields. We consider the Maxwell equation for a neutral medium, for we assume that the Universe is electrically neutral. These equations are not all independent. For an example, the Poisson equation, for gravitation (32), comes from the field equation (33), see [1].

Introducing a characteristic length R and a characteristic time T we can write these characteristic equations into an adimensional form:

The Schrödinger equation (30), with:

$$\mathbf{r} = R\xi \quad \nabla = \frac{1}{R}\,\delta \quad t = T\tau \tag{39}$$

$$U = \frac{h^2}{2mR^2}\,u \tag{40}$$

becomes:

$$-\frac{h^2}{2mR^2}\,(\delta^2\Psi + u\Psi) = i\,\frac{h}{2\pi T}\,\frac{\partial \Psi}{\partial \tau} \tag{41}$$

The Boltzmann equation (31), with:

$$\mathbf{v} = c\zeta \quad \mathbf{r} = R\xi \quad g = c\gamma \quad a = R\alpha \tag{42}$$

$$f = \frac{n}{\langle V \rangle^3}\,e^{\frac{-v^2}{\langle V \rangle^2}} \tag{43}$$

$$f = \frac{1}{R^3 c^3}\,\eta \tag{44}$$

$$f = \frac{1}{R^3 c^3}\,\eta \quad n = \frac{1}{R^3}\,\varpi \quad \phi = \frac{GM}{R}\,\varphi \tag{45}$$

becomes:

$$\frac{1}{T}\frac{\partial \eta}{\partial \tau} + \frac{c}{R}\frac{\partial \eta}{\partial \xi} - \frac{Gm}{R^2 c}\frac{\partial \phi}{\partial \xi}\cdot\frac{\partial \eta}{\partial \zeta} = \frac{c}{R}\int (\eta'\eta'_1 - \eta\eta_1)\,\gamma\alpha\, d\alpha\, d\omega\, d^3\zeta \tag{46}$$

The Poisson equation for the gravitational potential (32), with:

$$\phi = \frac{Gm}{R}\,\varphi \quad n = \frac{1}{R^3}\,\varpi \quad n^* = \frac{1}{R^3}\,\varpi^* \tag{47}$$

$$\frac{Gm}{R^3}\,\delta^2\varphi = 4\pi\,\frac{Gm}{R^3}\,(\varpi - \varpi^*) \tag{48}$$

becomes:

$$\delta^2\varphi = 4\pi(\varpi - \varpi^*) \tag{49}$$

The Maxwell equations (35), (36), (37), (38), with:

$$\nabla = R\delta \quad t = T\tau \quad \mathbf{B} = B^*\beta \quad E = E^*\varepsilon \quad \rho_e = \frac{e}{\varepsilon_0 R^3}\,\varpi_e \tag{50}$$

where e is the electric charge (we assume that the number of electric charges is conserved) become:

$$\frac{B^*}{R}\,\delta \times \beta = \frac{E^*}{c^2 T}\,\frac{\partial \varepsilon}{\partial \tau} \tag{51}$$

$$\frac{E^*}{R}\,\delta \times \varepsilon = -\frac{B^*}{T}\,\frac{\partial \beta}{\partial \tau} \tag{52}$$

$$\delta \cdot \beta = 0 \tag{53}$$

$$\frac{E^*}{R}\,\delta \cdot \varepsilon + \frac{e}{\varepsilon_0 R^3}\,\varpi_e = 0 \tag{54}$$

In these equations we find a certain number of physical constants:

$$h,\ m,\ c,\ G \tag{55}$$

The invariance of the Schrödinger equation is ensured if:

$$\frac{hT}{mR^2} = Cte \tag{56}$$

The Boltzmann equation is invariant if:

$$\frac{1}{T} \approx \frac{c}{R} \approx \frac{Gm}{R^2 c} \tag{57}$$

The Poisson equation for gravitation poses no particular problem and just becomes

$$\delta^2 \varphi = 4\pi(\varpi - \varpi^*) \tag{58}$$

From the Maxwell equations we get:

$$R \approx cT \tag{59}$$

$$\frac{E^*}{R} = \frac{e}{\varepsilon_0 R^3} \tag{60}$$

which is consistent with the definition of an electric field due to an electric charge.

From the Einstein equation, as pointed out earlier, we get:

$$G \approx c^2 \tag{61}$$

If not, the equation is no longer divergenceless.

If the quantities:

$$h,\ m,\ c,\ G,\ R,\ T \tag{62}$$

obey these relations, *it will not be possible to have evidence of their variations in any in-lab experiment.*

So what?

From (57) we get immediately:

$$\frac{Gm}{c^2} \approx R \tag{63}$$

which is nothing but the characteristic Schwarzschild length, so that:

$$R_s \approx R \tag{64}$$

Examine now the Jeans' length:

$$L_j = \frac{\langle V \rangle}{\sqrt{4\pi G \rho m}} \tag{65}$$

where:

$$\langle V \rangle = c \langle \zeta \rangle$$

$$L_j = \frac{cR^{3/2}}{\sqrt{Gm}} \frac{\langle \zeta \rangle}{\sqrt{4\pi \varpi}} \tag{66}$$

$$\frac{Gm}{c^2} \approx R \quad \frac{c}{\sqrt{Gm}} \approx \frac{1}{\sqrt{R}} \quad L_j = R\frac{\langle\zeta\rangle}{\sqrt{4\pi\varpi}}$$
$$L_j \approx R \tag{67}$$

Combine the equations (56) and (57), we get:

$$\frac{hT}{mR^2} = \frac{h}{mR^2}\frac{R}{c} = Cte$$
$$\frac{h}{mc} \approx R \tag{68}$$

The Compton Length varies like R:

$$R_c \approx R \tag{69}$$

The Planck length is:

$$L_p = \sqrt{\frac{hG}{c^3}} = \sqrt{\frac{h}{c}\frac{G}{c^2}}$$
$$L_p \approx R \tag{70}$$

The Planck time is:

$$t_p = \sqrt{\frac{hG}{c^5}} = \frac{R}{c} \approx T \tag{71}$$

The Jeans time is:

$$t_J = \frac{1}{\sqrt{4\pi G\rho}} \approx T \tag{72}$$

Combining (61) and (63) we get:

$$m \approx R \tag{73}$$

The variation of the constants does not conserve the mass.

If we conserve the number of species, the mass density ρ is found to obey:

$$\rho = nm = \frac{\varpi}{R^3}\, m \quad \rho \approx \frac{1}{R^2} \tag{74}$$

This is the same law for the contribution ρ_r of the radiation to the density ρ. The conservation of the radiative energy gives:

$$p_r R^3 = \text{constant} \tag{75}$$

Then:

$$p_r = \frac{\rho_r c^2}{3} \rightarrow \rho_r \approx \frac{1}{R^2} \tag{76}$$

7. The Variation of the Constants of Physics in the Framework of an Hypothesis of Generalized Conservation of the Energies

In the standard model of the Universe the mass is conserved, but not the energy. The energy-matter is composed both by massive particles and by photons. Each owns a mass-density, respectively: ρ_m and ρ_r. Today:

$$\rho_m \gg \rho_r \tag{77}$$

The pressure is a density of energy. Today we have:

$$p_m \ll p_r \tag{78}$$

In the standard model we impose the conservation of the mass mc^2, not the conservation of the energy of the cosmic background photons $h\nu$, which varies like $1/R$. Let us consider the opposite hypothesis and conserve the energy, but not the mass. Then:

$$mc^2 = Cte \tag{79}$$

whence:

$$c \approx \frac{1}{\sqrt{R}} \tag{80}$$

We get a model where the velocity of the light is no longer considered an absolute constant. *But, as we insist above, this cannot be evidenced in a laboratory*, for the 'constants' involved in the considered phenomena, according to our hypothesis, follow this process too.

In addition:

$$G \approx \frac{1}{R} \tag{81}$$

$$h\nu \approx \frac{h}{T} = Cte \quad h \approx T \tag{82}$$

Anyone of the considered parameters: G, c, m, R, h, T can be chosen as an evolution parameter. If we take T as an evolution parameter (V.S. Troistkii [28] considered c as an evolution parameter), we get:

$$R \approx t^{2/3}$$

$$G \approx t^{-2/3}$$

$$m \approx t^{2/3}$$

$$h \approx t \tag{83}$$

$$c \approx t^{-1/3}$$

$$\rho \approx t^{-4/3}$$

$$v \approx t^{-1/3}$$

$$L_j \text{ (Jeans)} \approx L_p \text{ (Planck)} \approx L_c \text{ (Compton)} \approx R_s \text{ (Schwarzschil)} \approx R$$

All these lengths vary like $t^{2/3}$.
In addition we have:

$$t_j \text{ (Jeans)} \approx t_p \text{ (Planck)} \approx T \tag{84}$$

And:

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = Cte \quad h\nu = Cte \tag{85}$$

We get a model in which all the characteristic lengths vary like R and all the characteristic times vary like T.

For an example, if we consider a two-bodies system, orbiting around their common center of gravity, we find that the radius of the orbit varies like R and the orbitation period varies like T.

Now, what could be the observational consequences of such a model?

This process cannot be evidenced in a lab. The only observable effect is the red shift.

8. The Red Shift as an Observational Effect due to the Secular Variation of the Constants of Physics. The Question of the Expansion of the Universe

We will identify the parameter T to the cosmic time t, running the events of the universe. According to our hypothesis, the energy of a travelling photon is conserved, but as

$$h \approx t$$

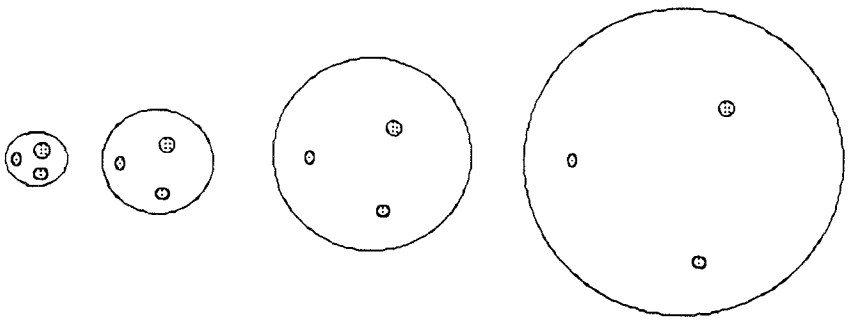


Fig. 15. Standard model: the containment expands, not the contents.

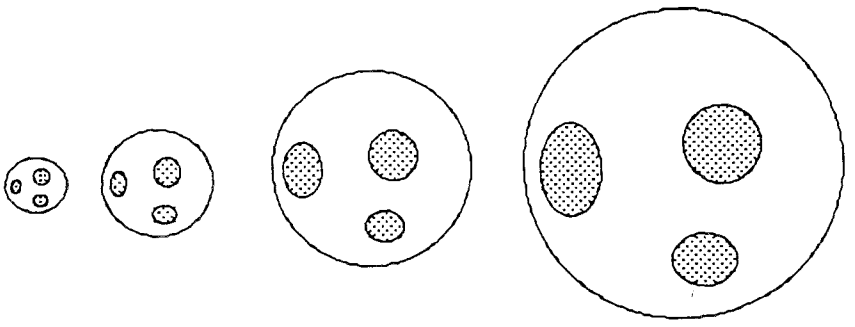


Fig. 16. Present model, both expand (the concept of an expansion looses its significance, for there is no longer a reference scale).

then

$$\nu \approx \frac{1}{t} \tag{86}$$

This idea was first introduced by E.A. Mine [15]. In an astronomical observatory we measure the frequency of the received photon and we find a red shift, that we interpret in terms of the variation of its energy, due to an expansion process. But if we consider that the energy is conserved over its long flight this red shift effect can be reinterpreted in terms of the secular variation of the Planck constant.

We then ask 'is the Universe expanding?' this questions falls, for all the objects of the Universe follow the above process: the massive bodies, the stars, the galaxies, and the Universe itself.

If we consider that the Universe is expanding, it implies that its content (the particles, the galaxies) does not expand in time, in order to have a reference scale.

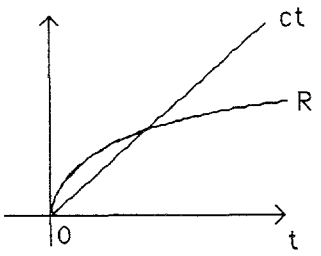


Fig. 17. Comparison of the evolution of the characteristic length of the Universe with the cosmological horizon, in an Eintein-de Sitter model.

If the contents of the Universe undergoes the same process as the Universe itself (a gauge process) the question of the expansion becomes irrelevant.

In the present model, in the laboratory, the instruments vary like any length we want to measure, for both follow the process. The question of the expansion becomes irrelevant and the only observable phenomenon, evidencing the cosmic evolutions is the red shift, which is no longer connected to the Doppler-Fizeau effect, but to the secular variation of h, combined to the hypothesis of the conservation of the photon energy $h\nu$ during its flight.

9. The Problem of the Cosmological Horizon

Classically this the cosmologic horizon is defined as ct, from which arises a paradox. The observed Universe is very homogeneous, at large scale. If we compare any characteristic distance $R(t)$ (for example, the mean distance between particles), with the horizon, we get:

In the present model the cosmological horizon becomes the following integral:

$$H = \int_0^t c(\tau)\, d\tau \approx t^{2/3} \approx R \tag{87}$$

If the Universe was homogeneous at the beginning, the collisional process, always present, tends to maintain this homogeneity. If it was not, it tends to smooth it. This constitutes an alternative to the theory of inflation.

This law between $R \approx t^{2/3}$ must not be considered as an expansion process but as a consequence of the secular variation of the constants of physics, a gauge process, whose single observable effect is the red shift.

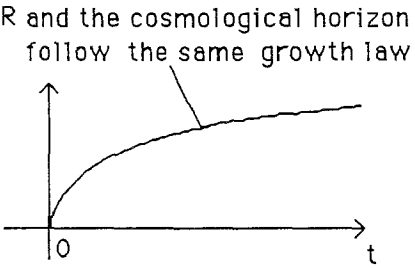


Fig. 18. Comparison of the evolution of the characteristic length R of the Universe with the cosmological horizon, in the present model. They have the same variation in time.

10. The Link with the Robertson-Walker Geometry

All this is compatible with the solution (34) if we give the following non-standard definition of the cosmic time:

$$t = \text{constant } (x^\circ)^{3/2} \tag{88}$$

The dimension of the constant is:

$$\text{constant} = \frac{\text{time}}{(\text{length})^{3/2}}$$

In the standard definition of the cosmic time from

$$t = \text{constant } x^\circ \quad (x^\circ = ct)$$

the dimension of the constant is

$$\text{constant} = \frac{1}{c} = \frac{\text{time}}{\text{length}}$$

11. Entropy as a Better Chronological Marker

The detailed calculation of the entropy per baryon, as defined by:

$$s = \frac{k}{n} \int_? f \operatorname{Log} f \, du \, dv \, dw \tag{89}$$

where f is the velocity distribution function, was given in a former paper, with 'variable constants'. See [13], Section 2. As a result, we found:

$$s \approx \frac{3}{2} k \operatorname{Log} R \approx \operatorname{Log} t \tag{90}$$

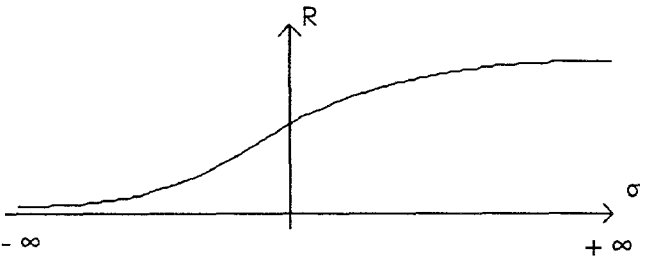


Fig. 19. The evolution of the curvature radius R of the Universe versus the entropy.

If $R(t)$ is an increasing function of t, the cosmic entropy grows like the cosmic time. In lab experiments we usually relate entropy with time and consider that, according to the second principle, there is no possible strictly isentropic phenomenon. We consider that the time flux depends on the entropy change. In the classical model it is somewhat paradoxal to notice that such enormous change in time would go with zero entropy variation. In the present model when the time t tends toward zero, s tends toward $-\infty$.

We have $s = \text{constant Log } t$. If we change the measure of the entropy (modifying the value of the constant) and write:

$$\sigma = \frac{2}{3} \operatorname{Log} t \tag{91}$$

we get:

$$dt = 3/2\, t\, d\sigma \tag{92}$$

Let us return to the Robertson Walker metric.

$$dS^2 = c^2 dt^2 - R^2 \frac{du^2 + u^2 d\theta^2 + \sin^2 \theta\, d\varphi^2}{\left(1 - \frac{u^2}{2}\right)^2}$$

We get, with $R = 3/2ct$:

$$dS^2 = R^2 \left\{ d\sigma^2 - \frac{du^2 + u^2\, d\theta^2 = \sin^2 \theta\, d\varphi^2}{\left(1 - \frac{u^2}{2}\right)^2} \right\} \tag{93}$$

In the representation {entropy, space variables} the metric becomes conformally flat and we have:

In the classical description (t, σ) the physicist, when t tends to zero, has some difficulty defining any material clock, for the velocities of the particles tend toward c. In a 'variable constant cosmological model' the entropy per baryon (99) is no longer constant and never fails to describe the events of the Universe. Notice that

in a (s, σ) description, the problem of the origin of the Universe falls down. In addition, if we describe the Universe in a phase space (position plus velocity) we find that the associate characteristic hypervolume R^3c^3 varies like t.

12. The Red Shift and the Robertson-Walker Metric with a Variable Light Velocity

The derivation of the distance from the red shift z, with 'variable constants', has already been presented. See reference [13], sections 3 to 7. The index refers to the emitter and the index 2 to the receiver. For an example c_2 is the today's value of the velocity of the light, as measuring in the observatory. It is assumed that the Rydberg constant (ionization energy of the hydrogen) follows

$$E_i \approx R^\gamma \tag{94}$$

Then we find:

$$1 + z = \left(\frac{R_2}{R_1}\right)^\gamma \tag{95}$$

The value $\gamma = 1$ is chosen in order to fit with the classical value.
Then, expanding the function $1/R(t)$ into a series with respect to

$$\varepsilon = \frac{c_2(t - t_2)}{R_2} \tag{96}$$

we get:

$$c_2 z \cong (2 - \gamma)\, \frac{R_2'}{R_2}\, d_2 \tag{97}$$

Which is nothing but the Hubble's red shift law, which still applies in this variable light velocity condition. From measurement of d_2, c_2 and z we derive the so-called Hubble's constant, i.e. the age of Universe.

$$t = \frac{2}{3}\, \frac{d_2}{c_2 z} \tag{98}$$

Thi is identical to the standard value. Then the distance to the object d_2 is evaluated:

$$d_2 = R_2 u = \frac{3}{2}\, c_2 t_2\, \frac{(1+z)^2 - 1}{(1+z)^2 + 1} \tag{99}$$

When z tends towards infinity we find the cosmological horizon $3/2c_2t_2$, which is twice as small as the standard value $3c_2t_2$. If we compare the present model to the EdS model, we get, for the distances, the ratio:

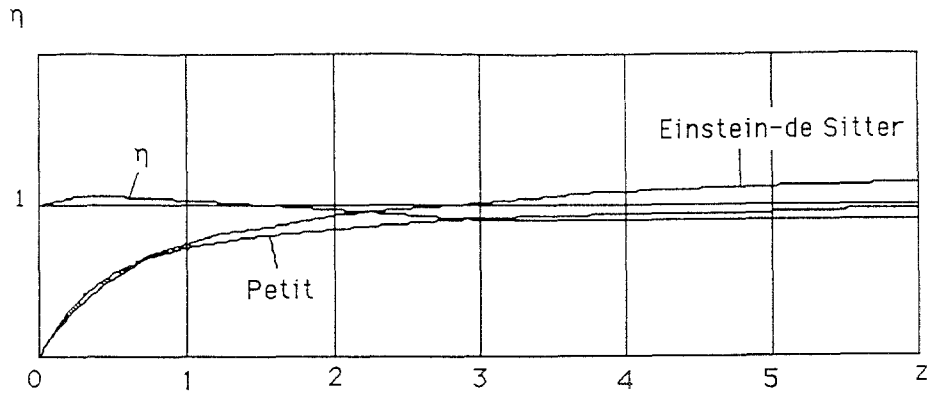


Fig. 20. The distances for the present model and for the Einstein-de Sitter model, and the ratio η of these distances, versus the red shift.

$$\eta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} \frac{1}{2 - \frac{2}{\sqrt{1+z}}} \tag{100}$$

They are similar for weak z values, as shown in the next figure. For weak z values, the distances, as derived from the present model, are a weakly larger. η is close to unity for $z = 1.5$. Then η tends toward 0.5 when z tends toward infinity. For $z < 2.5$ the difference between the two distance evaluations is less than 5%.

For the reference [14], Section 3, the evolution of the angular size of a distant object, versus z, was computed. For the EdS model and constant size objects, the law is:

$$\phi = \phi_0 \frac{(1+z)^2}{(1+z) - \sqrt{1+z}} \tag{101}$$

This function of z has a minimum for $z = 1.25$, and then ϕ tends to grow linearly versus z. The Figure 21 explains why it provides an overestimation of ϕ, for large z values.

In the present model, the situation is basically different, for the objects are supposed to expand with the Universe. See Figure 22.

The corresponding formula is:

$$\phi = \phi_0 \frac{(1+z)^2 + 1}{(1+z)^2 - 1} \tag{102}$$

When z tends toward infinity, ϕ tends to be constant.

Notice that in our model:

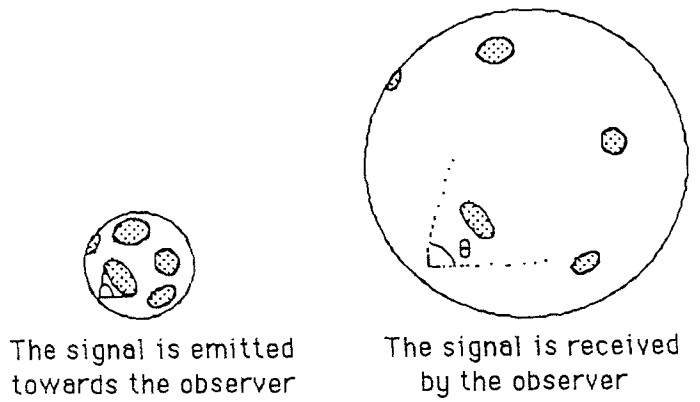


Fig. 21. Why the classical model overestimates the angular size of large red shift objects. The measure, at the reception time, corresponds to a 'fossil' angular size, when the object was closer.

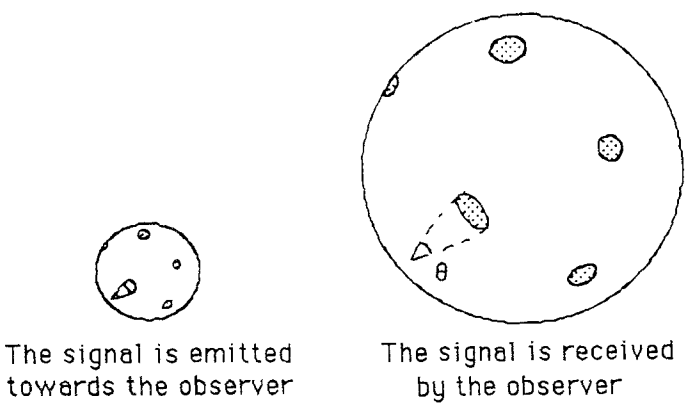


Fig. 22. Present model: The light moves along geodesics. The angular size is unchanged.

$$\phi \approx \frac{1}{d}$$

In the reference [14] this was used to compare the present model to the EdS model, applying to radio-QSO data (Barthel and Miley, 1988 [35]), giving a slight advantage to the first. Obviously, a single test, implying many assumptions about the nature of the observed objects, could not validate the model. See the discussion in reference [14].

13. The Light Emission Problem

Assume the energy production of light sources would proceed through collisions. The collision frequency may be written as:

$$\nu = nQv \tag{103}$$

n is the number density, Q is the collision cross-section and ν the thermal velocity. Assume all of these quantities follow our set of relations, i.e.:

$$n \approx \frac{1}{R^3} \quad Q \approx R^2 \quad V \approx \frac{1}{\sqrt{R}} \tag{104}$$

which gives:

$$\nu \approx R^{-\frac{3}{2}} \approx \frac{1}{t}$$

Assume now that the characteristic amount of energy E_i, for this energy production reaction would vary like $R(t)$.

The energy emission rate varies like:

$$P \approx \frac{R}{t} \approx c \approx \frac{1}{\sqrt{R}} \tag{105}$$

As such the emission rate would have been higher in the past. As, in this model, the energy is saved during the photon flight, the receiver would measure a higher luminosity, which would vary like $(1+z)^{1/2}$.

If we look at the data presented by Barthel and Miley and plot $\mathrm{Log}(P) - 0.5\,\mathrm{Log}\,(1+z)$ when find something quite constant.

14. Some Remarks about other Possible Comparison to Observational Material

14.1. Local relativistic effects

From the classical model of General Relativity have been imagined a large number of tests. The first were devoted to local tests, like the precession of the perihelia of Mercury or the time-delay of radar echos. There is no *a priori* incompatibility between these tests and the present model. In effect, according to the results of the numerical simulations, the matter-density in the region of the twin fold corresponding to the vicinity of the sun is highly rarefied, for the antipodal mass is pushed away by the mass. Then the second term of the second member of the equation (1) can be neglected:

$$\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T})) \approx \chi\mathbf{T} \tag{106}$$

Locally, the Einstein equation would become an approximate form of the equation (1). In such conditions, from the equation (1) we refind the classical local observational features, like the advance of the perihelia, etc.

14.2. About the strong field test from binary pulsars

A pulsar is supposed to be an object located in our galaxy. If we suppose again that the antipodal matter is very rarefied in the conjugated adjacent fold, the field equation becomes:

$$\mathbf{S} = \chi \mathbf{T} \tag{107}$$

i.e. the Einstein equation. Then the observed effects [30] fit both the equations (1) and (2).

15. The Problem of Electromagnetism and other Features of Physics

We propose a new cosmological model. As said before, basically, this model does not contain the electromagnetic nor strong or weak interaction phenomena and this is the same for the classical model. Something only a fully unified field theory could deal with. In such conditions is it licit to try to apply the gauge analysis to the charged particle, i.e. to see how could vary the Bohr radius versus R? This is questionable (this was examined by the author in the formal paper [13], Section 9). The same thing goes for the strong and weak interactions and their associated characteristic lengths (in order to give a new and complete description of the cosmic evolution, including the nucleosynthesis, one should introduce, in this constant energy model, corresponding time-dependant 'constants').

Personally I would think that the cosmological model is far to be achieved. For example, the so-called cosmological constant Λ could be added, through (suggestion of J.M. Souriau):

$$\mathbf{S} = \chi(\mathbf{T} + \Lambda \mathbf{g} - A(\mathbf{T}) - \Lambda A(\mathbf{g})) \tag{108}$$

or:

$$\mathbf{S} = \chi(\mathbf{T} + \Lambda \mathbf{g} - \mathbf{T}^* - \Lambda \mathbf{g}^*) \tag{109}$$

where $\mathbf{T}^*$ and $\mathbf{g}^* = A(\mathbf{g})$ are respectively the stress tensor and the metric tensor associated to the conjugated antipodal region.

This work suggests only that the geometry of the universe could be somewhat different from our standard vision. Perhaps a unified model (gravitation plus electromagnetism) could be built, by introducing complex tensors $\mathbf{S}$, $\mathbf{T}$ and $A(\mathbf{T})$ in the equation (1). On another hand, one can shift from a $S3 \times R1$ geometry towards a twin geometry based on the cover of a projective $P4$ by a sphere $S4$. Then it

could perhaps be possible to deal with CPT symmetry and then take account of the matter-antimatter duality (the antipodal matter would behave like antimatter and become the lost 'cosmological antimatter', as suggested by Andréi Sakharov and Novikov in 1967 [36,37] and the authors [38,39 and 40]). But this, we confess, is a hard mathematical task.

In a Kaluza model we consider a 5 dimensional manifold. Then the electromagnetism can be introduced, from what source or origin no-one knows what this fifth dimension represents exactly. Notice that, locally, the model is equivalent to a Kaluza model with a fifth dimension limited to the values ± 1.

In this model the statute of the Klein-Gordon equation is the same as in classical General Relativity.

Conclusion

Starting from the field equation presented in a former paper [1] we have presented new results, based on numerical simulations, performed by F. Lansheat. This provides a possible explanation of the spongy, very large structure of the Universe and is an alternative to the classical pancakes theory, for our structures are stable over a period of time comparable to the age of the Universe. Then we developed a theory of inverse gravitational lensing: the observed lensing effects could be due mainly to the effect of surrounding antipodal matter, acting like a distribution of negative mass, than to the action of the galaxy itself. This challenges the dark matter concept. Then, starting from the field equation $\mathbf{S} = \chi(\mathbf{T} - A(\mathbf{T}))$ we have developed a cosmological model with 'variable constants'. Because of the hypothesis of homogeneity ($\mathbf{T} = A(\mathbf{T} =$ constant over space) the metric must be a solution of the equation $\mathbf{S} = 0$, although the total mass of this closed universe is non-zero ($\mathbf{T} \neq 0$). In order to avoid the triviality of the classical subsequent solution $R \approx t$, we have built a solution with 'variable constants'. We have derived the laws linking the different constants of physics: G, c, h, m in order to keep the basic equations invariant, *so that the variation of these constants is not measurable in the laboratory. The only effect of this process is the red shift, due to the secular variation of these constants.*

All the energies are conserved, but not the masses. We have found that all the characteristic lengths (Schwarzschild, Jeans, Compton, Planck) vary like the characteristic length R, from where all the characteristic times vary like the cosmic time t.

As the energy of the photon $h\nu$ is conserved over its flight, the decrease of its frequency is due to the growth of the Planck constant $h \approx t$.

In such conditions the field equations have a single solution, corresponding to a negative curvature and to an evolution law: $R \approx t^{2/3}$.

The model is no longer isentropic and $s \approx \mathrm{Log}\, t$. The cosmologic horizon varies like R, so that the homogeneity of the Universe is ensured at any time,

which challenges the inflation theory. We refind, for moderate distances, Hubble's law. We find a new law: distance = $f(z)$, very close to the classical one for moderate red shifts.

An observational test is suggested, based on the values of the angular sizes of distant objects. Comparing the available data to the predictions of our model and to those of the (peculiar) Einstein-de Sitter model, we find a slight advantage in the first. Obviously, a single test cannot validate such a model.

Acknowledgement

The author thanks Professor Oaxiiboo F. for useful suggestions and critic.

References

1. Petit, J.P.: The missing mass effect. Il Nuovo Cimento B. Vol. 109, July 1994, pp. 697–710.
2. Zel'dovich, Ya.B.: 1970, *Astrofisica* **6**, 319; *MNRAS* **192**, 192.
3. Doroskhevitch, A.G.: 1980, *MNRAS* **192**, 32.
4. Klypin, A.A. and Shandarin, S.F.: 1983, *MNRAS* **204**, 891.
5. Centrella, J.M. and Mellot, A.L.: 1983, *Nature* **305**, 196.
6. Shandarin, S.F.: 1988, in: Large Scale Structures of the Universe, ed. J. Audouze, M.C. Peleton, and A. Szalay, 273. Dordrecht: Kluwer.
7. Kofman, L.A., Pogosyan, D., and Shandarin, S.: 1990, *MNRAS* **242**, 200.
8. Peebles, P.J.E.: 1993, Principles of Physical Cosmology, Princeton University Press.
9. R. Adler, M. Bazin, and M. Schiffer: 1975, Introduction to General Relativity. Mac-Graw Hill book company.
10. V.S. Troitskii: 1987, *Astrophysics and Space Science* **139**, 389–411.
11. J.P. Petit: 1988, *Mod. Phys. Lett.* **A3**, 1527.
12. J.P. Petit: 1988, *Mod. Phys. Lett.* **A3**, 1733.
13. J.P. Petit: 1988, *Mod. Phys. Lett.* **A4**, 2201.
14. E.A. Milne: 1948, Kinematic Relativity, Oxford.
15. P.A. Dirac: 1937, *Nature* **139**, 323.
16. P.A. Dirac: 1973, *Proc. Roy. Soc. London* **A333**, 403.
17. F. Hoyle and J.V. Narlikar: 1972, Cosmological models in conformally invariant gravitational theory, *Mon. Notices Roy. Astr. Soc.* **55**, 305–325.
18. V. Canuto and J. Lodenquai: 1977, Dirac cosmologiy, *Ap.J.* **211**, 342–356, January 15.
19. T.C. van Flandern: Is the gravitational constant changing?, *Ap.J.* **248**, 813–816.
20. V. Canuto and S.H.. Hsieh: 1978, The 3 K blackbody radiation, Dirac's large numbers hypothesis, and scale-covariant cosmology, *Ap.J.* **224**, 302–307, September 1.
21. A. Julg: 1983, Dirac's large numbers hypothesis and continuous creation, *Ap.J.* **271**, 9–10, August 1.
22. Brans and Dicke: 1961, *Phys. Rev.*, 124–925.
23. Ratra: 1992, *Astrophys. J. Lett.* **391**, L1.
24. Guth: 1981, *Phys. Rev.* **D23**, 347.
25. Sugiyama and Sato: 1992, *Astrophys. Jr.* **387**, 439.
26. Yoshii and Sato: 1992, *Astrophys. J. Lett.* **387**, L7.
27. H. Reeves: 1994, *Rev. Mod. Phys.* **66**, 193.
28. V.S. Troitskii: 1987, *Astrophysics and Space Science* **139**, 389–411.
29. J.M. Souriau: 1970, Souriau Structure des systèmes dynamiques, Ed. Dunod, France.
30. Taylor: 1994, *Rev. Mod. Phys.* **66**, 711.
31. Bahcall, N.A.: 1988, *Ann. Rev. of Astron. Ap.* **26**, 631 (19–20).

32. Bahcall, N.A. and Soneira, R.M.: 1992, *Ap.J.* **392**, 419.
33. Bahcall, N.A. and West, M.J.: 1992, *Ap.J.* **392**, 419.
34. Luo, X. and Schramm, D.N.: 1992, *Science* **256**, 313.
35. P.D. Barthel and G.K. Miley: 1988, Evolution od radio structure in quasars: a new probe of protogalaxies?, *Nature* **333**, 26 may.
36. A.D. Sakharov: 1967, *ZhETF Pis'ma* **5**, 32; 1967, *JETP Lett.* **5**, 24. trad. Preprint R2-4267, JINR, Dubna.
37. D. Novikov: 1966, *ZhETF Pis'ma* **3**, 223; 1966, *JETP Lett.* **3**, 142; 1966, *trad Astr. Zh.* **43**, 911; 1967, *Sov. Astr.* **10**, 731.
38. J.P. Petit: 'Univers énantiomorphes à temps propres opposés', CRAS du 8 mai 1977, t. 285, pp. 1217–1221.
39. J.P. Petit: 'Univers en interaction avec leur image dans le miroir du temps'. CRAS du 6 juin 1977, t. 284, série A, pp. 1413–1416.
40. J.P. Petit: 1983, Le Topologicon, Ed. Belin, France.

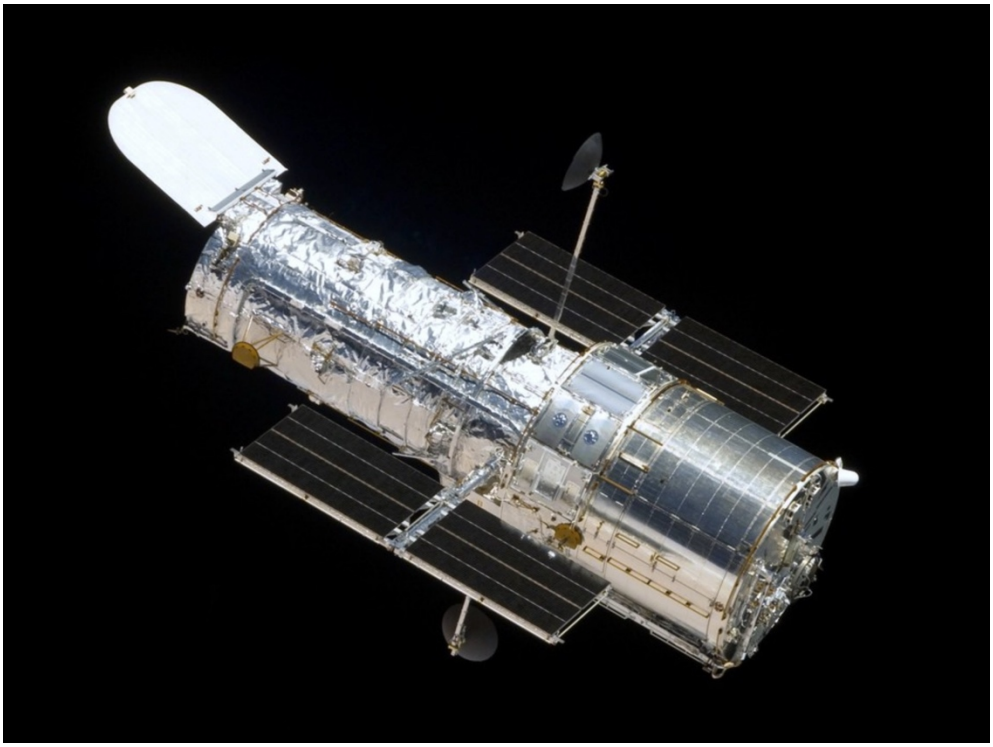
Finally, one thing becomes immediately apparent. This very large-scale structure emerges very rapidly. The positive mass, confined within flat plates, is violently "retro-compressed" by the two adjacent conglomerates of negative mass. This leads to a rapid rise in temperature, followed by a somewhat slower loss of energy through radiative dissipation.

A CRUCIAL finding, still unexploited, thirty years later.

Currently, the theory of galaxy formation draws on all available (or supposedly available) resources, such as dark matter. The scenario predicts that mini-galaxies with masses on the order of one million solar masses will form first, within 500 million years. The models did not anticipate the birth of the first stars until 300 million years later.

Then these mini-galaxies merge—again through gravitational instability—to form the galaxies we observe today, but not until one billion years later.

This scenario appeared to be confirmed by the first images obtained by the Hubble Space Telescope, equipped with a camera operating in the near-infrared (wavelength 1.7 microns)

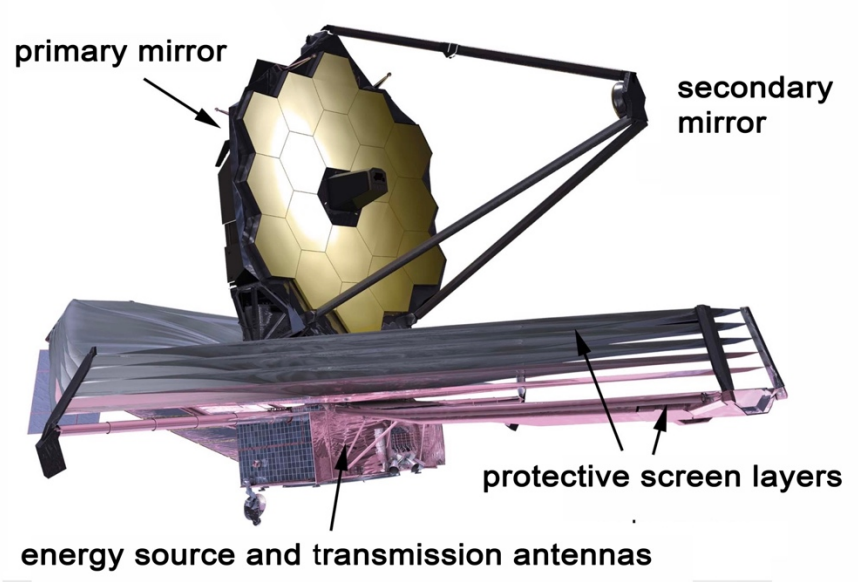


The Hubble Space Telescope

These cameras were equipped with cooling systems of limited capacity. This, however, made it possible to observe relatively small objects, which were then classified as "mini-galaxies."

But when the James Webb Space Telescope—equipped with much more efficient cooling systems, allowing it to operate at wavelengths up to 28 microns—becomes operational in 2022, everything changes.

Very quickly, the telescope will detect massive, fully formed spiral galaxies at distances such that the time when this light was emitted corresponds to less than 500 million years ago. And this time will rapidly decrease.



The James Webb Space Telescope

Currently, the JWST has revealed the existence of galaxies whose stars emit radiation not from primordial hydrogen and helium, but from heavy elements. These are therefore "second-generation stars," which formed from these elements produced—over periods of 3 to 10 million years—by first-generation[33] stars

Over the years, the JWST has contradicted all models of galaxy and star formation, even those taken to the extreme by incorporating the hypothetical presence of dark matter.

What is astonishing is that no one considers a faster formation of galaxies within the Janus model. Indeed, when gravitational instability gives rise to spheroidal clumps of matter, the entire problem lies in the time it will take for these clumps to dissipate the thermal energy resulting from their compression.

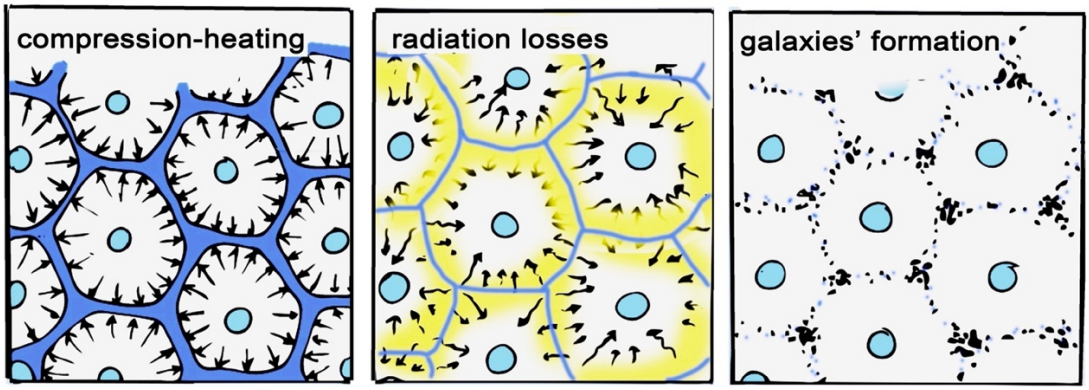
This time increases with the size of such objects. Indeed, the amount of energy to be dissipated increases with volume—that is, with the cube of the radius—while the surface

[33] Carniani et al., Spectroscopic confirmation of two luminous galaxies at a redshift of 14, *Nature* 633, 318–322 (2024).

area of the "radiator" increases only with the square of the radius. Thus, in the standard model, this makes the formation of massive objects impossible.

In the Janus model, everything changes. The flat-plate shape is, on the contrary, optimal for ensuring very rapid cooling.

Under these conditions:



All galaxies, as we know them, form
within the first hundred million years.

This constitutes one of the most compelling observational confirmations of the Janus model, which has been superbly ignored by its detractors[34].

Once again, no research team equipped with powerful computing resources is attempting to tackle the problem, still due to the scientific community's general skepticism toward any introduction of negative masses into the cosmological model.

Where things become truly absurd is with regard to the spiral structure of galaxies. For half a century—which theorists have devoted to trying to understand this—nothing has worked. When they attempt to use these spiral structures as initial conditions, they quickly disintegrate after just a few rotations.

François Combes, president of the French Academy of Sciences, who presents herself as a specialist in this spiral structure, concludes:

[34] JP Luminet: Twin Universe and Bimetric Gravity: From Sakharov's Cosmological Symmetry to Modern Speculations on Negative Mass. arXiv (gr-qc) 20609,26834

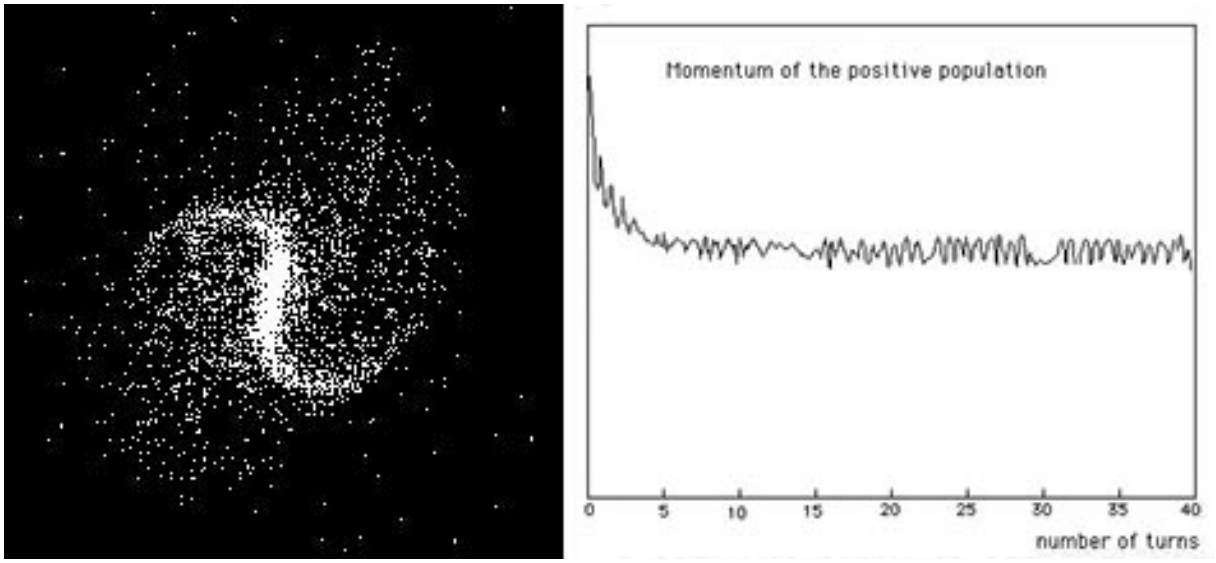


Françoise Combes, President of the Paris Academy of Sciences

- *We are still searching for the mechanism that allows this spiral structure to maintain itself.*

Yet we've had the answer since 1991, and no one wants to hear it. That same year—35 years ago—Descamps asked me what other simulation to run. I provided him with the initial conditions corresponding to the numerical data from a 2D simulation. He ran the calculation.

Immediately, a magnificent barred spiral structure appeared, which persisted for 40 rotations. It was an absolutely splendid result.



Simulation Results of a Galactic Spiral Structure

The curve on the right shows the evolution of the galaxy's angular momentum. The purpose of this structure—a density wave—becomes evident. In a collisional system, a rotating fluid mass (a vortex) exchanges momentum and energy with its environment through collisions and viscous friction. In a non-collisional system (galaxies), this viscosity disappears entirely. The exchange then takes place at a distance, via the gravitational field generated by these spiral structures, which interact with their negative images imprinted in their negative-mass environment. This appears to be a relatively ineffective braking mechanism. Conclusion: these formations will persist as long as there is gas in these galaxies—that is, on the scale of cosmic evolution: indefinitely.

The failure of the simulations conducted by theorists reflects their complete lack of understanding of the mechanism underlying this phenomenon. They can be compared to fluid mechanics engineers who attempt to understand what creates ocean waves and who run simulations using these waves as initial conditions. They would immediately see the waves disappear, simply because they overlook what creates and sustains them:

THE WIND

The formation of the bar is immediate, occurring at the very moment the galaxy forms.



The comparison with observational data speaks for itself.

In the 1990s, I tried in vain to publish this result in all the relevant journals. The responses were unanimous:

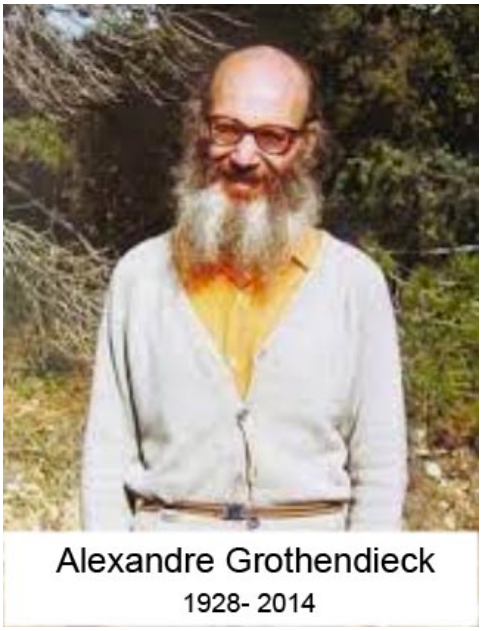
- *Sorry, we don't publish speculative works*[35]

And I wouldn't dare try to publish it today. You've seen the rejection expressed by *Astrophysics and Space Science* as soon as the topic turned to negative mass. It would be exactly the same there.

[35] We're sorry, but we do not publish speculative works.

→ *How many years—or even decades—will pass before scientists realize that this interaction with their negative-mass environment is the key to the spiral structure of galaxies?*

A number of years pass. The problem now is to model this scenario of two interacting universes in a relativistic context. The person who would play a decisive role in the events that followed was the mathematician and geometer Alexandre Grothendieck.



Grothendieck in Mormoiron

He was the son of two militant anarchists.



Hanka Grothendieck
1900–1957
Pictured here at age 17

Sacha Schapiro
1890–1942

Alexandre's mother, Hanka Grothendieck, came from a wealthy Protestant family in Hamburg. His father, Sacha Schapiro, was born into a Jewish family in Russia. At a very young age, he became involved in anarchist circles and, at age fifteen, took part in the Russian Revolution of 1905.

In 1907, at the age of seventeen, he was arrested along with several of his comrades and sentenced to death. His comrades were executed. Sacha escaped the firing squad because of his young age. His sentence was commuted to life imprisonment.

During his imprisonment, he lost his left arm following an escape attempt. In 1914, he reportedly spent an entire year in solitary confinement. He was not released until the 1917 Revolution.

Ten years of his youth were spent in Tsarist prisons.

Alexander was born on March 28, 1928, in Berlin, where his father worked as a street photographer and his mother worked occasionally as a journalist.

In May 1933, after Hitler came to power, Sacha left Berlin for Paris. Hanka joined him in December, after entrusting five-year-old Alexandre to Wilhelm and Dagmar Heydorn, an anti-Nazi couple who took in children in Hamburg.

In 1936, his father actively participated in the Spanish Civil War on the Republican side. Hanka joined him there.

In 1939, the Spanish Civil War ended with Franco's victory. Alexandre's parents returned to France. At age eleven, Alexandre took the train alone from Hamburg and was reunited with his father in Paris in May 1939. He then joined his mother in Nîmes. The family was briefly reunited.

But this reunion was short-lived.

On October 29, 1939—while France had been embroiled in the "Phoney War" since September, well before the German offensive of May 1940—French authorities decided to intern Sacha Schapiro because of his political background. He was arrested in Nîmes and interned on October 31 at the Vernet camp in Ariège.

He remained detained in French camps until 1942.

In August 1942, France was still divided by the demarcation line. The south, administered by the Vichy government, remained officially a zone not occupied by the German army.

Yet it was from this zone that the French authorities handed over to the Germans foreign Jews who were interned in their camps.

Sacha Schapiro was transferred to Drancy, then deported to Auschwitz on Convoy No. 19 on August 14, 1942, under the name Alexander Tanaroff. He was gassed there, most likely upon his arrival.

Alexandre was fourteen years old. He would never see his father again.

Such a family history did little to predispose Alexandre to conformism.

After the war, Alexandre Grothendieck began studying mathematics in Montpellier, then in Paris and Nancy. His exceptional talent quickly became apparent. He revolutionized algebraic geometry, rebuilding its foundations by introducing entirely new concepts. Beginning in 1958, he became the central figure at the Institut des hautes études scientifiques (IHÉS), near Paris. In 1966, he was awarded the Fields Medal, the most prestigious international distinction in mathematics, but refused to travel to Moscow to accept it.

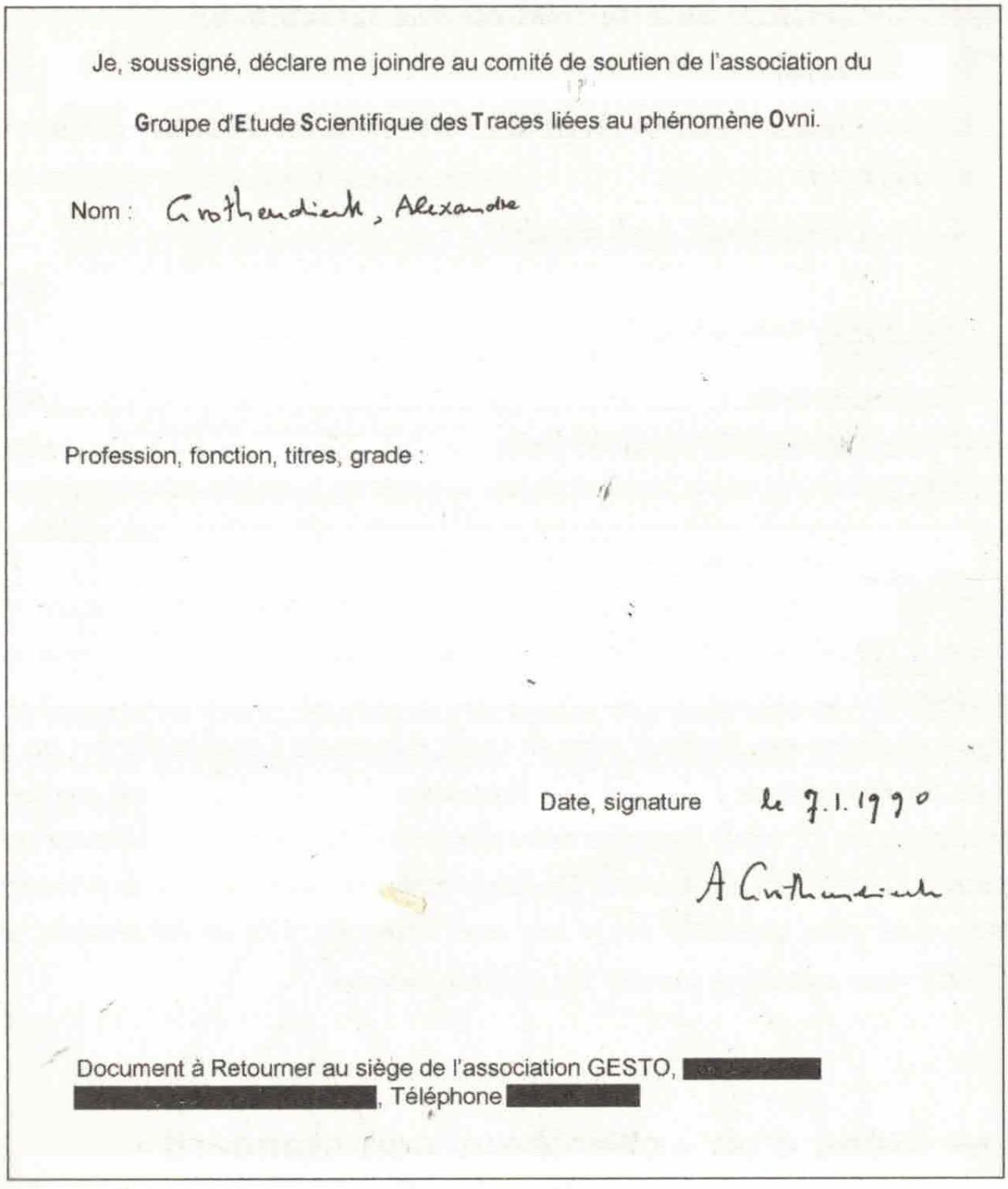
In 1970, at the age of forty-two, he left the IHÉS, particularly after discovering that the institute was receiving military funding. He became involved in environmental and peace movements. In 1973, he returned to a teaching position at the University of Montpellier.

In 1980, he disappeared without leaving an address.

When he contacted me, he said he was willing to give me the details of his retirement home if I promised not to share them with anyone.

It was therefore at his small farmhouse in Mormoiron, at the foot of Mont Ventoux, not far from Aix-en-Provence where I live, that our meetings took place, until 1991, when his hiding place was discovered, I lost track of him, and he took refuge in a village in the Pyrenees, where he lived until his death in 2014.

The following document will surprise many:



Je, soussigné, déclare me joindre au comité de soutien de l'association du

Groupe d'Etude Scientifique des Traces liées au phénomène Ovni.

Nom : Grothendieck, Alexandre

Profession, fonction, titres, grade :

Date, signature le 7.1.1990

A Grothendieck

Document à Retourner au siège de l'association GESTO, [redacted], Téléphone [redacted]

I, the undersigned, Alexandre Grothendieck, grant my support to the Association for the Scientific Study of the UFO Phenomenon.

GESTO's Endorsement by Grothendieck

For Alexandre, there is no subject that science does not have the right to investigate, and for him, the UFO phenomenon is one such subject. Through this document, he grants his support and sponsorship to the nonprofit organization I founded under the 1901 law, GESTO ("Scientific Study Group on UFO Traces"), alongside the late meteorologist

Christian Perrin de Brichambaut[36]. Furthermore, he was aware of my unwavering refusal to work on military applications of scientific research. These two points brought us closer together.

What is interesting, however, are not these anecdotal aspects. It is the mark left by this "Grothendieck method of approaching scientific questions"—uncompromising, boundless, and accepting from the outset the necessity of embracing the status of a "revolutionary."

- *If existing theoretical approaches don't suit you—no matter how prestigious their authors may be—don't impose any barriers or limits on yourself; create your own theory!*

I believe that without Alexandre's encouragement, I would not have had the courage to tackle, on my own, one of the foundations of contemporary physics: General Relativity.

But success on two fronts—variable-parameter modeling, which is more elegant than the concept of inflation, and the excellent results from numerical simulations—encouraged us to tackle the problem.

I am attempting to implement the following system of two field equations, with the goal of constructing a non-stationary solution in a universe where both entities are in a state of homogeneity and isotropy:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)} \right]$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + T_{\mu\nu}^{(+)} \right]$$

However, the mathematical consistency of this system then requires that the negative universe be nothing more than a copy of its positive counterpart, with its mass and energy having their signs reversed. The law of expansion corresponds to a model proposed in 1932–1937 by the cosmologist Milne[37], in the form of a time-linear law.

As early as 1922, the Russian Alexander Friedmann published a non-stationary solution to Einstein's equation[38], corresponding to a curvature index k = +1 (cyclic universe). Two years later, he extended this solution to include cases with negative curvature (k = −1) and zero curvature (k = 0)[39]. Milne, therefore, did not believe in these solutions proposed by Friedmann.

---

[36] After filing the association's articles of incorporation with the prefecture, Perrin and I considered using the acronym for what we had named the "**S**cientific **Study G**roup on **T**races **A**ssociated with the **UFO P**henomenon." We discovered that this formed **the acronym GESTAPO**, which was already taken.

[37] E. A. Milne, "World Structure and the Expansion of the Universe," Nature, 130, 9–10. (July 2, 1932).

[38] A. Friedmann, "Über die Krümmung des Raumes" (On the Curvature of Space), Zeitschrift für Physik, 10, 377–386 (1922).

[39] A. Friedmann, "Über die Möglichkeit einer Welt mit konstanter negativer Krümmung des Raumes" (On the Possibility of a World with Constant Negative Spatial Curvature), Zeitschrift für Physik, 21, 326–332 (1924).

The only similarity with Milne's model is an expansion with zero acceleration—and thus a zero gravitational field. This is an idea that Benoit-Lévy and Gabriel Chardin will attempt to propose in 2021[40]. The idea is different, however, and involves considering that the mass of antimatter could be negative[41], and that this elusive other half of the universe would thus combine with the visible mass to yield a net mass of zero.

But this doesn't sit well with me, because this elegant symmetry contradicts the results of simulations, which point to a dominant negative mass. So I'm trying a workaround by imagining that the "interaction tensors" could be derived from the "field tensors," modulo two functions of time $\varphi(t)$ and $\phi(t)$ as follows:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + \varphi(t)\ T_{\mu\nu}^{(-)} \right]$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + \phi(t) T_{\mu\nu}^{(+)} \right]$$

The solution must be consistent, which implies the emergence of a mathematical compatibility condition between the two equations.

The expansion laws of the two entities are expressed using space scale factors. $a^{(+)}(t)\ et\ a^{(-)}(t)$. The expansion of the solution, through the classical FLRW (Friedmann-Lemaître-Robertson-Walker) metric solutions, equation (5), yields two differential equations that must be satisfied:

(8a) $$3\ \frac{da^{(+)}}{a^{(+)}} + \frac{d\left[\rho^{(+)} + \varphi\ \rho^{(-)}\right]}{\rho^{(+)} + \varphi\ \rho^{(-)}} = 0$$

(8b) $$3\ \frac{da^{(-)}}{a^{(-)}} + \frac{d\left[\phi\rho^{(+)} + \rho^{(-)}\right]}{\phi\rho^{(+)} + \rho^{(-)}} = 0$$

These two equations must become … the same. Therefore, if we set:

$$\varphi = \left(\frac{a^{(-)}}{a^{(+)}}\right)^3 \quad \text{and} \quad \phi = \left(\frac{a^{(+)}}{a^{(-)}}\right)^3 = \varphi^{-1}$$

This yields, for both equations:

$$d\ Log\ \left(\rho^{(+)} {a^{(+)}}^3 + \rho^{(-)} {a^{(-)}}^3\right) = 0$$

That is, the conservation of mass:

---

[40] A. Benoit-Lévy & G. Chardin: Introducing the Dirac-Milne Universe. Astronomy and Astrophysics, Vol. 537 (January 2012), p. 78.

[41] However, measurements conducted at CERN in Geneva concluded that antimatter has a positive mass: E. K. Anderson et al. (ALPHA Collaboration), "Observation of the effect of gravity on the motion of antimatter," *Nature*, 621, 716–722 (2023). As early as 2014, the construction of the Janus symplectic group was moving in this direction (published in greater detail in 2025; see the article).

$$\rho^{(+)}a^{(+)^3} + \rho^{(-)}a^{(-)^3} = Cst$$

Or, by multiplying by , thec^2 conservation of energy:

(9)
$$E = \rho^{(+)}a^{(+)^3}c^2 + \rho^{(-)}a^{(-)^3}c^2 = Cst$$

Why the conservation of energy? Because one immediately thinks of extending the model to include two different speeds of light, which was done shortly thereafter in the following article, published that same year.

And here we exploit the model's profound asymmetry by setting

$$\left|\rho^{(-)}a^{(-)^3}\right| >> \rho^{(+)}a^{(+)^3}$$

In other words:

$$E \simeq \rho^{(-)}a^{(-)^3}c^2 < 0$$

We expand the solution and find:

(13a) $$\left(a^{(+)}\right)^2 \frac{d^2a^{(+)}}{dt^2} = -\frac{4\pi G}{c^2} E < 0$$

(13b) $$\left(a^{(-)}\right)^2 \frac{d^2a^{(-)}}{dt^2} = +\frac{4\pi G}{c^2} E > 0$$

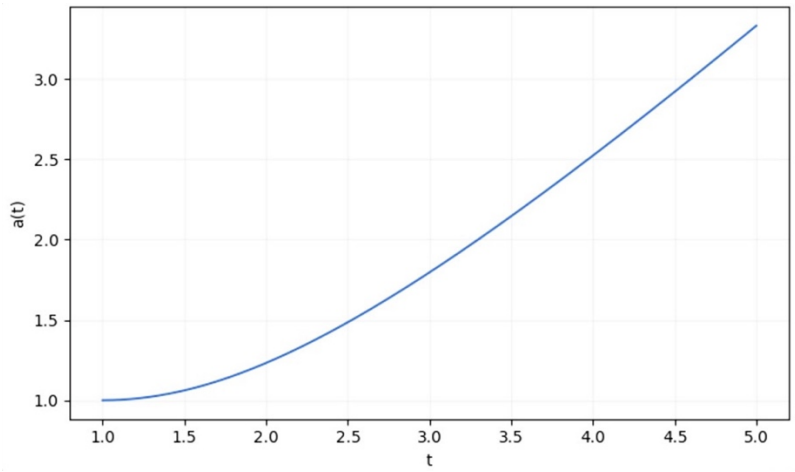
There it is—the dark energy that causes the expansion to accelerate. It is the energy of the negative components, the $\rho^{(-)}a^{(-)^3}c^2$!

Furthermore, an exact solution to equation (13a) exists; it was found by William Bonnor in 1989[42] and is expressed in parametric form:

(14)
$$a^{(+)}(u) = \alpha^2 ch^2 u$$
$$t(u) = \alpha^2\left(1 + \frac{sh\,2u}{2} + u\right)$$

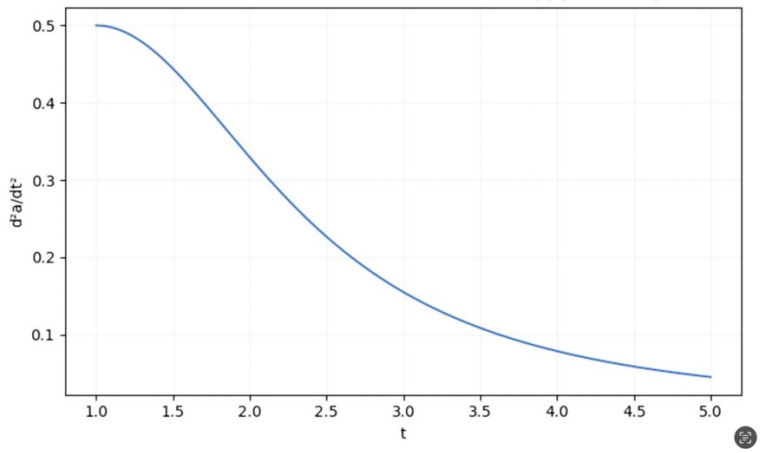
Here is the curve showing expansion with positive acceleration:

[42] H. Bonnor, Negative Mass and General Relativity, Gen. Relat. and Gravit., Nov. 1989, vol. 21, pp. 1143–1157



Expansion of the Universe

And here is the acceleration as a function of time (decreasing):



Variation of Acceleration Over Time

In comparison, the model predicts an exponential acceleration over time. Therefore, it was weaker in the past[43].

So, let's take stock. It is 2024—that is, as I write these lines, twelve years ago. We propose a preliminary model based on two coupled field equations. We propose the following system:

[43] However, the initial results from the DESI collaboration appear to refute this model. H. Chaudary, S. Capozziello, V.K. Shauma, I. Gomez, G. Mustafa. Evidence for evolving dark energy from DESI DR2 BAO and Pantheon+, DES-Dovekie, and Union3, European Physical Journal C, Vol. 86, Article 564 (2026).

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + \hat{T}_{\mu\nu}^{(-)} \right]$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + \hat{T}_{\mu\nu}^{(+)} \right]$$

These equations are written using tensors, which means they are covariant and independent of the choice of coordinates.

The problem data—which determine the form of the solution, namely the metrics— $g_{\mu\nu}^{(+)}$ $g_{\mu\nu}^{(-)}$ are tensor fields, whose terms all have the dimension of energy.

$T_{\mu\nu}^{(+)}$ is the tensor representing the contribution of the positive mass content.

is $\hat{T}_{\mu\nu}^{(-)}$ *est* the tensor representing the contribution of the negative mass content.

$\hat{T}_{\mu\nu}^{(-)}$ and $\hat{T}_{\mu\nu}^{(+)}$ are interaction tensors.

$\hat{T}_{\mu\nu}^{(-)}$ represents the contribution of the local negative mass content to the geometry of the positive sheet, to the metric $g_{\mu\nu}^{(+)}$

$\hat{T}_{\mu\nu}^{(+)}$ represents the contribution of the local negative mass content to the geometry of the positive sheet, to the metric $g_{\mu\nu}^{(-)}$

The mistake was to use the same uppercase letter T to denote these last two tensors, which, as we shall see later, was a source of confusion. It would have been better—and this is what I prefer to use today—to write:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + K_{\mu\nu}^{(+)} \right]$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + K_{\mu\nu}^{(-)} \right]$$

χ is the Einstein constant:

$$\chi = -\frac{8\,\pi\,G}{c^4}$$

This system allows for the introduction of negative masses in cosmology, which is impossible with a single field equation—Einstein's equation—with or without a cosmological constant:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

The first person to raise this issue, in 1957, was the cosmologist Hermann Bondi[44]



Hermann Bondi 1919–2005
Austrian Jew, naturalized British citizen

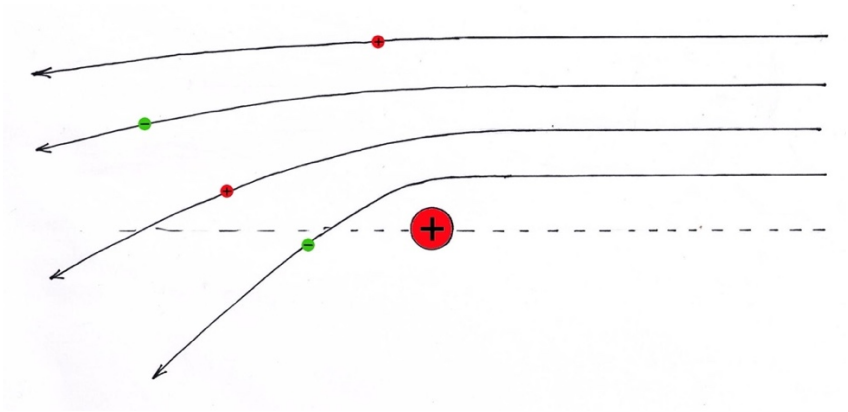
The cosmological constant is a term whose effect is only evident at very large distances. For our purposes, let's set it aside.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

Let's consider a region containing a concentration of positive mass.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T^{(+)}_{\mu\nu}$$

Outside such a mass, in the surrounding vacuum, the (unique) metric solution allows $g_{\mu\nu}$ us to construct the geodesics, the trajectories of the test masses:



Effect of a positive mass on test masses

[44] H. Bondi: Negative Mass in General Relativity: Negative Mass in General Relativity. Rev. of Mod. Phys., Vol. 29, No. 3, July 1957

Trajectory of test masses, whether positive or negative, in the field created by a positive mass. Einstein's equation provides only a single metric $g_{\mu\nu}$.

There is therefore only a single family of trajectories, which both positive (red) and negative test masses will follow indiscriminately.

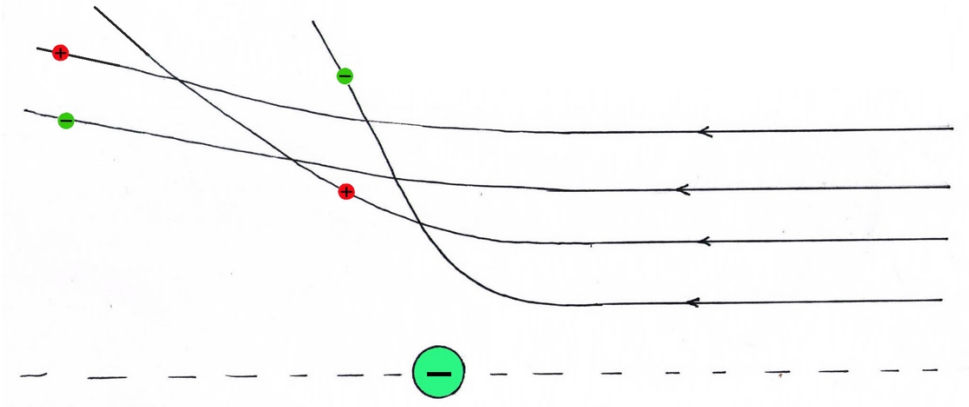
From this, we can deduce the direction of the interaction force:

- *Positive masses attract both their own kind and negative masses*

Let us now consider a region of space where a concentration of negative mass is located:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu}^{(-)}$$

We know how to calculate the metric describing the trajectories in the vacuum surrounding this mass, which are always derived from the metric solution $g_{\mu\nu}$. And from this, we can deduce the trajectory curves followed by the test masses, whether they are positive or negative.



Effect of a Negative Mass on Test Masses

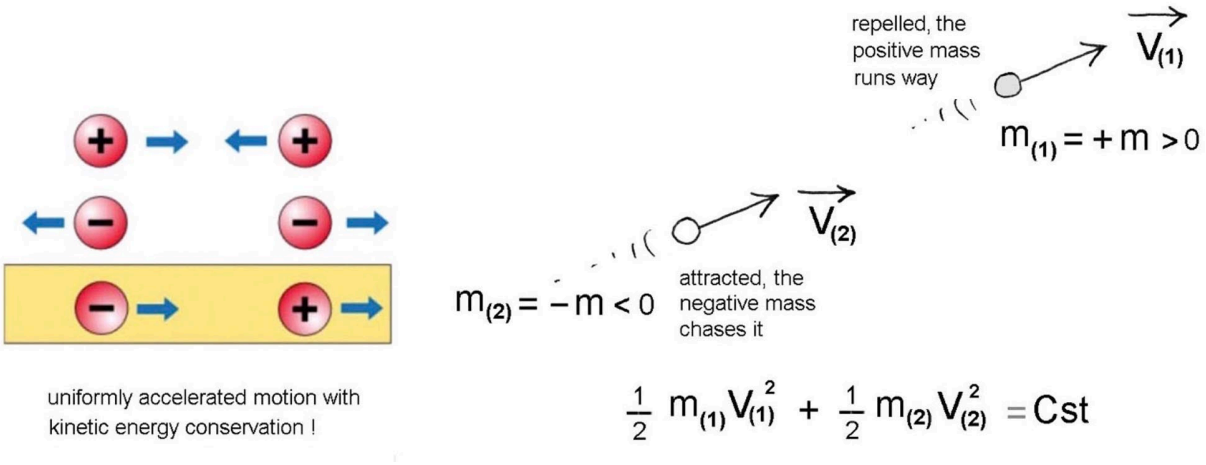
Trajectories of test masses, whether positive or negative, in the field created by a negative mass. Einstein's equation provides only a single metric $g_{\mu\nu}$.

There is therefore only a single family of trajectories, which both positive (red) and negative test masses will follow indiscriminately.

From this, we can deduce the direction of the interaction force:

- *Positive masses repel both their own kind and negative masses*

We thus have the following force diagram:



The Runaway Effect

The runaway effect.

This corresponds to the situation shown in the yellow box. When two masses of opposite signs are brought together, the positive mass flees, pursued by the negative mass. Both accelerate uniformly until they reach relativistic speeds, without any external force field acting on them.

And the total kinetic energy remains zero, since the negative mass acquires negative kinetic energy. This phenomenon has been given the name:

runaway effect

Note that this situation results in a violation of the action-reaction principle

It is impossible to do physics with this. Scientists concluded in 1957 that the cosmological model *could not contain negative masses.*

This is the point that, even today, effectively dissuades scientists from introducing these negative masses into their simulations.

The Janus bimetric model made it possible to circumvent this problem. How?

Prior to 2014, the German physicist Sabine Hossenfelder attempted to introduce a form of antigravity. Her first paper on the subject was published in 2006[45], followed by a second in 2008[46]. The notation, of course, is different. If we adapt it to our own, it corresponds to the following system:

[45] S. Hossenfelder, Antigravitation, Physical Letters B, vol. 636, issue 2, May 4, 2006, pp. 119–125
[46] S. Hossenfelder: A Bimetric Theory with Exchange Symmetry. Phys. Rev. D 78, 044015, 2008, and arXiv: 0807.2838v1 (gr-qc), July 17, 2008

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = T_{\mu\nu}^{(+)} + K_{\mu\nu}^{(+)}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = T_{\mu\nu}^{(-)} + K_{\mu\nu}^{(-)}$$

In her model, we find these field tensors and interaction tensors. If we compare them to the Janus equations, we would write them as follows:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi^{(+)} \left[T_{\mu\nu}^{(+)} + K_{\mu\nu}^{(+)} \right]$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = \chi^{(-)} \left[T_{\mu\nu}^{(-)} + K_{\mu\nu}^{(-)} \right]$$

These equations are thus the most general bimetric equations. Sabine Hossenfelder's system can then be summarized as:

$$\chi^{(+)} = \chi^{(-)} = 1$$

She set the Einstein constants equal to one, to "simplify" matters. A similar choice was made by T. Damour and I. Kogan in 2002, in a 40-page paper published in Physical Review D[47], which represents the first attempt at bimetric modeling.

The Janus model corresponds to the choice (which is perfectly legitimate mathematically, as we will see later):

$$\chi^{(-)} = -\chi^{(+)} = \chi$$

By setting the Einstein constants equal and opposite, we eliminate the intractable runway phenomenon. This is what neither Sabine Hossenfelder nor Thibaud Damour and Ian Kogan saw, and thus they missed the jackpot.

Let us conclude this commentary on the first article we published in 2014, which is reproduced below.

It provides an opportunity to "publish" the Janus group, shown below in its matrix representation (already presented on several occasions in various documents and at conferences).

(15)
$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_0 & C \\ 0 & 0 & 1 \end{pmatrix} \qquad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

As Souriau used to say:

[47] Damour, T., and Kogan, I.I.: Effective Lagrangians and Universality Classes of Nonlinear Bigravity. Phys. Rev. D 66 (2002) 104024. hep-th/0206042.

A space is a group; a⟷ group is a space

This Janus group (see my more detailed presentation from 2024[48]) is an extension of the restricted Poincaré group, used thus far by physicists:

$$\begin{pmatrix} L_0 & C \\ 0 & 1 \end{pmatrix}$$

Where L_0 is the subgroup of the Lorentz group, restricted to its orthochrone components (those that do not invert time), and *C is* the spacetime translation vector:

$$C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

ϕ represents translation in a fifth dimension.

This symplectic group acts on all possible motions in this five-dimensional space, building on the fundamental idea of Souriau and the symplectic geometry of which he is one of the authors[49].

A particle is defined by a set of specific motions in the n-dimensional space of physics. Souriau's statement:

- *Tell me how you move, and I'll tell you what you are.*

In its geometric quantification[50]. Souriau's geometric quantification involves adding a translation in a closed fifth dimension to 4D motion. In the group, the symmetry geometrically $\mu = -1$ reflects the matter-antimatter symmetry (C-symmetry). This corresponds to the group element:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Movement in this additional, angular dimension is accompanied by the conservation of a scalar, in this case the electric charge q (Emmy Noether's theorem).

Symplectic geometry (the action of the group on the dual of its Lie algebra) yields:

$$q' = \mu\, q = -q$$

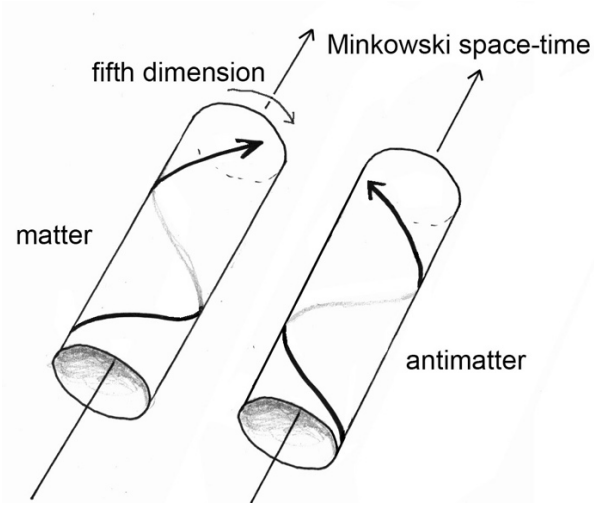
$\mu = -1$ represents matter-antimatter symmetry (inversion of the quantum charge q). This is the reversal of the direction of rotation along the fifth dimension:

---

[48] **J.P. Petit, H. Zejli: Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, No. 001, 2024. 1,315 downloads. Cited 8 times**

[49] With the American Bertram Kostant (1928–2017) and Alexandre Kirillov (born in 1936)

[50] J-M Souriau, *Structure of Dynamical Systems*, Ed. Birkhäuser, Chapter V



Geometric Interpretation of Matter-Antimatter Symmetry

The symmetry corresponds$\lambda = -1$ to the element:

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Acting on the motion of a matter particle, this group element causes a double inversion:

- Inversion of the 5th dimension, hence C-symmetry, inversion of the charge q.
- Inversion of time and space (PT-symmetry), hence inversion of mass.

In other words, CPT symmetry. This symmetry "creates" all motions, and thus the particles that evolve in the negative world.

The "CPT theorem" of quantum physics (which is not actually a true theorem, but rather a consequence of the assumptions made in quantum field theory regarding the operators C, P, and T) states:

- The CPT-symmetric counterpart of a particle (or its motion) is identical to that same particle.

If we add a C-symmetry by performing:

$$C \times CPT = PT$$

We arrive at Feynman's conception of antimatter: a particle of matter, viewed through a mirror and moving backward in time.

This is what happens when we start from the assumption made in quantum field[51] theory that T-symmetry reverses neither mass nor energy.

The Janus view is different.

- *The CPT-symmetric counterpart of a positive-mass matter particle is identical to that same particle, but with both mass and energy reversed.*

- *The PT-symmetric counterpart of a positive-mass matter particle is its antimatter counterpart, with opposite mass. We will call this "antimatter in the Feynman sense."*

- *The C-symmetric counterpart of a positive-mass matter particle is its antimatter, which we will call "antimatter in the Dirac sense."*

The matter-antimatter duality is therefore present in the negative world, which possesses its own antimatter, with negative mass.

Hence the origin of this prediction, which has been confirmed:

- *Antimatter created in the laboratory has a positive mass. It will react to the gravitational field just like matter particles.*

This perspective thus gives substance to Sakharov's theory, according to which the synthesis of baryons from (three) quarks would have been faster than the synthesis of antibaryons from three antiquarks. Hence the survival of one matter particle in a billion in our universe sheet.

The opposite situation occurs in its "twin universe" (for us, the sheet hosting negative masses).

The originality of the Janus model lies in:

- To give substance to Andrei Sakharov's idea (1967)

- To provide a precise definition of the particles constituting the negative world:

Hydrogen and helium antiatoms with negative mass.

Regarding the structure of the negative world and the nature of the spheroidal conglomerates that constitute it, we will present an article on this subject later.

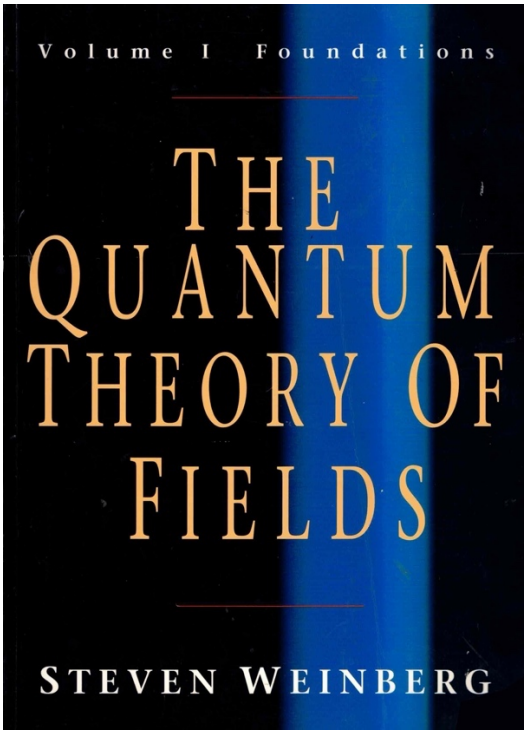
That concludes the section on groups.

This article contains the observations that served as the starting point for the work of Belgian mathematician Nathalie Debergh, who specializes in the mathematical foundations of quantum mechanics[52].

[51] That the quantum operator T is antilinear and antiunitary.

[52] 2026-. N. Debergh & J-P Petit. Beyond the Standard Model. Theoretical Exploration of Negative Mass Physics http://www.jp-petit.org/books/2026-Debergh-Petit.pdf

As soon as I delved into quantum field theory through Steven Weinberg's book *The Quantum Theory of Fields*[53]



Weinberg's book

I turned to page 74, to the section

2.6 **Space Inversion and Time Reversal**

Quantum physics also has its own time- and space-reversal operators, which offer two options. They can be

- linear and unitary
- Antilinear and anti-unitary

It's… up to you!

Weinberg points out that if we choose:

- Either an antilinear and antiunitary operator P
- Or a linear and unitary operator T

This leads to the existence of states with negative energy. Weinberg concludes

[53] S. Weinberg. The Quantum Theory of Fields. Cambridge University Press. 2009 edition.

Since these states do not belong to physics, we are forced to choose:

- *A linear and unitary P operator*

- *An antilinear and antiunitary operator T.*

On page 104, he revisits this choice and reinforces it, given that there is no known situation in physics that would result in the existence of such states.

This was true in 2009. Since then, the phenomenon of accelerating expansion has been demonstrated, for which a Nobel Prize was awarded in 2011[54].

In the 2014 article, I therefore point this out, which led to the work of Nathalie Debergh (see the book that compiles all of this).

We will now present a copy of the article published in 2014 in *Astrophysics and Space Science*. And before discussing the second article on this topic, published in *Modern Physics Letters A* that same year, we will take a detour to discuss the Dipole Repeller.

[54] S. Permutter, B. Schmidt, A. G. Riess, 2011 Nobel Prize in Physics

arXiv:submit/3658856 [gr-qc] 19 Mar 2021

Published in Astrophysics and Space Science, DOI 10.1007/s10509-014-2106-5

Negative mass hypothesis in cosmology and the nature of dark energy

Petit J.P.* and D'Agostini G.†

(Dated:)

The observed acceleration of the universe arises a puzzling question. What is the nature of a dark energy that would cause this phenomenon? We recall the arguments against the existence of negative matter on General Relativity grounds. They vanish if the universe is considered as a manifold M_4 associated to two coupled metrics, solutions of a coupled field equation system. We build a non-steady solution where the positive species accelerates while the negative one decelerates. So that dark energy is replaced by (dominant) negative matter action.

Keywords: Dark energy; Bimetric model; Negative mass.

INTRODUCTION

As pointed out by Riess et al. [24], Perlmutter et al. [17], reviewed by Riess [22], Filippenko and Riess [9], Leibundgut [16], observations of type Ia supernovae at redshift $z < 1$ rise a serious problem, showing that the universe accelerates, this phenomenon being attributed to some puzzling "dark energy". Expanding this investigation on SNe Ia comforts such conclusion (Knop et al. 15). This has been extended to larger redshifts samples (Tonry et al. 28; Barris et al. 2). See also Riess [23]. Nobody anticipated such startling phenomenon. The magnitude of the observed acceleration was not anticipated by theory and continues to defy a post facto explanation. What this newly discovered component of the universe could be made of?

CAN NEGATIVE MASS PLAY A ROLE IN PHYSICS?

The observed acceleration of the universe suggests the effect of a negative pressure, associated to a negative mass content. Is it possible that the Universe would contain negative energy particles? The dynamic group theory (Souriau 27) shows that such concept comes from the antichron components of the Poincaré group. This last contains four connex components. The first four form the orthochron subgroup, or restricted Poincaré group. The last four form the antichron subset, whose elements reverse time. The coadjoint action of this group on its momentum shows that time inversion goes with energy, and mass inversion. Quantum theory of fields analyzes the inversion of space and time too. In dynamical groups theory groups are built with real components. In QFT the T operator become complex, so that it can be either linear and unitary, either antilinear and antiunitary. Weinberg [29, pp. 75–76] writes, we quote:

"At this point we have not *decided* whether P and T are linear and unitary or antilinear and antiunitary. The *decision* is an easy one. Setting $\rho = 0$ in Eq. (2.6.4) gives $\mathrm{Pi}H\mathrm{P}^{-1} = -\mathrm{i}H$, where $H \equiv \mathrm{P}^0$ is the energy operator. If P were antiunitary and antilinear then it would anticommute with is, so $\mathrm{P}H\mathrm{P}^{-1} = -H$. But for any state Ψ of energy $E > 0$, there would be another state $\mathrm{P}^{-1}\Psi$ of energy $-E$. *There are no states of negative energy* (energy less than that of the vacuum), so we are *forced* to *choose* the other alternative: P is *linear* and *unitary* and commutes than anticommutes with H. On the other hand, setting $\rho = 0$ in Eq. (2.6.6) yields $\mathrm{Ti}H\mathrm{T}^{-1} = -\mathrm{i}H$. If we supposed that T is linear and unitary we could simply cancel the is, and find $\mathrm{T}H\mathrm{T}^{-1} = -H$, with the again *disastrous conclusion* that for any state Ψ of energy E there is another state $\mathrm{T}^{-1}\Psi$ of energy $-E$. *To avoid this, we are forced to conclude* that T is antilinear and antiunitary". We see than the quantum theory of fields is built from *a priori choices about fundamental operators*, which are done in order *to avoid the disastrous fact to include negative energy particles in the theory*".

* jppetit1937@yahoo.fr; BP 55 84122 Pertuis France
† dagostini.gilles@laposte.net

This absence is nothing but a pure hypothesis. A stronger critic comes from General Relativity theory and was pointed out in Bondi [3]. Newton's law emerges naturally from Einstein's equation, through Newtonian approximation. Considering the action of a particle (2) on a particle (1), Bondi defines three kinds of masses:

$$m_{\mathrm{i}}^{(1)} \frac{\mathrm{d}^2 r_1}{\mathrm{d}t^2} = \frac{G(r_2 - r_1) m_{\mathrm{p}}^{(1)} m_{\mathrm{a}}^{(2)}}{|r_2 - r_1|} \tag{1}$$

m_{i} is the inertial mass, m_{p} the passive gravitational mass and m_{a} the active gravitational mass. Immediately the equivalence principle gives $m_{\mathrm{i}} = m_{\mathrm{p}}$ so that (1) becomes:

$$\frac{\mathrm{d}^2 r_1}{\mathrm{d}t^2} = \frac{G(r_2 - r_1) m_{\mathrm{a}}^{(2)}}{|r_2 - r_1|} \tag{2}$$

which shows that positive mass attracts both positive and negative matter. Negative mass repels the two. So that two negative masses repel each other and cannot form clumps by gravitational instability. On another hand, if two masses with same absolute values and opposite signs encounter, the positive one escapes, while the negative one runs after it. The two experience a constant acceleration, but energy and momentum are conserved, because of the two masses are negative. Such preposterous features banned negative mass from cosmology during more than half a century.

REGIONS THAT BEHAVE AS IF THEY WERE FILLED BY NEGATIVE MATTER

Several authors suggested that the voids are a fundamental element of the large scale structure (LSS), with bubble-like structure (De Lapparent et al. 5; Kirshner et al. 14; Rood 25; Geller and Huchra 10; Petit 18, 19; Piran 21; El-Ad et al. 7, 8). El-Ad et al. [6] suggested that initial tiny perturbations, in the primordial density field, "acting as if the had a negative mass", could extend and play a role in the LSS. He made the hypothesis, without justification, that negative mass could own a negative active mass (from Bondi's definition), but positive inertial and passive masses, so that two negative masses would mutually attract. Negative material could form clusters. Positive and negative masses would repel each other, which was studied in references of Petit [18, 19]. Piran developed a Void Finder Algorithm which identifies voids in reshift surveys and measure their size and underdensity. In a more recent paper Izumi et al. [13] consider gravitational lensing shear by the magnifying lens models such as negative mass compact objects causing gravitational repulsion on light, like a concave lens. They suggest, as on reference of Knop et al. [15] that negative masses might attract each other, and form negative massive clumps, so that they could reside in cosmological voids (and there, they mention Piran's work). The demagnifying effect was previously predicted in Knop et al. [15], due to *negative lensing effect*. In addition they have shown that the images by the lens models are radially distorsed, due to the repulsive effect (instead tangentially, when due to positive masses).

Classically, the observed distorsion of the images of galaxies (weak lensing effect) is used to determine the three-dimensional distribution of some hypothetical CDM. Conversely, when decoding the data, if one assumes that the observed distorsion is radial, instead tangential, it could provide three-dimensional negative matter distribution. If this last is found as a regular set of clumps, located at the center of the observed big voids that would be in favor of the presence of such negative mass clumps in our universe.

TOWARDS A BIMETRIC DESCRIPTION OF THE UNIVERSE

General relativity is based on a M_4 manifold, associated to a single Riemannian metric, considered as solution of Einstein's field equation. On such grounds, negative matter cannot cohabit with positive matter. What about a different geometrical framewok? A first bimetric description was proposed in Petit [18, 19] and Petit et al. [20], then in Henry-Couannier [11] and Hossenfelder [12]. Let us consider the following coupled field equations system:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left(T_{\mu\nu}^{(+)} + \varphi T_{\mu\nu}^{(-)} \right) \tag{3a}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} = -\chi\left(\phi T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}\right) \tag{3b}$$

$g^{(+)}_{\mu\nu}$ metric refers to positive energy (and mass if the have one) particles. $g^{(-)}_{\mu\nu}$ refers to *negative* energy (and mass if the have one) particles. $R^{(+)}_{\mu\nu}$ and $R^{(-)}_{\mu\nu}$ are Ricci tensors, built from metrics $g^{(+)}_{\mu\nu}$ and $g^{(-)}_{\mu\nu}$. For sake of brevity we will sometimes use the notation: $f \in \{+,-\}$. In mixed form, we will write the tensors:

$$T^{(f)}{}_{\nu}{}^{\mu} = \begin{pmatrix} \rho^{(f)}c^2 & 0 & 0 & 0 \\ 0 & -p^{(f)} & 0 & 0 \\ 0 & 0 & -p^{(f)} & 0 \\ 0 & 0 & 0 & -p^{(f)} \end{pmatrix} \quad \text{with} \begin{cases} \rho^{(f)} > 0 \text{ and } p^{(f)} > 0 & \text{for } f = \text{“+”} \\ \rho^{(f)} < 0 \text{ and } p^{(f)} < 0 & \text{for } f = \text{“−”} \end{cases} \tag{4}$$

φ and ϕ are functions that will be defined further (to ensure energy conservation). Each metric produces its own geodesics family. The two are distinct, so that opposite energy particles cannot encounter, on pure geometric grounds. They interact only through gravitation. We assume that positive light, composed by positive energy photons, cruises along null geodesics of metric $g^{(+)}_{\mu\nu}$, while negative light, composed by negative energy photons, cruises along null geodesics of metric $g^{(+)}_{\mu\nu}$. We assume that structures made of positive masses can emit and capture positive energy photons. Similarly, structures made of negative masses can emit and capture negative energy photons. So that on geometric grounds, negative structures would be invisible to us, through our eyes or astronomic instruments.

NON-STEADY SOLUTION FOR DUST UNIVERSE

We assume this bimetric universe is homogeneous and isotropic, so that the Riemannian metrics become:

$$\left(\mathrm{d}s^{(f)}\right)^2 = c^2\,\mathrm{d}t^2 - \left(a^{(f)}\right)^2 \frac{\mathrm{d}u^2 + u^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)}{\left(1 + k^{(f)}\frac{u^2}{4}\right)^2} \tag{5}$$

Introducing these metrics in the system (3a) and (3b), with $p^{(+)} \cong 0$ and $p^{(-)} \cong 0$ we get classical systems:

$$\frac{3}{c^2\left(a^{(+)}\right)^2}\left(\frac{\mathrm{d}a^{(+)}}{\mathrm{d}t^2}\right)^2 + \frac{3k^{(+)}}{c^2\left(a^{(+)}\right)^2} = -\chi c^2\left[\rho^{(+)} + \varphi\rho^{(-)}\right] \tag{6a}$$

$$\frac{2}{c^2\left(a^{(+)}\right)^2}\frac{\mathrm{d}^2a^{(+)}}{\mathrm{d}t^2} + \frac{1}{c^2\left(a^{(+)}\right)^2}\left(\frac{\mathrm{d}a^{(+)}}{\mathrm{d}t}\right)^2 + \frac{k^{(+)}}{c^2\left(a^{(+)}\right)^2} = 0 \tag{7a}$$

$$\frac{3}{c^2\left(a^{(-)}\right)^2}\left(\frac{\mathrm{d}a^{(-)}}{\mathrm{d}t^2}\right)^2 + \frac{3k^{(-)}}{c^2\left(a^{(-)}\right)^2} = +\chi c^2\left[\phi\rho^{(+)} + \rho^{(-)}\right] \tag{6b}$$

$$\frac{2}{c^2\left(a^{(-)}\right)^2}\frac{\mathrm{d}^2a^{(-)}}{\mathrm{d}t^2} + \frac{1}{c^2\left(a^{(-)}\right)^2}\left(\frac{\mathrm{d}a^{(-)}}{\mathrm{d}t}\right)^2 + \frac{k^{(-)}}{c^2\left(a^{(-)}\right)^2} = 0 \tag{7b}$$

Applying classical mathematical methods (Adler et al. 1) the compatibility conditions of sets (6a) and (7a) and (6b) and (7b) gives:

$$3\frac{\mathrm{d}a^{(+)}}{a^{(+)}} + \frac{\mathrm{d}\left[\rho^{(+)} + \varphi\rho^{(-)}\right]}{\left[\rho^{(+)} + \varphi\rho^{(-)}\right]} = 0 \tag{8a}$$

$$3\frac{\mathrm{d}a^{(-)}}{a^{(-)}} + \frac{\mathrm{d}\left[\phi\rho^{(+)} + \rho^{(-)}\right]}{\left[\phi\rho^{(+)} + \rho^{(-)}\right]} = 0 \tag{8b}$$

The conservation of the energy (and mass):

$$E = \rho^{(+)}c^2\left(a^{(+)}\right)^3 + \rho^{(-)}c^2\left(a^{(-)}\right)^3 \tag{9}$$

is ensured if:

$$\varphi = \left(\frac{a^{(-)}}{a^{(+)}}\right)^3, \quad \phi = \left(\frac{a^{(+)}}{a^{(-)}}\right)^3, \quad \phi = \varphi^{-1} \tag{10}$$

So that coupled field equation system becomes:

$$R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi\left[T_{\mu\nu}^{(+)} + \left(\frac{a^{(-)}}{a^{(+)}}\right)^3 T_{\mu\nu}^{(-)}\right] \tag{11a}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(-)} = -\chi\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^3 T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right] \tag{11b}$$

Let:

$$\chi = -\frac{8\pi G}{c^4} \tag{12}$$

$$\left(a^{(+)}\right)^2\frac{\mathrm{d}^2a^{(+)}}{\mathrm{d}t^2} = -\frac{4\pi G}{c^4}E \tag{13a}$$

$$\left(a^{(-)}\right)^2\frac{\mathrm{d}^2a^{(-)}}{\mathrm{d}t^2} = +\frac{4\pi G}{c^4}E \tag{13b}$$

THE NATURE OF "DARK ENERGY"

Assume $E < 0$. Then $a''^{(+)} > 0$ and $a''^{(-)} < 0$. If we suppose that our visible part of the universe corresponds to positive mass, then it accelerates, while the negative species decelerates. Equation (13b) identifies to Friedman equation, while (13a) identifies to Bonnor's [1989] solution:

$$\begin{aligned} a^{(+)}(u) &= \alpha^2\cosh^2 u \\ t^{(+)}(u) &= \alpha^2\left(1 + \frac{\sinh 2u}{2} + u\right) \end{aligned} \tag{14}$$

From dynamic group theory (Souriau 27) all particles, including photons, may have negative mass. If L_o is the element of the orthochron restricted Lorentz group, let us consider the following extension of the Poincaré group.

$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with } \lambda = \pm 1 \text{ and } \mu = \pm 1 \tag{15}$$

which acts on a five-dimensional Kaluza space. The dimension of the group is 11. Then the momentum gets an extra scalar q, identified by J.M. Souriau as the electric charge (Souriau 26). The coadjoint action of the group gets an additional relation:

$$q' = \lambda\mu q \tag{16}$$

Matter-antimatter duality holds in the negative mass world. The negative matter, which acts as "dark energy" is composed by negative energy (and negative mass, if they have one) particles: electrons, protons, neutrons, neutrinos, quarks, an so on, plus their antiparticles, plus photons.

NEWTONIAN APPROXIMATION

To develop approximation we need to expand the metrics into series around Lorentzian metrics. The Universe is not homogeneous. When we develop classical Newtonian approximation we assume that the zeroth order term is Lorentzian, which means that this metric is solution of the field equation with a null second member, describing an empty portion of space. It is not necessary that this zeroth order solution corresponds to a description of the whole universe, only a restricted portion of space, small enough to encompass the interaction of particles, planets or stars, considered as mass-points. Similarly, whatever is the time-dependent model, it is possible to consider that the system is quasi steady. In our bimetric description, we consider a finite portion of quasi-steady empty space-time (null curvature, Lorentzian metrics).

$$g^{(+)} = \eta^{(+)} + \varepsilon\eta^{(+)}, \quad g^{(-)} = \eta^{(-)} + \varepsilon\eta^{(-)} \tag{17}$$

The two scale factor $a^{(+)}$ and $a^{(-)}$ are considered as constants. Then, field equations are expanded into a series. Neglecting the second order terms, we find:

$$\varepsilon\gamma^{(+)}_{00|\beta|\beta} = -\chi c^2 \left[\delta\rho^{(+)} + \left(\frac{a^{(-)}}{a^{(+)}} \right)^3 \delta\rho^{(-)} \right] \tag{18a}$$

$$\varepsilon\gamma^{(-)}_{00|\beta|\beta} = \chi c^2 \left[\left(\frac{a^{(+)}}{a^{(-)}} \right)^3 \delta\rho^{(+)} + \delta\rho^{(-)} \right] \tag{18b}$$

Defining the potentials:

$$\varphi^{(+)} = \varepsilon\gamma^{(+)}_{00}\frac{c^2}{2}, \quad \varphi^{(-)} = \varepsilon\gamma^{(-)}_{00}\frac{c^2}{2} \tag{19}$$

we get:

$$\frac{\mathrm{d}x^{(+)\alpha}}{\mathrm{d}t^2} = -\frac{\partial\varphi^{(+)}}{\partial x_\alpha{}^{(+)}}, \quad \frac{\mathrm{d}x^{(-)\alpha}}{\mathrm{d}t^2} = -\frac{\partial\varphi^{(-)}}{\partial x_\alpha{}^{(-)}} \tag{20}$$

$$\sum_{\alpha=1}^{3} \frac{\partial^2\varphi^{(+)}}{\partial x^{(+)\alpha}\partial x_\alpha{}^{(+)}} = 4\pi G \left[\delta\rho^{(+)} + \left(\frac{a^{(-)}}{a^{(+)}} \right)^3 \delta\rho^{(-)} \right] \tag{21a}$$

$$\sum_{\alpha=1}^{3} \frac{\partial^2 \varphi^{(-)}}{\partial x^{(-)\alpha} \partial x_\alpha{}^{(-)}} = -4\pi G \left[\left(\frac{a^{(+)}}{a^{(-)}} \right)^3 \delta\rho^{(+)} + \delta\rho^{(-)} \right] \tag{21b}$$

which are Poisson-like equations, associated to our bimetric model. Building the dynamics from this set of equations would perhaps require a more refined solution, involving different speeds of light $c^{(+)}$ and $c^{(-)}$, and possibly time scale factors $T^{(+)}$ and $T^{(-)}$. Anyway, it is possible to derive those laws in the simplified case $a^{(+)} = a^{(-)}$. Then:

$$\Delta\varphi^{(+)} = -\Delta\varphi^{(+)} = 4\pi G \left(\delta\rho^{(+)} + \delta\rho^{(-)} \right) \tag{22}$$

$$\frac{\mathrm{d}^2 x^\alpha}{\mathrm{d}t^2} = -\frac{\partial \varphi^{(+)}}{\partial x_\alpha}, \quad \frac{\mathrm{d}^2 x^{(-)\alpha}}{\mathrm{d}t^2} = -\frac{\partial \varphi^{(-)}}{\partial x_\alpha} = \frac{\partial \varphi^{(+)}}{\partial x_\alpha} \tag{23}$$

The subsequent interaction laws differ from GR's: positive masses attract together through Newton's law. Negative masses attract together through Newton's law. Opposite signs masses repel each other through an anti-Newton's law.

Referring to the previous works of Bondi [3] and Bonnor [4] the puzzling problem of runaway phenomenon is eliminated. By the way this model fits classical verification associated to GR. In effect, as the two species repel each other, where positive mass is present, negative mass is almost absent and the coupled field equations system reduces to:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} \simeq \chi T_{\mu\nu}^{(+)} \tag{24a}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} \simeq -\chi \left(\frac{a^{(+)}}{a^{(-)}} \right)^3 T_{\mu\nu}^{(+)} \tag{24b}$$

On another hand, considering portions of space containing a sphere filled by constant density matter (positive or negative), surrounded by void makes "internal" and "external" Schwarzschild solutions features clearer, as firstly presented in Petit [19], introducing negative gravitational lensing effect, evoked in Izumi et al. [13].

[1] Adler, R., Bazin, R., Schiifer, M.: Introduction to General Relativity. McGraw-Hill, New York (1967)
[2] Barris, B., et al.: Astrophys. J. **602**, 571 (2004)
[3] Bondi, H.: Negative mass in general relativity. Rev. Mod. Phys. **29** (3), 423 (1957)
[4] Bonnor, W.B.: Negative mass and general relativity. Gen. Relativ. Gravit. **21** (11) (1989)
[5] De Lapparent, V., Geller, M.H., Huchra, J.P.: Astrophys. J. Lett. **302**, L1 (1986)
[6] El-Ad, H., Piran, T.: Astrophys. J. **491**, 421 (1997)
[7] El-Ad, H., Piran, T., da Costa, L.N.: Astrophys. J. Lett. **462**, L13 (1996)
[8] El-Ad, H., Piran, T., da Costa L.N.: Mon. Not. R. Astro. Soc., **287**, 790 (1997)
[9] Filippenko, A.V., Riess, A.G.: In: AIP Conf. Proc., vol. 540, pp. 227–246 (2001), Particle
[10] Geller, M.J., Huchra, J.P.: Science **246**, 897 (1989)
[11] Henry-Couannier, F.: Int. J. Mod. Phys. A. **20**, 2341 (2005)
[12] Hossenfelder, S.: A Bimetric theory with exchange symmetry. Phys. Rev. D **78**, 044015 (2008)
[13] Izumi, K., Hagiwara, C., Nakajima, K., Kitamura, T., Asada, H.: Gravitational lensing shear by an exotic lens with negative convergence or negative mass. Phys. Rev. D, Part. Fields **88**, 024049 (2013)
[14] Kirshner, R.P., Oemler, A. Jr., Schlechter, P.L., Schectman, S.A.: Astrophys. J. **314**, 496, (1987)
[15] Knop, R., et al.: Astrophys. J. **598**, 102, (2003)

[16] Leibundgut, B.: Annu. Rev. Astron. Astrophys., **39**, 67, (2001)
[17] Perlmutter, S., et al.: Astrophys. J. **517**, 565 (1999)
[18] Petit, J.P.: The missing mass problem. Il Nuovo Cimento B **109**, 697–710 (1994)
[19] Petit, J.P.: Twin universe cosmology. Astrophys. Space Sci. **226**, 273–307 (1995)
[20] Petit, J.P., Midy, P., Landhseat, F.: Twin matter against dark matter. In: Conf. on Astr. and Cosm., "Where Is the Matter?", Marseille (2001).
[21] Piran, T.: On gravitational repulsion. Gen. Relativ. Gravit., **29** (11), 1363 (1997)
[22] Riess, A.G.: Publ. Astron. Soc. Pac. **112**, 1284 (2000)
[23] Riess, A.G.: Astrophys. J. **607**, 665—687 (2004)
[24] Riess, A.G. et al.: Astron. J. **116**, 1009 (1998)
[25] Rood, H.J.: Annu. Rev. Astron. Astrophys. **26**, 245 (1988)
[26] Souriau, J.M.: Géométrie et Relativité. Hermann, Paris (1964) (French edition only)
[27] Souriau, J.M.: Structure des Systèmes Dynamiques, Dunod, France (1970) (Structure of Dynamical Systems, Birkhaüser, Boston, 1997)
[28] Tonry, J.T., et al.: Astrophys. J. **594**, 1 (2003)
[29] Weinberg, S.: The Quantum Theory of Fields. Cambridge University Press, Cambridge (2005)

In 2017, an article appeared in the journal *Nature* that took an original[55] form:

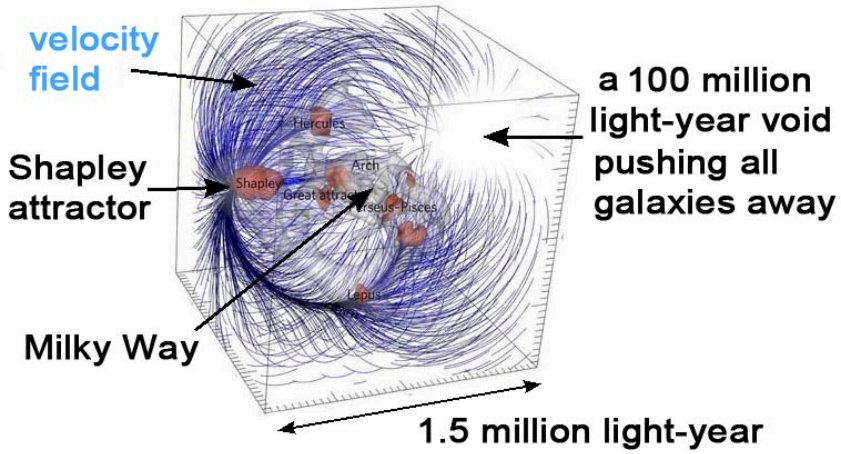
These four researchers analyzed data from hundreds of thousands of galaxies, located at distances of up to one and a half billion light-years, which can be summarized as (distance—redshift—position (θ, φ)in the sky).

To provide an educational illustration of cosmic expansion, imagine sticking decals representing galaxies onto a balloon as you inflate it. Hubble's law then becomes immediately apparent: the recession velocity is proportional to the distance.

But if we imagine that the galaxies are moving, we'll need to use decals that can slide along the balloon's surface.

The velocity field resulting from the balloon's inflation represents the way space itself is stretching out and expanding. But within this space, the galaxies are moving; they have their own motions.

Y. Hoffman, D. Pomarède, B. Tully, and H. Courtois therefore subtracted the expansion-related component from the galaxies' velocity measurements. They then constructed a remarkable 3D velocity field located within a cube with sides measuring one and a half billion.



The "Dipole Repeller": The observation point (our galaxy) is at the center of the cube.

The blue lines represent the velocity field organized between two poles. On the left is a cluster containing about a hundred thousand galaxies, the Shapley Attractor. But on the right, 600 million light-years away from the Milky Way—our vantage point—lies a vast void, 100 million light-years in diameter. This void repels the surrounding galaxies.

[55] Y. Hoffman, D. Pomarède, B. Tully, H. Courtois. The Dipole Repeller. *Nature Astronomy*. Vol. 17, Article 0036 (2017). DOI: 10.1038/s42550-016-0036 arXiv: 1792.02483

For us, this is the manifestation of the repulsive effect of an initial conglomerate of negative mass, which we locate at the center of this vast void. We immediately draft an article to this effect and submit it to "mainstream" journals.

General refusal to submit the paper for peer review.

No explanation was given. This is the "negative mass effect": Negative masses do not exist in the universe. Therefore, any explanation of this repulsive effect that attributes it to negative mass is a non-issue.

We refer the scientific reader to this article[56]. There, an attempt was made to deceive the editorial boards using two front people: Hicham Zejli and… his wife. It may have slipped through in an obscure Russian journal of… geology, which we will not name. I will resubmit this article under the name JP Petit to a journal that has agreed to have the articles peer-reviewed. And this reference will replace the one mentioned.

For we are faced with a completely implausible situation.

Later in this text, I will detail the content of an article titled "Alternatives to Black Holes: Gravastars and Plugstars[57]," published in 2025 in a peer-reviewed journal.

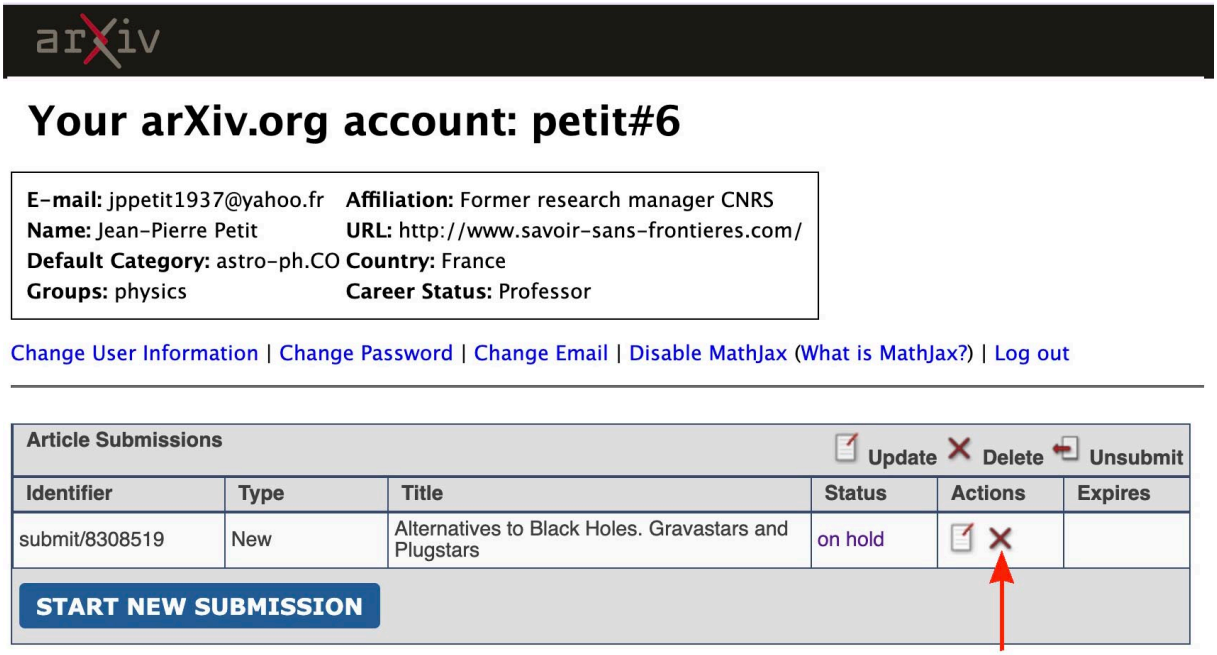
I will discuss later an article in which my name appears under a pseudonym and another in which the article was accepted because a French mathematician colleague agreed outright to substitute my name for his own. So the paper was accepted (and it is a major contribution to the analysis of the Schwarzschild solution!).

On October 31, 2026, I submitted this paper to arXiv via my account (gr-qc): General Relativity and Quantum Gravity.

The paper wasn't just put on hold—it was completely deleted!

[56] H. Zejli, J.-P. Petit, H. Zejli. Nature of the Dipole Repeller. https://hal.science/hal-04068080v1

[57] J.P. Petit, G. d'Agostini. Alternative to Black Holes, Gravastars, and Plugstars. Journal of Modern Physics. Vol. 16, No. 10. October 2025. 497 views. 3,610 downloads



arXiv

Your arXiv.org account: petit#6

E-mail: jppetit1937@yahoo.fr **Affiliation:** Former research manager CNRS
Name: Jean-Pierre Petit **URL:** http://www.savoir-sans-frontieres.com/
Default Category: astro-ph.CO **Country:** France
Groups: physics **Career Status:** Professor

Change User Information | Change Password | Change Email | Disable MathJax (What is MathJax?) | Log out

Article Submissions Update Delete Unsubmit

Identifier	Type	Title	Status	Actions	Expires
submit/8308519	New	Alternatives to Black Holes. Gravastars and Plugstars	on hold		

START NEW SUBMISSION

The article was put on hold by arXiv, then removed

Regarding the following document, before it is replaced by an article published in a peer-reviewed journal, note that it will be expanded to include the explanation that the Janus model offers for The Big Ring[58]. This is a ring-shaped formation discovered in 2024.

[58] A. Lopez, R.G. Clowes, G.M. Williger, "A Big Ring in the Sky." arXiv:20*24.07591, 2024



Nature of the Dipole Repeller

Hasnae Zejli, Jean-Pierre Petit, Hicham Zejli

▶ **To cite this version:**

Hasnae Zejli, Jean-Pierre Petit, Hicham Zejli. Nature of the Dipole Repeller. Russian Journal of Earth Science, In press, 8 (2). hal-04068080v1

HAL Id: hal-04068080

https://hal.science/hal-04068080v1

Submitted on 13 Apr 2023 (v1), last revised 17 Aug 2023 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Copyright

Nature of the Dipole Repeller

Hasnae ZEJLI - PhD in Physics & Mechanical Engineering, Hicham ZEJLI - Engineer,
Jean-Pierre Petit - Astrophysicist & Retired Research Director at CNRS

Keywords : dipole repeller, primeval antimatter, dark matter, dark energy, runaway paradox, Sakharov model, bimetric model, negative mass, Janus cosmological model

Abstract : It's shown through the Janus cosmological model framework that the object responsible for the phenomenon called "dipole repeller" is a spheroidal cluster made exclusively with anti-hydrogen and anti-helium of dark matter of negative mass. Emitting photons of negative energy, this object is not observable with optical instruments, but it plays a crucial role in the structure of the world of negative dark matters which contains neither galaxies, nor stars, nor heavy elements, nor planets, nor biomolecules. This model thus proposes an important extension of the theory of dark matter by including the existence of negative masses in the universe, and thus opens up new perspectives for understanding the structure and evolution of the universe as a whole.

1 – Introduction

In 2017 Yehudi Hoffman, B. Tully, H.Courtois and D.Pomarède published the first very large scale map of the universe (Hoffman et al. 2017), based not only on the position of galaxies, but also including the velocity field, subtracting the Hubble field from what emanated from the raw measurements of their redshift. The result is extremely impressive and will be, we believe, one of the major discoveries of observational cosmology today, comparable to that made by Edwin Hubble a century earlier. It was already known that the motions of a certain set of galaxies reflected a convergence toward a formation that had been given the name of Great Attractor. The analysis published in 2017 highlighted the action of another, larger formation, located beyond this one, to which the name Shapley Attractor was given. But the most singular result was to highlight a region, roughly diametrically opposed to these two formations, where no galaxies were found, only a large void around which the neighboring galaxies presented a flight motion, centered on this object. This formation was first given the name of dipole repeller. Then, deciding to couple it to the attractive formations, it was included in the name of Dipole Attractor. There is no doubt that the understanding of such a phenomenon, which cannot be qualified as an artifact, will require an important progress in our understanding of cosmic dynamics.

2 – Some attempts at interpretation.

Four years after this discovery, few attempts have been made to model this phenomenon. The article published by Neiser in 2020 does not focus on this question of the dipole repeller which follows from the hypotheses made. It is known that observations have highlighted a phenomenon of acceleration of the cosmic expansion (Perlmutter et al. 1999, Riess et al. 2004, Schmidt B.P. et al. 1998). This implies the action of a component associated with a

negative pressure. One of the models considered is to suppose the existence, within the cosmos, of elements of negative mass, which would then exert a repulsive action on the components of positive mass. This hypothesis is at the center of the works corresponding to the references (Petit et al. 2014b, Petit et al. 2014a, Petit et al. 2019, Petit et al. 2018, Petit et al. 2021b, Petit et al. 2021a). Neiser in 2020 makes numerous hypotheses in different fields, on the nature of the Big Bang, the quantum vacuum, the origin of the universe "which would have been created by the gravitational collapse of an antineutrino star". To the point that he recalls that the conclusions of his article remain speculative. Among these hypotheses is that antimatter would have a negative gravitational mass, which would allow, according to him, "the progressive formation of neutrino stars and stars made up of antineutrinos which would repel each other". This repulsive aspect of a primordial antimatter is also mentioned by Benoit-Lévy et al. in 2012, but without further justification. All these people are waiting for the result of the analysis of the behavior of the antimatter created in the LHC, slowed down and subjected to the earth gravitational field (Nieto et al. 1994), aspects discussed by Nieto et al. in 1991. This is a question that we will address later. Heald evokes in 2020 the situation of Laniakea, pushed by the dipole repeller and pulled by the Shapley Attractor. Here again, a repulsion between matter and antimatter is evoked, which would have given rise to the present large-scale structure of the universe, organized around large voids. But we do not find any real model of the object that would be located at the center of this great void, nor why we would not receive any light signal from it. Vuyk appeals in 2018 to a hypothetical 5th force. Hoffman et al. in 2018 through numerical simulations reconstruct the ad hoc distribution of dark matter consistent with the observational data (Hoffman et al. 2017). From this exploration we retain two explanatory schemes. That of the existence of hypothetical objects made of repulsive antimatter, unobservable and that of a gap in the dark matter distribution.

3 – About the interpretation through a gap in the dark matter.

One is entitled to ask the question: can a gap in the dark matter produce the observed repulsion effect? Conceptually one is then tempted to start by considering a spherical gap in a uniform dark matter distribution. We then think that this system should obey the Poisson equation.

$$\frac{d^2\Psi}{dr^2}+\frac{2}{r}\frac{d\Psi}{dr}=4\pi G\rho_{dm} \quad (1)$$

This equation is linear. We can therefore say that by superimposing two given density distributions ρ_1 and ρ_2, the resulting gravitational potential is the sum of the potentials associated with these two distributions $\Psi=\Psi_1+\Psi_2$.

Let us consider a uniform density distribution ρ_{dm}^{unif}, creating a potential Ψ1. We will create the gap by assuming that in a certain volume contained in a sphere we superimpose an equal and opposite density $-\rho_{dm}^{unif}<0$ creating a potential ψ2 .

Let us start by calculating this one, solution of

(2) $$\frac{d^2\Psi_2}{dr^2}+\frac{2}{r}\frac{d\Psi_2}{dr}=-4\pi G\rho_{dm}^{unif}$$

This solution is :

(3) $$\Psi_2=-\frac{4\pi G\rho_{dm}^{unif}}{3}r^2 \qquad \vec{g}_2=\frac{8\pi G\rho_{dm}^{unif}}{3}\vec{r}$$

We thus obtain a repulsive gravitational field, proportional to the distance to the center of the sphere. It remains to calculate the gravitational field created by the uniform distribution, still considering the Poisson equation (1). Its solution is then :

(4) $$\Psi_1=\frac{4\pi G\rho_{dm}^{unif}}{3}r^2 \qquad \vec{g}_1=-\frac{8\pi G\rho_{dm}^{unif}}{3}\vec{r}$$

We obtain the same force field, of opposite sign. Consequently, by superimposing, inside the gap the gravitational field is null:

(5) $$\vec{g}=\vec{g}_1+\vec{g}_2=0$$

In any case the solution (4), of the Poisson equation, corresponding to a uniform distribution is a paradox. We have placed ourselves in a stationary or quasi-stationary situation, considering a uniform distribution of density ρ_{dm}^{unif}. Then, whatever the point chosen as the origin of the coordinates, we find a non-zero gravitational field $\vec{g}$, whose modulus increases proportionally to the distance to this point. We are faced with a paradox.

We are forced to go back to the construction of this Poisson equation. Unlike electromagnetism, this equation cannot be constructed by calculating the flow of the gravitational field through a closed surface, because of the non-nullity at infinity. In gravitation we will be forced to consider the Poisson equation as the linearized version of Einstein's equation in a very restrictive case: in a stationary (or quasi-stationary) situation and when we can describe the metric as a perturbation of a Lorentz metric :

(6) $$g_{\mu\nu}=\eta_{\mu\nu}+\varepsilon\gamma_{\mu\nu}$$

The classical calculation gives :

(7) $$\varepsilon\sum_{i=0}^{3}\gamma_{00}{}_{|i}=-\chi\varepsilon\rho_1$$

The gravitational potential is thus defined by:

$$\Psi = \frac{c^2}{2}\varepsilon\gamma_{00} \tag{8}$$

and equation (7) is then identified with the Poisson equation. But, in this approach, it should be noted that everything is based on the fact that we can consider a stationary metric solution, in the zero order, expressed in the form of a Lorentz metric, immediately associated to a portion of empty space. In the above, the perturbation of the metric is due to a density of finite extension. It is not possible to reconcile this approach on the basis of a non-empty, uniform and infinite density of order zero. The conclusion is that it is simply impossible to define a gravitational potential in a uniform matter distribution. One can consider that this remark is of little interest. It remains that this question had never been raised before. However, the problem of the existence of large voids in the large scale structure of the universe is not solved. Indeed, the gravitational instability produces clusters, not vacuums, so that we do not have a scheme for their formation.

4 – The dipole repeller in the Janus cosmological model.

This translates into an introduction of negative masses in the cosmological model that satisfies the equivalence principle. This is impossible in the framework of classical general relativity, based on a single field equation, that of Einstein, with or without its cosmological constant:

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = \chi\,T_{\mu\nu} \tag{9}$$

Indeed the control particles, positive or negative, follow the same geodesics, whatever the source of the field. As a consequence, we get the following interaction scheme:

- The positive masses attract the positive and negative masses.
- The negative masses repel the positive and negative masses.

This leads to the runaway paradox (see Fig. 1), according to which a couple constituted by masses of opposite signs accelerates uniformly while keeping a constant kinetic energy.

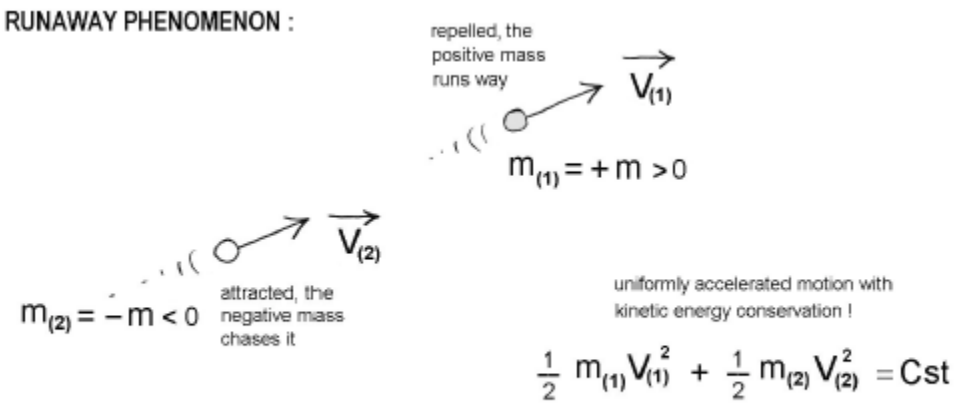


Fig.1 : The runaway paradox

When the authors Neiser in 2020, Benoit-Lévy et al. in 2012 and Nieto et al. in 1994 mention the existence of negative mass structures, these aspects are not considered, although they are fundamental. It is not possible to invoke the presence of negative masses without clarifying this point. In 2018, Farnes justifies what he calls himself a "toy model" by saying that this runaway effect could be the source of cosmic rays. To remain within the framework of a description using a single field equation, he is forced to invoke a hypothetical mechanism of continuous creation of negative mass so that this leads to the constancy of its value over time, which would then constitute an interpretation of the presence of the cosmological constant in the equation. But this scheme does not exclude the runway phenomenon which he then considers as the source of the existence of particles of very high energy, so-called cosmic rays. Because of this undescribed mechanism of constant creation of negative mass the author only obscures the situation even more.

The Janus cosmological model (Petit et al. 2014b, Petit et al. 2014a, Petit et al. 2019, Petit et al. 2018, Petit et al. 2021b, Petit et al. 2021a) takes up the idea of a bimetric description of the universe, initiated by Damour et al. in 2002, then taken up through articles published by Hossenfelder in 2006 and 2008. If the description done by Damour et al. in 2002 corresponds to branes whose points are connected by a hypothetical force field, the description done next by Hossenfelder in 2006 and 2008 is clearer and more constructed, geometrically. The system of the two field equations resulting from their construction, from a Lagrangian, satisfies the Bianchi conditions. The universe is then an M4 manifold equipped with two metrics, the first one producing the geodesics along which the positive masses and the positive energy photons move and the second one the geodesics along which the negative mass particles and the negative energy photons move. The disjoint character of these two systems of geodesics leads to the fact that each set of masses interacts with the other only through an antigravity force and that the negative masses are therefore invisible for an observer constituted of positive masses. The model was created by Hossenfelder in 2006 and 2008 at a time when the phenomenon of accelerating cosmic expansion was not recognized. In an effort to fit with what she considers as the standard model (of Friedmann) the author makes choices of signs which lead to a nonsatisfaction of the principle of equivalence for the negative masses, which are then endowed with a negative gravitational mass, but a positive

inertial mass. In the Janus model, on the contrary, we try to satisfy the equivalence principle within the two populations, which leads to the system of equations:

$$R_{\mu\nu}^{(+)} - \frac{1}{2} R^{(+)} g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + \sqrt{\frac{g^{(-)}}{g^{(+)}}}\, \widehat{T}_{\mu\nu}^{(-)} \right] \quad \textbf{(10a)}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2} R^{(-)} g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + \sqrt{\frac{g^{(+)}}{g^{(-)}}}\, \widehat{T}_{\mu\nu}^{(+)} \right] \quad \textbf{(10b)}$$

The tensors $\widehat{T}_{\mu\nu}^{(+)}$ and $\widehat{T}_{\mu\nu}^{(-)}$ correspond to "induced geometry", i.e. the way each population contributes to the geometry of the other. Their form is determined by the Bianchi conditions. In their mixed form :

$$\widehat{T}_{\mu}^{(+)\nu} = \begin{pmatrix} \rho^{(+)}c^{(+)2} & 0 & 0 & 0 \\ 0 & p^{(+)} & 0 & 0 \\ 0 & 0 & p^{(+)} & 0 \\ 0 & 0 & 0 & p^{(+)} \end{pmatrix} \quad \textbf{(11a)}$$

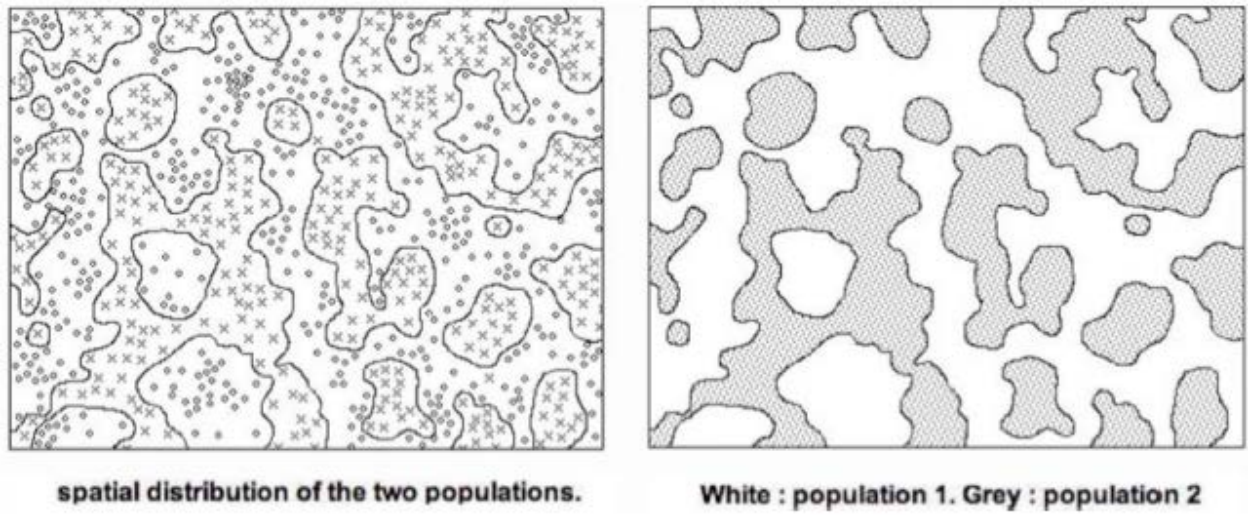
$$\widehat{T}_{\mu}^{(-)\nu} = \begin{pmatrix} \rho^{(-)}c^{(-)2} & 0 & 0 & 0 \\ 0 & p^{(-)} & 0 & 0 \\ 0 & 0 & p^{(-)} & 0 \\ 0 & 0 & 0 & p^{(-)} \end{pmatrix} \quad \textbf{(11b)}$$

These geometrical conditions are only problematic in regions where the densities of matter, of positive or negative mass, are non-zero. By explaining these conditions, in the conditions of the Newtonian approximation they simply translate the fact that inside these masses, where the density is assumed to be constant, the pressure force balances the gravity force.

Outside these regions it is the vacuum. The second members are zero and the Bianchi conditions are satisfied identically because of the form of the first members, themselves resulting from the presence of the terms $R^{(+)}\sqrt{-g^{(+)}}$ and $R^{(-)}\sqrt{-g^{(-)}}$ in action as first introduced by Damour et al. in 2002. The Janus model is based on a set of agreements with observation. What interests us here is a possible justification of the Great Repeller phenomenon. In its Newtonian approximation the system of the two coupled field equations leads to the following interaction laws:

- Masses of the same sign attract each other according to Newton's law
- Masses of opposite signs repel each other according to "anti-Newton"

The runaway phenomenon is thus eliminated and the principle of equivalence is satisfied in both populations. For negative masses, their gravitational masses and their inertial masses are negative and equal. It is then possible to do numerical simulations. If the absolute values of the two densities $\rho(+) > 0$ and $\rho(-) < 0$ the values of the thermal agitation velocities $<V(+)> = <V(-)>$ are equal we obtain a percolation, which does not fit with the observational data referring to the large scale structure of the universe (see Fig. 2).



spatial distribution of the two populations.

White : population 1. Grey : population 2

Fig.2 : Evolution of the system when $\left|\rho^{(-)}\right| = \rho^{(+)}$

On the other hand, if we assume that there is a strong dissymmetry between the two populations, so that $\rho(-) >> \rho(+)$ then the pattern changes completely. As the Jeans times vary as the inverse of the square root of the density, the negative masses are shorter. These then form a regular network of spheroidal conglomerates. The positive mass is forced to occupy the remaining space, which gives it a lacunar structure (El-Ad et al. 1997) comparable to a set of joined bubbles (see Fig. 3), a pattern repeated by Farnes in 2018.

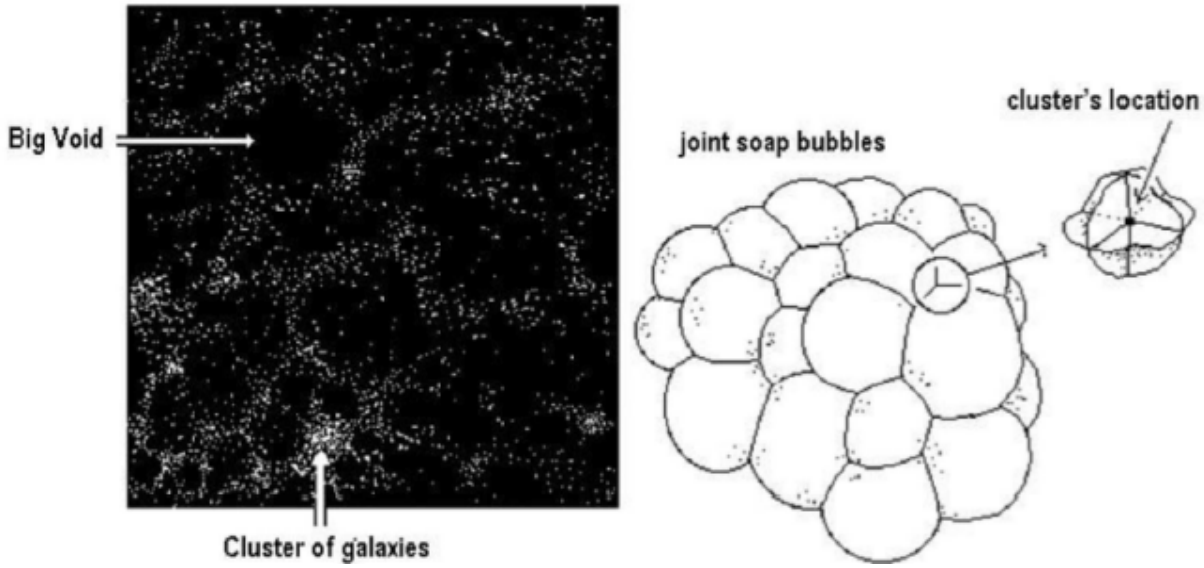


Fig.3 : Lacunar structure

On Fig. 4, there is the result of a 2D simulation (Petit 1995) :

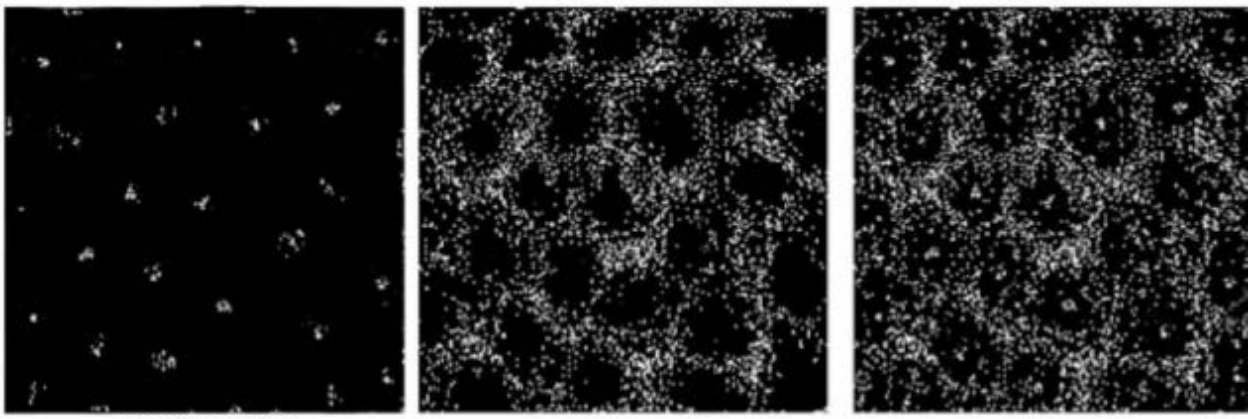


Fig.4 : Evolution when $\left|\rho^{(-)}\right| >> \rho^{(+)}$

Teams with adequate means will easily extend this to 3D. As developed by Petit et al. through articles referenced in 2014b, 2021b, 2021a, the negative mass content accounts for both the dark matter and the dark energy, which is identified with the contribution $\rho^{(-)} c^{(-)2} < 0$ and is therefore responsible for the acceleration of the expansion of positive masses. According to this model, the phenomenon of the dipole repeller translates the presence, within this great void, of a spheroidal conglomerate of negative mass, invisible, because it emits photons of negative energy that our eyes and our measuring instruments cannot capture.

5 – Nature of the objects of negative masses

This question has been examined by Petit et al. in 2014b and the detailed calculations can be found in the article published by Petit et al. in 2021a. In the theory of dynamical groups (Souriau 1970) the link between geometric structure and content is translated by the nature of the associated dynamical group. The General Relativity goes with the restricted Poincaré group, where we keep only the set of terms which do not reverse time. It is thus the orthochron subgroup of the complete Poincaré group, built from the orthochron Lorentz subgroup. Here after its matrix representation:

(12)
$$\begin{pmatrix} L_o & C \\ 0 & 1 \end{pmatrix}$$

C being the space-time translation vector. It is a group of dimension 10. By making this group act on the dual of its Lie algebra, or momentum space, one makes appear the energy E, the momentum p and the spin s, as attributes of the particle, of purely geometric essence. In order to endow the particles with n quantum charges, it is necessary to endow the space with n additional dimensions {ζ0 , ζ1, ... , ζn} and move on to the group below, where we have included its action.

(13)
$$\begin{pmatrix} 1 & 0 & \phi \\ 0 & L_o & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta + \phi \\ L_o \xi + C \\ 1 \end{pmatrix} \quad \text{with } \xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad \zeta = \begin{pmatrix} \zeta^0 \\ \zeta^1 \\ \cdots \\ \zeta^n \end{pmatrix} \quad \phi = \begin{pmatrix} \phi^0 \\ \phi^1 \\ \cdots \\ \phi^n \end{pmatrix}$$

The action on the momentum space adds to the particle n quantum numbers qi, including the electric charge, which are all constants. By introducing the group:

(14)
$$\begin{pmatrix} \mu & 0 & \phi \\ 0 & L_o & C \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{with} \quad \mu = \pm 1$$

We translate geometrically the concept of antimatter, the inversion of quantum charges (C-symmetry) going hand in hand with the inversion of movements in additional dimensions.

If we want to introduce negative energies, it is enough to pass from the restricted Poincaré group Lo, orthochronic, to the full group λLo with λ = ± 1 . Immediately we find that the time-inversion (λ = − 1) goes with energy and mass-inversion. The corresponding dynamic group is according to the article published by Petit et al. in 2014b :

(15)
$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_{\circ} & C \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

In this model the C-symmetry exists both in the world of positive and negative masses. There are thus two antimatter.

- The first one, C-symmetric ($\lambda = 1$; $\mu = -1$) of the ordinary matter, of positive mass
- The second, PT-symmetric ($\lambda = -1$; $\mu = 1$) of ordinary matter, of negative mass.

It is the first one that we make appear in laboratory. The model therefore predicts that this one will behave like ordinary matter in the earth's gravity field.

6 – About the lack of observation of primordial antimatter.

If we suppose, as A. Sakharov (1967, 1979, 1980), that the couple :

- Matter of positive mass
- Antimatter of positive mass

is formed from quarks and antiquarks of positive energy, and that a couple :

- Matter of negative mass
- Antimatter of negative mass

was formed from quarks and antiquarks of negative energy, and that in addition the synthesis of the matter of positive mass, in the first couple, was faster, whereas it is that of the antimatter of negative mass, in the second couple which was it one leads on the idea that the objects, in the center of the great voids of the large-scale structure of the universe, whose presence is betrayed by the phenomenon of the dipole repeller, would be constituted of antimatter, that is to say of antiprotons, anti-neutrons and antielectrons of negative mass. These would then form spheroidal objects made of antihydrogen (light elements) of negative mass comparable to immense protostars with a cooling time superior to the age of the universe, which could not be the seat of fusion reaction, thus generating neither stars, nor galaxies, nor planets.

7 – Observations specifically related to these objects.

As presented by Petit in 1995, these negative mass objects decrease the luminosity of the background sources by negative lensing. But the luminosities of galaxies at $z > 7$ are indeed weak. The classical approach is to consider them as dwarfs. But these could be galaxies of normal size and mass, so the luminosity would be affected by this phenomenon. Finer measurements, in large voids, could reveal a threshold that would allow access to the diameter of these formations.

8 – Conclusion.

We have examined the very few attempts to model the dipole repeller phenomenon. Then we have moved on to the interpretation of the Janus cosmological model. We recall the origin and the essential aspects of this model, based on the hypothesis that the universe has a content of negative masses, such that the principle of equivalence is satisfied in both populations and that eliminates the runaway phenomenon and produces the same effects attributed so far to dark matter and dark energy. In this model the dominant negative mass forms a regular set of spheroidal clusters, which repel the positive mass by confining it in the residual space, giving it a lacunar structure. One of these can then produce the phenomenon of dipole repeller.

Data availability

The data underlying this article are included within the article and in its online supplementary material.

References

Benoit-Lévy A., Chardin G., 2012, Introducing the Dirac-Milne universe, Astronomy and Astrophysics Vol. 537 A 78

Damour T., Kogan I., 2002, Effective Lagrangians and universality classes of nonlinear bigravity, Phys. Rev. D Vol. 66, Iss. 10

El-Ad H., Piran T., da Costa L.N., 1997, Mon. Not. R. Astro. Soc.

Farnes J., 2018, A unifying theory of dark energy and dark matter : Negative mass and matter creation within a modified ΛCDM framework, Astronomy and Astrophysics

Heald G., 2020, The Stronger Case for Gravitational Repulsion between Matter and Antimatter, Research Gate publication Num. 339339776

Hoffman Y., Carlesi E., Pomarede D., Tully R.B., Courtois H., Gottlober S., Libeskind N.I., Sorce J.G., Yepes G., 2018, The quasi-linear nearby Universe, Nature Astronomy, arxiv:1807.03724.pdf

Hoffman Y., Pomarède D., Tully R.B., Courtois H., 2017, The Dipole Repeller, Nature Astronomy 1.0036

Hossenfelder S., 2006, Antigravitation, Phys Letters B Vol. 636 issue 2

Hossenfelder S., 2008, A bimetric Theory with Exchange Symmetry, Phys. Rev. D78 044015, arXiv:0807.2838v1

Neiser T.F., 2020, Fermi Degenerate Antineutrino Star Model of Dark Energy. Advances in Astronomy, Volume 2020, Article ID 8654307

Nieto M.M., Goldman T., Anderson J.D., Lau E.L., Mercader J.P., 1994, Theoretical motivation for gravitation experiments on ultra-low energy antiprotons and antihydrogen, arxiv:9412234.pdf

Nieto M.M., Goldman T., 1991, The arguments against "antigravity" and the gravitational acceleration of antimatter, Physics Reports Vol. 205 Issue. 5, p. 221-281

Perlmutter S. et al., 1999, Measurements of Ω and Λ from 42 High-Redshift Supernovae, APJ, Vol. 517 Number. 2

Petit J.P., 1995, Twin Universe Cosmology, Astrophysics and Space Science Volume 226 Issue 2 pp.273-307

Petit J.P., D'Agostini G., 2014a, Cosmological Bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe, Modern Physics Letters A Vol. 29 Number. 34

Petit J.P., D'Agostini G., 2014b, Negative Mass hypothesis in cosmology and the nature of dark energy, Astrophysics And Space Science, A 354, pp. 611-615

Petit J.P., D'Agostini G., Debergh N., 2018, Evidence of negative energies and masses in the Dirac equation through a unitary time-reversal operator, J. Phys. Comm. 2 Number. 115012

Petit J.P., D'Agostini G., Debergh N., 2019, Physical and mathematical consistency of the Janus Cosmological Model (JCM), Progress in Physics 2019 Vol. 15 issue. 1

Petit J.P., D'Agostini G., Debergh N., 2021a, Bimetric models. When negative mass replaces both dark matter and dark energy. Excellent agreement with observational data. Solving the problem of the primeval antimatter, Database of the French National Center

Petit J.P., D'Agostini G., 2021b, Constraints on Janus Cosmological model from recent observations of supernovae type Ia, Astrophysics and Space Science, In press. hal-03426721f

Riess A. et al., 2004, Type Ia Supernova Discoveries at z > 1 from the Hubble Space Telescope, Evidence for Past Deceleration and Constraints on Dark Energy Evolution, APJ Vol. 607 Number. 2

Sakharov A.D., 1967, ZhETF Pis'ma 5 : 32, JETP Lett. 5 : 24

Sakharov A.D., 1979, ZhETF Pis'ma 76 : 1172, JETP 49 : 594

Sakharov A.D., 1980, Cosmological Model of the Universe with a Time Vector Inversion, ZhETF Tr. JETP 52, 349-351, 79 : 689–693

Schmidt B.P. et al., 1998, The high-Z supernova search. Measuring cosmic deceleration and global curvature of the universe using type Ia supernovae, APJ, Vol. 507 Number. 1

Souriau J.M., 1970, Structure des systèmes dynamiques, Dunod Ed. France, 1997, Structure of Dynamical Systems, Boston Birkhaüser Ed.

Vuyk V., 2018, A Fifth Force Vacuum Dipole Repeller, Needed to Create all Attraction Forces even around Black Holes in Quantum FFF Theory, Research Gate publication Num. 325995732

After this digression on the Dipole Repeller, we resume our presentation of the articles published in 2014, and now we turn to the second one, the content of which will be reproduced below.

Article length: 13 pages! Twice as long as the previous paper.

What does it contain? If you look at the previous article from 2014—a 7-page paper published in *Astrophysics and Space Science*—the condition for compatibility between the two equations, and thus for the mathematical consistency of the system, was expressed by the energy constant in a generalized form:

$$E = \rho^{(+)}a^{(+)^3}c^2 + \rho^{(-)}a^{(-)^3}c^2 = Cst$$

But I had assumed the same speed of light for both entities.

This paper therefore revisits that model by assuming that there are two limiting speeds for light;

- c in the positive world
- $\bar{c}$ in the positive world

The calculation proceeds in exactly the same way and this time leads to:

$$E = \rho^{(+)}a^{(+)^3}c^2 + \rho^{(-)}a^{(-)^3}\bar{c}^2 = Cst$$

However, in the article, we find, for example, at the beginning, this conclusion from the end of Andrei Sakharov's Nobel Peace Prize acceptance speech, delivered in Stockholm by his wife Helena Bonaire (since Sakharov was not permitted to leave the Soviet Union).

It seemed important to us to quote his own words:

> *Thousands of years ago, human tribes faced great difficulties in their struggle for survival. In this struggle, it was essential not only to know how to wield a club, but also to possess the ability to reason, to preserve the knowledge and experience accumulated by the tribe, and to forge bonds that enabled cooperation with other groups. Today, all of humanity faces a comparable challenge. In the vastness of space, there are inevitably many civilizations, some of which are wiser and more "advanced" than our own. I subscribe to the cosmological hypothesis that the development of the Universe repeats itself, in its fundamental features, an infinite number of times. Consequently, other civilizations—including more "advanced" forms—should exist an infinite number of times, on the "previous" and "following" pages of the Book of the Universe. This, however, must in no way diminish the significance of our noble aspirations in this world of ours, where, like faint glimmers in the darkness, we have emerged, for a moment, from the nothingness of an obscure material unconsciousness. It is up to us to meet the demands of reason and to build a life worthy of ourselves, as well as of the goals that we can only glimpse.*

Thousands of years ago, tribes of humans endured great hardships in their struggle to survive. In this struggle, it was important not only to be able to wield a club, but also to possess the ability to think rationally, to preserve the knowledge and experience accumulated by the tribe, and to forge the bonds that would enable cooperation with other tribes. Today, the entire human race faces a similar challenge. In infinite space, many civilizations are bound to exist, including some that are wiser and more "successful" than our own. I support the cosmological hypothesis, which states that the development of the universe repeats itself in its basic features an infinite number of times. Accordingly, other civilizations—including more "successful" ones—should exist an infinite number of times on the "preceding" and "following" pages of the Book of the Universe. Yet this should not diminish the sacred significance of our endeavors in this world of ours, where, like faint glimmers of light in the dark, we have emerged for a moment from the nothingness of the dark unconsciousness of material existence. We must live up to the demands of reason and create a life worthy of ourselves and of the goals we can only dimly perceive.

The text of the article, which follows, is in English. However, in the free version made available in languages other than English, the following passage appears:

This text explicitly mentions the possibility that Earthlings may not be the most scientifically and technologically advanced intelligent species. This is a logical position that any reasonable scientist should adopt. But this immediately raises a question: Have civilizations more advanced than ours visited us, including in the past, throughout our history? If such visits were technically and scientifically feasible, are they still occurring today? Faced with this question, skeptics put forward two arguments. First, like Enrico Fermi in his day, they are puzzled by the total absence of contact. Second, they cite the impossibility of interstellar travel, due to the strict limits imposed on a spacecraft's speed by the laws of special relativity—laws that have been verified time and again at all scales. Regarding the first argument, it should be noted that any contact between civilizations with vastly different levels of scientific and technical development is inevitably accompanied by a clash of cultures. Several scenarios are then conceivable. In one, the civilization with the lower scientific and technological level simply collapses. In another, the more advanced civilization absorbs the one with which it comes into contact, whether through assimilation or enslavement (colonization). In a third scenario, a major scientific and technological transfer takes place; however, as is often the case with any fundamental technological advance, this transfer is immediately and primarily diverted toward the creation of new weapons. All scientific progress is inevitably accompanied by an increase in our capacity for destruction. Thus, if sufficiently advanced technoscience were to enable interstellar travel, it would necessarily be accompanied by the development of colossal means of destruction. By comparison, these new weapons would make our thermonuclear warheads seem laughably insignificant—like mere matches. It is difficult to dispute the idea that wisdom is a rare commodity on Earth, where, despite past tragedies, one might wonder whether humanity will soon plunge into a third world war. Consequently, any transfer of technology would only exacerbate the deadly imbalance from which we suffer—a situation that, far from improving, is likely to worsen over time. This text focuses on the second argument: the feasibility of interstellar travel.

In this context, any movement can only occur by following the geodesics associated with the metric $g_{\mu\nu}$, which is a solution to the equation.

These geodesics are divided into two subsets: those of nonzero length, followed by particles with mass, and those of zero length, followed by photons. This observation imposes an insurmountable limit on all speeds: the speed of light, denoted by c. At relativistic speeds, considerable amounts of energy come into play; the limit imposed by the speed c implies, according to this view of the universe, that virtually infinite energy would be required to reach it. It follows that all attempts to model interstellar travel result in journey times incompatible with the span of a human life.

The conclusion is therefore clear: if interstellar travel is scientifically and technically feasible—that is, compatible with the span of a human lifetime—then a profound shift in the current cosmological paradigm, which excludes such a possibility, is essential. One could even say that the current situation calls for, in a sense, such a paradigm shift.

….

In our model, the Universe is a manifold associated $\mathcal{M}_4$ *not with a single metric, but with two metrics:* $g_{\mu\nu}$ *and* $\overline{g}_{\mu\nu}$*; the first being associated with entities possessing positive mass and energy, the second with those possessing negative mass and energy. A system consisting of two coupled field equations was then proposed*

(2a) $$R_{\mu\nu}^{(+)} - \frac{1}{2} R\, g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + \widehat{\boldsymbol{T}}_{\mu\nu}^{(-)} \right]$$

(2b) $$R_{\mu\nu}^{(-)} - \frac{1}{2} R\, g_{\mu\nu}^{(-+)} = -\chi \left[T_{\mu\nu}^{(-)} + \widehat{\boldsymbol{T}}_{\mu\nu}^{(+)} \right]$$

The capital letters ***T***, representing the "interaction tensors," have been set in bold. This will be explained later.

…

In both of these universes, the fact that and $a^{(-)} < a^{(+)}$ *implies that* $c^{(-)} > c^{(+)}$*:*

(25) $$c^{(+)} \sim \frac{1}{\sqrt{a^{(+)}}} \quad , \quad c^{(-)} \sim \frac{1}{\sqrt{a^{(-)}}}$$

That is to say:

(26) $$\frac{c^{(-)}}{c^{(+)}} \sim \sqrt{\frac{a^{(+)}}{a^{(-)}}} > 1$$

If mass inversion were possible, would the notion that interstellar travel is impossible (within a human lifetime) need to be reconsidered?

Indeed, the journey would then take place along the geodesics generated by the metric $g_{\mu\nu}^{(-)}$*, with the following benefits:*

- $A\left(c^{(-)} > c^{(+)}\right)$*higher speed limit (that of negative-energy photons)*

- *Shorter distances:* $\left(a^{(-)} < a^{(+)} \right)$

Two questions then arise:

- *How can the mass of a spacecraft and its passengers be inverted?*
- *How can the spacecraft be given a relativistic cruising speed?*

If a given technology allowed for the reversal of an object's mass, that object would appear to dematerialize to an observer. Particles, atoms, and molecules with negative mass would no longer interact with the positively massed molecules in the surrounding air, except through gravity (or, more precisely, antigravity). We assume that the mass-reversal process would be symmetrical: if the positive mass present in a given volume were reversed, the few negative masses found there would also undergo a reversal, thereby becoming positive masses. The result, within that volume, would be an extremely rarefied medium consisting of a few positive-mass hydrogen atoms. To an observer with positive mass, the space occupied by the device would appear empty. Air would then rush in instantly, causing a hydrodynamic disturbance.

(27) $$\lambda_C = \frac{h}{mc}$$

If mass inversion obeys the gauge laws as defined in Ref. 26, then:

(28) $$c \sim \frac{1}{\sqrt{a}} \qquad h \sim a^{3/2} \qquad m \sim a$$

This is "apparent mass," a concept that will be discussed in a future article. This leads to:

(29) $$\lambda_C \sim a$$

Mass inversion would be accompanied by a sort of "Gulliver effect." When transferred to the new reference frame, the inverted particles would have a Compton length greater than that of their negative-mass counterparts, since $a^{(-)} < a^{(+)}$*. The resulting energy is:*

(30) $$E = \frac{hc}{\lambda}$$

Such a transfer would be accompanied by a loss of energy. If we assume that the mass transfer implies the conservation of energy during the process of reversing the mass sign, the transferred particles must possess Compton wavelengths identical to those of their "counterparts" with negative mass. This would be feasible if, after the mass reversal, the particles acquired relativistic velocity, such that the reduction in their Compton wavelength would be ensured by Lorentz contraction. However, this process would have to direct the velocities in a single direction; otherwise, the

spacecraft and its passengers would be torn apart. If such a process were feasible, interstellar spacecraft would have no need for a propulsion system

The terms "acceleration" and "deceleration" would then lose all meaning.
Thus, although a considerable amount of energy is required to accelerate a positive mass to a relativistic velocity, an equivalent amount of energy would be needed, once the mass has been inverted, to bring its velocity back to zero while the spacecraft moves within this new frame of reference with negative mass.

After the inversion, the spacecraft would be moving at such a high speed that it would be impossible for it to slow down. However, upon arrival at its destination, a new mass inversion would restore its initial kinetic parameters.

Even in a very rudimentary form, these travel techniques have nothing to do with rocket propulsion. Everything relies on the manipulation of mass, combined with that of other parameters, such as spin (which can be easily controlled using a magnetic field). We suggest that the conservation of energy could be accompanied by a modification of the dynamic parameters that defies the imagination. After a mass inversion, a further modification of the parameters—while the spacecraft possesses negative mass—could endow it, in the realm of positive masses, with a velocity vector oriented in a different direction, while preserving the norm of the vector as well as the angular momentum.

If such a phenomenon is possible, an observer with positive mass would see the spacecraft make right-angle turns or even a complete reversal of its velocity—behaviors incompatible with the laws of conventional physics.

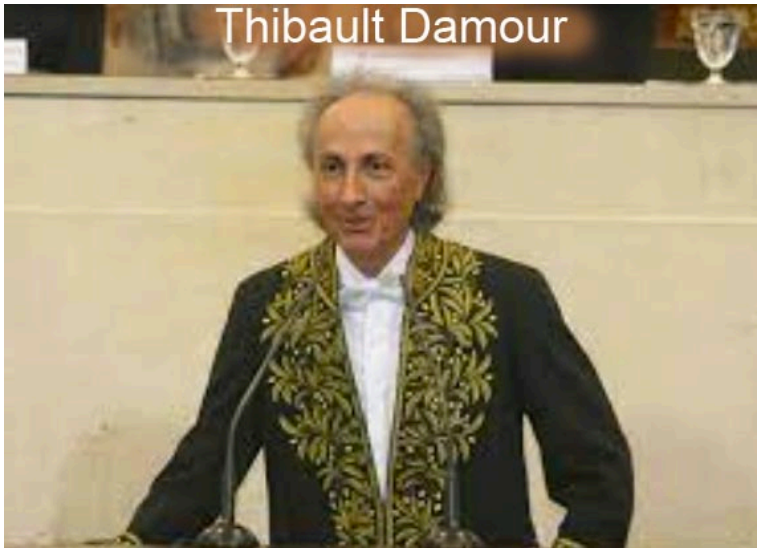
If, in the distant future, technological advances allowed a spacecraft to travel among the stars in timeframes comparable to a human lifetime, what would travelers see through the portholes as they passed through this world with negative mass? They would no longer be able to perceive the positive-mass universe; planets, stars, and galaxies would become invisible to them. On the other hand, since they would be able to capture images transmitted by photons of negative energy,

they might perceive, in the distance, diffuse spheroidal clusters composed of negative mass; these clusters would faintly emit—much like protostars—a reddish light as well as infrared radiation[59].

It is clear that in this second article I took advantage of the opportunity that presented itself to attempt to introduce elements outlining the technology of interstellar travel.

This text would have consequences and provoke extremely aggressive and violent reactions within the scientific community, centered on the response of France's "Mr. Cosmology," Academician Thibaud Damour.

[59] Described in an Ummite text, to be found.



Academician Thibault Damour (born in 1951)

Member of the Institut des Hautes Études Scientifiques.

arXiv:submit/3664506 [gr-qc] 23 Mar 2021

Cosmological bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe.

Published in *Modern Physics Letters A, Vol 29, No.34 (2014) 1450182*

J.P. Petit* G. D'Agostini†

March 23, 2021

Abstract

An extension of a previously published model of a bimetric Universe is presented, where the speeds of light associated to positive and negative mass species are different. As shown earlier, the asymmetry of the model explains the acceleration of the positive species, while the negative one slows down. Asymmetry affects scale factors linked to lengths, times and speeds of light; so that if a mass inversion of a craft can be achieved, then interstellar travels would become non-impossible at a velocity less than the speed of light of the negative sector, and possibly much higher than that of the positive sector.

Keywords: Janus cosmological model; dark energy; bimetric model; negative mass; interstellar travel.
PACS No.: 98.80

1 Introduction

Only a few decades ago there were many scientists who doubted that intelligent life could exist in the Universe elsewhere than on Earth. Some of them doubted about the possibility of life being present in other planets on the sole grounds of probabilities. This position can be summed up by the famous Drake formula which is expressed as a product of multiple probabilities. And skeptics say: "Suppose that one of those factors is zero, and then the probability would fall to nothing". Even further, there were scientists who wondered if a planetary system around a star could ever be a common thing in the Universe.

Observations in the last decade have completely upset this view. But in fact we are witnessing a rearguard action aimed at challenging the existence of planets with intelligent life with the same level of organization than that found on Earth. When the first exoplanet was discovered in 1995 by Mayor and Queloz, 40 light-years away from Earth, it was evidently a giant planet unsuitable for hosting life. Skeptics asked whether planetary systems with telluric planets could ever exist. They thought that even if telluric planets were to exist in extrasolar systems, it was unlikely they might bear surface conditions conducive to the emergence of life. In the line of arguments against this eventuality was the requirement of the existence of a satellite around the planet, allowing the latter to stabilize its rotation movement. In fact, proponents of the theory of deterministic chaos had shown, using computer simulations, that the axis of rotation of a lonely planet would be unstable and topple completely, making the emergence of life very problematic. But these simulations do consider the Earth as a rigid sphere, which she is far from being at the scale of the phenomena considered.

*CNRS, BP 55, 84122 Pertuis, France, e-mail: jppetit1937@yahoo.fr
†dagostini.gilles@laposte.net

Returning to the so now called exoplanets, their number has rapidly grown, and it will still grow with increasing acceleration, as it happens for any existing object in the cosmos, as a function of the progress of detection systems. Telluric planets, including those lying "within the band of water", were soon discovered next to the stars they orbit.

All the prejudices on which one could rely to defy the existence of life on a planet collapsed one after another. And it became not as evident as before the fact that not only Sun-like stars may be conducive to the emergence of life in its associated planetary system.

Nowadays the view of a scientist like Drake has been totally reversed, and it is likely that his formula will soon be forgotten. According to Ref. 41, the NASA, based on the fact that 1500 planets were detected within a radius of 50 light-years from the Earth, used this figure to estimate 100 billions of the minimum number of planets in the Milky Way. This number should be weighted by a coefficient of habitability on which we should still come to an agreement. But that makes us to think that the probability of the Earth not being the only inhabited planet in our galaxy cannot be zero. An estimation of this number being made, if it is multiplied by the number of galaxies in the observable Universe, a considerable figure is anyway obtained.

One can sum this up in one sentence: "The probability that there does not exist, not only in the Universe but in our own galaxy, a planet other than ours, which is carrier of organized and intelligent life, must be considered as zero". So it was compelling to recognize that the belief of the skeptics was nothing more than the manifestation of a ridiculous geocentrism.

You can still find however scientists that doubt that the level of organization of life, such as it exists on Earth, can be found elsewhere in other planets. Some of them say: "Life elsewhere, yes, but only in bacterial form".

Humans are indeed facing an unprecedented awareness in Earth's history, with the obvious impacts on all their religious beliefs. Some precursors, such as Andrei Sakharov, have expressed a very advanced view on this, which is summarized in the last lines of his acceptance speech of the Nobel Peace Prize in 1975. Given the stature of this scientist and the outstanding quality of his contributions in various scientific fields, ranging from hot plasma physics (he was the designer of the first Soviet hydrogen bomb) to magnetohydrodynamics (where he was, during the '50s, one of the pioneers) and cosmology, it seemed important to us to quote his own words:

> *Thousands of years ago tribes of human beings suffered great privations in the struggle to survive. In this struggle, it was important not only to be able to handle a club, but also to possess the ability to think reasonably, to take care of the knowledge and experience garnered by the tribe, and to develop the links that would provide cooperation with other tribes. Today the entire human race is faced with a similar test. In infinite space many civilizations are bound to exist, among them civilizations that are also wiser and more "successful " than ours. I support the cosmological hypothesis which states that the development of the Universe is repeated in its basic features an infinite number of times. In accordance with this, other civilizations, including more "successful" ones, should exist an infinite number of times on the "preceding" and the "following" pages of the Book of the Universe. Yet this should not minimize our sacred endeavors in this world of ours, where, like faint glimmers of light in the dark, we have emerged for a moment from the nothingness of dark unconsciousness of material existence. We must make good the demands of reason and create a life worthy of ourselves and of the goals we only dimly perceive.*

In this text is explicitly mentioned the fact that earthlings might not be the most advanced intelligent species in terms of science and technology. It is the logical view every reasonable scientist should assume. But it leads immediately to one question: More advanced civilizations that our own, have they been able to visit us, including ancient times of our history? If such visits were technically and scientifically possible, do they currently continue?

Faced with this question, skeptics exhibits two arguments. First, as once did Enrico Fermi, they are amazed that there has not been any contact at all. Second, they evoke the impossibility of interstellar travel due to the severe limitation imposed on the speed of a craft by the laws, repeatedly checked at all scales, of special relativity.

On the first argument, let us say that any contact between civilizations with very different technical and scientific levels of development is unavoidably accompanied by a shock of cultures. Different scenarios are then possible. In one of them, the low-level scientific and technological civilization quite simply collapses. In a second one the most advanced civilization does swallows up the one which it comes in contact with. It can assimilate it, or enslave it (colonization). In a third scenario, a powerful scientific and technological transfer occurs, which is immediately converted with priority, as in any fundamental progress of technology, into new weapons.

Any progress in science is inevitably accompanied by a subsequent progress in destructive capacity. So that if some advanced sophisticated technoscience arose which made interstellar travel possible, the corollary would be the emergence of huge means of destruction. In comparison, such new weapons would make our thermonuclear warheads look like match sticks.

I do not think I would be contradicted by saying that wisdom is far from Earth where, despite past dramas, we wonder if humanity would not soon switch into a third world war. Thus, any technology transfer would not do other but accentuate the deadly imbalance which we suffer and that is not going to get better with time.

This paper focuses on the second argument, the feasibility of interstellar travel.

2 A Bimetric Model

The cosmological model of General Relativity represents an extension of the revolution represented by the appearance of Special Relativity. It is worth to think for a moment about the essence of these profound changes of paradigm. The transition from Newtonian physics to relativistic physics is based on a modification of the geometric paradigm. Special Relativity can be summed up in the sentence: "We live in a spacetime which is an M_4 manifold with a hyperbolic Lorentz metric of signature $(+---)$". General Relativity can be stated accordingly: "The Universe is an M_4 manifold with a Riemannian metric of signature $(+---)$" which is a solution of the Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \chi T_{\mu\nu} \tag{1}$$

Every travel in this context can proceed only by moving along the geodesics associated with the metric $g_{\mu\nu}$ that is solution of the equation. These geodesics include two subsets: those of nonzero length followed by particles that own a mass, and those of zero length, followed by photons. This statement implies an unavoidable limit for all velocities, that is the speed of light c. In the domain of relativistic velocities, an enormous energy must be involved, and the limitation to a speed c implies the fact that, according to this vision of the Universe, an almost infinite energy is necessary to reach it.

Everything else follows and all those who have tried to model the interstellar travel are led to travel times that are incompatible with the duration of human life. The conclusion is therefore the following: if interstellar travel is scientifically and technically feasible, that is, compatible with the duration of human life, then a deep change in the current cosmological paradigm, which prohibits such an eventuality must be operated.

One could say that the current situation somehow calls for such a change of paradigm. Theoretical astrophysics virtually stopped in the early '70s when theorists, disregarding the approach initiated by Chandrasekhar in 1941,[1] abandoned any attempt to build a self-consistent model of galaxies and their spiral structure, whenever they possess one, in favor of numerical computer simulations on which high hopes were deposited and which have strictly produced nothing in half

a century. The improvement of observational data on the velocity curves of rotation of gas in galaxies, and the discovery of gravitational arcs, soon attributed to gravitational lens effects and incompatible with the amount of matter in galaxies and clusters of galaxies from photometric observations, gave birth to a *deus ex machina*: the dark matter. But all attempts to identify this new component, being it MACHOs or astroparticles, have failed. And it comes that since ten years ago it is confirmed, against all odds, that the Universe, rather than decelerate as a result of the attractive forces of visible matter and dark matter, accelerates instead (see Refs. 2–10). Faced with this new dilemma, astrophysicists have not done anything but to create a new word: dark energy. And have done so without having the slightest idea of the nature of this new component, which would represent 70% of the cosmic content. Many attempts were made in order to explain the nature of such phenomenon. Among them, let us cite Refs. 11 to 14.

In 1967, Sakharov proposed a twin description of the Universe with the aid of two entities linked by a Big Bang singularity, and with antiparallel arrows of time.[15–17] If we exclude our own work, which was initiated in 1977[18,19] using the tools of non-relativistic Newtonian cosmology, and ignore the existence of his own, this approach had no continuators. In 1994,[20] we clarified the approach by offering a bimetric description of the Universe. Though it has nothing to do with the bimetric models of Refs. 21 and 22 where the second metric refers to gravitons with nonzero mass. Strictly speaking, these models have not produced anything.

In our model, the Universe is an M_4 manifold associated not to one single metric, but to two: $g^{(+)}_{\mu\nu}$ and $g^{(-)}_{\mu\nu}$, the former linked to species of positive mass and energy, the latter to species of negative mass and energy. From these metrics, one can build the associated Ricci tensors, $R^{(+)}_{\mu\nu}$ and $R^{(-)}_{\mu\nu}$. A system of two coupled field equations was then proposed:[20]

$$R^{(+)}_{\mu\nu} - \frac{1}{2} R^{(+)} g^{(+)}_{\mu\nu} = \chi \left(T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right), \tag{2a}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2} R^{(-)} g^{(+)}_{\mu\nu} = -\chi \left(T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right). \tag{2b}$$

where the tensors $T^{(+)}_{\mu\nu}$ and $T^{(+)}_{\mu\nu}$ represent positive and negative energy contents (and positive and negative mass contents as well). Previously, in 1957, Bondi[23] did study the possibility of introducing negative masses into the Einsteinian model with a single metric. As a result, positive masses attracted everything and negative masses repelled everything. It then led to a phenomenon that was called "runaway": when a mass $+m$ met a mass $-m$, the positive mass fled, pursued by the negative mass, both undergoing a continuous acceleration, with conservation of energy, since one of the energies was positive and the other negative.

This idea of cohabitation of masses of opposed signs was therefore abandoned during 57 years.

Quantum Field Theory (QFT) rejects the existence of negative energy states on the basis that no state could exist with energy less than that of the vacuum. This argument is developed by Weinberg in his book (Sec. 2.6 of Ref. 24). But it is more a hypothesis than a demonstration in the sense that everything depends on the choice of the T operator, that is, time inversion. If it is to be considered as unitary and linear, then QFT enables the existence of negative energies, time inversion being synonymous of energy inversion (which is the case in the Theory of Dynamical Groups, build with real coefficients[25]). In order to avoid this "disastrous" situation, QFT imposes the choice of an anti-unitary and anti-linear T operator, which does not invert energy.

The paper of 1994[20] was completed the following year with another publication[26] and with a communication in an international workshop.[27] This theoretical description was also recently revisited.[28] The main lines of this model are as follows:

– Particles of positive mass follow non-null geodesics of the $g^{(+)}_{\mu\nu}$ metric.

– Photons of positive energy follow null geodesics of the $g_{\mu\nu}^{(+)}$ metric.

– Particles of negative mass follow non-null geodesics of the $g_{\mu\nu}^{(-)}$ metric.

– Photons of negative energy follow null geodesics of the $g_{\mu\nu}^{(-)}$ metric.

– The two families of geodesics, one followed by particles of positive energy and the other followed by particles of negative energy, are *a priori* disjoint.

– Particles of positive mass emit positive energy photons that can in turn be captured by observational devices constituted also by positive masses.

– Particles of negative mass emit negative energy photons that cannot be captured by our observational devices.

References 26 and 27 contain results of 2D numerical simulations carried out in the Daisy Laboratory in Germany with the computing capabilities of that time. They give an idea of the fertility of such approach when it comes to explain the large-scale structure of the Universe, galaxy confinement and durable spiral structures. The initial results could not be pursued because of the lack of access to adequate means of calculation. Beyond that, the goal is obviously to build a complete cosmological model, with a radiation dominated era and a matter dominated era. The schema includes therefore a radiative phase "with constant variables" and free of redshift. We published the first paper referring to a cosmological model with variable light speed in this same journal in 1988,[29,30] and then in 1995.[26] Subsequently, we modified the model[28] on the assumption that the secular drift of the constants ceased when the radiation was no longer the dominant component. Laws of evolution which are not semi-empirical, but are derived from a universal gauge relationship assuring the invariance of the equations of physics (field equation, Maxwell equations, quantum physics equations) were provided for the different constants. If a is the scale factor associated with length, the evolution of c is:[26,29]

$$c \sim \frac{1}{\sqrt{a}} \tag{3}$$

It leads to a cosmological horizon that varies with a, which ensures cosmic homogeneity and makes the appeal to inflation theory unnecessary, what was already stated in the conclusions of Ref. 30. Moreover, this evolution was "Lorentz invariant". But this is simply a remark. The formulation of the cosmic evolution in its radiative phase and in the regime of "constant variables" will be the subject of a future paper. It is at the end of this phase that the two cosmic sets, one of positive energy, the other of negative energy, are provided with constant but very different $c^{(+)}$ and $c^{(-)}$ light speeds.

We mention here the work of Couannier,[31] Hossenfelder[32] and Milgrom[33] which also aim at the construction of a bimetric description of the Universe.

3 A Model with Two Different Light Speeds

This section deals with a regime with two different light speeds. Some previous work was presented in Ref. 28. We take the system of two coupled field equations:

$$R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi^{(+)}\left(T_{\mu\nu}^{(+)} + \varphi T_{\mu\nu}^{(-)}\right), \tag{4a}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(-)} = \chi^{(-)}\left(\phi T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right). \tag{4b}$$

For the sake of brevity, we will sometimes use the notation: $f \in \{+,-\}$. In mixed form, we will write the tensors:

$$T_\nu{}^{(f)\mu} = \begin{pmatrix} \rho^{(f)} \left(c^{(f)}\right)^2 & 0 & 0 & 0 \\ 0 & -p^{(f)} & 0 & 0 \\ 0 & 0 & -p^{(f)} & 0 \\ 0 & 0 & 0 & -p^{(f)} \end{pmatrix} \quad \begin{array}{l} \text{with } \rho^{(f)} > 0 \text{ and } p^{(f)} > 0 \text{ for } f = \text{“+”} \\ \text{and } \rho^{(f)} < 0 \text{ and } p^{(f)} < 0 \text{ for } f = \text{“−”} \end{array} \tag{5}$$

φ and ϕ are functions determined from conservation energy requirements. Positive energy photons cruise along null geodesics of the metric $g^{(+)}_{\mu\nu}$ and negative energy photons along null geodesics of the metric $g^{(-)}_{\mu\nu}$. Note that the speeds $c^{(+)}$ and $c^{(-)}$ on one hand, and the constants $\chi^{(+)}$ and $\chi^{(-)}$ on another hand, may be different. We assume this bimetric Universe is homogeneous and isotropic, so that the Riemannian metric becomes:

$$\left(\mathrm{d}s^{(f)}\right)^2 = \left(c^{(f)}\right)^2 \mathrm{d}t^2 - \left(a^{(f)}\right)^2 \frac{\mathrm{d}u^2 + u^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)}{\left(1 + k^{(f)}\frac{u^2}{4}\right)^2} \tag{6}$$

Introducing these metrics in the system (3a)+(3b), we get classical systems:

$$\frac{3}{\left(c^{(+)}\right)^2\left(a^{(+)}\right)^2}\left(\frac{\mathrm{d}a^{(+)}}{\mathrm{d}t}\right)^2 + \frac{3k^{(+)}}{\left(c^{(+)}\right)^2\left(a^{(+)}\right)^2} = -\chi^{(+)}\left[\rho^{(+)}\left(c^{(+)}\right)^2 + \varphi\rho^{(-)}\left(c^{(-)}\right)^2\right], \tag{7a}$$

$$\frac{2}{\left(c^{(+)}\right)^2\left(a^{(+)}\right)^2}\frac{\mathrm{d}^2a^{(+)}}{\mathrm{d}t^2} + \frac{1}{\left(c^{(+)}\right)^2\left(a^{(+)}\right)^2}\left(\frac{\mathrm{d}a^{(+)}}{\mathrm{d}t}\right)^2 + \frac{k^{(+)}}{\left(c^{(+)}\right)^2\left(a^{(+)}\right)^2} = 0 \tag{8a}$$

$$\frac{3}{\left(c^{(-)}\right)^2\left(a^{(-)}\right)^2}\left(\frac{\mathrm{d}a^{(-)}}{\mathrm{d}t}\right)^2 + \frac{3k^{(-)}}{\left(c^{(-)}\right)^2\left(a^{(-)}\right)^2} = +\chi^{(-)}\left[\phi\rho^{(+)}\left(c^{(+)}\right)^2 + \rho^{(-)}\left(c^{(-)}\right)^2\right], \tag{7b}$$

$$\frac{2}{\left(c^{(-)}\right)^2\left(a^{(-)}\right)^2}\frac{\mathrm{d}^2a^{(-)}}{\mathrm{d}t^2} + \frac{1}{\left(c^{(-)}\right)^2\left(a^{(-)}\right)^2}\left(\frac{\mathrm{d}a^{(-)}}{\mathrm{d}t}\right)^2 + \frac{k^{(-)}}{\left(c^{(-)}\right)^2\left(a^{(-)}\right)^2} = 0 \tag{8b}$$

Applying classical mathematical methods[34] the compatibility conditions of sets (7a) + (7b), and (8a) + (8b) give:

$$3\frac{\mathrm{d}a^{(+)}}{a^{(+)}} + \frac{\mathrm{d}\left[\rho^{(+)}\left(c^{(+)}\right)^2 + \varphi\rho^{(-)}\left(c^{(-)}\right)^2\right]}{\left[\rho^{(+)}\left(c^{(+)}\right)^2 + \varphi\rho^{(-)}\left(c^{(-)}\right)^2\right]} = 0, \tag{9a}$$

$$3\frac{\mathrm{d}a^{(-)}}{a^{(-)}} + \frac{\mathrm{d}\left[\phi\rho^{(+)}\left(c^{(+)}\right)^2 + \rho^{(-)}\left(c^{(-)}\right)^2\right]}{\left[\phi\rho^{(+)}\left(c^{(+)}\right)^2 + \rho^{(-)}\left(c^{(-)}\right)^2\right]} = 0. \tag{9b}$$

The conservation of energy:

$$E = \rho^{(+)}\left(c^{(+)}\right)^2\left(a^{(+)}\right)^3 + \rho^{(-)}\left(c^{(-)}\right)^2\left(a^{(-)}\right)^3 \tag{10}$$

is ensured if:

$$\varphi = \left(\frac{a^{(-)}}{a^{(+)}}\right)^3 \quad \phi = \left(\frac{a^{(+)}}{a^{(-)}}\right)^3 \quad \phi = \varphi^{-1}. \tag{11}$$

So that our coupled field equation system becomes:

$$R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi^{(+)}\left[T_{\mu\nu}^{(+)} + \left(\frac{a^{(-)}}{a^{(+)}}\right)^3 T_{\mu\nu}^{(-)}\right], \tag{12a}$$

$$R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(-)} = -\chi^{(-)}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^3 T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right] \tag{12b}$$

as previously established in Ref. 28. Let:

$$\left(a^{(+)}\right)^2 \frac{\mathrm{d}^2 a^{(+)}}{\mathrm{d}t^2} = \frac{\chi^{(+)}}{2}E, \tag{13a}$$

$$\left(a^{(-)}\right)^2 \frac{\mathrm{d}^2 a^{(-)}}{\mathrm{d}t^2} = -\frac{\chi^{(-)}}{2}E, \tag{13b}$$

$$\chi^{(+)} = -\frac{8\pi G}{c^4} \quad \text{with } c^{(+)} = c \text{ and } G^{(+)} = G. \tag{14}$$

Let us assume $E < 0$. Then $a^{(+)\prime\prime} > 0$ and $a^{(-)\prime\prime} < 0$. If we assume further that our visible part of the Universe corresponds to positive mass, then it accelerates, while the negative species decelerates. As in Ref. 28, we find that the evolution equation (13b) is identical to Friedmann's equation, while (13a) is identical to Bonnor's solution:[35]

$$\begin{aligned} a^{(+)}(\mu) &= \alpha^2 \operatorname{ch}^2 u, \\ t^{(+)}(\mu) &= \alpha^2\left(1 + \frac{\operatorname{sh} 2u}{2} + u\right). \end{aligned} \tag{15}$$

Let us now develop the Newtonian approximation. The Universe owns a global nonzero curvature. In steady-state or quasi steady-state condition, the Universe can be considered as a large 3D hypersurface where very small regions contain matter surrounded by large empty spaces where the metric corresponds to the Lorentz solution of the $R_{\mu\nu} = 0$ equation. Consider a portion of empty space with a finite extension. If we consider phenomena occurring over a characteristic time that is small compared to the one corresponding to the evolution of the Universe, the description by time-independent metric holds. Introduce a finite space extension perturbation, corresponding to the terms $\gamma_{\mu\nu}^{(+)}$ and $\gamma_{\mu\nu}^{(-)}$.

$$g^{(+)} = \eta^{(+)} + \varepsilon\gamma^{(+)}, \quad g^{(-)} = \eta^{(-)} + \varepsilon\gamma^{(-)}. \tag{16}$$

More explicitly

$$\left(\mathrm{d}s^{(+)}\right)^2 = \left(c^{(+)}\right)^2 \mathrm{d}t^2 - \left(a^{(+)}\right)^2\left[\left(\mathrm{d}\xi^1\right)^2 + \left(\mathrm{d}\xi^2\right)^2 + \left(\mathrm{d}\xi^3\right)^2\right], \tag{17a}$$

$$\left(\mathrm{d}s^{(-)}\right)^2 = \left(c^{(-)}\right)^2 \mathrm{d}t^2 - \left(a^{(-)}\right)^2\left[\left(\mathrm{d}\xi^1\right)^2 + \left(\mathrm{d}\xi^2\right)^2 + \left(\mathrm{d}\xi^3\right)^2\right]. \tag{17b}$$

In quasi steady-state conditions, the two scale factors are considered as constants. Then, the field equations are expanded into a series. Neglecting second-order terms in the expansion, we find:

$$\varepsilon\gamma_{00|\beta|\beta}^{(+)} = -\chi^{(+)}\left[\delta\rho^{(+)}\left(c^{(+)}\right)^2 + \left(\frac{a^{(-)}}{a^{(+)}}\right)^3 \delta\rho^{(-)}\left(c^{(-)}\right)^2\right], \tag{18a}$$

$$\varepsilon\gamma^{(-)}_{00|\beta|\beta} = \chi^{(-)}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^3 \delta\rho^{(+)}\left(c^{(+)}\right)^2 + \delta\rho^{(-)}\left(c^{(-)}\right)^2\right], \tag{18b}$$

Defining the potentials:

$$\varphi^{(+)} = \varepsilon\gamma^{(+)}_{00}\frac{\left(c^{(+)}\right)^2}{2}, \quad \varphi^{(-)} = \varepsilon\gamma^{(-)}_{00}\frac{\left(c^{(-)}\right)^2}{2} \tag{19}$$

we get:

$$\sum_{\alpha=1}^{3}\frac{\partial^2\varphi^{(+)}}{\partial\xi^\alpha\partial\xi_\alpha} = -\frac{\chi^{(+)}\left(a^{(+)}\right)^2}{2}\left[\delta\rho^{(+)}\left(c^{(+)}\right)^2 + \left(\frac{a^{(-)}}{a^{(+)}}\right)^3\delta\rho^{(-)}\left(c^{(-)}\right)^2\right], \tag{20a}$$

$$\sum_{\alpha=1}^{3}\frac{\partial^2\varphi^{(-)}}{\partial\xi^\alpha\partial\xi_\alpha} = \frac{\chi^{(-)}\left(a^{(-)}\right)^2}{2}\left[\left(\frac{a^{(+)}}{a^{(-)}}\right)^3\delta\rho^{(+)}\left(c^{(+)}\right)^2 + \delta\rho^{(-)}\left(c^{(-)}\right)^2\right] \tag{20b}$$

which are Poisson-like equations that represent an extension of Ref. 28 when the two light speeds are different. Using the two forms of geodesic equations we get for the positive mass particles:

$$\frac{\mathrm{d}^2\xi^\alpha}{\mathrm{d}t^2} = -\frac{1}{\left(a^{(+)}\right)^2}\frac{\partial\varphi^{(+)}}{\partial\xi_\alpha}. \tag{21a}$$

And for the negative mass particles:

$$\frac{\mathrm{d}^2\xi^\alpha}{\mathrm{d}t^2} = -\frac{1}{\left(a^{(-)}\right)^2}\frac{\partial\varphi^{(-)}}{\partial\xi_\alpha}. \tag{21b}$$

Building the dynamics from this set of equations would require a more refined solution, involving time scale factors $T^{(+)}$ and $T^{(-)}$. This will be developed in a future paper. Anyway, it is possible to derive the laws in the simplified case $a^{(+)} = a^{(-)}$, $c^{(+)} = c^{(-)}$. Then:

$$\Delta\varphi^{(+)} = -\Delta\varphi^{(+)} = 4\pi G\left(\delta\rho^{(+)} + \delta\rho^{(-)}\right), \tag{22}$$

$$\frac{\mathrm{d}^2x^\alpha}{\mathrm{d}t^2} = -\frac{\partial\varphi^{(+)}}{\partial x_\alpha}, \quad \frac{\mathrm{d}^2x^{(-)\alpha}}{\mathrm{d}t^2} = -\frac{\partial\varphi^{(-)}}{\partial x_\alpha} = \frac{\partial\varphi^{(+)}}{\partial x_\alpha} \tag{23}$$

Referring to the previous works of Bondi[23] and Bonnor,[35] the puzzling problem of runaway phenomenon is eliminated. This model fits classical verification associated with General Relativity. As the two species repel each other, where positive mass is present, negative mass is almost absent and the system of coupled field equations reduces to:

$$R^{(+)}_{\mu\nu} - \frac{1}{2}R^{(+)}g^{(+)}_{\mu\nu} \approx \chi^{(+)}T^{(+)}_{\mu\nu} = -\frac{8\pi G}{c^4}T^{(+)}_{\mu\nu}, \tag{24a}$$

$$R^{(-)}_{\mu\nu} - \frac{1}{2}R^{(-)}g^{(-)}_{\mu\nu} \approx -\chi^{(-)}\left(\frac{a^{(+)}}{a^{(-)}}\right)^3 T^{(+)}_{\mu\nu}. \tag{24b}$$

On the other hand, considering portions of space containing a sphere filled with constant density matter (positive or negative) surrounded by vacuum, makes "internal" and "external" Schwarzschild solutions features clearer, as first presented in Ref. 26 introducing a negative gravitational lensing effect.

To give the relative size of the parameters involved, we need to develop the non-steady solution applying to the radiative era with variable speeds of light. This will also be done in a future paper.

The basic features of the bimetric model, interaction laws and negative lensing effect, were developed in Refs. 20, 26 and 27. In Ref. 28, from dynamical group theory,[25] the particles of the second sector are CPT-symmetrical with respect to positive energy particles (protons, neutrons, electrons, photons and so on), which gives them negative energy and mass. So the matter–antimatter duality holds in the two sectors.

Given that masses of opposite signs repel each other, the component responsible for the strong effects of gravitational lensing around galaxies and clusters of galaxies is in an ultra-rarified state in the galaxies and also in the solar system (where it could be responsible for the deceleration of the Pioneer space probes, a subject to be examined). If the proposed model is correct, then attempts to find evidence for the existence of astroparticles would be doomed to failure.

Reference 28 was focused on the explanation of the acceleration of the Universe, caused by a negative pressure associated to the negative energy component. The present paper has shown that a bimetric model can be associated with two different light speeds.

4 Back to the Problem of Interstellar Travel

The present work needs completion. A study of the radiation dominated era with "variable constants" must be carried out in order to justify the difference between $c^{(+)}$ and $c^{(-)}$. As evoked above, time scale factors should be added, which we have not done in the present paper.

Species with positive and negative mass behave differently, the whole being fully asymmetric. Sakharov[15–17] was the first to imagine an asymmetry in the characteristic times of production of baryons from quarks, and of antibaryons from antiquarks in our Universe. He suggested that the two twin Universes owned opposite arrows of time (notice that according to dynamical group theory[25] time inversion goes with mass inversion). He suggested that different rates of production of baryons and antibaryons could explain the absence of cosmological antibaryons in our orthochron Universe of positive masses and energies, while "primeval baryons" would be absent in the twin Universe. This would go with a remnant of free antiquarks in our fold and a remnant of quarks with negative energy in the twin.

The absence of primeval antimatter is therefore the first smoking gun of such asymmetry.

The second one is the Very Large Structure of the Universe, which according to that model looks like joint soap bubbles (see Refs. 36–39). Negative mass would form a set of clumps located at the center of each bubble.

As evoked in Ref. 28 a new cosmic cartography could be built on the basis of a recent idea published by a Japanese group,[40] which would make it possible to size the ratio between averaged densities of the two species $\rho^{(+)}$ and $\rho^{(-)}$. Further data could come from the analysis of magnitude of distant galaxies with strong redshifts, considered classically as dwarfs. We believe they are "normal" galaxies whose signal is weakened by negative gravitational lensing effects. Notice that those galaxies emit positive energy photons which can cross negative mass clumps.

As shown in former 2D simulations,[26] macroscopic structures are different in positive and negative worlds. Negative mass forms spheroidal clumps, similar to huge proto-stars whose cooling time is so great with respect to the age of the Universe that fusion cannot occur. In the negative world there are no stars, no galaxies, no heavy elements, no planets. Life is absent.

Scale factors are also different: $a^{(+)} > a^{(-)}$, so that in order to go from a point A to a point B, two ways are possible along two geodesic paths whose lengths are different depending on the mass of the object, positive or negative.

The evolution of the bimetric Universe in a radiation dominated era with variable constants and $a^{(-)} < a^{(+)}$ implies $c^{(-)} > c^{(+)}$:

$$c^{(+)} \sim \frac{1}{\sqrt{a^{(+)}}}, \quad c^{(-)} \sim \frac{1}{\sqrt{a^{(-)}}} \tag{25}$$

that is:

$$\frac{c^{(-)}}{c^{(+)}} \sim \sqrt{\frac{a^{(+)}}{a^{(-)}}} > 1. \tag{26}$$

If mass inversion could be achieved, then the feasibility of interstellar travel should be reconsidered. In effect, a transferred vehicle would cruise along geodesics of the metric $g^{(-)}$, with several gains:

- The speed limit ($c^{(-)} > c^{(+)}$) would be higher.
- Distances would be shorter: $a^{(-)} < a^{(+)}$.
- The addition of a different time gauge could reduce travel duration (to be developed in a future work).

Two questions remain:

- Is it possible to achieve mass inversion of a craft and its passengers?
- How to give to the craft a relativistic velocity with respect to the new value $c^{(-)}$?

Mass inversion is accompanied by a local modification of the geometry, which implies a deconnection of the geodesics of one system and a reconnection with the geodesics of the other system. We believe that such process occurs in nature and will give a theoretical description of it in a future paper, showing subsequent observations. As we will show, if a mass is inverted, it does not imply that it goes backwards in time and could emerge at a point of its own past. This is so because the proper time is not modified by the process.

The theoretical physicist Michio Kaku suggests a classification of civilizations of "types I, II, III, etc.". He imagines that the last of them would be capable of using energies defying imagination, which would make interstellar travel possible. Would it be necessary, for example, in order to ensure mass inversion of a given mass M, to use an energy comparable to Mc^2? Should we create a "wormhole" and jump in? We believe that the energy we can handle today through nuclear technology and, in the future, matter–antimatter annihilation, could be enough. We believe also that our present science and technology has reached the required threshold if used in a new geometric context.

If a given technology makes a craft's mass inversion possible, it would seem to dematerialize if observed by a witness. Negative mass particles, atoms and molecules no longer interact with the positive molecules of the surrounding air except for gravitation (in fact, anti-gravitation). We conjecture that the mass inversion process would be symmetrical. If a positive mass is inverted in a given volume, this would affect the few negative masses present in the volume, whose mass would be inverted too, and become positive. As a consequence, a highly rarefied medium would appear in the volume, composed by few positive mass hydrogen atoms. For a positive mass observer, the volume occupied by the craft would look empty. The air would fill it immediately, the process producing a hydrodynamical perturbation. If a plane is flying too close to a large craft that is inverting its own mass, the subsequent gas perturbation could break it.

We will elaborate below, on a very conjectural basis, on a possible modification of the craft kinetic parameters after the hypothetic mass inversion process. Anyway, an inverted mass would be repelled by the Earth, so that "it would fall upward". An appropriate cyclic mass inversion could cancel weight. In effect, values of the craft weight would be alternatively $P = Mg$ and $P = -Mg$, that is zero on average.

A witness would say that the force of gravity is cancelled. Combination with appropriate modifications of the kinetic parameters would ensure the crafts movement.

Particles with mass m own a characteristic length, the Compton's length λ_c:

$$\lambda_c = \frac{h}{mc} \tag{27}$$

If mass inversion follows gauge laws as defined in Ref. 26, then:

$$c \sim \frac{1}{\sqrt{a}}, \quad h \sim a^{\frac{3}{2}}, \quad m \sim a. \tag{28}$$

It is an "apparent mass", concept that will be developed in a future paper. This leads to:

$$\lambda_c \sim a. \tag{29}$$

Mass inversion would be accompanied with some kind of "Gulliver effect". Transferred in the new frame of reference, the inverted particles would have a Compton's length greater than that of their sisters of negative mass because $a^{(-)} < a^{(+)}$. The subsequent energy is:

$$E = \frac{hc}{\lambda}. \tag{30}$$

Such transfer would occur with loss of energy. If we assume that mass transfer implies energy conservation during the process of mass sign inversion, the transferred particles must have Compton's lengths equal to that of their "sisters" owing negative mass. This would be possible if, after mass inversion, particles gain relativistic velocity so that their Compton's length shortening is ensured by the Lorentz contraction. But the process should orient velocities in the same direction. If not, the craft and the passengers would be splitted. Would the process be possible, interstellar crafts would not need any propeller at all. The words "acceleration" and "deceleration" would lose meaning. Thus, if a huge amount of energy is necessary to accelerate a positive mass at a relativistic velocity, once its mass is inverted, a comparable amount of energy is required to bring its speed to zero when the craft cruises in the new negative frame of reference. After mass inversion, a craft would go so fast that it could not slow down. But, arriving at its destination, a new mass inversion would give it again its former kinetic parameters.

Even in a very primitive way, these travel techniques have nothing to do with rocket propulsion. All relies on mass manipulation, together with the manipulation of other parameters as the spin (quite easy to handle with a magnetic field). We suggest that energy conservation could be accompanied by a modification of the dynamical parameters that defies imagination. After mass inversion, a new modification of parameters, when the craft owns a negative mass, could give it in the positive realm a velocity vector oriented in another direction, while the length of the vector and the kinetic momentum would be conserved. If such thing is possible, a positive mass observer would see the craft performing turns at right angles, or a total inversion of its velocity, both incompatible with the conventional laws of physics.

If, in a distant future, technical progress makes it possible for a craft to travel between stars in times comparable with human life duration, what would the travelers see through the windows when they go over the negative world? They could no longer see the Universe of positive mass. They could not see planets, stars or galaxies. Instead, being able to capture images mediated by photons of negative energy, it would be possible for them to see the distant fuzzy spheroidal clusters of negative mass emitting weakly, like proto-stars, reddish and infrared light.

References

1. S. Chandrasekhar, *Principles of Stellar Dynamics* (Dover publications, 1941).
2. A. G. Riess *et al., Astron. J.* **116**, 1009 (1998).

3. S. Perlmutter *et al., Astrophys. J.* **517**, 565 (1999).

4. A. G. Riess, *Publ. Astron. Soc. Pac.* **112**, 1284 (2000).

5. A. V. Filippenko and A. G. Riess, *AIP Conf. Proc.* 540, Particle (2001).

6. B. Leibundgut, *Annu. Rev. Astron. Astrophys.* **39**, 67 (2001).

7. R. Knop *et al., Astrophys. J.* **598**, 102 (2003).

8. J. T. Tonry *et al., Astrophys. J.* **594**, 1 (2003).

9. B. Barris *et al., Astrophys. J.* **602**, 571 (2004).

10. A. G. Riess, *Astrophys. J.* **607**, 665 (2004).

11. P. H. Frampton and K. J. Ludwick, *Mod. Phys. Lett. A* **28**, 1350125 (2013).

12. S. del Campo, V. H. Cárdenas and R. Herrera, *Mod. Phys. Lett. A* **27**, 1250213 (2012).

13. D. Savickas, *Mod. Phys. Lett. A* **28**, 1330025 (2013).

14. I. Haranas and I. Gkigkitzis, *Mod. Phys. Lett. A* **28**, 1350077 (2013).

15. A. D. Sakharov, *ZhETF Pis'ma* **5**, 32; *JETP Lett.* **5**, 24 (1967).

16. A. D. Sakharov, *ZhETF Pis'ma* **76**, 1172 (1979); *JETP* **49**, 594 (1979).

17. A. D. Sakharov, *ZhETF Tr. JETP* **52**, 349; 689 (1980).

18. J. P. Petit, Univers énantiomorphes à flèches du temps opposés, CRAS du 8 mai t. **285**, pp. 1217–1221 (1977).

19. J. P. Petit, Univers en interaction avec leur image dans le miroir du temps, CRAS du 6 juin, t. 284, série A, pp. 1413–1416 (1977).

20. J. P. Petit, *Nuovo Cimento* B **109**, 697 (1994).

21. T. Damour and I. I. Kogan, *Phys. Rev. D* **66**, 104024 (2002).

22. T. Damour, I. I. Kogan and A. Papazoglou, *Phys. Rev. D* **66**, 104025 (2002).

23. H. Bondi, *Rev. Mod. Phys.* **29** (1957).

24. S. Weinberg, *The Quantum Theory of Fields*, Vol. 1 (Cambridge University Press, 2005).

25. J. S. Souriau, *Structure des Systèmes Dynamiques* (Dunod, 1970); *Structure of Dynamical Systems* (Birkhaüser, 1997).

26. J. P. Petit, *Astrophys. Space Sci.* **226**, 273 (1995).

27. J. P. Petit, P. Midy and F. Landhseat, Twin matter against dark matter, in *Conf. on Astrophysics and Cosmology, Where is the Matter?, Tracing Bright and Dark Matter with the New Generation of Large-Scale Surveys* (Marseille, France, June 2001).

28. J. P. Petit and G. D'Agostini, *Astrophys. Space Sci.*, DOI:10.1007/s10509-014-2106-5.

29. J. P. Petit, *Mod. Phys. Lett. A* **3**, 1527 (1988).

30. J. P. Petit, *Mod. Phys. Lett. A* **3**, 1733 (1988).

31. F. Henry Couannier, *Int. J. Mod. Phys. A* **20**, 2341 (2005).

32. S. Hossenfelder, *Phys. Rev. D* **78**, 044015 (2008).

33. M. Milgrom, Matter and twin matter in bimetric MOND, arXiv:1001.4444v3 [astrop-ph.CO].

34. R. Adler, R. Bazin and M. Schiffer, *Introduction to General Relativity* (McGraw Hill Book, 1967).

35. W. B. Bonnor, *Gen. Relat. Gravit.* **21**, 11 (1989).

36. T. Piran, *Gen. Relat. Gravit.* **29**, 11 (1997).

37. H. El-Ad, T. Piran and L. N. da Costa, *Astrophys. J. Lett.* **462**, L13 (1996).

38. H. El-Ad, T. Piran and L. N. da Costa, *Mon. Not. R. Astro. Soc.* **287**, 790 (1997).

39. H. El-Ad and T. Piran, *Astrophys. J.* **491**, 421 (1997).

40. K. Izumi, C. Hagiwara, K. Nakajima, T. Kitamura and H. Asada, *Phys. Rev. D* **88**, 024049 (2013)

41. W. Clavin, http://www.nasa.gov/topics/Universe/features/micro20120111.html.

I am sorting through all the elements that may have emerged in connection with this Ummo affair.

I will now discuss a phone message. These were always brief, limited to the bare minimum. None were recorded.

This must have taken place in 2015. At that time, I was beginning to work on the topic of black holes, which led me to examine the stationary solution to Einstein's equation in spherical symmetry, published in January 1916 by Karl Schwarzschild[60], who was stationed on the Russian front and died a few months later from an infection.

The form (wrongly) considered standard is the bilinear form:

$$ds^2 = \left(1 - \frac{R_s}{r}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{R_s}{r}} - r^2(\,d\theta^2 + sin^2\theta\, d\varphi^2\,)$$

When the variable r reaches the positive value known $R_s$as the "Schwarzschild radius," a term in the denominator becomes zero. This results in a singularity, which was pointed out by the mathematician David Hilbert as early as December 1916. [61]

This is not a "true" singularity, but rather a singularity resulting from an inappropriate choice of coordinates. Various coordinate transformations were proposed and implemented to eliminate this singularity.

The late-night phone call can be summarized as follows:

- You're starting to work on the Schwarzschild inner metric solution and are interested in the singularity on the Schwarzschild sphere. To eliminate it, set:

  let *$r = Rs$*, where 1 plus the logarithm of the hyperbolic cosine of ρ.

End of transmission.

In mathematical terms, this change of variables is written as:

$$r = R_s\,(\,1 + Log\,ch\,\rho\,) = R_s\left[1 + Log\left(\frac{e^{\rho} + e^{-\rho}}{2}\right)\right]$$

---

[60] K. Schwarzschild: On the Gravitational Field of a Mass Point According to Einstein's Theory. Website of the German Academy of Sciences, 1916
English translation by S. Antoci and A. Loinger. On the gravitational field of a mass point according to Einstein's theory. arXiv:physics/9912033v1 [physics.hist-ph] Dec. 16, 1999.
http://www.jp-petit.org/papers/cosmo/1916-Schwarzschild-exterior-en.pdf

[61] D. Hilbert. Die Grundlagen der Physik. (Second Communication) in Proceedings of the Royal Society of Göttingen. Math-Phys. Class 1917, pp. 53–76. Presented at the session of December 23, 1916. English translation: The Foundations of Physics (Second Communication)

This is not a simple concept that would immediately occur to a theorist. Any calculation involving this metric is then written as:

$$ds^2 = \frac{ln\ ch\rho}{1 + ln\ ch\rho} c^2 dt^2 - {R_s}^2 \left[\frac{1 + ln\ ch\rho}{ln\ ch\rho} th^2\rho\ d\rho^2 + (1 + ln\ ch\rho)^2 (d\theta^2 + sin^2\theta\ d\varphi^2) \right]$$

The advantage of this new variable is that it takes into account the topology of the four-dimensional geometric object associated with this solution. There are then two sheets, depending on the values of ρ.

- $\rho > 0$: our own universe leaf, that of positive masses
- $\rho < 0$: The second leaf, that of negative masses.
- $\rho = 0$: The throat sphere, with area $4\ \pi\ {R_s}^2$

With this formalism, the variable ρ can take on all real values of

$$-\infty\ to\ +\infty$$

It's… "magical." But the question that then arises is, "Where does this rather complicated change of variables come from?"

The question remains open today, along with its corollary:

- Who was on the other end of the line that night?

I'm publishing a new article in *Modern Physics Letters A*[62]

Details of this calculation can also be found in one of the 40 videos dedicated to my Janus[63] Cosmological Model

[62] J.P. Petit & G. D'Agostini: Cancellation of the singularity of the Schwarzschild solution with a natural mass inversion process. Mod. Phys. Lett. A, vol. 30, no. 9, 2015
arXiv: gr-qc/2103.12845/13 Feb 2001 (March 23, 2021) https://arxiv.org/abs/2103.12845
http://www.jp-petit.org/papers/cosmo/2015-ModPhysLettB.pdf

[63] Video: Janus 22-7 (with English (sub)titles) "Mathematics from Another World"
https://www.youtube.com/watch?v=FwwScatz6Z

Modern Physics Letters A
Vol. 30, No. 9 (2015) 1550051 (13 pages)
© World Scientific Publishing Company
DOI: 10.1142/S0217732315500510

World Scientific
www.worldscientific.com

Cancellation of the central singularity of the Schwarzschild solution with natural mass inversion process

Jean-Pierre Petit* and G. D'Agostini†
National Center for Scientific Research, CNRS, France
**jppetit1937@yahoo.fr*
†*dagostini.gilles@laposte.net*

Received 4 December 2014
Accepted 6 January 2015
Published 27 February 2015

We reconsider the classical Schwarzschild solution in the context of a Janus cosmological model. We show that the central singularity can be eliminated through a simple coordinate change and that the subsequent transit from one fold to the other is accompanied by mass inversion. In such scenario matter swallowed by black holes could be ejected as invisible negative mass and dispersed in space.

Keywords: Black hole; space bridge; Schwarzschild metric; Kerr metric; central singularity; Janus cosmological model; Gaussian coordinates; mass inversion process.

PACS Nos.: 98.80.Bp, 98.80.Qc

1. Introduction

When Schwarzschild published his solution[1] of Einstein equation, in 1916, the basic hypothesis was just time-independence and spherical symmetry. Nobody suspected that there was an additional one: null-homotopy. The consequence of this last one was that, in the proposed solution, r was a radial coordinate:

$$ds^2 = \left(1 - \frac{R_s}{r}\right)c^2\,dt^2 - \frac{dr^2}{\left(1 - \frac{R_s}{r}\right)} - r^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,. \tag{1}$$

A first problem at $r = R_s$, considered as coordinate singularity, was eliminated by various coordinates changes. See Fig. 1.

But $r = 0$ was considered as a *true singularity*, a point of space where the geodesics end. The goal of this paper is to show, through a very simple coordinate change, that this so-called "central singularity" is due to a wrong choice of local topology.

We must introduce the concept of *representation space* which is the space in which we build a mental image of an object. As an example we believe we live in

Coordinates	Line element	
Eddington–Finkelstein (ingoing)	$\left(1-\frac{r_s}{s}\right)dv^2 - 2\,dv\,dr - r^2\,d\Omega^2$	regular at horizon, extends across future horizon
Eddington–Finkelstein (outgoing)	$\left(1-\frac{r_s}{s}\right)du^2 + 2\,du\,dr - r^2\,d\Omega^2$	regular at horizon, extends across past horizon
Gullstrand–Painlevé	$\left(1-\frac{r_s}{r}\right)dT^2 - 2\sqrt{\frac{\tau_s}{r}}dT\,dr - dr^2 - r^2\,d\Omega^2$	regular at horizon
Isotropic	$\frac{\left(1-\frac{r_s}{4R}\right)}{\left(1+\frac{\tau_s}{4R}\right)^2}dt^2 - \left(1+\frac{r_s}{4R}\right)^4(dx^2+dy^2+dz^2)$	isotropic lightcones on constant time slices

Fig. 1. Schwarzschild's geometry. Alternative coordinates.[2–7]

R3 and that space and time are separated. This old belief is useful and enough for today's life. But if the speed of light was much smaller we should shift to Minkowski's spacetime. Similarly, when we send a probe to Jupiter or predict an eclipse we neglect space curvature. If this last was not neglectible, we should use Einstein's equation instead of Newton's. If we intend to describe an object through a line element it is easy to show, through 2D examples that inadequate coordinate choice may induce wrong image of a geometric object. For example, consider the following metric:

$$ds^2 = \frac{dr^2}{1-\frac{r^2}{R_s^2}} + r^2\,d\varphi\,, \tag{2}$$

whose signature $(+,+)$ changes into $(-,+)$ when $r > R_s$. But, through the simple following coordinate change

$$r = R_s \sin\theta\,, \tag{3}$$

this object becomes a sphere:

$$ds^2 = R_s^2(d\theta^2 + \sin^2\theta\,d\varphi^2)\,. \tag{4}$$

Another example. Consider the metric:

$$ds^2 = \frac{dr^2}{-r^2+2rR+r_0^2-R^2} + r^2\,d\varphi^2\,. \tag{5}$$

The signature is $(+,+)$ if

$$R - r_0 < r < R + r_0\,, \tag{6}$$

but through the new coordinate change

$$r = R + r_0\cos\theta\,, \tag{7}$$

we get the well-known metric of the torus:

$$ds^2 = r_0\, d\theta^2 + (R + r_0 \cos\theta) d\varphi^2\,. \tag{8}$$

Now, let us focus on the space part of Schwarzschild's line element, limited to $\{r, \varphi\}$ coordinates:

$$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2\, d\varphi^2\,. \tag{9}$$

For $r < R_s$, the signature $(+,+)$ is changed into $(-,+)$. Let us make the change of variable:

$$r = R_s(1 + \text{Log ch}\,\rho)\,, \tag{10}$$

which gives:

$$d\Sigma^2 = R_s^2 \left[\frac{(1 + \text{Log ch}\,\rho)}{\text{Log ch}\,\rho}\, \text{th}^2\,\rho\, d\rho^2 + (1 + \text{Log ch}\,\rho)^2\, d\varphi^2 \right]. \tag{11}$$

All singularities disappear, $r = R_s$ corresponds to $\rho = 0$. In this point the determinant of the metric

$$\det\, g = R_s^4 \frac{(1 + \text{Log ch}\,\rho)^2}{\text{Log ch}\,\rho}\, \text{th}^2\,\rho\,,$$

is no longer zero. The metric is well defined for all values of ρ. If we embed the surface in a 3D-Euclidean space we can define the meridians, corresponding to

$$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + dz^2\,. \tag{12}$$

And we immediately get the meridians as

$$z = \pm 2R_s \sqrt{\frac{r}{R_s} - 1}\,, \qquad r^2 = R_s + \frac{z^2}{4R_s}\,. \tag{13}^{\star}$$

The surface is a *space bridge*, a "2D diabolo" linking two 2D-Euclidean surfaces.

The problem of the signature has disappeared. From Lagrange equations we can calculate the geodesics in the $[\rho, \varphi]$ coordinate system. If embedded, the surface owns a throat circle whose perimeter is $2\pi R_s$. We can shape the surface as a twofold $F^{(+)}$ and $F^{(-)}$ cover of a M_2 manifold with a 1D common circular border, and create induced mapping between adjacent points $M^{(+)}$ and $M^{(-)}$.

Figure 4 shows the projection $\alpha\,\beta\,\gamma$ of a geodesic ABC (see Fig. 2) tangent to the common boundary, which ensures the continuity of the geodesics of the fold $F^{(+)}$ with the ones of the fold $F^{(-)}$ (dotted lines). Radial line $\lambda\,\mu\,\nu$ is the image of a geodesic meridian curve L M N of Fig. 2. This will be important for the following.

Figure 5 shows how vicinities of adjacent points are linked by enantiomorphy relationship.

* Erratum page 13

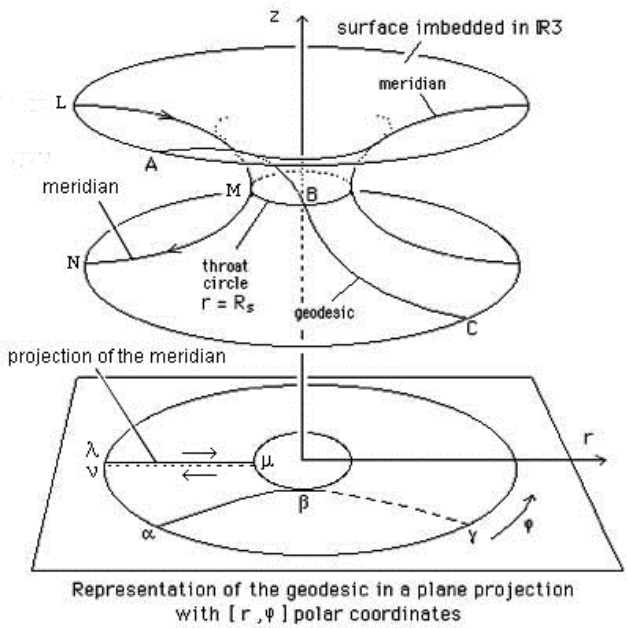


Fig. 2. The 2D diabolo embedded in R^3.

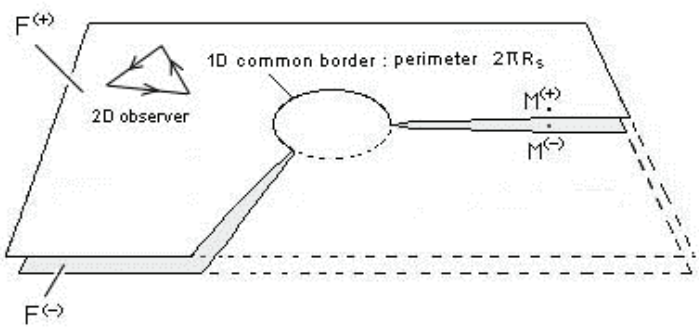


Fig. 3. Twofold cover of a manifold with a circle as common boundary.

Figures 3–5 correspond to a reduction of our representation space to a 2D Euclidean representation space, through a non-isometric imbedding which artificially transforms the throat into a pleat and couples points through an adjacent and enantiomorphic relationship. "The circle owns no center and there is nothing inside", because we are out of the considered 2D surface, the 2D diabolo.

Now introduce the 3D metric

$$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{14}$$

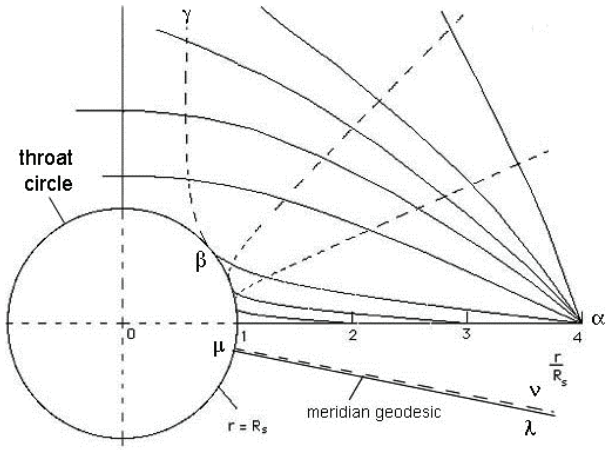


Fig. 4. Projection of geodesics.

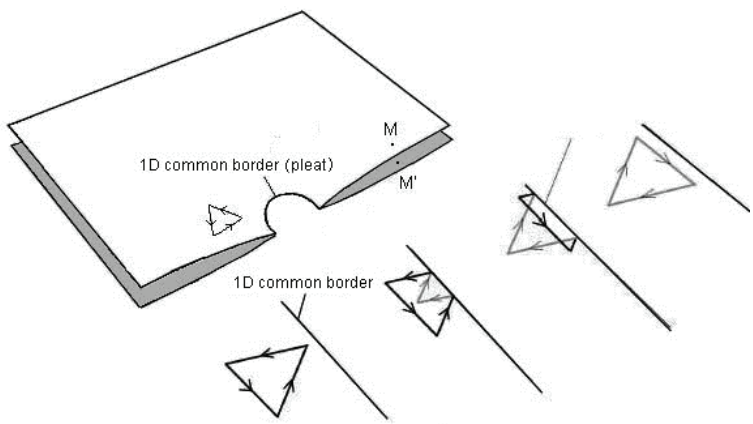


Fig. 5. When the triangle crosses the common boundary, its orientation is reversed.

which is Euclidean at infinity. When $r < R_s$ the signature $(+++)$ is changed into $(-++)$. Applying (10) we get

$$ds^2 = R_s^2 \left[\frac{(1 + \text{Log ch}\,\rho)}{\text{Log ch}\,\rho}\,\text{th}^2\,\rho\,d\rho^2 + (1 + \text{Log ch}\,\rho)^2(d\theta^2 + \sin^2\theta\,d\varphi^2) \right]. \quad (15)$$

Its determinant never vanishes. The metric is well defined for all values of ρ and is Euclidean at infinite. It is a *3D space bridge* linking to 3D Euclidean spaces. Its throat, corresponding to $\rho = 0$, is a S2 sphere.

Unfortunately we do not own a 4D Euclidean representation space to operate an isometric imbedding. So that, similarly as in Fig. 3, we will use a non-isometric imbedding in 3D Euclidean space. Then the throat is converted into a "pleat"

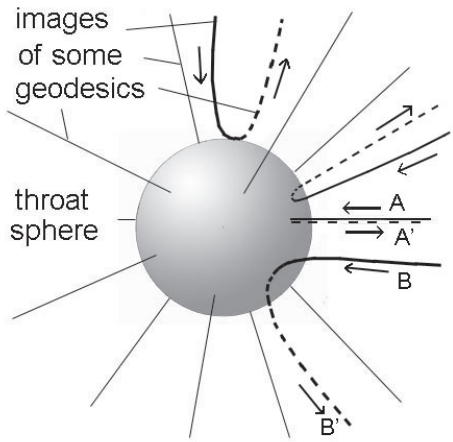


Fig. 6. When geodesics of the 3D hypersurface cross the throat sphere they appear tangent to it in this 3D non-isometric imbedding in a 3D Euclidean space.

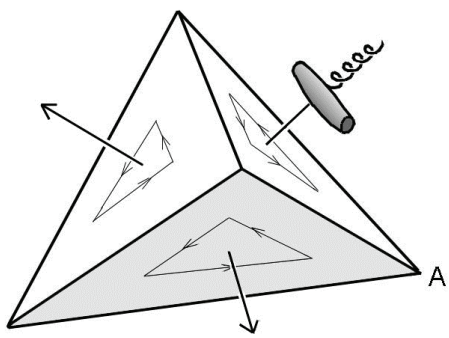


Fig. 7. Oriented tetrahedron.

along the projected S2 sphere. Similarly, some geodesics, continuous in this 3D hypersurface, seem to turn back on the sphere (see Fig. 6).

To show 3D space orientation we will use a tetrahedron (see Fig. 7). To illustrate the enantiomorphy relationship we need to project this object through the S2 throat sphere, each vertice following a geodesic, as showed in Fig. 8.

In the 2D $\{\rho, \varphi\}$ representation the adjacent points are defined by the relation:

$$M : (\rho, \theta) \to M' : (-\rho, \theta)\,.$$

In the 3D $\{\rho, \theta, \varphi\}$ representation the adjacent points in 2D and 3D are defined by the relation:

$$M : (\rho, \theta, \varphi) \to M' : (-\rho, \theta, \varphi)\,.$$

The association of points M and M' goes hand in hand with an enantiomorphic relation between their corresponding neighborhoods.

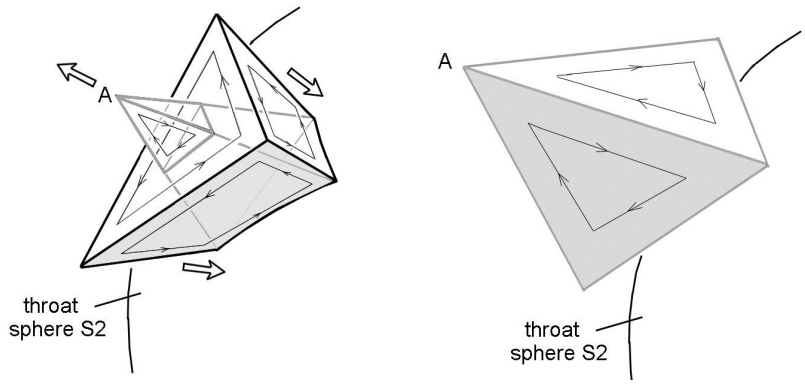


Fig. 8. By crossing the throat sphere, the tetrahedron is inverted.

Now let us go back to (2) and apply (10). We get

$$ds^2 = \frac{\text{Log ch}\,\rho}{1+\text{Log ch}\,\rho}c^2\,dt^2 - R_s^2\left[\frac{(1+\text{Log ch}\,\rho)}{\text{Log ch}\,\rho}\,\text{th}^2\,\rho\,d\rho^2\right.$$

$$\left. + (1+\text{Log ch}\,\rho)^2(d\theta^2+\sin^2\theta\,d\varphi^2)\right]. \tag{16}$$

When ρ tends to $\pm\infty$, $\text{Log ch}\,\rho \to \rho$ and $\text{th}\,\rho \to 1$. The metric tends to Lorentz metric. Space is extended to ($\rho > 0$; $\rho < 0$) domain. The hypersurface becomes a *spacetime bridge*, linking two Lorentz spaces through a throat surface S2. When we calculate the geodesics in the plane $\theta = \frac{\pi}{2}$ in the $\{t, r, \theta, \varphi\}$ representation we find the following (Eq. (6.90) in Ref. 8):

$$d\varphi = \pm\frac{1}{r^2}\frac{dr}{\sqrt{\frac{c^2l^2-1}{h^2}+\frac{R_s}{h^2r}-\frac{1}{r^2}+\frac{R_s}{f^3}}}\,, \tag{17}$$

where l and h are the classical parameters of the quasi-Keplerian trajectory (Eqs. (6.80) and (6.81) in Ref. 8). On the Schwarzschild's sphere ($r = R_s$) we get:

$$\text{tg}\,\alpha = R_s\left|\frac{d\varphi}{dr}\right|_{r=R_s} = \frac{h}{R_s cl}\,. \tag{18}$$

In a $\{t, \rho, \theta, \varphi\}$ representation, with $\theta = \frac{\pi}{2}$, we obtain curves $\rho = f(\varphi)$ that will be inscribed in two (adjacent) folds $F^{(+)}$ and $F^{(-)}$.

With

$$(\text{tg}\,\beta)_{\rho\to 0} = \frac{\rho\,d\varphi}{d\rho} = \frac{r\,d\varphi}{dr}\frac{\rho}{r}\frac{dr}{d\rho} = \frac{h}{R_s^2 cl}\rho^2 \to 0\,. \tag{19}$$

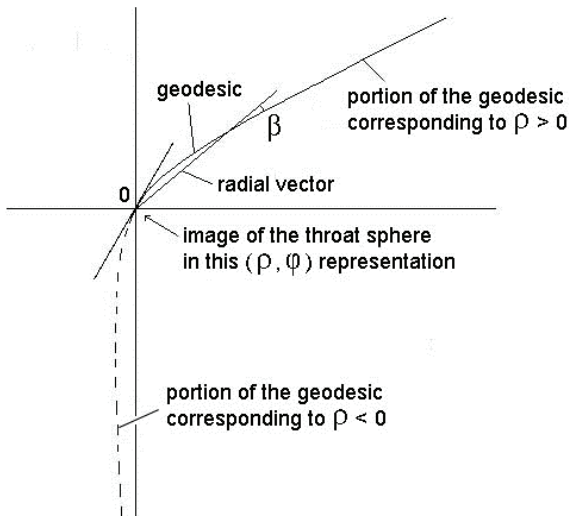


Fig. 9. Geodesical path in a $[\rho, \varphi]$ representation (plane, $\theta = \pi/2$).

The throat sphere $r = R_s$ is reduced to a point. But the geodesics of the fold $F^{(+)}$ can be prolonged continuously in the adjacent fold $F^{(-)}$. *The central singularity disappears.* Now let us deal with spacetime structures.

Referring to *Introduction to General Relativity*, Sec. 2.6 of Ref. 8:

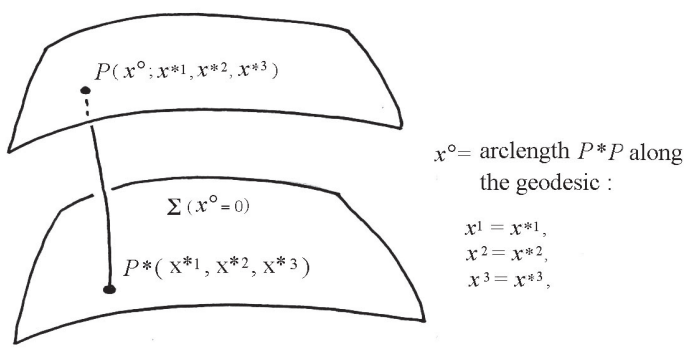


Fig. 10.

By letting a family of geodesics play a particular role among the coordinates lines Gauss introduced a useful coordinate system. Consider a 4D space with hyperbolic metric with signature $(1, -1, -1, -1)$. Assume we can imbed a 3D hypersurface Σ_3, imbedded in the 4D space Σ_4. Assume, in some place, we can define a vector n normal to Σ_3, which satisfies:

$$n^0 n_0 + (n^1 n_1 + n^1 n_1 + n^1 n_1) > 0$$

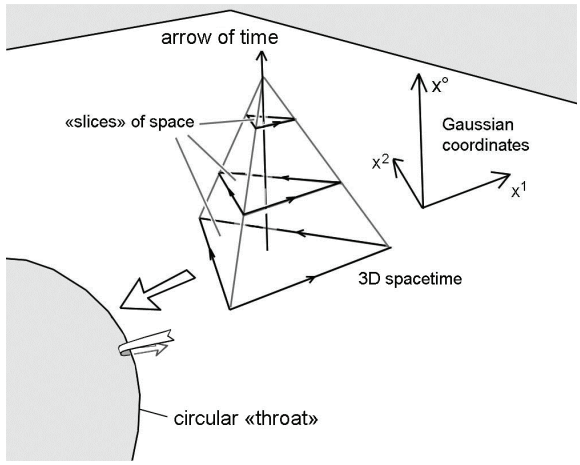


Fig. 11. Building a 3D spacetime.

which, in the familiar language of special relativity theory, implies that Σ_3 is "oriented in space" (whereas the vector n, normal to s, is "oriented in time").

We introduce in the surface Σ_3 three coordinates x^{*1}, x^{*2}, x^{*3} which serve to characterise the point $P* \in \Sigma_3$. Through each point $P*$ of the 3D surface Σ_3 we draw the geodesic which is orthogonal to Σ_3 at $P*$. These geodesic will form a non-intersecting curves in some neighborhood M of Σ_3 such that, through each point P of M there will be exactly one of the geodesics constructed. We introduce now, in the entire 4D domain M, coordinates as follows: Given P, we consider the geodesic passing through P and its original point $P* \in \Sigma_3$. We define the coordinate x^i of P in terms of the arc length $P * P$ of the geodesic and of the coordinate x^{*i} of $P*$.

In this manner, the three coordinates x^1, x^2, x^3 remain constant along any geodesic perpendicular to Σ_3. It follows that, along such a geodesic,

$$ds^2 = (d\dot{x})^2\,, \qquad g_{00} = 1\,.$$

This is the classical way Gaussian coordinates are defined. This is possible if and only if $g_{00} \neq 0$, if the term of the line element related to time-marker is nonzero. Note that, in Fig. 1, through various choices of coordinate systems, in Schwarzschild solution this term g_{00} (called g_{tt}) vanishes on the so-called horizon ($r = R_s$), or throat surface ($\rho = 0$). It means that on this peculiar portion of the hypersurface, normal vector and space orientation cannot be defined.

Let us build a 3D spacetime on a 2D surface, as the twofold cover of a manifold M_2, as shown in Fig. 11.

As shown in Fig. 12 we will find some problem at the common boundary, where it is impossible to define space orientation and arrow of time (identified to normal

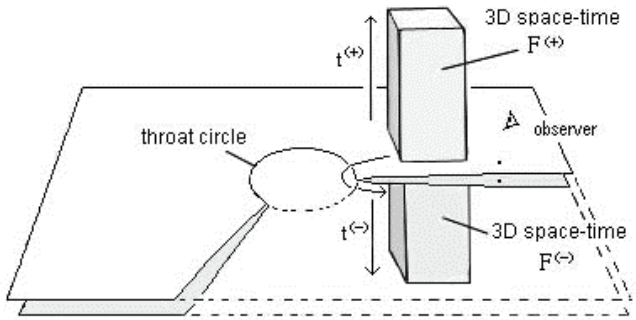


Fig. 12. In turning the throat circle, the arrow of time is inverted.

vector). But, for adjacent regions we have imbricated PT-symmetrical spacetime structures. Such coupling concept, called at that time "twin universe theory", was first presented by Sakharov in 1967[12–15] and later in Ref. 11. In addition, this goes with recent works[9,10] (Janus Cosmological Model).

We have to deal with 4D spacetime, not 3D spacetime, which is just a didactic image of such geometric structure.

Back to the expression of the Schwarzschild metric in the new coordinates (16) we see that, on the throat surface ($\rho = 0$), g_{tt} vanishes (as well as in expressions shown in Fig. 1). According to Sec. 2.6 of Ref. 8, Gaussian coordinates cannot be defined, which means that on the throat surface, time and space cannot be oriented. This fits time and space inversion from fold $F^{(+)}$ to fold $F^{(-)}$, so that we must write joint metrics:

$$ds^{(+)2} = \frac{\text{Log ch}\,\rho}{1 + \text{Log ch}\,\rho} c^2\, dt^{(+)2} - R_s^2 \left[\frac{(1 + \text{Log ch}\,\rho)}{\text{Log ch}\,\rho} \,\text{th}^2\,\rho\, d\rho^2 \right.$$

$$\left. + (1 + \text{Log ch}\,\rho)^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \right], \tag{20a}$$

$$ds^{(-)2} = \frac{\text{Log ch}\,\rho}{1 + \text{Log ch}\,\rho} c^2\, dt^{(-)2} - R_s^2 \left[\frac{(1 + \text{Log ch}\,\rho)}{\text{Log ch}\,\rho} \,\text{th}^2\,\rho\, d\rho^2 \right.$$

$$\left. + (1 + \text{Log ch}\,\rho)^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \right], \tag{20b}$$

with

$$t^{(-)} = -t^{(+)}\,. \tag{21}$$

In the neighborhood of $\rho = 0$ it is possible to write the nearby expressions:

$$(ds^{(+)})^2 = \frac{\rho^2}{2} (dt^{(+)})^2 - 2\, d\rho^2 - R_s^2\, d\varphi^2\,, \tag{22a}$$

$$(ds^{(-)})^2 = \frac{\rho^2}{2}(dt^{(-)})^2 - 2\,d\rho^2 - R_s^2\,d\varphi^2\,. \tag{22b}$$

The inversion of the time variable does not imply a change in the sign of the proper time, and from one fold to the other $ds^{(-)}$ takes the reins of $ds^{(+)}$. It is not possible to have an inversion in the length measure along a geodesic. In other words we have $(ds^{(+)})(ds^{(-)}) > 0$. But $t^{(+)}$ and $t^{(-)}$ are nothing more than "time markers", simple coordinates, and thus one will have: $dt^{(-)} = -dt^{(+)}$. So here one finds again the central idea of differential geometry: the length element has an only intrinsic reality. Lagrange equations give always in the vicinity of $\rho = 0$ the relations:

$$\ddot{\varphi} = \ddot{\rho} = 0\,. \tag{23}$$

These functions are linear and monotonic as a function of the proper times (lengths $s^{(+)}$ and $s^{(-)}$) that enchain themselves in passing from one fold to the other without inversion.

$$\frac{dt^{(+)}}{ds^{(+)}} = \frac{C}{\rho^2}\,, \qquad \frac{dt^{(-)}}{ds^{(-)}} = \frac{-C}{\rho^2}\,, \qquad C = Cst\,. \tag{24}$$

The sign of the constant C depends on the sense adopted for the passage from one fold to the other. The object is thus a "black hole" and a "white fountain" at the same time. But, if measured with variable $t^{(+)}$ the passage is achievable only in an infinite time. If the object results from the implosion of a neutron star, its mass would be transferred to the negative energy region. But for the observers located in one of the folds such phenomena of implosion–explosion will appear to be "freezed in time".

As shown in Ref. 16 time-inversion goes with mass and energy inversion. In Refs. 11 and 12, according to Janus Cosmological Model the universe is composed by positive and negative energy (and mass if they own) particles, respectively described by metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$, solutions of a coupled field equation system. Spacetime bridges operate mass inversion. From Ref. 11 we know that masses with opposite signs repel each other. On another hand negative matter does not interact with positive matter by electromagnetic, strong or weak interaction.

2. Conclusion and Discussion

If black holes exist and swallow matter this last, instead to be crushed in a central singularity, would be discretely rejected as an invisible negative matter.

The main feature of the theory presented here is the mass (and space) inversion process. The advantage is to avoid the puzzling problem of a "central singularity" and to explain the fate of matter swallowed by black holes. But it implies injection of negative energy (and mass if the own) particles in spacetime, considered as a manifold plus two metrics $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$.

As precised in Ref. 9 we assumed that particles of opposite masses do not interact neither by electromagnetic forces nor strong or weak forces, they could not enter into a collision. Some colleagues have criticized this idea arguing that "the particles

are on the same spacetime". The answer to this question is: if one considers the problem on purely geometrical grounds, those encounters would be "geometrically impossible" because the two subsets move along disjoint families of geodesics.

A second criticism may rely on the immediate instability of a quantum vacuum which could create pairs $(+m, -m)$. But it is based on the theoretical framework of quantum gravity that, however, still today remains purely hypothetical. The creation and annihilation of pairs of particles of opposite mass has not been described till today.

A third criticism may issue from Quantum Field Theory, which excludes straight away states of negative energy "*because a particle could not have an energy less than that of the vacuum*" (p. 76 of Ref. 17). We quote:

> *If we suppose that* T *is linear and unitary then we should face the disastrous conclusion that for any state* Ψ *of energy* E *there is another state* $\mathrm{T}^{-1}\Psi$ *of energy* $-E$*. To avoid this we are forced here to conclude that* T *is anti-linear and anti-unitary.*

In order to refute this statement we would say that it is bootstrap talk and that the conclusion is contained in the hypothesis, as occurs with the "CPT theorem".

In his book Weinberg,[17] let us quote his sentence on p. 104:

> *No examples are known of particles that furnish unconventional representation of inversions, so these possibilities will not be pursued here. From now on, the inversions will be assumed to have the conventional action assumed in Sec.* 2.6.

This sentence refers of course to the hypothesis expressed on p. 76 of Ref. 17 about the anti-unitary and anti-linear character of the T operator. However, cosmic acceleration implies the action of a negative pressure and hence of negative energy (pressure is an energy density by unit volume). The discovery of such quite unforeseen phenomenon[18–27] makes it compelling for Quantum Field Theory to be extended in order to include negative energy states.

References

1. K. Schwarzschild, *Sitzber. Preuss. Akad. Wiss. Berlin* 189 (1916).
2. D. Hilbert, *Math. Ann.* **92**, 1 (1924).
3. P. Painlevé, *C. R. Acad. Sci.* (*Paris*) **173**, 677 (1921).
4. A. Gullstrand, *Arkiv. Mat. Astron. Fys.* **16**, 1 (1922).
5. A. S. Eddington, *Nature* **113**, 192 (1924).
6. D. Finkelstein, *Phys. Rev.* **110**, 965 (1958).
7. G. Lemaître, *Ann. Soc. Sci. Bruxelles A* **53**, 51 (1933).
8. R. Adler, M. Bazin and M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, 1965).
9. J. P. Petit and G. D'Agostini, *Astrophys. Space Sci.* **354**, 611 (2014).
10. J. P. Petit and G. D'Agostini, *Mod. Phys. Lett. A* **29**, 34 (2014).
11. J. P. Petit, *C. R. Acad. Sci.* **284**, 1217 (1977).

12. A. Sakharov, *ZhETF Pis'ma* **5**, 32 (1967) (in Russian); *JETP Lett.* **5**, 24 (1967).
13. A. Sakharov, *ZhETF Pis'ma* **76**, 1172 (1979) (in Russian); *JETP* **49**, 594 (1979).
14. A. Sakharov, *ZhETF Tr. JETP* **52**, 349 (1980); **79**, 689 (1980) (in Russian).
15. A. Sakharov, *Collected Scientific Works* (Library of Congress, 1982).
16. J. M. Souriau, *Structure Des Systèmes Dynamiques* (Dunod France, 1970) (in French); *Structure of Dynamical Systems*, Chap. III (Birkhaüser, 1997).
17. S. Weinberg, *The Quantum Theory of Fields* (Cambridge Univ. Press, 2005), Vol. 1.
18. A. G. Riess *et al.*, *Astron. J.* **116**, 1009 (1998).
19. S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999).
20. A. G. Riess, *Publ. Astron. Soc. Pac.* **112**, 1284 (2000).
21. A. V. Filippenko and A. G. Riess, *AIP Conf. Proc.* **540**, 227 (2000).
22. B. Leibundgut, *Annu. Rev. Astron. Astrophys.* **39**, 67 (2001).
23. R. Knop *et al.*, *Astrophys. J.* **598**, 102 (2003).
24. J. T. Tonry *et al.*, *Astrophys. J.* **594**, 1 (2003).
25. B. Barris *et al.*, *Astrophys. J.* **602**, 571 (2004).
26. A. G. Riess, *Astrophys. J.* **607**, 665 (2004).
27. B. Schmidt, *Rev. Mod Phys.* **84**, 987 (2012).

Erratum

Equation (13) page 3 has a typo (extra square) and should be written:

$$r = R_s + \frac{z^2}{4R_s}$$

We will now attempt to summarize the main features of the Janus model through a series of relatively recent articles.

In 2024, a significant milestone was reached with the publication of a 24-page[64] article in a highly regarded mainstream journal, European Physical Review C (EPJC), considered one of the most reputable in physics today. It has a "rating" comparable to that of the journal considered the benchmark in Physics, Particles, and Fields:

> *Physical Review D*, which has an impact factor (2025) of 5.3
>
> The impact factor of *European Physical Review D* is 4.9. This is very comparable.

This article provides an almost complete overview of the ins and outs of the Janus Model.

- Foundations in Dynamic Group Theory
- Topology
- Construction of the system of coupled field equations from an action
- Exact solution in the non-stationary regime and assumptions of homogeneity and isotropy
- Mathematical constraints, Bianchi conditions. Newtonian solutions
- Observational Confirmations

[64] JP Petit, F. Margnat, H. Zejli: A Bimetric Cosmological Model Based on Andrei Sakharov's Twin Universe Approach. Eur Phys Jr (2024) 84:1226
https://doi.prg/10.1140/epjc 10052-024-13560-v http://www.jp-petit.org/papers/cosmo/2024-11-EPJC.pdf

Eur. Phys. J. C (2024) 84:1226
https://doi.org/10.1140/epjc/s10052-024-13569-w

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article

Theoretical Physics

A bimetric cosmological model based on Andreï Sakharov's twin universe approach

Jean-Pierre Petit[1,a], **Florent Margnat**[2,b], **Hicham Zejli**[1,c]

[1] Manaty Research Group, Glanon, France
[2] University of Poitiers, Poitiers, France

Received: 28 June 2024 / Accepted: 1 November 2024
© The Author(s) 2024

Abstract The standard cosmological model, based on Cold Dark Matter and Dark Energy (ΛCDM), faces several challenges. Among these is the need to adjust the scenario to account for the presence of vast voids in the large-scale structure of the universe, as well as the early formation of the first stars and galaxies. Additionally, the observed matter–antimatter asymmetry in the universe remains an unresolved issue. To address this latter question, Andrei Sakharov proposed a twin universe model in 1967. Building upon this idea and introducing interactions between these two universe sheets through a bimetric model, we propose an alternative interpretation of the large-scale structure of the universe, including its voids and the acceleration of cosmic expansion.

Contents

1 Introduction . . . . . .
2 The physical interpretation of time inversion (T-symmetry) . . . . . .
3 Geometrical interpretation of electric charge . . . . .
4 Matter–antimatter symmetry (C-symmetry) . . . . . .
5 Group associated with A. Sakharov's model: the *Janus group* . . . . . .
6 Topology of the Janus model . . . . . .
7 Introducing negative masses: first approach . . . . . .
8 A paradigm shift to escape the crisis of today's cosmology
9 Foundation of the Janus cosmological model . . . . .
10 Construction of a time-dependent, homogeneous and isotropic solution . . . . . .
11 Interaction laws and observational consequences . . .
12 The mathematical and physical consistency of the model
12.1 Newtonian approximation of the field generated by a positive mass M . . . . . .
12.2 Newtonian approximation of the field generated by a negative mass $\bar{M}$ . . . . . .
13 Dipole repeller prediction . . . . . .
14 Beyond the Newtonian approximation . . . . . .
15 Conclusion . . . . . .
References . . . . . .

1 Introduction

Between 1967 and 1980, the physicist Andreï Sakharov published several papers [18–20] in which he presented a cosmological model with two universes, connected by an initial singularity: the Big Bang. The first universe corresponds to ours, while the second is described by Sakharov as a twin universe. The *"arrows of time"* of these two universes are antiparallel, and they are *"enantiomorphic"*, that is, mirrored. Through this model, Sakharov proposed a possible explanation for the apparent absence of primordial antimatter in our universe.

For more than half a century, cosmology has been unable to solve one of its greatest enigmas: not only has no convincing explanation been found as to why one particle of matter in a million escaped total annihilation with antimatter, but no significant observation of a corresponding amount of primordial antimatter has been made.

Sakharov was interested in the violation of CP-symmetry, a fundamental property of the laws of physics, and hypothesized that a twin universe, where these violations would be reversed, could exist. This model would thus restore a generalized symmetry on a large scale. Based on the fact that matter is formed from the assembly of quarks and antimatter from antiquarks, he supposed that in our universe, the

[a] e-mail: jean-pierre.petit@manaty.net
[b] e-mail: florent.margnat@univ-poitiers.fr (corresponding author)
[c] e-mail: hicham.zejli@manaty.net

Published online: 26 November 2024

reaction leading to the formation of matter would have been slightly faster than the one leading to the formation of antimatter, while the inverse situation would occur in the twin universe.

Thus, in our matter-dominated universe, there would remain a small surplus of matter, accompanied by an equivalent amount of free antiquarks. Symmetrically, in the twin universe, one would find antimatter with a corresponding surplus of free quarks. Although this model may seem exotic, it nevertheless offers the only theoretical explanation proposed so far to account for the disappearance of half of the predicted cosmic content. Consequently, it seems legitimate to examine in detail the aspects and implications of such a model.

This article revisits the pioneering work of Andreï Sakharov and proposes a new cosmological model, inspired by his approach, in which two folds of the universe are connected by the same initial singularity, folded over one another and interacting through gravitational effects. It puts this work in perspective with modern concepts to address some of the challenges posed by the standard cosmological model, particularly those of the ΛCDM model. This model offers potential explanations for phenomena such as the acceleration of cosmic expansion or the existence of large-scale structures like cosmic voids.

Our paper is structured around several key sections. The first explores T-symmetry, which corresponds to time reversal, based on the mathematical framework of the Poincaré group. This symmetry is related to the existence of particles with negative mass and energy, at the core of the bimetrical Janus model, inspired by the work of J.-M. Souriau. It plays a central role in the dynamics of this double spacetime, where time reversal opens the door to a new interpretation of physical phenomena [22,24].

Next, C-symmetry, associated with charge conjugation, is extended within the framework of an additional dimension through the Kaluza–Klein model. This extension allows the interpretation of electric charge as a geometric component, in accordance with Noether's theorem. This connection between the extra dimension and charge conservation offers a new perspective on charged particles in a five-dimensional spacetime, where charge naturally emerges from geometry [24].

The model is enriched by the introduction of the *Janus restricted group*, which extends the Kaluza space to several compactified dimensions. This dynamic group links the matter–antimatter symmetry (C-symmetry) to the inversion of quantum charges in a multidimensional framework. Through this extension, the group's geometry allows for the understanding of the quantization of several charges, including electric charge, and opens the way to the emergence of new quantum charges. This section establishes a connection between Souriau's work and the Kaluza–Klein formalism to explain complex physical phenomena in a higher-dimensional spacetime [9,21,23,24].

The *Janus dynamic group*, which combines PT-symmetry (simultaneous inversion of energy, time, and spatial coordinates) and C-symmetry (charge conjugation), allows the modeling of interactions between matter, antimatter, and negative mass particles. Thanks to Noether's theorem, this group associates scalar invariants with the observed symmetries, thus clarifying the interactions between these different entities within a bimetrical framework, and allowing the extension of Sakharov's model by adding compactified dimensions for each quantum charge [16,18,24].

To illustrate this concept, in the context of our study on bimetric models, we proposed a model of wormhole linking two PT-symmetric folds of the universe via a modified Einstein–Rosen bridge [10]. This model includes a cross term $dr\,dt$ in the corresponding metric, leading to a finite free-fall time to the wormhole's throat for an external observer. The two folds are CPT-symmetric for photons, which are neutral particles. This wormhole model allows for unidirectional traversal through its throat, inducing a space-time inversion. This opens the possibility of interactions between matter and antimatter, arising from the PT symmetry observed during the transition between the two universe folds. Thus, the congruent identification of points on the two universe folds and the reversal of the arrow of time induce an inversion of energy, offering new insights into the structure of space-time and the potential inversion of particle mass while crossing this bridge.

The Janus model will also be studied from a topological perspective, with a closed universe geometry where P and T symmetries naturally emerge. Spacetime is modeled by a compact universe with the topology of a 4-dimensional sphere $\mathbb{S}^4$, which forms a two-fold cover of the projective space $\mathbb{P}^4$. In this structure, the antipodal points, representing the Big Bang and the Big Crunch, coincide. By replacing these singularities with a tubular structure, they disappear, allowing P and T symmetries to emerge as natural consequences of this closed projective geometry ($\mathbb{P}^4$) and be interpreted in a purely topological framework [3,13,15].

One of the earliest attempts to introduce negative masses into a cosmological model, explored by H. Bondi in 1957, showed that the coexistence of positive and negative masses, which respectively induce attraction and repulsion, leads to the *"runaway effect"* [2]. In this effect, a positive mass and a negative mass attract gravitationally while moving away from each other, thus violating the action-reaction principle. This effect has remained a major challenge for integrating negative masses into standard cosmology.

Thus, to resolve the crisis of modern cosmology, the Janus model proposes a paradigm shift. Since the 1970s,

the ΛCDM model has failed to explain certain observed phenomena, such as the rotation speeds of galaxies and the acceleration of cosmic expansion. The Janus model, based on a bimetrical geometry with positive and negative masses evolving on distinct geodesics, offers an alternative. It proposes a new approach to solving anomalies such as the rapid formation of galaxies after the Big Bang and discrepancies in the measurement of the Hubble constant [6,8].

The Janus model proposes a bimetrical system where gravitational interactions between positive and negative masses are described by distinct field equations, each associated with its own metric. The construction of a homogeneous, isotropic, and time-dependent solution in the Janus model relies on FLRW-type metrics, respectively describing the universes of positive and negative masses. A common energy conservation relation is established, proposing an exact solution for dust universes, where the observed cosmic acceleration is interpreted as a negative total energy. Numerical comparisons confirm the model's compatibility with observations, as illustrated by the magnitude-redshift curve. The interaction laws in the Janus model reveal that masses of the same sign attract, while those of opposite signs repel, thus eliminating the *"runaway effect"*. The model reproduces local observations of general relativity while replacing dark matter and dark energy with invisible negative masses. These negative masses form void-like structures that confine positive mass, accelerating star and galaxy formation in the first few hundred million years, in agreement with data from the James Webb telescope and observations of large cosmic voids [4,12,14,16,17].

Finally, the mathematical consistency of the Janus model is demonstrated in the weak field limit, thanks to the generalized conservation of energy and the Bianchi identities. The calculation of Schwarzschild metrics for positive and negative masses shows that masses of the same sign attract, while those of opposite signs repel. The model satisfies the Tolman–Oppenheimer–Volkoff equations in the Newtonian approximation, while remaining compatible with cosmological observations. It is also valid in regions dominated by negative masses, such as the *dipole repeller*, where it predicts a negative gravitational lensing effect, dimming the luminosity of background objects [1,8,10,11].

In summary, the Janus cosmological model proposes an extension of general relativity by introducing two distinct metrics, each associated with a type of mass, allowing for the explanation of both the acceleration of the universe's expansion and certain large-scale structures, while remaining compatible with local observations of general relativity. This analysis opens new perspectives and places the Janus model among the approaches that can be tested by modern cosmological observations.

2 The physical interpretation of time inversion (T-symmetry)

The T-symmetry refers to the inversion of the time coordinate. In 1970, contributing to the development of symplectic geometry and its application to physics, mathematician J.-M. Souriau provided the physical interpretation of this inversion of the time coordinate [24]. The Gram matrix defining the *Minkowski space* is:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

Its isometry group is the *Poincaré group*:

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}, \tag{2}$$

where L is the matrix representing the *Lorentz group* $\mathcal{L}or$ which describes how spacetime coordinates change between different inertial frames. These transformations include rotations in space as well as Lorentz transformations (boosts), which are changes of reference frames moving at a constant speed relative to each other. It is axiomatically defined by:

$$L^{\mathsf{T}} G L = G, \tag{3}$$

and C is the quadrivector of space-time translations in $\mathbb{R}^{1,3}$ as follows:

$$C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}. \tag{4}$$

It acts on points in *Minkowski space*:

$$\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}. \tag{5}$$

This Lie group with 10 independent parameters[1] is the isometry group of this space, defined by its metric:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2. \tag{6}$$

The *Lorentz group* $\mathcal{L}or$ has four connected components:

- $\mathcal{L}or_n$ is the neutral component (its *restricted subgroup*), does not invert either space or time and is defined by:

$$\mathcal{L}or_n = \{L \in \mathcal{L}or,\ \det(L) = 1 \ \cap\ [L]_{00} \geq 1\}$$

[1] Including the 6 independent parameters of the Lorentz group (3 rotations and 3 boosts) and 4 independent transformations, which are translations in the 4 directions of *Minkowski space*.

- $\mathcal{L}or_s$ inverts space and is defined by:

$$\mathcal{L}or_s = \{L \in \mathcal{L}or,\ \det(L) = -1\ \cap\ [L]_{00} \geq 1\}$$

- $\mathcal{L}or_t$ inverts time but not space and is defined by:

$$\mathcal{L}or_t = \{L \in \mathcal{L}or,\ \det(L) = 1\ \cap\ [L]_{00} \leq -1\}$$

- $\mathcal{L}or_{st}$ inverts both space and time and is defined by:

$$\mathcal{L}or_{st} = \{L \in \mathcal{L}or,\ \det(L) = -1\ \cap\ [L]_{00} \leq -1\}$$

And we have:

$$\mathcal{L}or = \mathcal{L}or_n \cup\ \mathcal{L}or_s \cup\ \mathcal{L}or_t \cup\ \mathcal{L}or_{st}. \tag{7}$$

The first two components are grouped to form the so-called *"orthochronous"* subgroup:

$$\mathcal{L}or_o = \mathcal{L}or_n \cup\ \mathcal{L}or_s. \tag{8}$$

It includes P-symmetry, which poses no problem for physicists who know that there are photons of *"right"* and *"left"* helicity whose motions are derived from this symmetry. This corresponds to the phenomenon of the polarization of light.

The last two components form the subset *"retrochronous"* or *"antichronous"*, whose components invert time:

$$\mathcal{L}or_a = \mathcal{L}or_t \cup\ \mathcal{L}or_{st}. \tag{9}$$

Thus, we have:

$$\mathcal{L}or = \mathcal{L}or_o \cup\ \mathcal{L}or_a. \tag{10}$$

Note that:

$$\mathcal{L}or_t = -\mathcal{L}or_s \quad \mathcal{L}or_{st} = -\mathcal{L}or_n. \tag{11}$$

The *Poincaré group* inherits the properties of the *Lorentz group* and thus has four connected components, it is defined by:

$$g := \left\{ \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix},\quad L \in \mathcal{L}or \cap C \in \mathbb{R}^{1,3} \right\}, \tag{12}$$

acting on *Minkowski space* as follows:

$$g(X) = L.X + C. \tag{13}$$

The action of the group on its space of moments is the action on the dual of the Lie algebra of the group[2]. The element of the Lie algebra is obtained by differentiating the ten components of the group. Souriau designates by the Greek letter Λ the differential of the square matrix Z representing the element of the *Poincaré group*, and by the Greek letter Γ the element of the subgroup of spatio-temporal translations[3]:

$$Z := \left\{ \begin{pmatrix} \Lambda & \Gamma \\ 0 & 0 \end{pmatrix}, \bar{\Lambda} = -\Lambda \cap \Gamma \in \mathbb{R}^{1,3} \right\}. \tag{14}$$

The elements of the *Lorentz group* act on points in space-time, transforming one motion into another. By applying an element L of the *Lorentz group* to a given motion, we obtain a new motion. The neutral component $\mathcal{L}or_n$ is a subgroup containing the identity matrix that inverts neither space nor time.

Let's consider the 4-component matrix ω made up of two parameters λ_1 and λ_2:

$$\omega_{(\lambda_1,\lambda_2)} = \begin{pmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & 0 & 0 & \lambda_2 \end{pmatrix} \quad \text{with} \quad \begin{cases} \lambda_1 = \pm 1 \\ \lambda_2 = \pm 1 \end{cases} \tag{15}$$

Thus, the four components of the *Lorentz group* can be easily expressed using the four possible combinations of these two parameters applied to its neutral component, of which an element $L_n \in \mathcal{L}or_n$ is expressed according to the expression $L = \omega L_n$:

$$\omega_{(1,1)} \times L_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathcal{L}or_n$$

$$\omega_{(1,-1)} \times L_n = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \in \mathcal{L}or_s$$

$$\omega_{(-1,1)} \times L_n = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathcal{L}or_t$$

$$\omega_{(-1,-1)} \times L_n = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \in \mathcal{L}or_{st} \tag{16}$$

We note that $\lambda_1 = -1$ inverts time while $\lambda_2 = -1$ inverts space. The four components are grouped into two

[2] Souriau's approach, thanks to the Poincaré group which is the isometry group of *Minkowski space* encompassing the *Lorentz group* (with its four connected components), allows the parameters associated with each of these motions, whose representative points belong to a vector space, *the space of moments*, to emerge. The dimension of this space is equal to that of the group: ten. Indeed, the *Lorentz group* is made up of transformations that preserve the quadratic form of space-time. It consists of the orthochronous Lorentz transformations and the translation group. The transformations of the orthochronous *Lorentz group* have 6 degrees of freedom, while the translation group has 4 degrees of freedom. This structure leads to 10 independent parameters of the Poincaré group. By combining them into an antisymmetrical matrix called a *torsor*, the parameters of the space of motions can thus be defined.

[3] (13.54) of [24]. He then writes μ, an element of the space of motions, in the form (13.57) and expresses the invariance in the form of the constancy of the scalar (13.58), where M is an antisymmetric matrix.

subsets *"orthochronous"* and *"retrochronous"* according to the respective expressions (8) and (9).

The *Poincaré group* can then be written according to these four connected components as follows:

$$g := \left\{ \begin{pmatrix} \omega L_n & C \\ 0 & 1 \end{pmatrix}, \omega L_n \in \mathcal{L}or \cap C \in \mathbb{R}^{1,3} \right\}. \tag{17}$$

Thus, the action of this *Poincaré group* on the spacetime coordinates yields the following space of motions:

$$\begin{bmatrix} \omega L_n & C \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} \xi \\ 1 \end{bmatrix} = \begin{bmatrix} \omega L_n \xi + C \\ 1 \end{bmatrix}. \tag{18}$$

Indeed, this describes the action of the *Poincaré group* on its space of moments μ, consisting of ten independent scalar quantities:

- The energy E,
- The momentum $p = \{p_x, p_y, p_z\}$,
- The passage $f = \{f_x, f_y, f_z\}$,
- The spin $s = \{l_x, l_y, l_z\}$.

If we consider the motion of an object in space, such motion is also defined by its moment μ. The physicist can then apply an element $\mathcal{G}$, for example from the Galilean group, to this moment μ. This produces a new moment μ'. This action can be written as follows:

$$\mu' = \mathcal{G}\mu\mathcal{G}^{\mathsf{T}}, \tag{19}$$

where $\mathcal{G}^{\mathsf{T}}$ represents the transpose of this matrix $\mathcal{G}$. μ is an antisymmetric moment matrix of size 5×5[4] where the more compact form is defined as follows:

$$\mu = \begin{pmatrix} M & -P \\ P^T & 0 \end{pmatrix}, \tag{20}$$

with[5]:

$$M = \begin{pmatrix} 0 & -l_z & l_y & f_x \\ l_z & 0 & -l_x & f_y \\ -l_y & l_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix}, \quad P = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}. \tag{21}$$

Then, by applying the action of the *Poincaré group* (12) on the dual of its Lie algebra, i.e., on its space of moments, we obtain the following action according to (19):

$$\mu' = \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \begin{pmatrix} M & -P \\ P^{\mathsf{T}} & 0 \end{pmatrix} \begin{pmatrix} L^{\mathsf{T}} & 0 \\ C^{\mathsf{T}} & 1 \end{pmatrix}, \tag{22}$$

$$\mu' = \begin{pmatrix} LML^{\mathsf{T}} - LPC^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}} & -LP \\ P^{\mathsf{T}}L^{\mathsf{T}} & 0 \end{pmatrix}. \tag{23}$$

We can deduce[6]:

$$M' = LML^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}} - LP^{\mathsf{T}}C, \tag{24}$$

and

$$P' = LP. \tag{25}$$

Therefore, the torsor of *Poincaré group* is given by the different components of the space of moments[7] as follows:

$$\mu = \{M, P\} = \{l, g, p, E\}, \tag{26}$$

where l is the angular momentum of M, g is the relativistic barycenter of M, p is the linear momentum of P and E is the energy of P.

Now, let's consider for example the symmetry T, where there is only a time inversion ($\lambda_1 = -1$), without space inversion ($\lambda_2 = 1$), in a case where there is also no translation in spacetime ($C = 0$). We thus have:

$$\omega_{(-1,1)} \times L_n = L_t. \tag{27}$$

Hence:

$$L_t \times \xi = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \boldsymbol{-t} \\ x \\ y \\ z \end{pmatrix}. \tag{28}$$

Thus, we obtain the action of time inversion in the space of trajectories or in spacetime.

The second Eq. (25) sheds light on the physical significance of this inversion of the time coordinate. Indeed, the application of the L_t component of the *Lorentz group* to the motion of a particle gives:

$$P' = L_t P = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \boldsymbol{-E} \\ p_x \\ p_y \\ p_z \end{pmatrix}. \tag{29}$$

Therefore, we can deduce that the application of the L_t component of the *Lorentz group* to the motion of a particle induces an inversion of its energy from E to $-\boldsymbol{E}$.

The T symmetry applied to the motion of a particle inverts its energy which leads to mass inversion[8] following the definition of the mass[9] as:

$$m = \sqrt{P^{\mathsf{T}} \cdot P}\, \mathrm{sgn}(E). \tag{30}$$

A very detailed commentary on the work can be found in reference [22]. The approach is based on the introduction of the space of motions as a dual of the Lie algebra of the group.

[4] Meaning the symmetric elements with respect to the main diagonal have opposite signs. The elements on the main diagonal are equal to zero, as each is its own opposite.

[5] M is the moment matrix associated with μ with dimensions 4×4, and P, a four-vector energy-momentum with dimensions 4×1.

[6] (13.107) of [24].

[7] (13.57) of [24].

[8] page 198–199 of [24].

[9] (14.57) on page of [24].

In this context, we uncover the physical interpretation of the model proposed by A. Sakharov: the second universe in his framework could consist of particles possessing both negative energy and negative mass.

To further extend the interpretation of fundamental symmetries, we now turn our attention to C-symmetry, which is associated with charge conjugation. By introducing a higher-dimensional framework inspired by Kaluza–Klein theory, we can offer a geometrical interpretation of electric charge, according to Noether's theorem. This will allow us to explore the relationship between spacetime transformations and the emergence of electrically charged particles.

3 Geometrical interpretation of electric charge

The geometrical interpretation of C-symmetry, which is synonymous with charge conjugation and matter–antimatter duality, was provided by J.-M. Souriau in 1964 in chapter V of reference [24].

Let's apply an extension of the *Poincaré group* to form the following dynamic group:

$$g := \left\{ \begin{pmatrix} 1 & 0 & \phi \\ 0 & L & C \\ 0 & 0 & 1 \end{pmatrix}, \phi \in \mathbb{R} \cap L = \lambda L_o \in \mathcal{L}or \cap \lambda = \pm 1 \cap C \in \mathbb{R}^{1,3} \right\}. \tag{31}$$

Starting from *Minkowski space*:

$$\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ r \end{pmatrix}, \tag{32}$$

let's introduce Kaluza space[10] that incorporates a 5×5 Gram matrix:

$$\Gamma = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} G & 0 \\ 0 & -1 \end{pmatrix} \quad \text{where}$$

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{33}$$

In the considered group, we just add a translation ϕ to the fifth dimension ζ. Thus, the dimension of the group becomes 11. It is the isometry group of Kaluza space, defined by its metric:

$$ds^2 = dX^{\mathsf{T}} \Gamma dX = dt^2 - dx^2 - dy^2 - dz^2 - d\zeta^2, \tag{34}$$

with:

$$X = \begin{pmatrix} \xi \\ \zeta \end{pmatrix} = \begin{pmatrix} t \\ x \\ y \\ z \\ \zeta \end{pmatrix}. \tag{35}$$

According to Noether's theorem,[11] this new symmetry is accompanied by the invariance of a scalar that we will call q. The torsor of this *Kaluza group* then incorporates an additional parameter:

$$\mu = \{M, P, q\} = \{l, g, p, E, q\}. \tag{36}$$

Let's introduce the action of the group on its Lie algebra:

$$Z' = g^{-1} Z g. \tag{37}$$

If we consider an element of the Lie algebra of this group:

$$Z = \begin{pmatrix} 0 & 0 & \delta\phi \\ 0 & G\omega & \gamma \\ 0 & 0 & 0 \end{pmatrix} \quad Z' = \begin{pmatrix} 0 & 0 & \delta\phi' \\ 0 & G\omega' & \gamma' \\ 0 & 0 & 0 \end{pmatrix}, \tag{38}$$

we obtain:

$$Z' = \begin{pmatrix} 0 & 0 & \delta\phi' \\ 0 & G\omega' & \gamma' \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \delta\phi \\ 0 & L^{-1}G\omega L & L^{-1}G\omega C + L^{-1}\gamma \\ 0 & 0 & 0 \end{pmatrix}. \tag{39}$$

This allows us to deduce the action of the following group:

$$q' = q, \tag{40}$$

$$M' = LML^{\mathsf{T}} - LPC^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}}, \tag{41}$$

$$P' = LP. \tag{42}$$

If we identify q as the electric charge, this would show that the motion of a massive particle in a five-dimensional space would transform it into an electrically charged particle.

The interpretation of C-symmetry within a higher-dimensional framework, as explored, leads naturally to a broader geometric understanding of symmetries in the Janus model. Specifically, the notion of charge conjugation extends to encompass the duality between matter and antimatter. To develop this further, we now introduce the *Janus restricted group*, which provides a formal structure to describe these

[10] Kaluza space is a hyperbolic Riemannian manifold with signature $(+----)$.

[11] Noether's theorem states that for every continuous symmetry of a physical action, there exists a conserved quantity. In our context, if a new symmetry ensures the invariance of a scalar q, this scalar is the conserved quantity. This means that q remains constant when the symmetry is applied to the system's action.

symmetries. This group allows us to explore how quantum charges can be inverted by compactified dimensions, linking the symmetry properties of spacetime to the emergence of quantized charges and new quantum numbers.

4 Matter–antimatter symmetry (C-symmetry)

Let's introduce the *Janus restricted group* as follows:

$$g := \left\{ \begin{pmatrix} \mu & 0 & \phi \\ 0 & L & C \\ 0 & 0 & 1 \end{pmatrix}, \mu = \pm 1 \cap \phi \in \mathbb{R} \cap L = \lambda \right.$$
$$\left. L_o \in \mathcal{L}or \cap \lambda = \pm 1 \cap C \in \mathbb{R}^{1,3} \right\}. \tag{43}$$

The action of the group on the coordinates of the 5-dimensional spacetime defined by (35) yields the space of the following motions:

$$\begin{pmatrix} \mu & 0 & \phi \\ 0 & L & C \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \mu\zeta + \phi \\ L\xi + C \\ 1 \end{pmatrix}. \tag{44}$$

A similar calculation to the previous one yields the action of the group:

$$q' = \mu q, \tag{45}$$
$$M' = LML^{\mathsf{T}} - LPC^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}}, \tag{46}$$
$$P' = LP. \tag{47}$$

This group acts on the five-dimensional Kaluza space. We observe that $\mu = -1$ reverses the fifth dimension ζ and the scalar q.

Through a dynamic interpretation of the group, we find the idea suggested by J.-M. Souriau [24]: the inversion of the fifth dimension is associated with the inversion of electric charge. However, this is only one of the quantum charges. Indeed, the C-Symmetry translating the *"matter–antimatter"* symmetry introduced by Dirac [5], reverses all quantum charges. This inversion operation is only obtained by adding as many compactified dimensions as there are quantum charges. The action of the group on the coordinates of n-dimensional spacetime yields the space of the following motions:

$$\begin{pmatrix} \mu & 0 & 0 & \cdots & 0 & \phi^1 \\ 0 & \mu & 0 & \cdots & 0 & \phi^2 \\ 0 & 0 & \ddots & \cdots & 0 & \vdots \\ \vdots & \vdots & \cdots & \mu & 0 & \phi^p \\ 0 & 0 & \cdots & 0 & L & C \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \zeta^1 \\ \zeta^2 \\ \vdots \\ \zeta^p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \mu\zeta^1 + \phi^1 \\ \mu\zeta^2 + \phi^2 \\ \vdots \\ \mu\zeta^p + \phi^p \\ L\xi + C \\ 1 \end{pmatrix}. \tag{48}$$

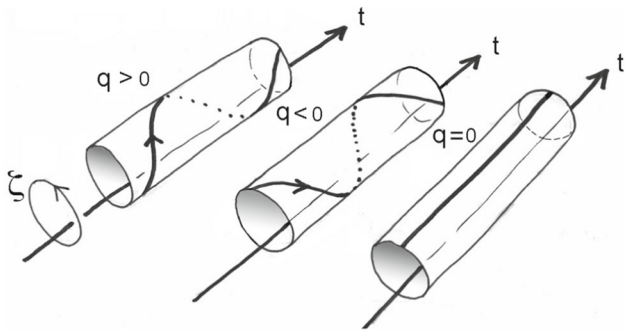


Fig. 1 Inversion of the winding direction of a particle's motion reflecting the C-symmetry

The torsor of this group incorporates several additional scalars q^p:

$$\mu = \{M, P, \sum_{1}^{p} q^i\} = \{l, g, p, E, q^1, q^2, \ldots, q^p\}. \tag{49}$$

This allows us to obtain the action of the group on its momentum space:

$$q'^1 = \mu q^1, \tag{50}$$
$$q'^1 = \mu q^1, \tag{51}$$
$$\cdots \tag{52}$$
$$q'^p = \mu q^p, \tag{53}$$
$$M' = LML^{\mathsf{T}} - LPC^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}}, \tag{54}$$
$$P' = LP. \tag{55}$$

Moreover, Souriau considers that electric charge can be geometrically quantized into discrete values $(+e, 0, -e)$ when the associated fifth dimension is closed.

Imagine representing motion in *Minkowski space* along a simple straight line oriented in time. At each point, we add a closed dimension, which extends *Minkowski space* into a bundle. In Fig. 1, it is represented as a cylinder.

But in approach [21], these transformations no longer a priori preserve the electric charge q, which then becomes dependent on the chosen coordinate system. In reference [21], taking up the approach initiated in [23], the author opts for a closed fifth dimension, in which the radius of this *"universe tube"* becomes very small, of the order of Planck's length. He then rediscovers the invariance of electric charge and concludes [21], we quote:

> In this paper, we revisit the Kaluza–Klein theory from the perspective of the classification of elementary particles based on the coadjoint orbit method. The keystone conjecture is to consider the electric charge as an extra momentum on an equal footing with the mass and the linear momentum. We study the momentum map of the

corresponding symmetry group $\hat{\mathbb{G}}_1$ which conserves the hyperbolic metric. We show that the electric charge is not an invariant, i.e. it depends on the reference frame, which is in contradiction with the experimental observations. In other words, it is not the symmetry group of the Universe today as we know it. To avert this paradox, we scale the fifth coordinate and consider the limit when the cylinder radius ω vanishes. For the corresponding group $\hat{\mathbb{G}}_0$ also of dimension 15, the charge is an invariant then independent of the frame of reference and the observer. On this ground, we propose a cosmological scenario in which the elementary particles of the early Universe are classified from the momenta of the group $\hat{\mathbb{G}}_1$, next the three former dimensions inflate quickly while the fifth one shrinks, leading to the 4D era in which as today the particles are characterized by the momenta of the group $\hat{\mathbb{G}}_0$. By this mechanism, the elementary particles can acquire electric charge as a by-product of the $4+1$ symmetry breaking of the Universe. This work opens the way to the geometric quantization of charged elementary particles.

The expression for this characteristic dimension of this universal tube is given in [23] on page 412:

$$e\frac{\hbar}{e}\sqrt{\frac{\chi}{2\pi}}, \tag{56}$$

χ being the Einstein constant taken equal to [23]:

$$\chi = -\frac{8\pi G}{c^2} = 1.856 \times 10^{-27}\text{cm g}^{-1}. \tag{57}$$

By introducing numerical values, this characteristic length is 3.782×10^{-32}cm. Dividing by 2π gives us the order of magnitude of Planck's length. In this view, the quantization of electric charge and its constancy are derived from the closure of the extra dimension associated with the decrease in the characteristic dimension associated with it.

This group refers to an extension of the *Poincaré group*, i.e. to a field-free, curvature-free universe. This construction of a five-dimensional relativity was suggested in 1964 in reference [23] and has been taken up again more recently in [21]. Note that it is in [23], page 413, that the link between charge conjugation and fifth-dimensional inversion is first mentioned.

By generalizing [9], we can envisage an extension of space-time to a space with $4+p$ dimensions, all of which may see their characteristic dimensions reduced, like that of this fifth dimension, each of these collapses leading to the emergence and quantization of new quantum numbers, baryonic, leptonic, unique etc., the electric charge being only the first of these.

Thus, the *Janus restricted group* has provided us with a framework for understanding the matter–antimatter symmetry (C-symmetry) and the inversion of quantum charges through additional compactified dimensions. We can now extend it to a broader symmetry group associated with A. Sakharov's model, the *Janus group*, which incorporates both C-symmetry and PT-symmetry. This extension allows us to explore a dynamic group structure that includes negative masses and antimatter within the framework of Sakharov's twin universe model.

5 Group associated with A. Sakharov's model: the *Janus group*

If we want to construct a group that translates the T-symmetry invoked by Sakharov, we'll replace L_o by λL_o with $\lambda = \pm 1$. But, as proposed in [16], we can translate what had already been proposed [18], we quote:

> All phenomena corresponding to $t < 0$ are, in this hypothesis, assumed to be CPT images of phenomena corresponding to $t > 0$.

Then, by introducing a new symmetry to the previous *Janus restricted group*, which we can call PT Symmetry allowing the conversion of matter into antimatter with negative mass[12], we thus combine C-symmetry and PT-symmetry to form the *Janus dynamic group* as follows:

$$g := \left\{ \begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix}, \right.$$
$$\left. \lambda, \mu \in \{-1, 1\} \cap \phi \in \mathbb{R} \cap L_o \in \mathcal{L}or_o \cap C \in \mathbb{R}^{1,3} \right\} \tag{58}$$

We can consider that particles of matter and antimatter can coexist in the same space fold. However, no coexistence is possible for the motion of particles deduced by T-symmetry (or PT-symmetry). This space is of dimension $4+p$ (for p quantum charges). We will therefore consider the two-fold covering of this manifold M_{n+p}. In each of these two folds, there remains a possibility to perform the symmetry corresponding to $\mu = -1$, that is, the inversion of all quantum charges. In other words, the *"matter–antimatter"* duality exists in both folds.

To understand the nature of the different components of these folds, we will consider the motion of a particle of matter with energy and mass:

- By acting on this motion with elements of the group corresponding to $(\lambda = 1; \mu = 1)$, we will obtain other motions of particles of matter with positive mass and energy.

[12] A concept we could call *antimatter in the sense of Feynman* [7].

- By acting on this motion with elements of the group corresponding to ($\lambda = 1; \mu = -1$), we will obtain other motions of antimatter particles with positive mass and energy[13].
- By acting on this motion with elements of the group corresponding to ($\lambda = -1; \mu = 1$), we will obtain other motions of particles of matter with negative mass and energy.[14]
- By acting on this motion with elements of the group corresponding to ($\lambda = -1; \mu = -1$), we will obtain other motions of antimatter particles with negative mass and energy[15].

Its isometry group is that of Janus space, defined by the same metric as structuring Kaluza space (34), and its dimension is 11.[16] The torsor of the group is also the same as (36). However, if we consider an element of the Lie algebra of this group:

$$Z = \begin{pmatrix} 0 & 0 & \delta\phi \\ 0 & \lambda G\omega & \gamma \\ 0 & 0 & 1 \end{pmatrix}, \tag{59}$$

we can then calculate Z' according to the relation (37) as follows:

$$\begin{aligned} Z' &= \begin{pmatrix} 0 & 0 & \delta\phi' \\ 0 & \lambda G\omega' & \gamma' \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & (\lambda\mu)\delta\phi \\ 0 & \lambda^3 L_o^{-1} G\omega L_o & \lambda^2 L_o^{-1} G\omega C + \lambda L_o^{-1}\gamma \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{60}$$

Thus, by identification, we can deduce:

$$\delta\phi' = \lambda\mu\delta\phi, \tag{61}$$
$$\omega' = \lambda^2 G L_o^{-1} G\omega L_o, \tag{62}$$
$$\gamma' = \lambda^2 L_o^{-1} G\omega C + \lambda L_o^{-1}\gamma. \tag{63}$$

We know that:

$$L_o^{-1} = G L_o^{\mathsf{T}} G. \tag{64}$$

Then:

$$\begin{aligned} \delta\phi' &= \lambda\mu\delta\phi, \\ \omega' &= \lambda^2 L_o^{\mathsf{T}}\omega L_o, \\ \gamma' &= \lambda^2 G L_o^{\mathsf{T}}\omega C + \lambda G L_o^{\mathsf{T}} G\gamma. \end{aligned} \tag{65}$$

However, inspired by J.-M. Souriau, we could add as many additional closed dimensions as quantum charges and write the dynamic group as follows:

$$\begin{pmatrix} \lambda\mu & 0 & 0 & \cdots & 0 & \phi^1 \\ 0 & \lambda\mu & 0 & \cdots & 0 & \phi^2 \\ 0 & 0 & \ddots & \cdots & 0 & \vdots \\ \vdots & \vdots & \cdots & \lambda\mu & 0 & \phi^p \\ 0 & 0 & \cdots & 0 & \lambda L_o & C \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}. \tag{66}$$

The isometry group of this space can be defined by the following metric:

$$\begin{aligned} \mathrm{d}s^2 = {} & (\mathrm{d}t)^2 - (\mathrm{d}x)^2 - (\mathrm{d}y)^2 - (\mathrm{d}z)^2 - (\mathrm{d}\zeta^1)^2 \\ & -(\mathrm{d}\zeta^2)^2 - \cdots - (\mathrm{d}\zeta^p)^2. \end{aligned} \tag{67}$$

With:

$$X = \begin{pmatrix} \xi \\ \zeta \end{pmatrix} = \begin{pmatrix} t \\ x \\ y \\ z \\ \zeta^1 \\ \zeta^2 \\ \vdots \\ \zeta^p \end{pmatrix}. \tag{68}$$

The action of this *Janus group* on the coordinates of $10 + p$ independant parameters then yields the space of the following motions:

$$\begin{pmatrix} \lambda\mu & 0 & 0 & \cdots & 0 & \phi^1 \\ 0 & \lambda\mu & 0 & \cdots & 0 & \phi^2 \\ 0 & 0 & \ddots & \cdots & 0 & \vdots \\ \vdots & \vdots & \cdots & \lambda\mu & 0 & \phi^p \\ 0 & 0 & \cdots & 0 & \lambda L_o & C \\ 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \zeta^1 \\ \zeta^2 \\ \vdots \\ \zeta^p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda\mu\zeta^1 + \phi^1 \\ \lambda\mu\zeta^2 + \phi^2 \\ \vdots \\ \lambda\mu\zeta^p + \phi^p \\ \lambda L_o\xi + C \\ 1 \end{pmatrix}. \tag{69}$$

According to Noether's theorem, this new symmetry is accompanied by the invariance of additional scalars q^p. Therefore, the torsor of the group integrates them according to this relation:

$$\mu = \{M, P, \sum_1^p q^i\} = \{l, g, p, E, q^1, q^2, \ldots, q^p\}. \tag{70}$$

Thus, the duality relation[17] gives us:

$$\frac{1}{2} T_r(M \cdot \omega) + P^{\mathsf{T}} \cdot G\gamma + \delta\phi \sum_1^p q^i$$

[13] These are *"antimatter in the sense of Dirac"* (C-symmetry).

[14] CPT-symmetry.

[15] These are *"antimatter in the sense of Feynman"* (PT-symmetry).

[16] 10 + 1 dimension associated with the fifth space dimension ζ that J.-M. Souriau identifies with the electric charge q.

[17] (13.58) from [24].

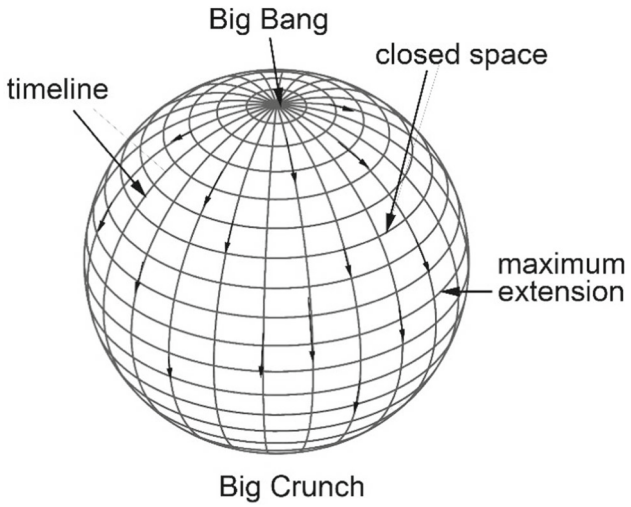


Fig. 2 A simplified 2D representation of a closed universe with a spherical topology $\mathbb{S}^2$, illustrating the temporal progression from the Big Bang to the Big Crunch, with the universe reaching maximum spatial extension in between

$$= \frac{1}{2}T_r(M' \cdot \omega') + P'^{\mathsf{T}} \cdot G\gamma' + \delta\phi \sum_1^p q'^i. \tag{71}$$

This allows us to deduce the action of the group by identification with (65):

$$\sum_1^p q'^i = \lambda\mu \sum_1^p q^i, \tag{72}$$

$$M' = LML^{\mathsf{T}} - LPC^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}}, \tag{73}$$

$$P' = LP. \tag{74}$$

Having established the *Janus dynamic group* as a natural extension of Sakharov's model, which incorporates both PT-symmetry and C-symmetry, we now shift our focus to the topological implications of the Janus model. In particular, we will explore how the symmetries discussed earlier can emerge from a closed, higher-dimensional universe. This section delves into the topological structure of the model, illustrating how P and T symmetries can arise naturally from the geometry of a closed universe, modeled as a projective space $\mathbb{P}^4$.

6 Topology of the Janus model

Let's consider a universe closed in all its dimensions, including space and time (see Fig. 2).

Diametrically opposed, antipodal points can be brought into coincidence. The image is then that of a $\mathbb{P}^2$ projective. The north and south poles, one representing the Big Bang and the other the Big Crunch, come into coincidence. The sphere cannot be paved without the presence of these two singularities. The same applies to any sphere $\mathbb{S}^{2n}$ if n is even, especially if this dimension is 4. This geometry was proposed in [13].

The Fig. 3 shows how this coincidence of antipodal regions generates this T-symmetry. On the $\mathbb{S}^2$ sphere, the direction of time is given by the orientation of the meridian curves. This orientation is shown on the left at the new state of maximum expansion, when space is identified with the sphere's equator. During this folding of the $\mathbb{S}^2$ sphere, described in reference [15] page 65, the vicinity of this equator is configured as the two-folds cover of a Möbius strip with three half-turns (see Fig. 3 on the right).

In Fig. 4, we evoke the appearance of T-symmetry by manipulating the vicinity of a meridian line. In addition, we evoke the possible elimination of the Big Bang - Big Crunch double singularity by replacing them with a tubular passage, which then gives this geometry the nature of the two-fold cover of a Klein bottle (Fig. 4).

For enantiomorphy and P-symmetry to appear, the operation would have to be performed on a larger sphere. This aspect can be highlighted by considering the conjunction of antipodal regions in the vicinity of a meridian line, which is then configured according to the two-fold covering of a half-turn Möbius strip. The Fig. 5 illustrates this enantiomorphic situation (Fig. 5).

By bringing the antipodal points of even-dimensional spheres into coincidence, we locally create a configuration associating two T-symmetrical folds. By adding further dimensions, the coincidence of the antipodes creates a two-fold CPT-symmetric coating configuration of a projective space. In the case of the sphere $\mathbb{S}^2$, which corresponds only to a 2D didactic image, the image of the projective $\mathbb{P}^2$ is its immersion in, which corresponds to the surface described in 1903 by the German mathematician Werner Boy [3], see Fig. 6. In this figure, we show how the coincidence of the antipodal points of the equator of the sphere $\mathbb{S}^2$ gives the two-fold covering of a Möbius ribbon with three half-turns (Fig. 6).

In this section, we have demonstrated that the P and T symmetries invoked by A. Sakharov can arise as consequences of a purely topological structure, specifically the covering of a projective space $\mathbb{P}^4$.

After exploring the topological structure of the Janus model, we now address a major consequence of T-symmetry: the introduction of negative masses. According to Souriau, the application of T-symmetry to the motion of a particle inverts its energy, which leads to the inversion of its mass,[18] in accordance with the definition of mass.[19] Although this idea is elegant, it presents significant challenges when integrated into the framework of general relativity. In the follow-

[18] pages 198–199 of [24].

[19] (14.57) on page 346 of [24].

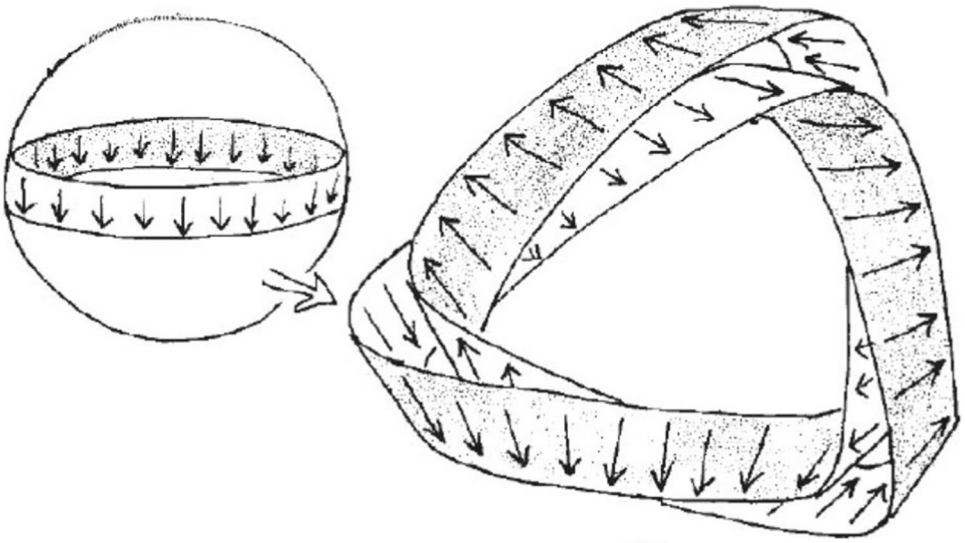


Fig. 3 How the coincidence of antipodal regions creates T-symmetry. Drawing extracted from [15], page 65

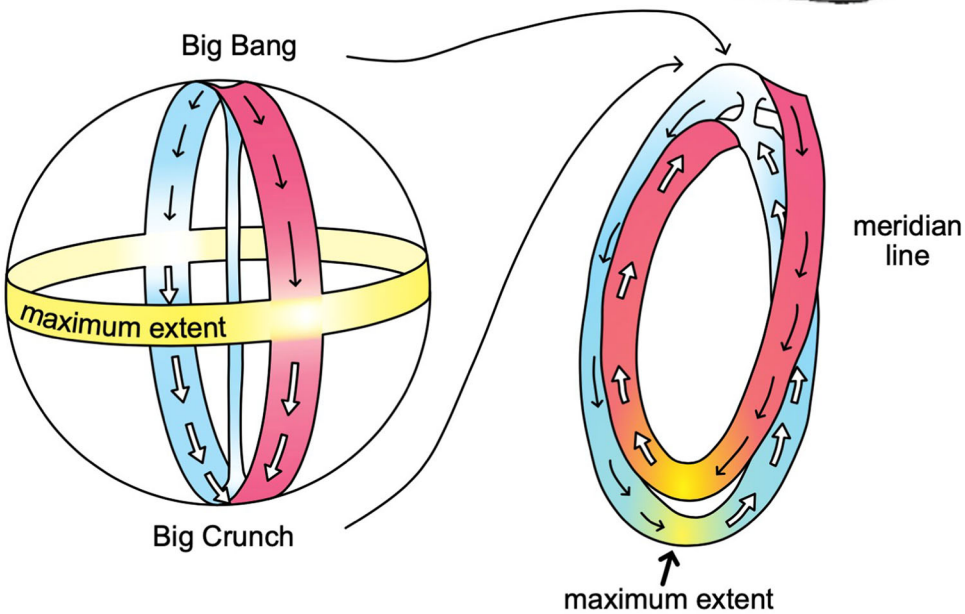


Fig. 4 Coincidence of antipodal regions on a sphere $\mathbb{S}^2$, according to the two-folds cover of a half-turn Möbius strip, with the appearance of T-symmetry

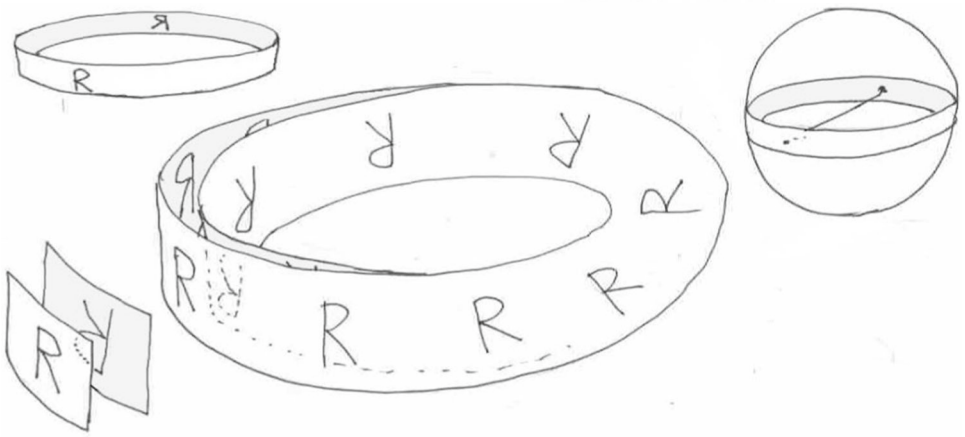


Fig. 5 P-symmetry as a consequence of contacting antipodal region neighborhoods on an $\mathbb{S}^2$ sphere

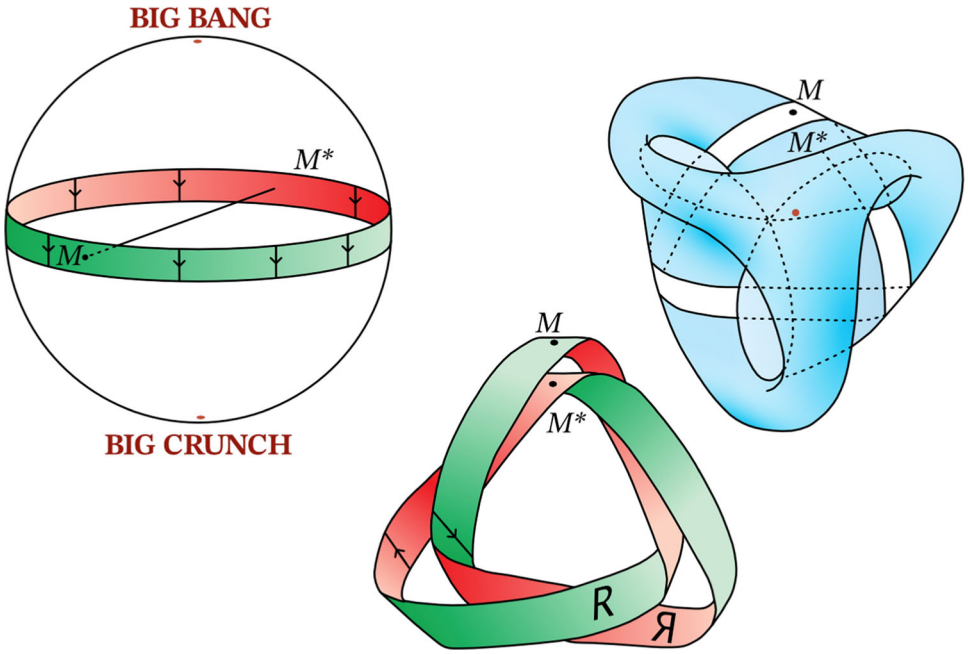


Fig. 6 Boy's surface, immersion of the $\mathbb{P}^2$ projective in $\mathbb{R}^3$

ing section, we will propose an initial approach to incorporating negative masses into the cosmological model, analyzing the implications of their interaction with positive masses and the resulting geodesics.

7 Introducing negative masses: first approach

Using dynamical group theory, we showed that this T-symmetry was synonymous with the introduction of negative masses into the cosmological model. A. Sakharov's primordial antimatter would therefore be endowed with negative mass. This first step is far from anecdotal since, if we neglect it, we admit to losing nothing less than half the universe from the outset. Is it then possible to introduce negative masses into the standard model of general relativity?

A first idea would be to consider that the field comes from two sources, represented by two tensors, the first referring to a positive mass content and the second to a negative mass content:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi\left[T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu}\right]. \tag{75}$$

We can then consider the metric solution corresponding to a region where the field is created, firstly by a positive mass content:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T^{(+)}_{\mu\nu}. \tag{76}$$

Geodesics are given by a solution in the form of an external metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM^{(+)}}{c^2 r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - \dfrac{2GM^{(+)}}{c^2 r}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right). \tag{77}$$

The geodesics evoke an attraction (see Fig. 7).

Now consider the field created by a negative mass $M^{(-)}$, the field equation becomes then:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T^{(-)}_{\mu\nu}. \tag{78}$$

And the solution corresponds to the metric:

$$\mathrm{d}s^2 = \left(1 + \frac{2G|M^{(-)}|}{c^2 r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 + \dfrac{2G|M^{(-)}|}{c^2 r}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right). \tag{79}$$

The geodesics then represent a repulsion (see Fig. 8).

In this context, our single field equation provides only a single family of geodesics, which the test particles, with both positive and negative masses, must follow. We deduce that:

- Positive masses attract both positive and negative masses.
- Negative masses repel both positive and negative masses.
- Two masses of identical absolute values but opposite signs are brought together, the positive mass flees, pursued by the negative mass. Both then accelerate uniformly, but without any energy input, since the energy of the negative mass is itself negative. This result was illustrated in 1957 by H. Bondi [2]. This phenomenon is known as *"runaway effect"*. What's more, this scheme violates the action-reaction principle. In 1957, the conclusion was reached that it was physically impossible

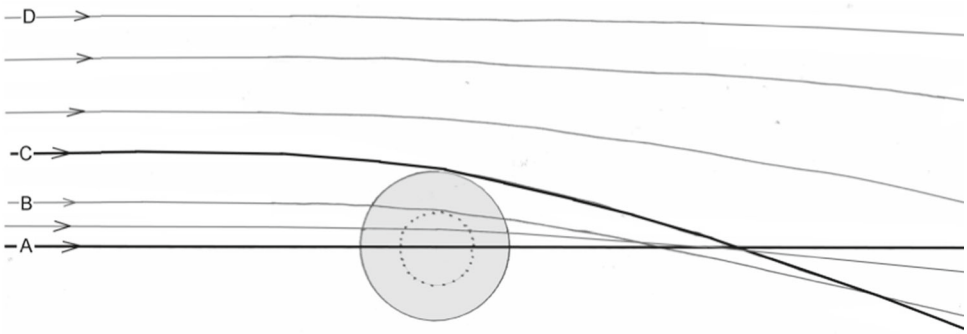


Fig. 7 Deflection of positive-energy neutrinos by a positive mass. The trajectories, when passing near the mass, are deflected more strongly due to the gravitational effect. The angle of deflection reaches its maximum (C) when the neutrinos graze the edge of the mass. Trajectories further away, such as (D), experience a weaker deflection, and the deflection angle becomes null for trajectories passing at a very large distance from the mass. The trajectory passing through the center of the mass (A) remains undeflected due to the symmetry of the configuration

to include negative masses in the cosmological model. This would only be possible at the price of a profound paradigmatic shift, not by denying the achievements of general relativity, but by considering its extension in a wider geometric context.

After examining the introduction of negative masses and their implications within the framework of general relativity, we now turn to a broader cosmological context. The discovery of anomalies, such as the dipole repeller and the accelerating expansion of the universe, has revealed significant shortcomings in the standard model ΛCDM. Recent observations, particularly those made with the James Webb Space Telescope, have intensified the crisis in cosmology by challenging long-held assumptions about galaxy formation. In the following section, we will explore how the Janus cosmological model offers a paradigm shift capable of resolving these issues by proposing a bimetrical structure for the universe, integrating both positive and negative masses into a broader and more innovative geometric framework.

8 A paradigm shift to escape the crisis of today's cosmology

In the mid-1970s, the excessive rotation speeds of stars in galaxies had already led specialists to propose the existence of dark matter, ensuring their cohesion. In 2011, the discovery that the cosmic expansion was accelerating was attributed to a new, unknown ingredient known as dark energy. Over the decades, all attempts to assign an identity to these new components ended in failure.

In 2017 [8], Hélène Courtois, Daniel Pomarède, Brent Tully and Yeudi Hoffman produced the first very-large-scale mapping of the universe, in a cube of one and a half billion light-years across, with the Milky Way, our observation

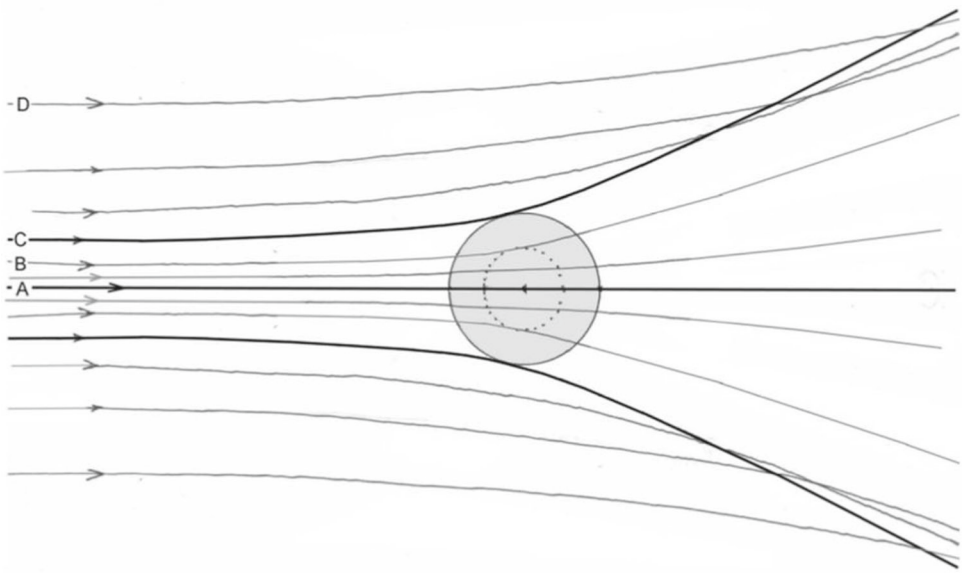


Fig. 8 Deviation of positive-energy photons by a negative mass. The trajectories, when the curvature remains moderate, are very close to hyperbolas. The angle of deviation reaches a maximum (C) when the geodesic is tangent to the limit of the mass. It then decreases steadily to zero at very large distances (D). The angle of deviation is null, due to symmetry, when the geodesic passes through the center of the mass (A)

point, at the center (see Fig. 9). By subtracting the radial component of the velocity linked to the expansion motion, they indicate the trajectories followed by the masses. A dipolar structure appears. One formation, the Shapley attractor, comprising hundreds of thousands of galaxies, attracts galaxies to itself. But, symmetrically to this formation, 600 million light-years from the Milky Way, there is an immense void, some one hundred million light-years across, which, on the contrary, repels galaxies, and to which we give the name of dipole repeller. To date, no theory has been able to explain the existence of this vast void. While the idea of a gap in dark matter, positive and attractive, has been evoked, it does not hold water, as no mechanism has been found to give rise to it. Since 2017, several other such voids have been detected and located.

The launch of the James Webb Space Telescope has only added to this crisis [6]. The Standard Model ΛCDM proposes a hierarchical mechanism for the birth of stars and galaxies. Gravitational instability appears as soon as matter and radiation are decoupled. The scenarios for the formation of both stars and galaxies in this model make use of the attributes conferred on hypothetical dark matter. But even with these parameters, it is impossible to imagine galaxies forming before a billion years. The Hubble Space Telescope was already able to obtain images in the near infrared. Early images of distant objects appeared to show groups of minigalaxies. But the James Webb Space Telescope showed that these objects were nothing other than HII regions belonging to barred spiral galaxies, fully formed, hosting old stars, only 500 million years old.

For decades, the Standard Model ΛCDM has relied on its ability to account for CMB fluctuations as gravitoacoustic oscillations, by adapting the numerous parameters relating to dark matter, dark energy and, in particular, the value of the Hubble constant. This desire to match observational data has resulted in a Hubble constant value of 67 Km s^{-1} Mpc^{-1}. This is significantly lower than the value of 70 Km s^{-1} Mpc^{-1} deduced from direct observation of standard candles.

All these factors are creating a deep crisis within the specialist community, and some voices are beginning to be heard, suggesting the need to consider a profound paradigm shift. This is what the Janus cosmological model[20] proposes.

Since we are unable to introduce negative masses into the general relativity model, let's consider a profound change of geometric paradigm, already evoked in the previous sections under the aspect of group theory and topology. The motion of positive masses, immersed in the gravitational field, takes place according to geodesics that we consider to be derived from a first metric $g_{\mu\nu}$. We will therefore describe the motion of negative masses using a second set of geodesics, derived from a second metric $\overline{g}_{\mu\nu}$. We thus have a manifold, whose points are marked by the coordinates $\{x^0, x^1, x^2, x^3\}$, equipped with a pair of metrics $(g_{\mu\nu}, \overline{g}_{\mu\nu})$. We shall neglect the action of electromagnetic fields and consider only the field of gravity. From the metrics and we can construct Ricci tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ and their associated Ricci scalars R and $\overline{R}$.

As the Janus model proposes a paradigm shift by introducing a bimetric structure to account for both positive and negative masses, we will now focus on the foundational mathematical structure underlying this model. The Janus cosmological model builds upon the interaction between two entities, i.e. positive and negative mass populations, each associated with its respective metric. In the following section, we will explore the formulation of the action and field equations governing this interaction, and how these coupled systems lead to a coherent description of cosmic phenomena, offering an alternative to the limitations of the standard model ΛCDM.

9 Foundation of the Janus cosmological model

To build this model, let us now consider the interaction between two entities: ordinary matter with positive mass interacting with negative mass through gravitational effects. This model involving negative mass takes into account the influence of both dark matter and dark energy.

We can describe this system of two entities with respective metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. Let R and $\overline{R}$ be the corresponding Ricci scalars. We then consider the following two-layer action[21]:

$$A = \int_{\mathscr{E}} \left(\frac{1}{2\chi} R + S + \mathcal{S} \right) \sqrt{|g|}\, \mathrm{d}^4x + \int_{\mathscr{E}} \left(\frac{\kappa}{2\bar{\chi}} \overline{R} + \overline{S} + \overline{\mathcal{S}} \right) \sqrt{|\bar{g}|}\, \mathrm{d}^4x. \tag{80}$$

The terms S and $\overline{S}$ will give the source terms related to the populations of the two entities, while the terms $\mathcal{S}$ and $\overline{\mathcal{S}}$ will generate the interaction tensors. χ and $\bar{\chi}$ are the Einstein gravitational constants for each entity. g and $\bar{g}$ are the determinants of the metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. For $\kappa = \pm 1$, we apply the principle of least action. The Lagrangian derivation

[20] See Sect. 9 where this model is developped.

[21] Integration over $\mathscr{E}$ using the element d^4x is a method for computing the total action in the bimetric spacetime, reflecting the four-dimensional nature of this bimetric universe. This implies considering the entire spacetime as the domain of integration, integrating the contributions from each point to the action. The term d^4x represents an infinitesimal element of hypervolume of this bimetric spacetime, used to measure each segment during integration. Thus, it is a multiple volume integral performed over the four dimensions of spacetime, accumulating contributions to the total action from each four-dimensional volume segment corresponding to each metric.

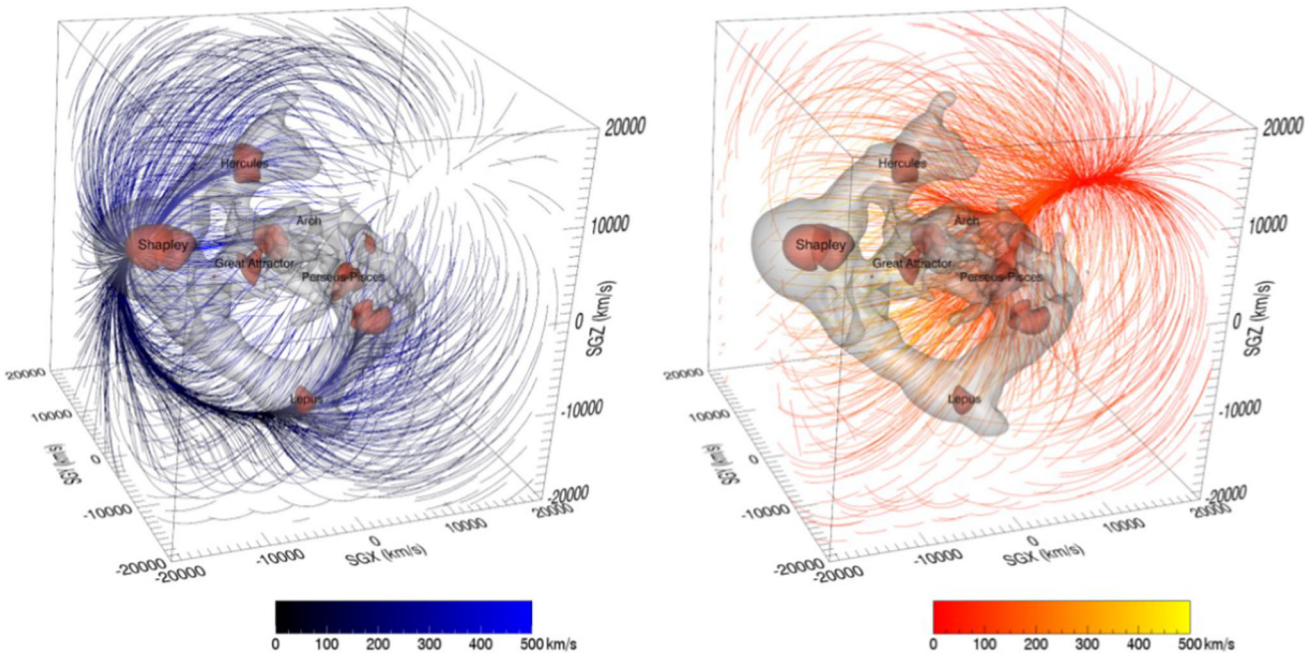


Fig. 9 Location of the dipole repeller within the large-scale structure of the universe [8]. The dipole repeller is a hypothesized region of space where galaxies are pushed away from, counteracting the attractive force of the Shapley Supercluster

of this action gives us:

$$\begin{aligned}
0 &= \delta A, \\
&= \int_{\mathscr{E}} \delta\left(\frac{1}{2\chi}R + S + \mathcal{S}\right)\sqrt{|g|}\,\mathrm{d}^4x \\
&\quad + \int_{\mathscr{E}} \delta\left(\frac{\kappa}{2\bar{\chi}}\overline{R} + \overline{S} + \overline{\mathcal{S}}\right)\sqrt{|\bar{g}|}\,\mathrm{d}^4x, \\
&= \int_{\mathscr{E}} \delta\left[\frac{1}{2\chi}\left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{|g|}}\frac{\delta\sqrt{|g|}}{\delta g^{\mu\nu}}\right)\right. \\
&\quad \left. + \frac{1}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}S)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}}\right]\delta g^{\mu\nu}\sqrt{|g|}\,\mathrm{d}^4x \\
&\quad + \int_{\mathscr{E}} \delta\left[\frac{\kappa}{2\bar{\chi}}\left(\frac{\delta\overline{R}}{\delta\bar{g}^{\mu\nu}} + \frac{\overline{R}}{\sqrt{|\bar{g}|}}\frac{\delta\sqrt{|\bar{g}|}}{\delta\bar{g}^{\mu\nu}}\right)\right. \\
&\quad \left. + \frac{1}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{S})}{\delta\bar{g}^{\mu\nu}} + \frac{1}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{\mathcal{S}})}{\delta\bar{g}^{\mu\nu}}\right]\delta\bar{g}^{\mu\nu}\sqrt{|\bar{g}|}\,\mathrm{d}^4x.
\end{aligned} \tag{81}$$

For any variation $\delta g^{\mu\nu}$ and $\delta\bar{g}^{\mu\nu}$, we locally obtain:

$$\begin{aligned}
&\frac{1}{2\chi}\left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{|g|}}\frac{\delta\sqrt{|g|}}{\delta g^{\mu\nu}}\right) \\
&\quad + \frac{1}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}S)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} = 0, \\
&\frac{\kappa}{2\bar{\chi}}\left(\frac{\delta\overline{R}}{\delta\bar{g}^{\mu\nu}} + \frac{\overline{R}}{\sqrt{|\bar{g}|}}\frac{\delta\sqrt{|\bar{g}|}}{\delta\bar{g}^{\mu\nu}}\right) + \frac{1}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{S})}{\delta\bar{g}^{\mu\nu}} \\
&\quad + \frac{1}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{\mathcal{S}})}{\delta\bar{g}^{\mu\nu}} = 0.
\end{aligned} \tag{82, 83}$$

Let us then introduce the following tensors:

$$T_{\mu\nu} = -\frac{2}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}S)}{\delta g^{\mu\nu}} = -2\frac{\delta S}{\delta g^{\mu\nu}} + g_{\mu\nu}S, \tag{84}$$

$$\overline{T}_{\mu\nu} = -\frac{2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{S})}{\delta\bar{g}^{\mu\nu}} = -2\frac{\delta\overline{S}}{\delta\bar{g}^{\mu\nu}} + \bar{g}_{\mu\nu}\overline{S}, \tag{85}$$

$$\mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}}, \tag{86}$$

$$\overline{\mathcal{T}}_{\mu\nu} = -\frac{2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{\mathcal{S}})}{\delta\bar{g}^{\mu\nu}}. \tag{87}$$

We obtain then from Eqs. (86) and (87):

$$\begin{aligned}
\sqrt{\frac{|\bar{g}|}{|g|}}\mathcal{T}_{\mu\nu} &= \sqrt{\frac{|\bar{g}|}{|g|}}\frac{-2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} \\
&= \frac{-2}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} = -2\frac{\delta\mathcal{S}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{S},
\end{aligned} \tag{88}$$

$$\begin{aligned}
\sqrt{\frac{|g|}{|\bar{g}|}}\overline{\mathcal{T}}_{\mu\nu} &= \sqrt{\frac{|g|}{|\bar{g}|}}\frac{-2}{\sqrt{|g|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{\mathcal{S}})}{\delta\bar{g}^{\mu\nu}} \\
&= \frac{-2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\overline{\mathcal{S}})}{\delta\bar{g}^{\mu\nu}} = -2\frac{\delta\overline{\mathcal{S}}}{\delta\bar{g}^{\mu\nu}} + \bar{g}_{\mu\nu}\overline{\mathcal{S}}.
\end{aligned} \tag{89}$$

Introduced into Eqs. (82) and (83), we can thus deduce the coupled field equations describing the system of the two entities. To obtain the desired interaction laws under the New-

tonian approximation, we must choose $\kappa = -1$. The system of equations then becomes:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \chi \left(T_{\mu\nu} + \sqrt{\frac{|\bar{g}|}{|g|}} \mathcal{T}_{\mu\nu} \right), \tag{90}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} \overline{R} = \kappa \bar{\chi} \left(\overline{T}_{\mu\nu} + \sqrt{\frac{|g|}{|\bar{g}|}} \overline{\mathcal{T}}_{\mu\nu} \right). \tag{91}$$

The tensor $T_{\mu\nu}$ is the energy-momentum tensor, which represents the source of the field acting on positive mass entities and positive-energy photons. The term $\sqrt{\frac{|\bar{g}|}{|g|}}$ is the source of this field attributed to the action of negative masses on these positive masses. The tensor $\overline{T}_{\mu\nu}$ is the energy-momentum tensor, which represents the source of the field acting on negative mass entities and negative-energy photons, and the term $\sqrt{\frac{|g|}{|\bar{g}|}}$ is the source of this field attributed to the action of positive masses on these negative masses. $\mathcal{T}_{\mu\nu}$ and $\overline{\mathcal{T}}_{\mu\nu}$ are the interaction tensors of the system of the two entities corresponding to the *"induced geometry"*, meaning how each matter distribution on one layer of the universe contributes to the geometry of the other.[22]

General relativity produces only a limited number of exact solutions. We will follow the same logic.

Having established the foundation of the Janus cosmological model, with its bimetric structure and the corresponding field equations, we turn to constructing explicit solutions under the assumption of homogeneity and isotropy. By considering the FLRW form for both metrics, we aim to derive a time-dependent solution that accounts for the interaction between positive and negative mass populations. The next section will focus on obtaining these solutions, exploring their compatibility with observational data, and providing a theoretical framework for the accelerated cosmic expansion.

10 Construction of a time-dependent, homogeneous and isotropic solution

Given the symmetry assumptions, the metrics then have the FLRW form. The variable x^0 is the common chronological coordinate (time marker).

$$g_{\mu\nu} = \mathrm{d}x^{0^2} - a^2 \left[\frac{\mathrm{d}u^2}{1 - ku^2} + u^2 \mathrm{d}\theta^2 + u^2 \sin^2\theta \mathrm{d}\varphi^2 \right], \tag{92a}$$

$$\overline{g}_{\mu\nu} = \mathrm{d}x^{0^2} - \bar{a}^2 \left[\frac{\mathrm{d}u^2}{1 - \bar{k}u^2} + u^2 \mathrm{d}\theta^2 + u^2 \sin^2\theta \mathrm{d}\varphi^2 \right]. \tag{92b}$$

[22] Interaction between populations of positive and negative masses.

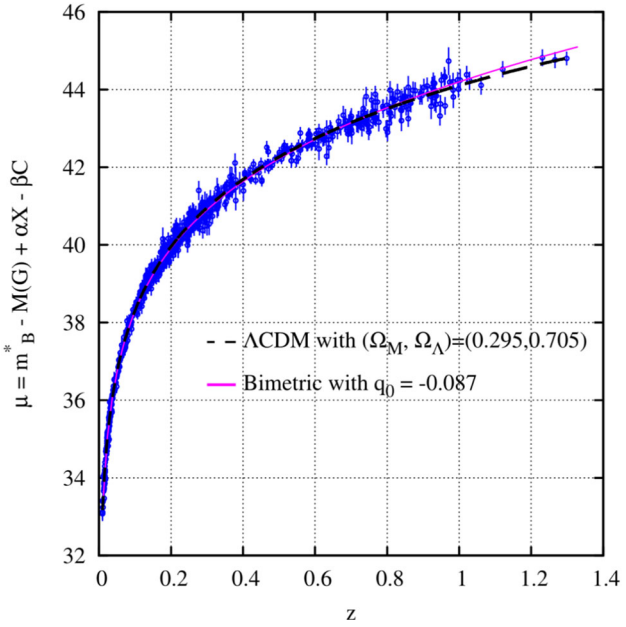


Fig. 10 Comparison of observed and theoretical magnitudes as a function of z redshift [4]

The determinants of the two metrics are

$$g = -a^6 \sin^2\theta, \qquad \bar{g} = -\bar{a}^6 \sin^2\theta. \tag{93}$$

As shown in reference [16] the treatment of the two equations leads to the compatibility relation:

$$\rho c^2 a^3 + \bar{\rho} \bar{c}^2 \bar{a}^3 = E = \mathrm{cst}. \tag{94}$$

This translates into conservation of energy, extended to both populations. The exact solution, referring to two dust universes, corresponds to:

$$k = \bar{k} = -1 \tag{95}$$

and:

$$a^2 \frac{\mathrm{d}^2 a}{\mathrm{d}x^{0^2}} = -\frac{4\pi G}{c^2} E, \tag{96a}$$

$$\bar{a}^2 \frac{\mathrm{d}^2 \bar{a}}{\mathrm{d}x^{0^2}} = +\frac{4\pi G}{\bar{c}^2} E. \tag{96b}$$

A theoretical model loses interest if it cannot be compared with observational data. The evolution of the positive species will correspond to an acceleration if the energy E of the system is negative. This provides a physical interpretation of the acceleration of the cosmic expansion [12,17], which then follows from the fact that the energy content is predominantly negative. Numerical data have been successfully compared with observational data [4]. The corresponding curve is shown in Fig. 10.

To complete the model, we now need to provide exact stationary solutions. We will restrict ourselves to $\mathfrak{so}(3)$ symmetry.

We now focus our attention on the interaction laws and their observational consequences. These interaction laws, derived from the coupled field equations of the Janus model, govern how positive and negative mass entities influence each other. The next section explores these laws in detail and examines how they provide explanations for various cosmological phenomena, including the formation of large-scale structures and the resolution of issues related to dark matter and dark energy.

11 Interaction laws and observational consequences

In the system of coupled field equations (**??**) and (**??**), the terms on the left-hand side involve the Ricci tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ and the corresponding Ricci scalars R and $\overline{R}$. These terms are calculated from the two metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. Using these two metrics, we then calculate the form of two operators known as *covariant derivatives* ∇_μ and $\overline{\nabla}_\mu$. It turns out that, due to their form, the two left-hand sides of both equations identically satisfy the following relation:

$$\nabla_\mu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = 0, \tag{97}$$

$$\overline{\nabla}_\mu \left(\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \bar{g}_{\mu\nu} \right) = 0. \tag{98}$$

The corresponding covariant derivatives of the two second members must therefore also be zero, which corresponds to the Bianchi identities, implying:

$$\nabla_\mu T_{\mu\nu} = 0, \tag{99}$$

$$\overline{\nabla}_\mu \overline{T}_{\mu\nu} = 0. \tag{100}$$

We should also have:

$$\nabla_\mu \left[\sqrt{\frac{\bar{g}}{g}} \mathcal{T}_{\mu\nu} \right] = 0, \tag{101}$$

$$\overline{\nabla}_\mu \left[\sqrt{\frac{g}{\bar{g}}} \overline{\mathcal{T}}_{\mu\nu} \right] = 0. \tag{102}$$

In stationary conditions, the square roots of the ratios of the determinants behave like constants, reflecting an *"apparent mass effect"*. Conditions (101) and (102) can therefore be replaced by:

$$\nabla_\mu \mathcal{T}_{\mu\nu} = 0, \tag{103}$$

and

$$\overline{\nabla}_\mu \overline{\mathcal{T}}_{\mu\nu} = 0. \tag{104}$$

Let's write the system of equations in mixed notation, replacing the square roots, which have become constant, by the positive constants b^2 and $\bar{b}^2$:

$$R^\nu_\mu - \frac{1}{2} R g^\nu_\mu = \chi \left[T^\nu_\mu + b^2 \mathcal{T}^\nu_\mu \right], \tag{105a}$$

$$\overline{R}^\nu_\mu - \frac{1}{2} \overline{R} \bar{g}^\nu_\mu = -\chi \left[\overline{T}^\nu_\mu + \bar{b}^2 \overline{\mathcal{T}}^\nu_\mu \right]. \tag{105b}$$

Using the Newtonian approximation, in both populations the non-zero tensor terms reduce to:

$$\begin{aligned} T^0_0 &= \rho c^2 > 0 \qquad \mathcal{T}^0_0 = \bar{\rho} \bar{c}^2 < 0 \\ \overline{T}^0_0 &= \bar{\rho} \bar{c}^2 < 0 \qquad \overline{\mathcal{T}}^0_0 = \rho c^2 > 0. \end{aligned} \tag{106}$$

In our system of coupled field equations, the presence of a minus sign in front of the second member of the second equation gives the following interaction laws:

- Masses of the same sign attract each other;
- Masses of opposite signs repel each other.

We have thus eliminated the runaway effect.

The first conclusion to be drawn is that where one of the two types of mass is present, the other is absent, as immediately confirmed by simulations [14]. This is the case in the vicinity of the Sun, and under these conditions the first equation is identified with Einstein's 1915 equation. The model is therefore in line with all the classical local observational data of general relativity: Mercury's perihelion advance, deflection of light rays by the Sun. The model therefore does not invalidate that of general relativity, but presents itself as its extension, made essential to integrate the new observational data, which can no longer be managed by introducing the hypothetical components of dark matter and dark energy.

We have seen, in our construction of the unsteady solution, that negative energy dominates. The model is thus profoundly asymmetrical. The negative mass component is proposed as a substitute for the combined roles traditionally attributed to dark matter and dark energy. By the way, going back to the original idea, inspired by the work of Andreï Sakharov, this allows us to attribute a well-defined identity to these components. They are invisible, insofar as negative masses emit photons of negative energy that our optical instruments cannot capture. They are therefore simply copies of our own antimatter, assigned a negative mass. We then have a new distribution of contents (see Fig. 11).

At the moment of decoupling, when the gravitational instability can play its role (we must then speak of joint gravitational instabilities), the characteristic Jeans time is shorter for negative masses:

$$\bar{t}_\mathrm{J} = \frac{1}{\sqrt{4\pi G |\bar{\rho}|}} \ll t_\mathrm{J} = \frac{1}{\sqrt{4\pi G \rho}}. \tag{107}$$

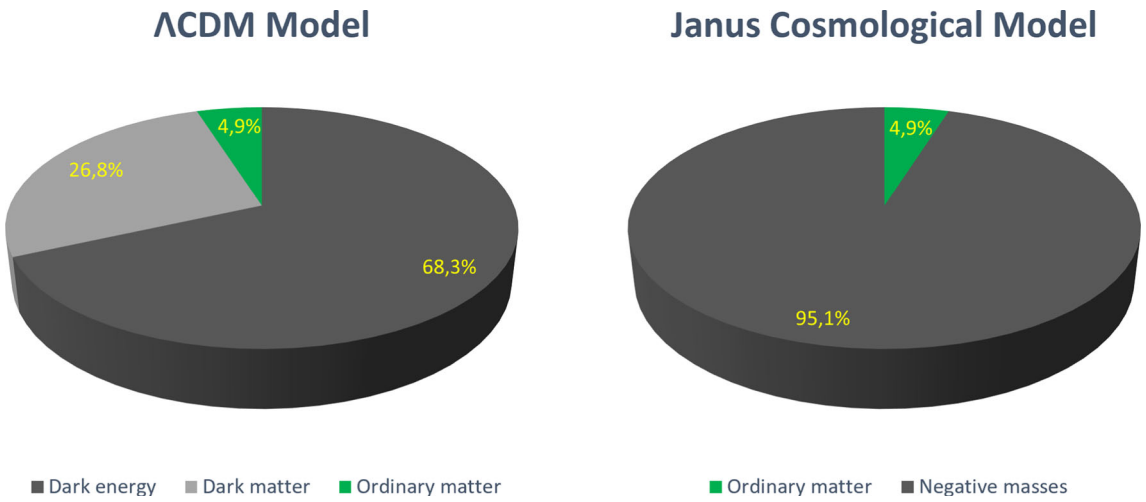


Fig. 11 Comparative contents of the ΛCDM and Janus models

The result will be a regular distribution of negative-mass conglomerates of spheroidal antihydrogen and negative-mass antihelium. These will behave like immense negative-mass protostars. As soon as their temperature causes hydrogen reionization, their contraction will cease. These formations will then radiate in the red and infrared wavelengths. But their cooling time is then large compared to the age of the universe, which means that these objects will no longer evolve. The history of this universe fold associated with negative masses is totally different from our universe fold of ordinary matter. It will not give rise to stars, galaxies or planets. It will contain no atoms heavier than negative-mass antihelium. And there will be no life. And, as we will see later: these negative formations are deliberately situated within the Newtonian approximation.

But there is another very important point. When these spheroidal conglomerates form, they confine the positive mass to the residual space, giving it a lacunar structure, comparable to joined soap bubbles. The positive mass is thus distributed in the form of thin plates, sandwiched between two negative conglomerates that exert a strong back pressure on it. The positive mass is thus violently compressed and heated. However, due to its plate-like arrangement, it can cool down very quickly through the emission of radiation (see Fig. 12).

The result is a pattern of first-generation star and galaxy formation totally different from the standard one. This configuration had been the subject of simulations [14] since the first, heuristic, approach to the model, and the fact that objects all form within the first hundred million years was one of its predictions, largely confirmed by JWST data.

The lacunar structure, advocated as early as 1995 [14], predicted the existence of large voids, which the discoveries of the dipole repeller and other similar large voids have also confirmed. Once this lacunar structure has been formed, matter tends to concentrate along the segments common to three gaps, forming filaments (see Fig. 13 on the next page). The nodes of this distribution will only develop into galaxy clusters.

After establishing the interaction laws and exploring their observational consequences, it is essential to verify the mathematical and physical consistency of the Janus model. This requires demonstrating that the system of coupled field equations respects the Bianchi identities and provides consistent solutions in the weak field limit. In the following section, we will examine the necessary conditions to ensure this consistency, particularly in regions dominated by ordinary matter, such as near the Sun, as well as in regions dominated by negative masses, such as near the dipole repeller.

12 The mathematical and physical consistency of the model

This is ensured in an isotropic, homogeneous and unsteady situation, the required condition being the generalized conservation of energy expressed by Eq. (94). We now turn to the case of stationary solutions, limiting ourselves to those that satisfy $\mathfrak{so}(3)$ symmetry. Bianchi identities must then be satisfied, i.e. relations (99), (100), (103) and (104).

First, we will show the existence of asymptotic consistency in Newtonian approximation situations. The key aspects of this approximation are as follows:

- Velocities must be negligible compared to the speed of light. This is the case for velocities $\langle v\rangle$ and $\langle \bar{v}\rangle$ of thermal agitation in both media, which are involved in the definition of pressures. After decoupling:

$$\varepsilon p = \frac{\varepsilon \rho \langle v\rangle}{3} \quad \text{and} \quad \varepsilon \bar{p} = \frac{\varepsilon \bar{\rho} \langle \bar{v}\rangle}{3}. \tag{108}$$

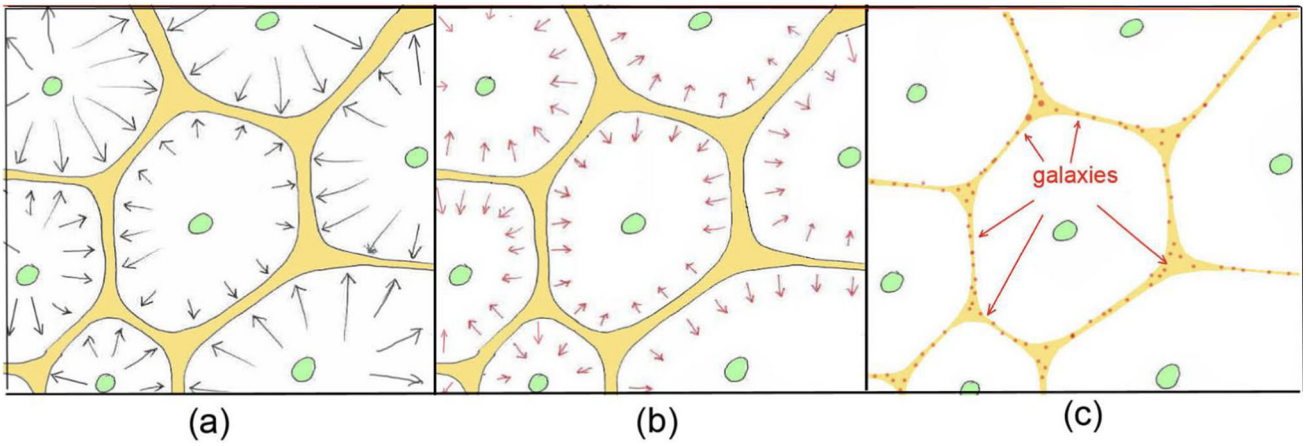
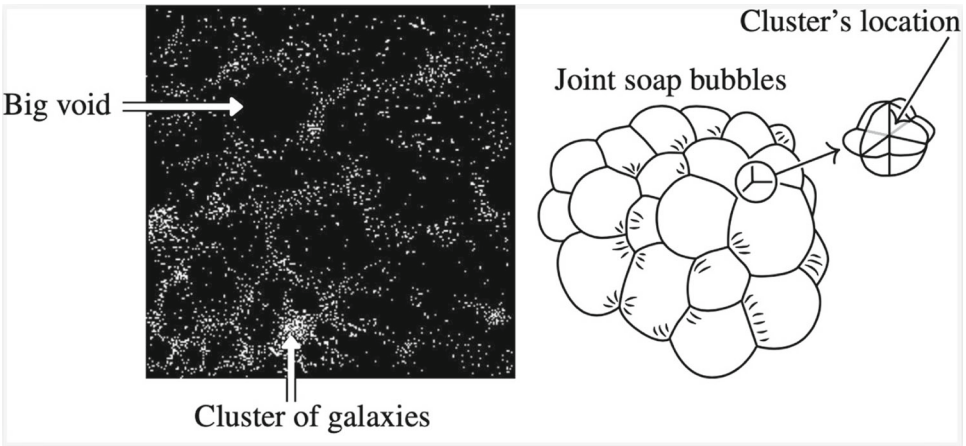


Fig. 12 Early rapid star and galaxy formation

Fig. 13 Structure of positive mass in contiguous bubbles



- Curvature effects must be neglected, meaning that the radial coordinate must be much larger than the characteristic length scale associated with curvature, i.e., the Schwarzschild radius.

12.1 Newtonian approximation of the field generated by a positive mass M

Let's introduce the Schwarzschild radius R_S as follows:

$$\varepsilon R_S = \varepsilon \frac{2GM}{c^2}, \tag{109}$$

where ε being a small parameter. $\mathfrak{so}(3)$ symmetry imposes the shapes of the two metrics:

$$ds^2 = e^{\nu} dx^{0^2} - e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2, \tag{110a}$$

$$d\bar{s}^2 = e^{\bar{\nu}} dx^{0^2} - e^{\bar{\lambda}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2. \tag{110b}$$

The construction of a stationary solution then requires to calculate the functions:

$$\nu(r), \quad \lambda(r), \quad \bar{\nu}(r), \quad \text{and} \quad \bar{\lambda}(r). \tag{111}$$

To locate this solution, we need to consider the shapes of the field source tensors:

$$T_\mu^\nu, \quad \mathcal{T}_\mu^\nu, \quad \overline{T}_\mu^\nu, \quad \text{and} \quad \overline{\mathcal{T}}_\mu^\nu. \tag{112}$$

Let's start by considering a situation where only positive mass is present. The tensors $\mathcal{T}_\mu^\nu$ and $\overline{T}_\mu^\nu$ are then null and the two field equations (**??**) and (**??**) become in mixed-mode form:

$$R_\mu^\nu - \frac{1}{2} g_\mu^\nu R = \chi T_\mu^\nu, \tag{113}$$

$$\overline{R}_\mu^\nu - \frac{1}{2} \bar{g}_\mu^\nu \overline{R} = -\bar{\chi} \sqrt{\frac{|g|}{|\bar{g}|}} \overline{\mathcal{T}}_\mu^\nu. \tag{114}$$

The form of the tensor T_μ^ν in its classical mixed-mode form is given by[23]:

$$T_\mu^\nu = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -\varepsilon p & 0 & 0 \\ 0 & 0 & -\varepsilon p & 0 \\ 0 & 0 & 0 & -\varepsilon p \end{pmatrix}. \tag{115}$$

[23] (13.1) p.425 of [1].

As we are in the Newtonian approximation, ε is very small. With the introduction of the metric (110a) and the tensor (115) in the first field equation, we are led to introduce the function $m(r)$ such that:

$$\mathrm{e}^{-\lambda} = 1 - \frac{2m(r)}{r} \implies 2m(r) = r\left(1 - \mathrm{e}^{-\lambda}\right). \tag{116}$$

Similarly to equation (14.18) from [1], the classic calculation leads to the relationship:

$$m(r) = \frac{G\rho}{c^2}\int_0^r 4\pi r^2 \mathrm{d}r = \frac{4}{3}\pi r^3 \rho \frac{G}{c^2}. \tag{117}$$

We then obtain the classical Tolman–Oppenheimer–Volkoff (TOV) equation [11]. Relation (117) places the small quantity in front of any quantity that will be neglected in the Newtonian approximation:

$$\frac{1}{c^2}\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{m + \frac{4\pi\varepsilon G p r^3}{c^4}}{r(r - 2m\varepsilon)}\left(\rho + \varepsilon\frac{p}{c^2}\right). \tag{118}$$

When ε tends to zero (or c tends to infinity) we get:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\rho m c^2}{r^2} = -\frac{G\rho}{r^2}\frac{4\pi r^3 \rho}{3}. \tag{119}$$

The quantity $\frac{4\pi r^3\rho}{3}$ represents the amount of matter $\mu(r)$ contained inside a sphere of radius r. We know that the force of gravity exerted inside a mass of constant density is equivalent to that exerted by the mass located at the center of the sphere, and that the mass located outside this sphere gives a force of zero. So the quantity $-\frac{G\rho\mu(r)}{r^2}$ is the force of gravity, per unit volume, acting on the matter contained in an elementary volume around a point at distance r from the center. Thus the relation (115), which follows from the Newtonian approximation, expresses that the force of gravity balances the force of pressure. This is the classic Euler relationship.

Hence, the Schwarzschild interior metric built is given by:

$$\begin{aligned}\mathrm{d}s^2 = &\left[\frac{3}{2}\sqrt{\left(1 - \frac{r_n{}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1 - \frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1 - \frac{r^2}{\hat{r}^2}} \\ &-r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right).\end{aligned} \tag{120}$$

This metric connects with the Schwarzschild exterior metric:

$$\begin{aligned}\mathrm{d}s^2 = &\left(1 - \frac{2GM}{c^2 r}\right)c^2\mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1 - \frac{2GM}{c^2 r}} - r^2 \\ &\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right),\end{aligned} \tag{121}$$

where r_n is the radius of the star and $\hat{r}$ is a stellar constant as a function of its density ρ. It is the characteristic radius of a neutron star, defined under the assumption of constant density ρ. It establishes a critical threshold for the star's radius, beyond which the internal pressure becomes infinite at the center, indicating a physical singularity or instability. This radius is derived from the balance between gravitational forces and the internal pressure gradients within the star [10]. It is given by:

$$\hat{r} = \sqrt{\frac{3c^2}{8\pi G\rho}}. \tag{122}$$

We can thus deduce, according to the classical theory of general relativity, that a particle of ordinary matter will undergo an attractive gravitational field due to the effect of a distribution of positive masses.

To ensure the mathematical consistency of the system of two field equations (113) and (114), we therefore need to consider a form of the tensor $\overline{\mathcal{T}}^{\nu}_{\mu}$ that gives back this same Euler relation when the Newtonian approximation is also applied to this solution. This is guaranteed with the form of the interaction tensor $\overline{\mathcal{T}}^{\nu}_{\mu}$ of the field equation (114) as this choice can stem from a Lagrangian derivation:

$$\overline{\mathcal{T}}^{\nu}_{\mu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & +\varepsilon p & 0 & 0 \\ 0 & 0 & +\varepsilon p & 0 \\ 0 & 0 & 0 & +\varepsilon p \end{pmatrix}. \tag{123}$$

On the right-hand side of the second field equation (114), the ratio of determinants will be considered almost unity insofar as we perform this calculation within the Newtonian approximation.

Then, if we consider that:

$$\sqrt{\frac{|g|}{|\bar{g}|}} = \sqrt{\frac{\mathrm{e}^{\nu}\mathrm{e}^{\lambda} r^4 \sin^2\theta}{\mathrm{e}^{\bar{\nu}}\mathrm{e}^{\bar{\lambda}} r^4 \sin^2\theta}} \approx 1, \tag{124}$$

the calculation leads to the Tolman–Oppenheimer–Volkoff (TOV) solution for the population of negative masses managed by the second field equation:

$$\frac{1}{c^2}\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{m - \frac{4\pi\varepsilon G p r^3}{c^4}}{r(r + 2m\varepsilon)}\left(\rho - \varepsilon\frac{p}{c^2}\right). \tag{125}$$

The two solutions, (118) and (125), asymptotically approach the Euler equation in the Newtonian approximation as ε tends to zero. This also corresponds to the asymptotic satisfaction of the Bianchi identities in the same context[24].

Consequently, it is possible to build the Schwarzschild interior metric associated with the population of negative masses by applying the same calculation scheme as for the population of positive masses, thus constituting the solution

[24] The inequality $r \gg 2\,m$ (where m is often replaced by $\frac{GM}{c^2}$ to obtain a dimension of length, M being the mass of the object and G the gravitational constant) indicates that we are sufficiently far from the gravitational source for the effects of general relativity to be negligible. Indeed, at great distances, the length $\frac{2GM}{c^2}$ is completely negligible.

to the second field equation (114) as follows:

$$\mathrm{d}\bar{s}^2 = \left[\frac{3}{2}\sqrt{\left(1+\frac{{r_n}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1+\frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1+\frac{r^2}{\hat{r}^2}} - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{126}$$

This metric must join the Schwarzschild exterior metric:

$$\mathrm{d}\bar{s}^2 = \left(1+\frac{2GM}{c^2r}\right)c^2\mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1+\frac{2GM}{c^2r}} - r^2 \times \left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{127}$$

We can deduce that a particle with negative mass will undergo a repulsive gravitational field due to the effect of a distribution of positive masses.

The Janus model presents a new paradigm, extending general relativity by describing the universe as a four-dimensional manifold M_4, endowed with two distinct metrics. These metrics are solutions to the system of coupled field equations (**??**) and (**??**).

Let's now consider the case, still in the Newtonian approximation, where the geometry is determined by the presence of negative mass, corresponding to regions of space dominated by negative masses, such as near the dipole repeller [8].

12.2 Newtonian approximation of the field generated by a negative mass $\bar{M}$

In regions where negative masses dominate, the system becomes in mixed-mode form:

$$R_\mu^\nu - \frac{1}{2}g_\mu^\nu R = \chi\sqrt{\frac{|\bar{g}|}{|g|}}\mathcal{T}_\mu^\nu, \tag{128}$$

$$\overline{R}_\mu^\nu - \frac{1}{2}\bar{g}_\mu^\nu \overline{R} = -\bar{\chi}\overline{T}_\mu^\nu. \tag{129}$$

If we consider the impact of the presence of negative masses on the geometry of spacetime structured by the metric tensor of the first field equation (128) associated with the population of positive masses, we can define the corresponding interaction tensor (130) as follows:

$$\mathcal{T}_\mu^\nu = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & -\bar{p} & 0 & 0 \\ 0 & 0 & -\bar{p} & 0 \\ 0 & 0 & 0 & -\bar{p} \end{pmatrix}. \tag{130}$$

Thus, the impact of the pressure gradient of negative masses on the geodesics followed by ordinary matter and positive-energy photons according to the field equation (128) translates into the following Tolman–Oppenheimer–Volkoff equation:

$$\frac{\bar{p}'}{\bar{c}^2} = -\frac{m - \frac{4\pi G\bar{\rho}r^3}{\bar{c}^4}}{r(r+2m)}\left(\bar{\rho}-\frac{\bar{p}}{\bar{c}^2}\right). \tag{131}$$

Therefore, it is possible to build the Schwarzschild interior metric solution in this manner:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{\left(1+\frac{{r_n}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1+\frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1+\frac{r^2}{\hat{r}^2}} - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{132}$$

This metric can be connected to the Schwarzschild exterior metric:

$$\mathrm{d}s^2 = \left(1+\frac{2GM}{c^2r}\right)c^2\mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1+\frac{2GM}{c^2r}} - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{133}$$

We can deduce that a particle of ordinary matter will undergo a repulsive gravitational field due to the effect of a distribution of negative masses.

Then, when the source of the gravitational field of the second field equation (129) is created by a negative mass, we can freely define the following energy-momentum tensor as follows:

$$\overline{T}_\mu^\nu = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & \bar{p} & 0 & 0 \\ 0 & 0 & \bar{p} & 0 \\ 0 & 0 & 0 & \bar{p} \end{pmatrix}. \tag{134}$$

We can therefore deduce the following Tolman–Oppenheimer–Volkoff equation:

$$\frac{\bar{p}'}{\bar{c}^2} = -\frac{\overline{m} + \frac{4\pi G\bar{\rho}r^3}{\bar{c}^4}}{r(r-2\overline{m})}\left(\bar{\rho}+\frac{\bar{p}}{\bar{c}^2}\right). \tag{135}$$

Hence, the interior Schwarzschild metric can be constructed as follows:

$$\mathrm{d}\bar{s}^2 = \left[\frac{3}{2}\sqrt{\left(1-\frac{\bar{r_n}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1-\frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1-\frac{r^2}{\hat{r}^2}} - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{136}$$

This metric matches the exterior Schwarzschild metric:

$$\mathrm{d}\bar{s}^2 = \left(1-\frac{2G\bar{M}}{\bar{c}^2r}\right)\bar{c}^2\mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1-\frac{2G\bar{M}}{\bar{c}^2r}} - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right). \tag{137}$$

We can deduce that a particle of negative mass will undergo an attractive gravitational field due to the effect of a distribution of negative masses.

Both solutions (131) and (135) reduces to the Euler equation approximately equal to $-\frac{G\bar{M}(r)\bar{\rho}(r)}{r^2}$ in the Newtonian limit, reflecting hydrostatic equilibrium.[25]

The form of these two source tensors satisfies the Bianchi identities. This would obviously not be the case if the negative mass were to fall outside of this framework. For that, there would need to exist neutron stars of negative mass. However, the characteristic time of evolution of conglomerates of negative mass, their *"cooling time"*, exceeds the age of the universe. These spheroidal conglomerates cannot evolve, so the content of this negative spacetime will be limited to a mixture of negative mass anti-hydrogen and anti-helium. Since nucleosynthesis cannot occur, there can be no anti-galaxies or anti-stars, regardless of their mass. Consequently, there cannot exist anti-neutron stars.

Moreover, in the case where this negative spacetime would generate hyperdense stars through an as-yet-unknown mechanism, it would then be necessary to reconsider the form of these tensors. However, the current configuration satisfies all currently available and potentially available observational data.

After verifying the mathematical and physical consistency of the Janus model, we now turn to its predictive capabilities. One of the most striking predictions concerns the existence of large voids and structures such as the dipole repeller. The Janus model not only accounts for these features but also offers novel predictions regarding the effect of negative gravitational lensing on the magnitudes of background sources. In the following short section, we explore the observational signatures of this phenomenon, with a particular focus on the implications for the dipole repeller.

13 Dipole repeller prediction

The Janus model is essentially falsifiable in Popper's sense. It predicted a large-scale twin structure with large voids. This has been confirmed [8]. It predicted a very early birth of first-generation stars and galaxies. A new prediction this time concerns the magnitude of sources located in the background of the large void. According to the model, the magnitude of the light emitted by these distant sources will be attenuated by the negative gravitational lensing effect. This is a novel aspect, since it has been assumed that the two entities, positive and negative, interact only through antigravitation. Photons from these distant sources can then freely pass through the negative-mass conglomerates. This means that both external and internal geodesics must be used. The deflection effect of light rays will be greatest when they graze the surface of the object, with radius $\bar{r}_0$. This effect weakens as you move deeper into the object, becoming zero when the photons pass through its center (see Fig. 8). Eventually, we will be able to map the magnitudes of objects in the background of the dipole repeller. Schematically, their luminosity will be attenuated in a ring-shaped pattern (see Fig. 14). This measurement will immediately give us the value of the radius $\bar{r}_0$ of this formation.

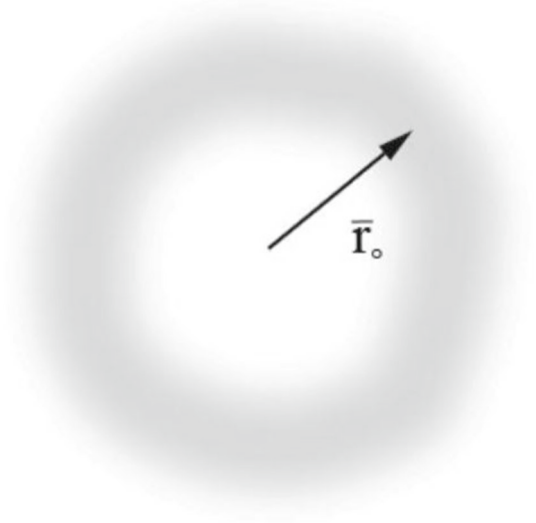


Fig. 14 Attenuation of the magnitude of objects in the background of the dipole repeller

After exploring the implications of the Janus model in the Newtonian approximation and its predictions for large-scale structures, such as the dipole repeller, we now move beyond these limitations. In a universe dominated by positive masses, certain astrophysical objects, such as neutron stars and supermassive black holes, exhibit strong gravitational effects that require a relativistic treatment. The following section addresses the challenges of extending the model to these extreme cases.

14 Beyond the Newtonian approximation

These objects are absent in the universe fold associated with negative masses. In our universe fold of ordinary matter, objects that deviate from the Newtonian approximation are neutron stars and hypermassive objects located at the center of galaxies, which early images show to be the seat of a strong gravitational redshift effect, darkening their central part. These objects are a priori manageable using the classic pair of outer and inner metrics, taking rotation into account. It should be remembered that we are under no obligation to

[25] Where the pressure at the center of this negative mass spheroid is balanced by the negative gravitational force depending on density and mass.

provide the form of the source tensor of the other sector, in this case an interaction tensor, whose form would be precisely imposed by the Bianchi identities. It is conceivable that one day someone will provide the exact form of this tensor.

But even in the absence of such an object, there is no a priori inconsistency.

15 Conclusion

The genesis of the Janus model spanned several decades. The starting point, in 1967, was Andreï Sakharov's attempt to provide an initial explanation for the absence of observations of primordial antimatter, which remains a significant flaw in the Standard Model ΛCDM. This model offers no explanation for the loss of half of the universe's content. Sakharov therefore proposed a universe structure with two sectors, the second being T-symmetrical to our own. A few years later, in 1970, through the application of symplectic geometry, mathematician Jean-Marie Souriau demonstrated that this inversion of the time coordinate, i.e., T-symmetry, is synonymous with the inversion of energy and mass. Pushing this idea of global symmetry further, Sakharov envisioned a twin universe that is CPT-symmetrical to ours. In this scenario, the invisible components of the universe reduce to negative-mass antimatter.

In 1994, we proposed that this universe structure corresponds to a two-fold cover of a projective $\mathbb{P}^4$, by a compact universe with the topology of a $\mathbb{S}^4$ sphere. The two singularities of this spherical universe, the Big Bang and the Big Crunch, then coincide. By introducing a tubular structure, these singularities disappear. This configuration consists of two PT-symmetrical folds. These adjacent sectors are assumed to interact solely through gravity. Therefore, the interaction between positive masses in one sector and negative masses in the other sector must be taken into account.

However, the introduction of negative masses is not feasible within the framework of general relativity, as it would result in interaction laws that are incompatible with known physical principles. Thus, a bimetric model is proposed. A system of coupled field equations is then constructed from an action, whose form eliminates the problematic runaway effect. The interaction laws in the model dictate that masses of the same sign attract each other according to Newton's law, while masses of opposite signs repel each other following an anti-Newtonian law. Since these masses are mutually exclusive, the negative mass can be neglected in the vicinity of the Sun, and the first field equation then aligns with Einstein's equation.

In this way, the model remains consistent with local relativistic observations, such as the advance of Mercury's perihelion and the deflection of light by the Sun. Therefore, the Janus model can be considered an extension of general relativity. An exact, time-dependent solution is constructed, revealing a generalized energy conservation law that applies to both sectors. When adapting the model to observations, it becomes evident that an accelerating expansion is present, imposing a fundamental dissymmetry between the two entities involved.

In this framework, the vast majority of negative mass replaces the hypothetical components of dark matter and dark energy. As a result, the matter distribution is approximately 5% visible matter and 95% negative mass, which is invisible because it emits photons of negative energy that elude detection by our observation instruments. This dissymmetry implies that, following decoupling, the negative masses form a regular network of spheroidal conglomerates, while the positive mass, confined to the remaining space, adopts a patchy distribution.

The model also accounts for the existence of large voids, with the dipole repeller being the first identified among them. At the centers of these large voids are invisible spheroidal conglomerates that behave like giant protostars, with cooling times exceeding the age of the universe. These objects, which emit negative-energy photons corresponding to light in the red and infrared regions, do not evolve and do not give rise to stars, galaxies, or atoms heavier than helium. Life, therefore, is absent from this negative sector, which consists of a mixture of negative-mass antihydrogen and antihelium.

Furthermore, the model explains the very early formation of first-generation stars and galaxies, as recently demonstrated by the James Webb Space Telescope. We then examine the issue of the model's mathematical consistency, specifically whether the Bianchi identities are satisfied. We show that they can be asymptotically satisfied under conditions corresponding to the Newtonian approximation.

Lastly, we address the question of objects that do not fit within this approximation, primarily located on the positive-mass side. We assert that we are not required to provide the exact form of the interaction tensor in such cases, as it is determined by the zero-divergence condition. The lack of definition of this tensor does not invalidate the consistency of a non-linear solution.

Author contributions JPP designed the model, developed the theoretical formalism, drafted the manuscript and prepared the figures. FM managed the submission and peer-review process. HZ corrected many typos from the draft and improved the layout of the manuscript.

Data Availability Statement My manuscript has no associated data. [Authors' comment: This is a theoretical study and no experimental data has been listed.]

Code Availability Statement My manuscript has no associated code/software. [Authors' comment: Code/Software sharing not applicable to this article as no code/software was generated or analysed during the current study.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.
Funded by SCOAP[3].

References

1. R. Adler, M. Bazin, M. Schiffer, *Introduction to General Relativity* (McGraw-Hill, New York, 1975)
2. H. Bondi, Negative mass in general relativity. Rev. Mod. Phys. **29**(3), 423–428 (1957). https://doi.org/10.1103/RevModPhys.29.423
3. W. Boy, Über die Curvatura integra und die Topologie geschlossener Flächen. Mathematische Annalen **57**(2), 151–184 (1903). https://doi.org/10.1007/BF01444342
4. G. D'Agostini, J.-P. Petit, Constraints on Janus cosmological model from recent observations of supernovae type Ia. Astrophys. Space Sci. **363**(7), 139 (2018). https://doi.org/10.1007/s10509-018-3365-3
5. P.A.M. Dirac, The quantum theory of the electron. Proc. Roy. Soc. Lond. Ser. A Contain. Pap. Math. Phys. Character **117**(778), 610–624 (1928)
6. L. Ferreira et al., Panic! at the disks: first rest-frame optical observations of galaxy structure at $z>3$ with JWST in the SMACS 0723 field. Astrophys. J. Lett. **938**(1), L2 (2022). https://doi.org/10.3847/2041-8213/ac947c
7. R.P. Feynman, The theory of positrons. Phys. Rev. **76**, 749–759 (1949). https://doi.org/10.1103/PhysRev.76.749
8. Y. Hoffman, D. Pomarède, R.B. Tully, H.M. Courtois, The dipole repeller. Nat. Astron. **1**(2), 0036 (2017). https://doi.org/10.1038/s41550-016-0036
9. R. Kerner, Generalization of the Kaluza–Klein theory for an arbitrary non-abelian gauge group. Annales de l'institut Henri Poincaré Sect. A Physique Théorique **9**(2), 143–152 (1968). http://www.numdam.org/item/AIHPA_1968__9_2_143_0/
10. P. Koiran et al., PT-symmetry in one-way wormholes. Ann. Phys. (2024). https://doi.org/10.1016/j.aop.2024.169765. arXiv:2409.02951 [gr-qc]. ISSN: 0003-4916
11. J.R. Oppenheimer, H. Snyder, On continued gravitational contraction. Phys. Rev. **56**(5), 455–459 (1939)
12. S. Perlmutter et al., Measurements of Ω and Λ from 42 high-redshift supernovae. Astrophys. J. **517**(2), 565–586 (1999). https://doi.org/10.1086/307221
13. J.-P. Petit, The missing-mass problem. Nuovo Cimento B Serie **109**(7), 697–709 (1994). https://doi.org/10.1007/BF02722527
14. J.-P. Petit, Twin universes cosmology. Astrophys. Space Sci. **226**(2), 273–307 (1995). https://doi.org/10.1007/BF00627375
15. J.-P. Petit, *Topo The World (The Adventures of Archibald Higgins)*
16. J.-P. Petit, G. D'Agostini, Negative mass hypothesis in cosmology and the nature of dark energy. Astrophys. Space Sci. **354**(2), 611–615 (2014). https://doi.org/10.1007/s10509-014-2106-5
17. A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. **116**(3), 1009–1038 (1998). https://doi.org/10.1086/300499
18. A.D. Sakharov, Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe. ZhETF Pisma Redaktsiiu **5**, 32 (1967)
19. A.D. Sakharov, Baryon asymmetry of the universe. Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki **76**, 1172–1181 (1979)
20. A.D. Sakharov, Cosmological models of the universe with reversal of time arrow. Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki **79**, 689–693 (1980)
21. G. de Saxcé, Which symmetry group for elementary particles with an electric charge today and in the past?. (2024). arXiv:2403.14846 [math-ph]
22. G. de Saxcé, C.-M. Marle, Presentation of Jean-Marie Souriau's book "Structure des systèmes dynamiques". (2023). arXiv:2306.03106 [math.DS]
23. J.-M. Souriau, *Géométrie et Relativité* (Hermann, Paris, 1964)
24. J.-M. Souriau, Structure des systè mes dynamiques: maîtrises de mathématiques. Collection Dunod Université. Dunod. (1970). ISSN: 9782040107529

Following this, there is a series of articles that constitute the result of an extremely thorough examination of the mathematical and geometric foundations of the black hole model. These are:

- J-P. Petit, G. D'Agostini. Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I. Journal of Modern Physics, 2026, Vol. 17, No. 2. 3,223 views. 423 downloads. Cited 4 times.

- J-P Petit, G. D'Agostini, J-M Pechinot: The True Schwarzschild Solution and Its Subsequent Geometry. Journal of Modern Physics, 2026, 17(8), 997–1027. https://content.scirp.org/pdf/jmp_7506149.pdfAugust 2026 31 downloads 329 views cited 2 times.

- J-P Petit. Kruskal coordinates, semi-complex extension of spacetime. Journal of Modern Physics, 2026. 17(9), 1197–1128 39 downloads 197 views

- J-P Petit. Mathematical and Geometrical Inconsistency of the Black Hole Model. Journal of Modern Physics. Part II 2026. Forthcoming.

These articles are accessible only to mathematicians and geometers, or to specialists in general relativity who have refreshed their knowledge of geometry. We will, however, attempt to summarize the content. This is a central issue in the history of science.

In 1905, Einstein established the principle of the invariance of the speed of light in a vacuum, which is the same for all inertial observers. Combined with the principle of relativity, this postulate leads to the Lorentz transformations and revolutionizes classical conceptions of space and time.

This revolution found its geometric expression in Hermann Minkowski's famous lecture, *Raum und Zeit* (Space and Time), delivered in Cologne on September 21, 1908, and published in 1909.

Minkowski unified space and time into a single four-dimensional geometric entity: spacetime. But he died prematurely on January 12, 1909, at the age of 44, from complications of a ruptured appendix.

His geometry is characterized by the invariant interval element, which is written as:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \geq 0$$

The length element defines the proper time element.

In 1915, from June 28 to July 5, Einstein delivered a series of six lectures in Göttingen devoted to his theory of gravitation, which was still under development at the time.

These lectures aroused keen interest in the great mathematician David Hilbert, already famous for his work on the foundations of mathematics and geometry.

The search for a geometric and variational formulation of physical laws was part of a much older tradition.

In the 18th century, Euler and Lagrange had developed the methods of the calculus of variations, which made it possible to deduce the equations of motion from a principle of stationary action.

In the 19th century, Hamilton and Jacobi had expanded on this approach, while Riemann had introduced, in 1854, a general conception of curved spaces, characterized by the local definition of a quadratic differential form.

More recently, in 1912–1913, Gustav Mie proposed an electromagnetic theory of matter based on a variational principle.

Einstein and Hilbert were then engaged in the same line of research—that of the gravitational field equations—which would become the foundation of general relativity. But they approached this question using two profoundly different strategies.

Einstein began with physical requirements: the equivalence principle, the covariance of the equations, and the need to reproduce Newtonian gravity in the weak-field approximation. His friend Marcel Grossmann introduced him to tensor calculus, giving him access to the tools of differential geometry. Einstein thus sought to construct the geometric term of an equation whose right-hand side must describe the material sources of the gravitational field, represented by the energy-momentum tensor.

Hilbert takes a different approach. Building on Gustav Mie's variational approach, he constructs an action that combines the Ricci curvature scalar with a Lagrangian describing matter and electromagnetism. The variation of this action with respect to the components of the metric yields the gravitational equations, which include both a geometric and a physical contribution.

The contrast is therefore striking. Einstein sought the geometric structure capable of representing a physics whose fundamental requirements he understood. Hilbert sought to derive the equations of this physics from a single variational principle.

Einstein's genius lay in his extraordinary physical intuition. Whereas Hilbert prioritized the power and coherence of mathematical constructions, Einstein sought first and foremost to grasp the fundamental physical principles underlying observable phenomena. These are two forms of scientific creativity, but it was Einstein's physical intuition that served as the initial driving force behind the relativistic revolution.

In November 1915, both men derived the field equation almost simultaneously.

Let us introduce today's translation of two space qualifiers defined by bilinear forms$g_{\mu\nu}dx^{\mu}dx^{\nu}$.

In a Riemannian space, they all have the same sign. In a pseudo-Euclidean space, all signs are positive. It is Euclidean geometry plus curvature.

In a Lorentzian space, one of the signs is different from all the others. A space with the signature (+ - - -) can be called "Minkowskian."

In 1915–1916, for Einstein, Schwarzschild, Droste, Weyl, Flamm, and Lorentz, cosmology was "Minkowskian."

But Hilbert believed and wrote that the cosmos has a "pseudo-Euclidean" geometry with only pluses, but in which one of the coordinates—the fourth—is purely imaginary.

For decades, the definitions remained somewhat vague. Before Hilbert, it was the Minkowskian signature (+ - - -). It is to Hilbert—due to the profound influence that Euclidean geometry exerted on him—that we owe the shift to the signature (- + + +) and the emergence of "time-like trajectories." We abandon a form of "principle of reality," implicitly followed by the others (real coordinates and real length).

Second point: Schwarzschild does not express his solution using his initial real coordinates and their polar variants:

$$(t, x, y, z) \longrightarrow (t, r, \theta, \varphi) \quad r = \sqrt{x^2 + z^2 + z^2}$$

Instead, he uses an "intermediate quantity" and coordinates:

$$(t, R, \theta, \varphi) \leftarrow R = \left(r^3 + {R_s}^3\right)^{1/3} \geq R_s$$

His solution is therefore:

$$ds^2 = \left(1 - \frac{\alpha}{R}\right)c^2dt^2 - \frac{dR^2}{1 - \frac{\alpha}{R}} - R^2(d\theta^2 + sin^2\theta\, d\varphi^2) \qquad R = (r^3 + \alpha^3)^{1/3}$$

Act I: Following the popularization of this relativistic geometrization of astrophysics and cosmology—primarily due to the English-language works of Arthur Eddington in 1923[65] and Richard Tolman in 1934[66]—this distinction was lost sight of, and a "standard expression of the Schwarzschild metric" became established:

$$ds^2 = \left(1 - \frac{\alpha}{r}\right)c^2dt^2 - \frac{dr^2}{1 - \frac{\alpha}{r}} - r^2(d\theta^2 + sin^2\theta\, d\varphi^2)$$

Act Two: This position is reinforced by Hilbert's[67] calculation; when confronted with the metric term—depending on how he introduces it—he performs the "simplification"[68]

---

[65] A. Eddington. http://www.jp-petit.org/books/#1923-Eddington.pdf

[66] R.C. Tolman: Relativity, Thermodynamics, and Cosmology. Published in England in 1946 by the Clarendon Press, Oxford; first edition: 1934. http://www.jp-petit.org/books/1934-Tolman.pdf

[67] D. Hilbert. The Foundations of Physics (Second Communication). English translation. Boston Series in Philosophy and Science. Copyright Springer, Vol. 250, 2007
https://link.springer.com/chapter/10.1007%2F978-1-4020-4000-9_45
http://www.jp-petit.org/papes/cosmo/1916-Hilbert-en.pdf

[68] Understandable as a physical approximation

$$G(r)\,(\,d\theta^2 + \,sin^2\theta\,d\varphi^2)\;plus\;G(r)\;\approx\;r^2\;\longrightarrow\;\;r^2\,(\,d\theta^2 + \,sin^2\theta\,d\varphi^2)$$

Sans se rendre which is strictly equivalent to:

$$r\;\;\longrightarrow\;\;\;R\;=\;(\,r^2 + \,\alpha^3)^3$$

In 1989, seventy-three years later, the mathematician Abrams realized this and pointed[69] it out in an article that went completely unnoticed (it was already too late).

Act Three: 1939: Oppenheimer and Snyder[70], based on the infinite values of the free-fall and escape times of test particles, proposed using the Schwarzschild stationary solution to describe a rapidly collapsing stellar body. We are just before the curtain rises on this play, "The Black Hole."

Act Four: John Archibald Wheeler, working at the Institute for Advanced Study in Princeton, where he rubs shoulders with Einstein—also a member of this prestigious IAS—is searching for a description of such an "object" $0 \leq r \leq \alpha$when… "inside the Schwarzschild sphere."

In 1947, the Irish physicist Synge proposed moving from the "standard" formulation of the Schwarzschild metric—using (*t, r*) coordinates—to real (*u, v*) coordinates. He was the first to propose their form, for r > $\alpha$and for r <α. He was also the first to publish what would later be called "the Kruskal diagram[71]."

In 1960, the Hungarian G. Szekeres also derived the form of these coordinates inside the Schwarzschild sphere[72].

The young mathematician M. Kruskal, who was working in the plasma physics laboratory at Princeton, became aware of all these findings.

He met A. Wheeler in 1959 at the Princeton cafeteria, where Wheeler shared this information with him on a paper napkin provided by a waiter. Wheeler left for a cosmology conference held in Royaumont, France, organized by Lichnérowicz. There, it finally seemed possible to shed light on this matter of real coordinates, which would allow one to penetrate the interior of the Schwarzschild sphere.

Upon his return to Princeton, he wrote a paper titled:

---

[69] L.S. Abrams: Black Hole: The Legacy of Hilbert's Error. Can. Jr. Phys. **67**, 9191 (1989), accessed Feb. 13, 2001 (The Black Hole: The Legacy of Hilbert's Error)

[70] J.R. Oppenheimer and H. Snyder: On Continued Gravitational Contraction. Phys. Rev. Vol. 56, pp. 455–459, Sept. 1939 https://journals.aps.org/pr/pdf/10.1103/PhysRev.56.455
http://www.jp-petit.org/papers/cosmo/1939-Oppenheimer-Snyder.pdf

[71] J.L. Synge. The Gravitational Field of a Particle. Proceedings of the Irish Academy, Section A, Mathematical and Physical Science, 53, pp. 83–114 (1950)
https://www.jstor.org/stable/ffd0dec8-d9d5-3ebf-a636-e9299aa13403?googleloggedin=true&seq=1#page_scan_tab_contents

[72] G. Szekeres. On the singularities of a Riemannian manifold. Publicationes Mathematicae Debrecen 7, 285 (1960). http://www.jp-petit.org/papers/cosmo/1960-Szekeres.pdf

Maximal Extension of the Schwarzschild Metric

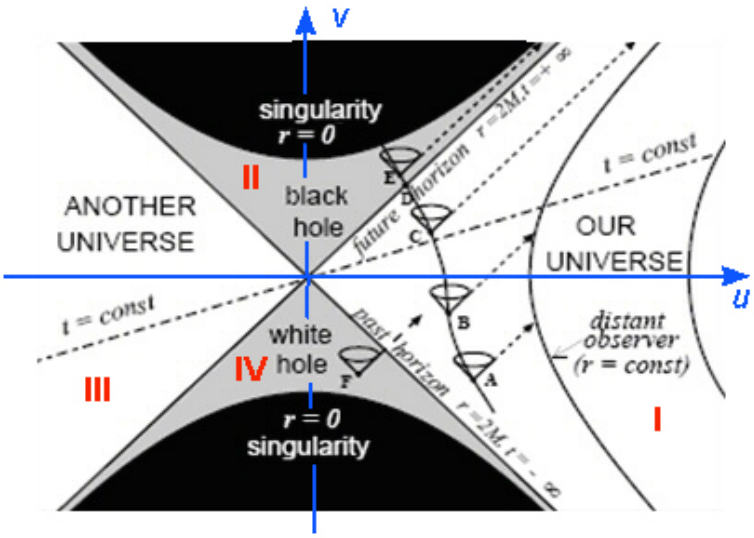
He submitted it to the journal *Physical Review*, listing Martin Kruskal as the sole author.

Kruskal discovered the existence of this paper—of which he had not written a single word—when the editor sent him the proofs for final approval!

Kruskal went to Wheeler's office to demand an explanation and asked him, at the very least, to co-author the paper. Wheeler replied that he alone should be the author. Kruskal managed to convince him, signed the paper, and it was published[73].

Kruskal thus became "famous against his will." Indeed, his name would be cited countless times in connection with "the Kruskal–Szekeres coordinates" and, above all, "the Kruskal diagram," even though he was not the author of either.

However, after signing, he returned to his work in plasma physics and would henceforth take no further interest in cosmology or in what J.A. Wheeler would come to call "black holes."



The "Kruskal diagram"
Shown in black is the region "that does not belong to physics" (r < 0)

Below is a document that is likely very little known:

[73] M.D. Kruskal: Maximal extension of the Schwarzschild metric. Phys. Rev. 119, 1734 (Sept. 1, 1960).http://www.jp-oetit.org/papers/cosmo/1960-Kruskal.pdf

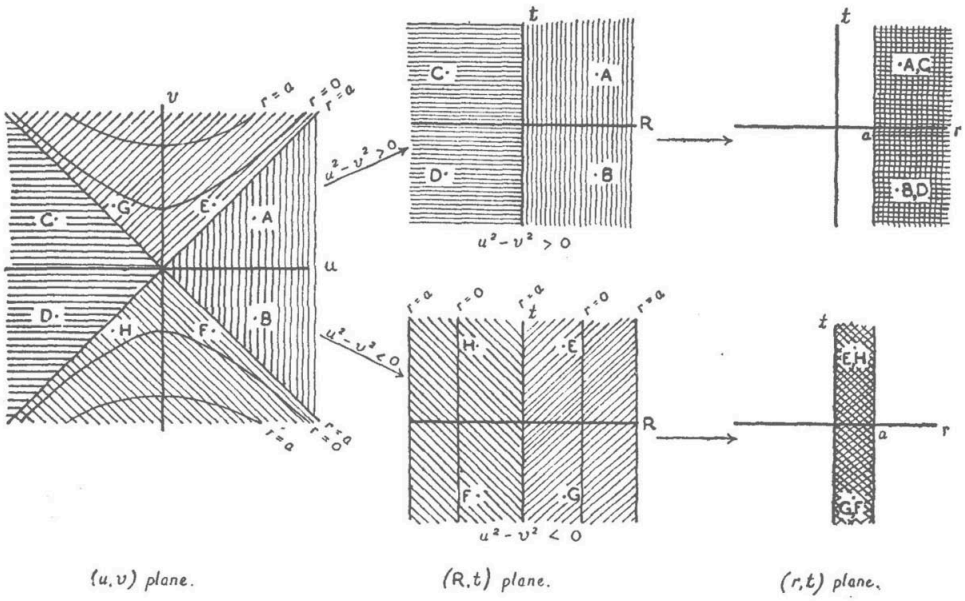


FIG. 8.—Mapping $(u, v) \longleftrightarrow (R, t) \longleftrightarrow (r, t)$, by (4.4).

The plane (u, v) is mapped twice on the plane (R, t).

(top right): The quadrant $u^2 - v^2 > 0$, $u > 0$ is mapped uniquely on $r > a$, and so is the quadrant $u^2 - v^2 > 0$, $u < 0$.

(bottom right): The pair of quadrants $u^2 - v^2 < 0$ is mapped infinitely often on the strip $0 < r < a$, each curvilinear strip in the (u, v) plane corresponding to the straight strip in the (r, t) plane.

The same diagram, 11 years earlier, from Synge's article (1949)

Experts will note that Synge includes in his description the portion shown in black in the previous diagram—which was "excluded from physics"—and which represents values where the "radial coordinate" *r* becomes … negative. Below is the plate detailing this region:

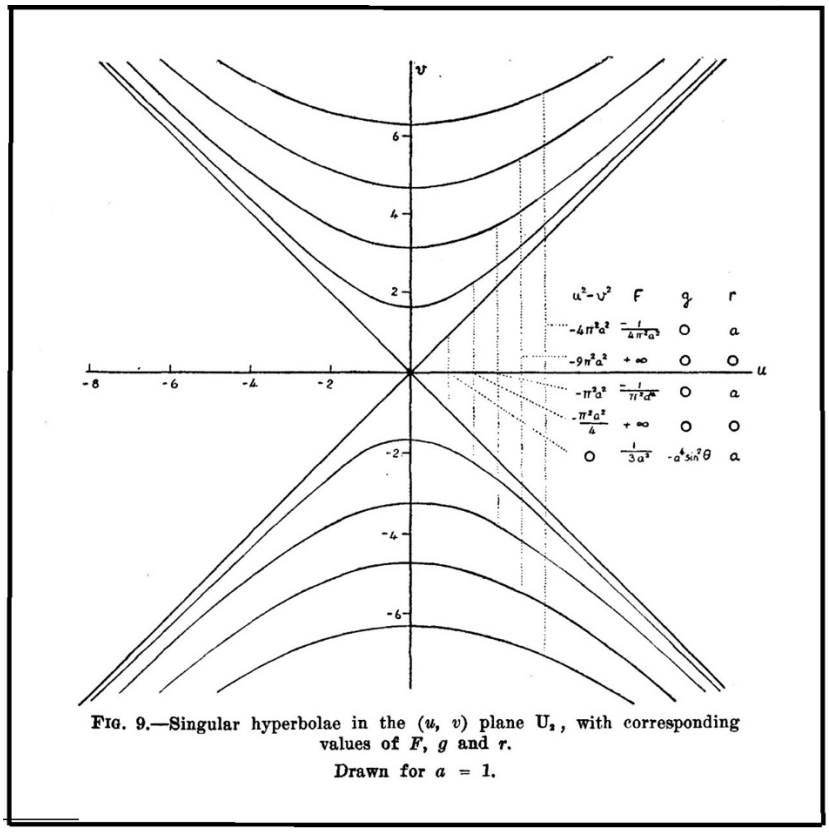


FIG. 9.—Singular hyperbolae in the (u, v) plane U_2, with corresponding values of F, g and r.
Drawn for $a = 1$.

The "black" region, as acknowledged by Synge

Now, let us recall that:

$$r = \sqrt{x^2 + y + z^2}$$

If we assume that *r* is negative, there are no complex values of *x, y, and z* that satisfy this equation. To do so, we would have to go so far as to "tinker" with the very definition of this "radial coordinate," changing it to:

$$r = \pm\sqrt{x^2 + y + z^2}$$

Why not? But then… where is the physics in all of this?

It remains a mystery: how do we manage—through what magical change of variables—to describe the interior of this sphere using real coordinates *u* and *v*, when, topologically speaking, this part of the geometric object lies outside the real domain?

Over the years, "solutions" have emerged, in which problematic algebraic quantities—such as:

$$\sqrt{1 - \frac{\alpha}{r}} \quad when\ r < \alpha \quad \text{replaced by} \quad \sqrt{\left|1 - \frac{\alpha}{r}\right|}$$

$$log\left(1-\frac{\alpha}{r}\right) when\ r<\alpha \quad \text{replaced by} \quad log\left|1-\frac{\alpha}{r}\right|$$

In the article[74]:

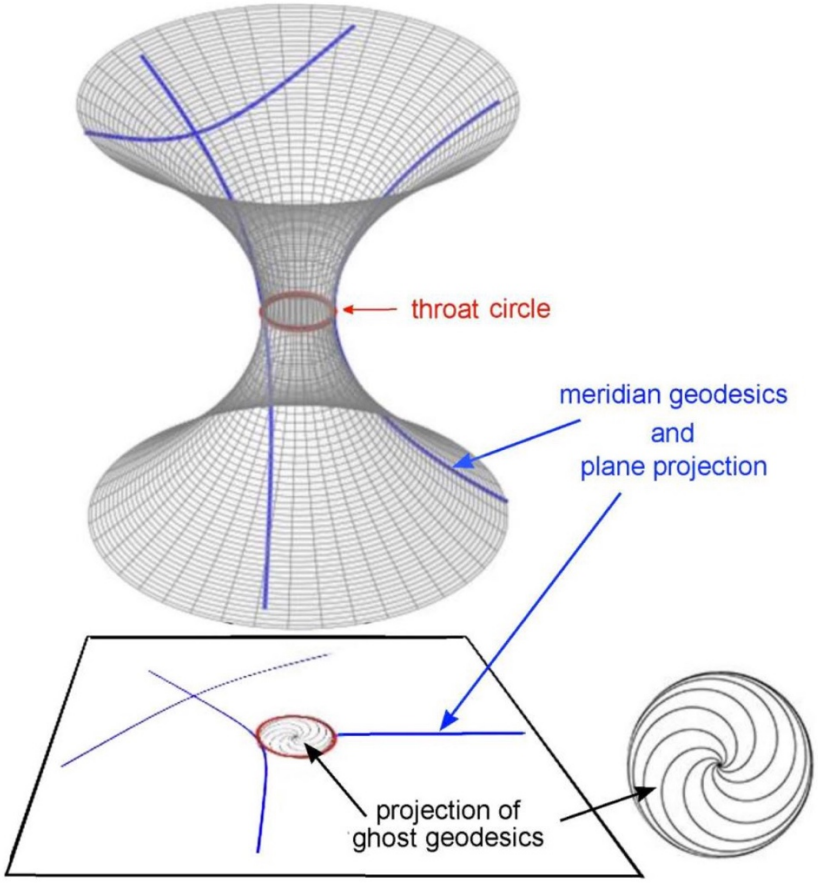
J-P Petit. Kruskal coordinates, semi-complex spacetime expansion. Journal of Modern Physics. 2026 17(9), 1197–1228 https://doi.org/10.4236/jmp.2026.179053

We show that this artifice conceals the extension of the geometric object beyond the real world.

In the same way that it is mathematically possible to venture beyond the Flamm surface, which is topologically equivalent to the hypersurface representing the geometry associated with the Schwarzschild solution. But… we are leaving the real world to venture into the… imaginary world.

All articles published on this topic are included in this lengthy, richly illustrated[75] review paper, which follows up on an initial article published a few months earlier[76].

[74] http://www.jp-petit.org.papers/cosmo/2026-JMP-Petit-Kruskal-coordnates-semi-complex-extension-ofspacetime.pdf
[75] http://www.jp-petit.org/papers/cosmo/2026-JMP-Petit-Physical-and-Geometrical-Inconsistency-of-the-Black-Hole-Model.PartII.pdf
[76] http://www.sp-petit.org/papers/cosmo/2026-JMP-Petit-Physical, and Geometrical Inconsistencies of the Black Hole Model. Part I. PDF



189 *Semi-complex extension of the Flamm surface*

When looking at the figure above, the objects "reside" in a three-dimensional (and, moreover, Euclidean) space. The blue curves inscribed on this "diabolo"-shaped surface (a "Flamm surface") are real and project onto the plane as black curves. If these blue curves do not penetrate "inside the diabolo," their projections do not penetrate inside the circle, which is the projection of the throat circle of this Flamm surface.

But the mathematical tool used to construct these "geodesics" (the "Lagrange equations") also produces "ghost geodesics," which reside in a "semi-complex" three-dimensional space ,(x , y , ζ) where the first two dimensions are real but the third dimension, represented by the Greek letter ζ, is "purely imaginary."

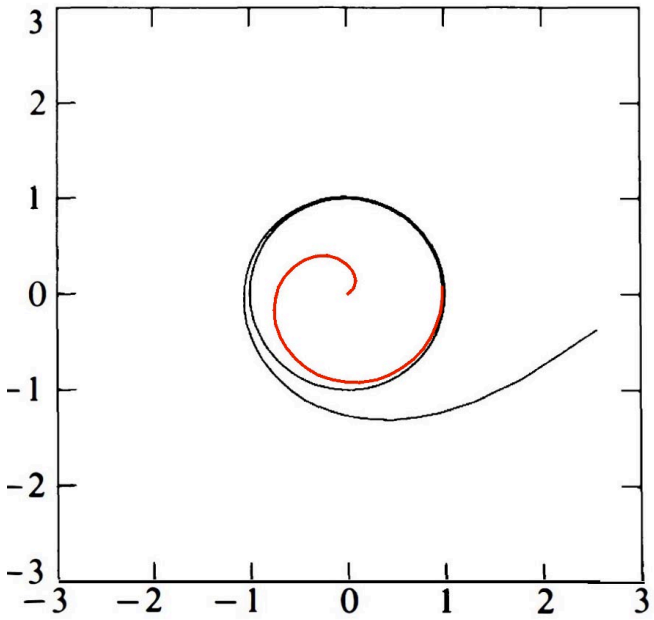
A true "mathematical marvel," these 3D curves—which are "only half-real"—project onto the plane as beautiful, very real, yet "ghostly" curves, spiraling toward "a central singularity."

Thus, when the mathematician and Nobel Prize winner in Physics, S. Chandrasekhar[77], constructs the trajectories followed by masses "penetrating inside the Schwarzschild

[77] S. Chandrasekhar: The Mathematical Theory of Black Holes. Oxford University Press, 1992. http://www.jp-petit.org/books/1992-Chandrasekhar.pdf

sphere" into "the interior of the black hole," spiraling inward, these curves do not belong to the real world! These curves are only… semi-real.

But no one noticed this

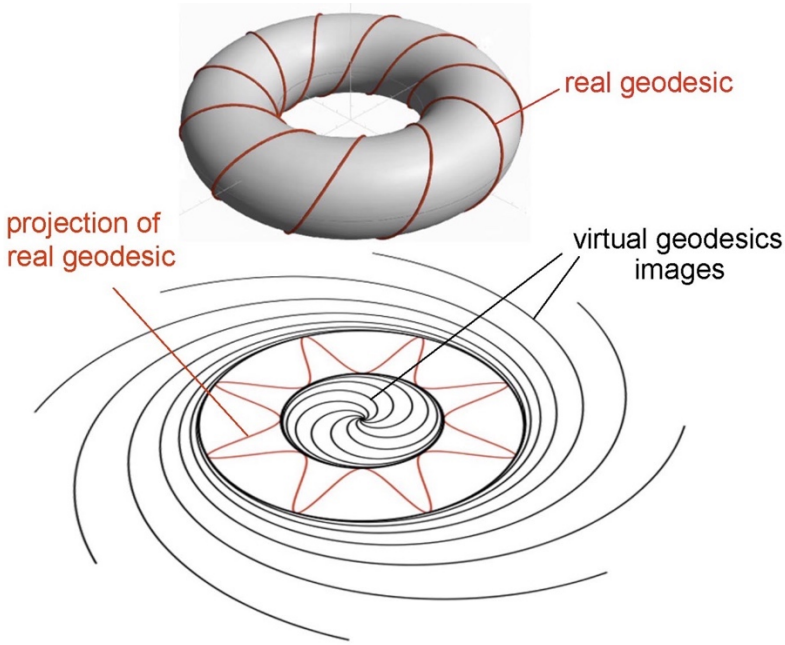


(d) $e = 0.001i$, $\ell = 1$, $M = 0.3$

Chandrasekhar, 1992, The Mathematics of Black Holes.
Trajectory of a falling mass inside a black hole
toward the "central singularity"

To illustrate this "phantom physics," the article[78] constructs projections of these "phantom geodesics of the torus," derived from (Lagrange's) equations—curves that are "half-real, half-imaginary"—which also yield planar projections in the form of curves spiraling until they meet "the axis of the torus."

[78] http://www.jp-petit.org/papers/cosmo/2026-Kruskal-coordinates-semi-complex-spacetime-extension.pdf



The phantom geodesics of the torus

No one noticed anything for 60 years. This led a reviewer, upon first reading the article, to express his skepticism by writing in his initial report:

- *How can one imagine that so many people never noticed this for six decades!*

I would be willing to defend these articles at any seminar[79]. At the moment, my efforts are focused on presenting this to mathematicians specializing in geometry, particularly those versed in Lorentzian geometries, to regain their endorsement, and to supplement these analyses with an article published this time in a … mathematics journal.

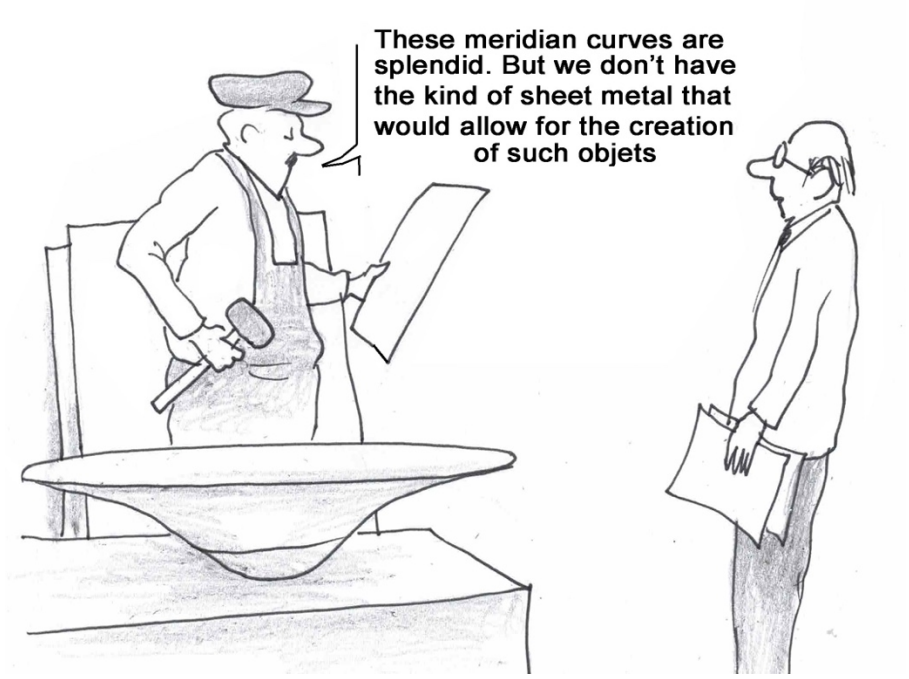
This first aspect deals a devastating blow to that mathematical chimera known as the black hole model. This renders obsolete thousands of articles and hundreds of doctoral dissertations, including the Nobel Prize awarded to the affable English mathematician Roger Penrose "for showing that the black hole model inevitably follows from Einstein's equation, from General Relativity." And, together with Stephen Hawking, he authored theorems about a central singularity that exists only in… their imagination—a world that is purely imaginary.

My latest article features three cartoons, the French versions of which are provided here, and which illustrate the idea:

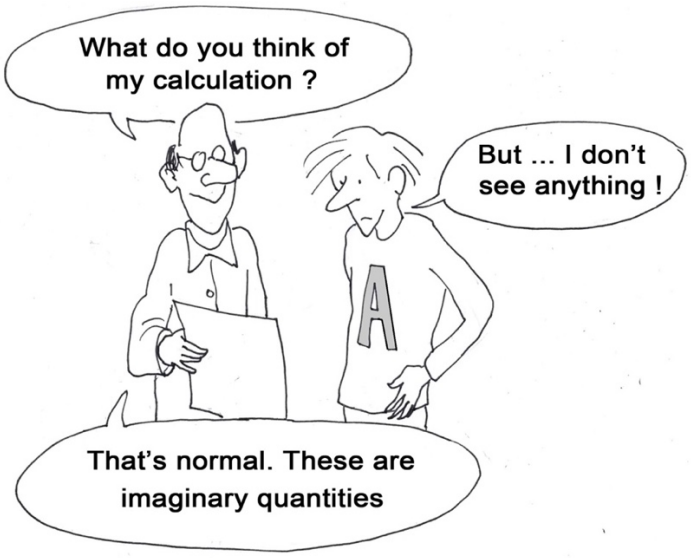
[79] However, in France, the organizers of seminars at leading institutions such as the Paris Institute of Astrophysics, the Paris-Meudon Observatory, the École Normale Supérieure seminar, the Institute for Advanced Scientific Studies, etc., have simply failed to respond to my proposals for seminars.



How to Build a Basket with Imaginary Reeds?



How to create a surface with an imaginary sheet of metal?



An ... imaginary calculation

This chimera known as the Black Hole Model originated in the 1960s and 1970s from a misinterpretation of calculation results by the leading proponents of this new field of knowledge called "cosmology"—primarily Eddington and Tolman—in the English language.

Let's add that only those comfortable with the language of Goethe could venture into the original texts, produced by people who, for the most part, passed away shortly after writing them—generally mathematicians.

Hermann Minkowski, who died in 1909 of peritonitis.

Werner Boy, as early as 1914, killed in the Battle of the Marne.

Karl Schwarzschild, who died on the Russian front in May 1916, three months after his publication

Félix Klein, who died in 1925

None of this would have happened if, in January 1916, Karl Schwarzschild had published his famous solution with its true chrono-polar coordinates t, r, θ, φ :

$$ds^2 = \frac{(r^3+\alpha^3)^{1/3} - \alpha}{(r^3+\alpha^3)^{1/3}} c^2 dt^2 - \frac{r^4\, dr^2}{(r^3+\alpha^3)[(r^3+\alpha^3)^{1/3} - \alpha]} - (r^3+\alpha^3)^{2/3}(d\theta^2 + sin^2\theta d\varphi^2)$$

Schwarzschild's True Solution

This is the first time anywhere in the world that this solution, formulated in this way, has appeared in a scientific article, even though any first-year student could have derived it in just a few lines.

And this 116 years after the January 1916 publication!

What is simply astounding is that no one took it upon themselves to restore this solution to its true, real coordinates. Its signature (+ - - -) would then be "set in stone," unchanging regardless of the (real) values of the four coordinates.

The chimera could never have arisen from such fertile ground.

I have presented the entire derivation, equation by equation, line by line, in the article:

J-P Petit G. D'Agostini: The True Schwarzschild Solution and Its Subsequent Geometry[80]. The Journal of Modern Physics. August 2026. Vol. 17, No. 8.

We saw earlier that the "birth certificate" of the black hole was the 1939 proposal by Oppenheimer and Snyder. In the "standard" presentation of the Schwarzschild solution, the coordinate *t* is merely a means of locating points on the hypersurface. With this arbitrary choice of coordinates, the free-fall and escape times are infinite.

Oppenheimer therefore suggested using this stationary solution to describe a process unfolding over a few… days! The implosion of a massive star. And this is the second key to the black hole model.

Building on analyses that have existed for many years, mathematician Pascal Koiran adapted this stationary solution in spherical symmetry of Einstein's equation to its formulation "in Eddington coordinates." This change of coordinates involves time. Let's call t_S the time coordinate (the "Schwarzschild time") in the "standard form" presentation. We switch to "Eddington time" with the change of variables, where α is the circumference p of the throat sphere, divided by 4π, since the term "radius" no longer makes sense, given that this Schwarzschild sphere no longer has a "center":

$$r = \alpha (1 + ln\, ch\, \rho)$$

The metric becomes:

$$ds^2 = \left(1 - \frac{\alpha}{r} \right) c^2 {dt_E}^2 - \left(1 + \frac{\alpha}{r} \right) dr^2 - \frac{2\alpha c}{r}\, dr\, dt_E - r^2 (d\theta^2 + sin^2\theta d\varphi^2)$$

Eddington introduced this in 1924 to eliminate the coordinate singularity at $r = \alpha$

This introduces a cross-term in $dr\, dt_E$

[80] http://www.jp-petit.org/papers/cosmo/2026-JMP-The-True-Schwarzschild-Solution-and-its-Subsequent-Geometry.pdf

Some sharp minds will tell you that Birkhoff's theorem [81]excludes the presence of this term. This is a misinterpretation of its conclusions. Birkhoff's theorem states that the spherically symmetric solution is automatically invariant under time translation and *can* be expressed in the standard form we are familiar with. Do not confuse *can* with *"must*."

In the form above, the escape time remains infinite, but the free-fall time becomes finite—and brief[82]. Thus, Oppenheimer's hypothesis becomes "coordinate-dependent." In a subsequent article, Koiran and Lévy[83] (me… under a pseudonym) devised a second metric in which the sign of the cross term is reversed; this is also a solution and can be derived by a change of variables applied to the standard form:

$$r = \alpha\,(1 - ln\,ch\,\rho)$$

We obtain:

$$d\bar{s}^2 = \left(1 - \frac{\alpha}{r}\right)c^2 {dt_E}^2 - \left(1 + \frac{\alpha}{r}\right)dr^2 + \frac{2\alpha c}{r}\,dr\,dt_E - r^2(\,d\theta^2 + sin^2\theta d\varphi^2\,)$$

In the inverse situation, the escape time is finite and the free-fall time is infinite. We obtain a

One-way membrane

A passage "through a throat sphere," which can only occur in one direction. We can combine these two metrics into a single one by applying the "magical change of variables" to ρ. This yields the "Petit metric":

$$ds^2 = \left(\frac{ln\,ch\rho}{1 + ln\,ch\rho}\right)c^2 {dt_E}^2 - \left(\frac{2 + ln\,ch\rho}{1 + ln\,ch\rho}\right)\alpha^2 th^2\rho\,d\rho^2 - 2c\alpha\left(\frac{th\rho}{1 + ln\,ch}\right)d\rho dt$$

$$- \alpha^2(1 + ln\,ch\rho)^2(\,d\theta^2 + sin^2\theta d\varphi^2\,)$$

When moving from one sheet to the other, the cross-term changes sign due to the property

$$th\,(-\rho) = -\,th\,\rho$$

At this point, I think the matter is settled. Nothing remains of the chimera. Some will say that the black hole model is not based on the Schwarzschild solution, but on the Kerr metric, which takes rotation into account.

---

[81] G.D. Birkhoff: Relativity and Modern Physics. Cambridge: Harvard University Press, 1923

[82] P. Koiran: Fall Time in the Eddington-Finkelstein Metric, with Application to Eddington-Rosen Bridges. International Journal of Modern Physics D, vol. 30, no. 15, 2150106 (2021) https://arxiv.org/abs/2110.05938

[83] P. Koiran, J-P Levy, F Margnat, M-F Duval, H. Zejli. PT-symmetry in one-way wormholes. Annals of Physics. 470 (2024) 169765

This external metric has a cross-term in $dtd\varphi$. We can modify it using a change of variables with respect to time, introducing a cross-term in *dtdr.* And once again, we will eliminate this marvelous "freeze of time"—the key to the black hole—where the free-fall time is infinite. Rotation will only slightly alter the scenario.

No, invoking this Kerr metric, as Luminet does, will not keep this chimera alive.

How many years will pass before we decide to tear down this house of cards? I won't venture a guess.

One is reminded of the English expression:

- "Too big to fail[84]."

So we have all this research, which illustrates the first sentence heard in Madrid by the technician Dominguez, in response to his question:

- ¿Qué es un agujero negro?[85]

The answer had been:

- "That doesn't exist[86]."

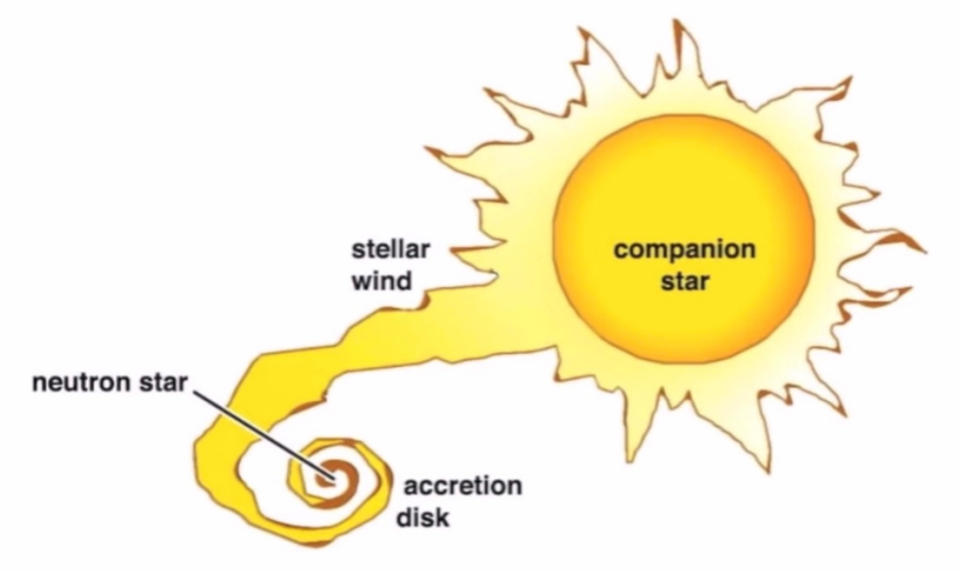
Now we must address the second statement:

- When a neutron star becomes destabilized due to the capture of matter from a companion star, it expels that matter into the twin universe[87].

Let's try to think this through.

The case of the neutron star is interesting because, unlike the problem caused by the collapse of a massive star at the end of its life—which lasts only a few days—a neutron star's progression toward criticality is very slow. A companion star emits only bursts of matter. The Sun emits solar wind. Stars emit stellar wind. There are many "close binaries"—pairs where the two stars are very close to each other.
Reaching criticality can take hundreds of millions of years. But one thing is clear: there are inevitably a significant number of neutron stars that have met this fate—they have reached these critical thresholds.

[84] An expression used to describe companies or banks that are "too big to fail."
[85] What is a black hole?
[86] They don't exist.
[87] "When a neutron star is destabilized by capturing matter sent by a companion star, it gets rid of it by sending it into the twin universe."



A neutron star capturing solar wind from a companion star

We can describe the geometry both outside and inside a neutron star (ignoring rotation) using the two metric solutions published by Karl Schwarzschild in January and February 1916.

Many cosmologists—or at least black hole specialists—are unaware of the content of this second paper[88] and sometimes even of its very existence.

The exterior metric already exhibited a criticality; let's call it

geometric criticality when $r = \alpha\ Rs$

In a neutron star, these are packed tightly together. It is more like a liquid drop—incompressible—than a gaseous mass. We can therefore envision the slow growth of an object in which the density remains at a nearly constant value. The typical value is:

$$\rho = 10^{18}\ \text{kg}/m^3$$

The mass of the star increases as the cube of its radius. Given that the Schwarzschild radius is

$$R_s = \frac{2\,G\,M}{c^2} = \frac{8\,\pi\,G\,\rho}{3\,c^2}\,{R_e}^3$$

The Schwarzschild radius therefore increases as the cube of the R_e star's radius. When this radius equals the star's radius, this corresponds to a mass of:

2.2 to 2.4 solar masses.

---

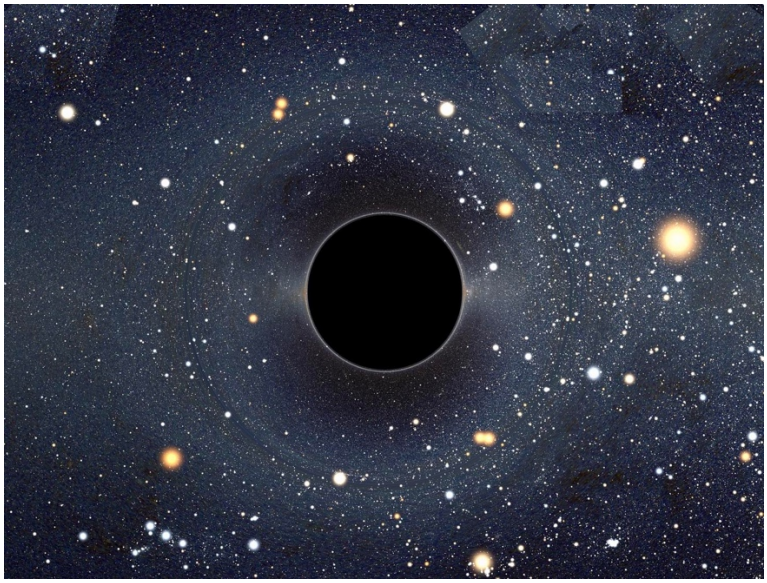
[88] K. Schwarzschild**:** On the Gravitational Field of a Sphere of Incompressible Fluid According to Einstein's Theory. Meeting of the Physical-Mathematical Class on March 23, 1916. English translation by S. Antoci. On the gravitational field of a sphere of incompressible fluid according to Einstein's theory. arXiv:physics/9905030v1 [physics.hist-ph] May 12, 1999.

This corresponds to a radius of 6.8 km for a mass of 2.3 solar masses.

It is estimated that rotation can increase this mass value by 20%, meaning it could likely reach 3 solar masses.

Thus, according to the “Black Hole” scenario, the mass of the neutron star increases slowly. Its central region exhibits a gravitational redshift effect.

The web is full of computer-generated images showing black holes from every angle. Without its accretion disk, a black hole would look like this:



A “naked” black hole

But no one has created all the intermediate images that would depict a “naked” neutron star, devoid of any accretion disk. Assuming rotation is negligible, we would obtain an image (perhaps someone will create it) in which, as the mass increases, the wavelength of the radiation emitted by the central region gradually increases to infinity.

A black hole could be likened to a neutron star exhibiting an infinite gravitational redshift, where the dark region at the center grows increasingly darker to yield the final image shown above: a “naked” neutron star with a mass of 3 solar masses.

In the “black hole” model, any additional mass disappears behind this “event horizon” and falls indefinitely toward a central singularity, located “at the center of the black hole.” This influx of matter would then simply cause the diameter of this “black disk” to increase in proportion to the total mass, since its diameter is then:

$$\phi = \frac{4\,G\,M}{c^2}$$

But is this “realistic”?

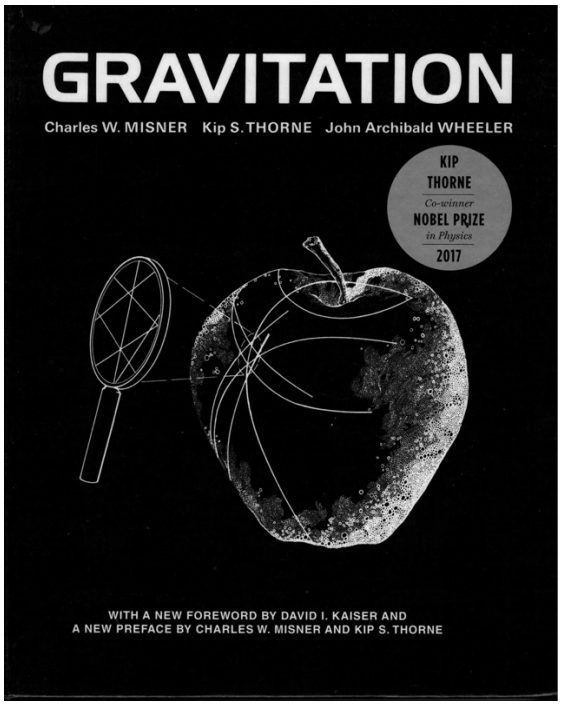
Given that this influx of matter can be considered to occur very gradually, we can regard each state of the object as capable of being described by a stationary geometric solution, with the exterior of the object described by the Schwarzschild metric, a solution to the field equation without a right-hand side:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 0$$

And the interior by the equation with a right-hand side:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

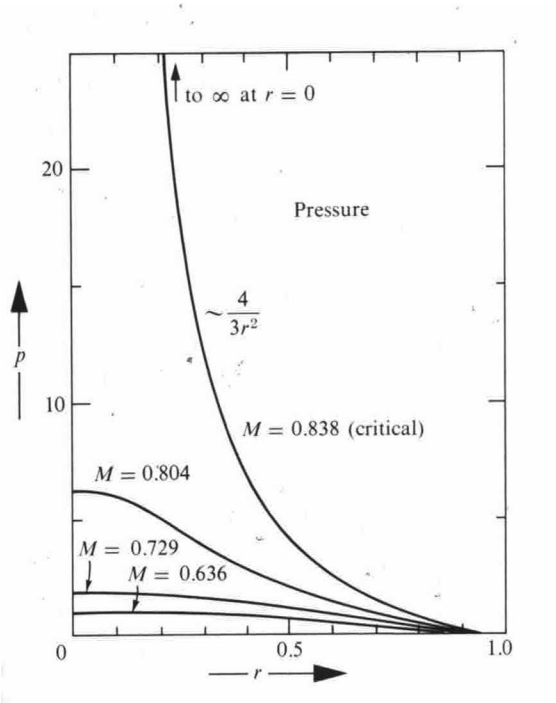
Karl Schwarzschild provided us with this "inner metric" solution as early as February 1916[89], assuming a constant density. In the "Bible" of cosmology, 2017 edition—the book *Gravitation* by J.A. Wheeler, K.S. Thorne, and C.W. Misner



The "Bible" of cosmologists

Here, on page 611, is the evolution of pressure as the mass increases.

[89] K. Schwarzschild**:** On the Gravitational Field of a Sphere of Incompressible Fluid According to Einstein's Theory. Meeting of the Physical-Mathematical Class on March 23, 1916. English translation by S. Antoci. On the gravitational field of a sphere of incompressible fluid according to Einstein's theory. arXiv:physics/9905030v1 [physics.hist-ph] May 12, 1999.



Evolution of pressure at the center of the star as a function of mass M

If the value of the "geometric critical mass" is$M_{cr\ geom}$ taken to be 3 solar masses, the pressure at the center of the star becomes infinite for a value of one

Physical critical mass:

$$M_{cr\ phys} = M_{cr\ geom} \times 0.838 \cong 2.5\ masses\ solaires.$$

For six decades, regardless of our analysis of the interpretation of the so-called "standard" presentation of the Schwarzschild metric, all theorists have remained silent on this point.

This is the application of the principle:

- *What I can't handle, I sweep under the rug*

If we return to this inner metric solution, it tells us how the proper time of a "stationary observer" varies:

$$dr = d\theta = d\varphi = 0$$

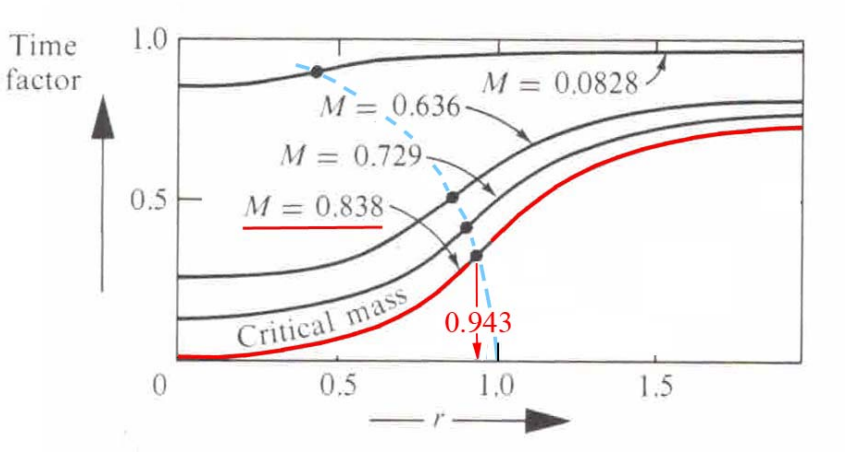
The local value of mass—the intensity of the gravitational field—alters the flow of time. This is a proven fact. We must take this into account to prevent the satellites used by our GPS systems from drifting to the point of rendering these systems unusable. Thus, time passes more slowly on the Earth's surface than in the surrounding space where these satellites orbit.

In the case of the Schwarzschild inner metric solution, not only does time pass more slowly at the surface of the celestial body, but as one moves deeper into its interior, the phenomenon

becomes more pronounced, with the time dilation reaching its maximum at the center. If τ is the proper time of the "observer" located at that point and *t is* the time at infinity, this *time* dilation is expressed as multiplication by a function known as *the time factor*, *f*:

$$d\tau = f(r)\, dt$$

Here is this function $f(r)$, again taken from the book *Gravitation*, same page;



The time factor as a function of the radius t

The blue curve represents the growth of the "Schwarzschild radius" as a function of the reduced mass:

$$M = masse\ /\ M_{cr\ geom} = M/\ 3\ masses\ solaires.$$

And as a function of the reduced radius:

$$rayon\ réduit\ r = rayon\ de\ l'astre/R_{cr\ geom} = rayon\ de\ l'astre/6.8\ km$$

When $M = 1$, the radius $r = 1$.

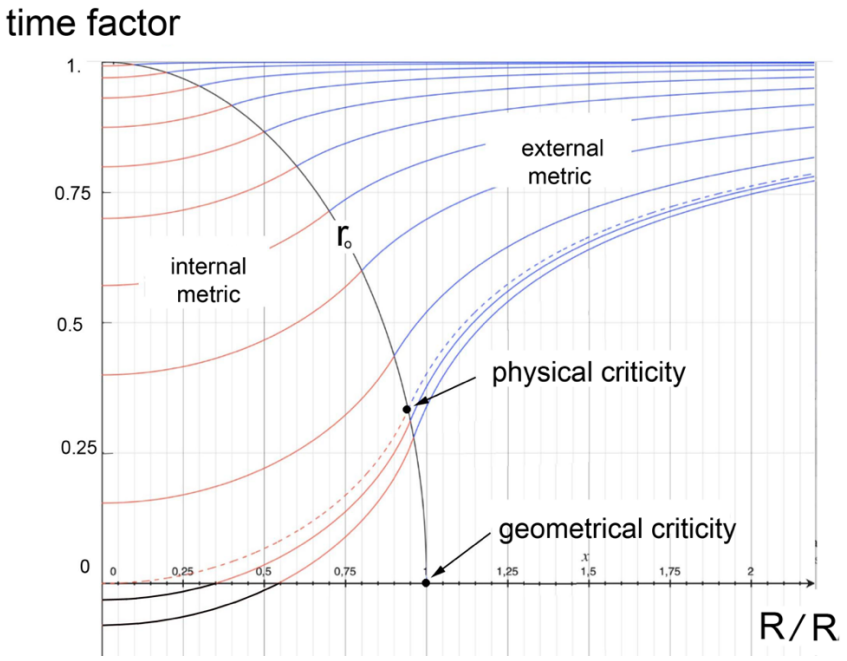
But when $M = M_{cr\ phys} = 0.838\ M_{cr\ geom}$ the time factor f corresponds to the red curve, it vanishes at $r = 0$, at the center of the object.

Proponents of the black hole model have ignored this phenomenon for six decades.

What happens if, considering this internal metric as a "quasi-stationary" solution, we introduce a small perturbation by giving this mass a $M_{cr\ phys}$ very small increase:

$$M = M_{cr\ phys} + \Delta M$$

The function *f* becomes negative in a very small portion of space around the center:



Evolution of the time factor "beyond physical criticality"

At the slightest increase in my mass, this time factor becomes negative in the central part of the star. However, we have:

$$d\tau = f(r)\, dt$$

Proper time is also the way in which the point representing the observer within the geometric object progresses along its "*timeline*." Geometrically, one cannot turn back along a geodesic. This proper time is the time that elapses on "the wristwatch the observer wears on his wrist," whose movement cannot be reversed. That is to say:

$$d\tau > 0$$

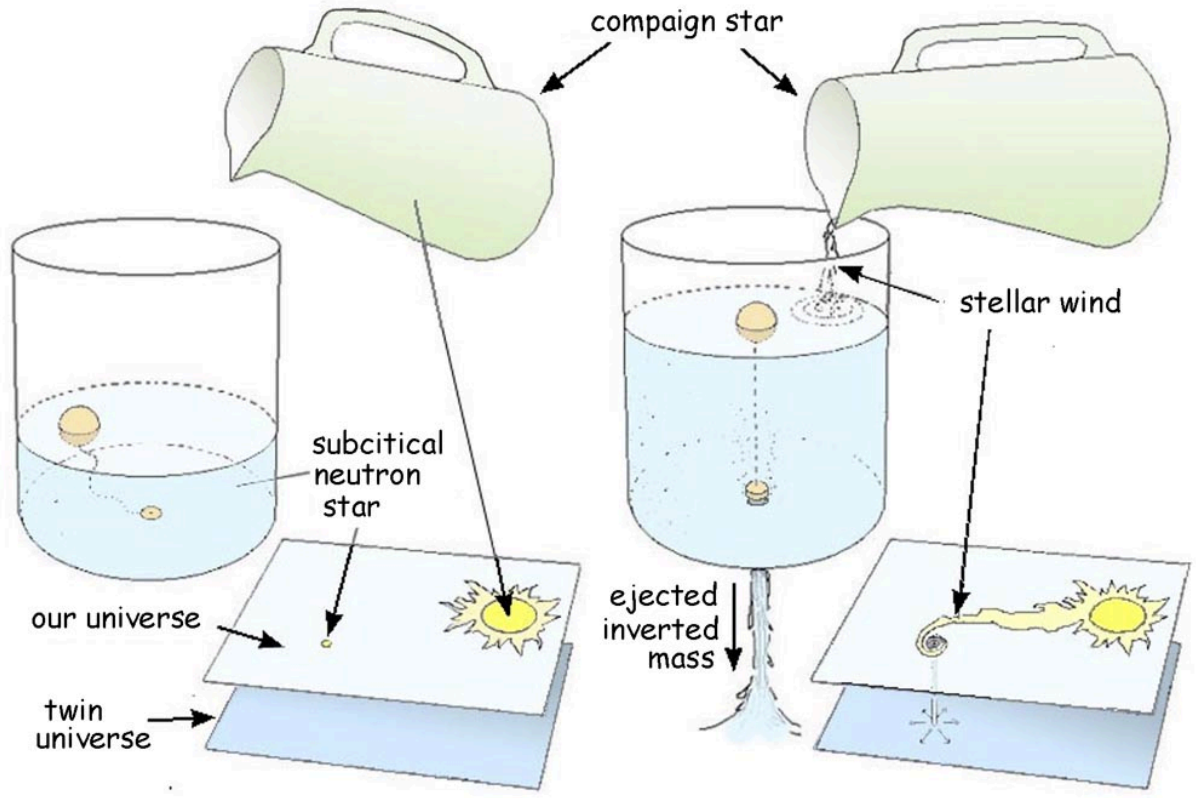
So, if *f* is reversed, it means that in this region the time coordinate *t* is reversed, and we have a local *T-symmetry*.

And here, we return to Souriau's theorem:

$$T - symmetry \quad \Leftrightarrow \quad mass\ and\ energy\ inversion$$

In this central region, the mass of the elements is reversed.

In the framework of general relativity, none of this makes any sense. But within the framework of the Janus Cosmological Model, it makes sense. This inverted mass then immediately ceases$\bar{m} < 0$ to interact with the mass of the neutron star, no matter how dense it may be. Since it is repelled by the positive masses, as soon as it moves even slightly away from the geometric center—where, by symmetry, the gravitational field is zero—it is violently ejected from the star. We thus have a self-stabilizing mechanism reminiscent of the flush mechanism in our toilets:



Educational model of a plugstar.

The drawings speak for themselves without the need for commentary.

So, what should such an object look like? We have seen that physical criticality must result in a maximum gravitational redshift effect. All of this was noted as early as 1959 by H.A. Buchdahl[90]. If the term "Plugstar" is unappealing, it can be replaced with "Buchdahl object," and this limitation of the gravitational redshift to a "Buchdahl effect" is of no consequence.

Regardless of the mechanism by which the light is emitted—which our telescopes would then detect—we would encode this increase in wavelength using colors, ranging from black to light blue.

What our measuring instruments detect (in this case, our radio telescopes operating at millimeter wavelengths: 1.3 mm). At each point on the sensors (equivalent to the cells of a retina), what is detected is an energy density.

What we want to highlight is a variation in the detected energy density emitted by each point of the source—an energy density per steradian. And what interests us is describing the variation in this intensity. We then convert it to "equivalent temperature" or "equivalent wavelength" using Boltzmann's constantk_B , according to:

[90] H.A. Buchdahl, General Relativistic Fluid Spheres, *Physical Review*, Vol. 116, No. 4, November 15, 1959

$$E = k_B T_r = \frac{h\,c}{\lambda}$$

This is the meaning of the coding shown in the images of the two supermassive objects located at the centers of the M87 and Milky Way galaxies.

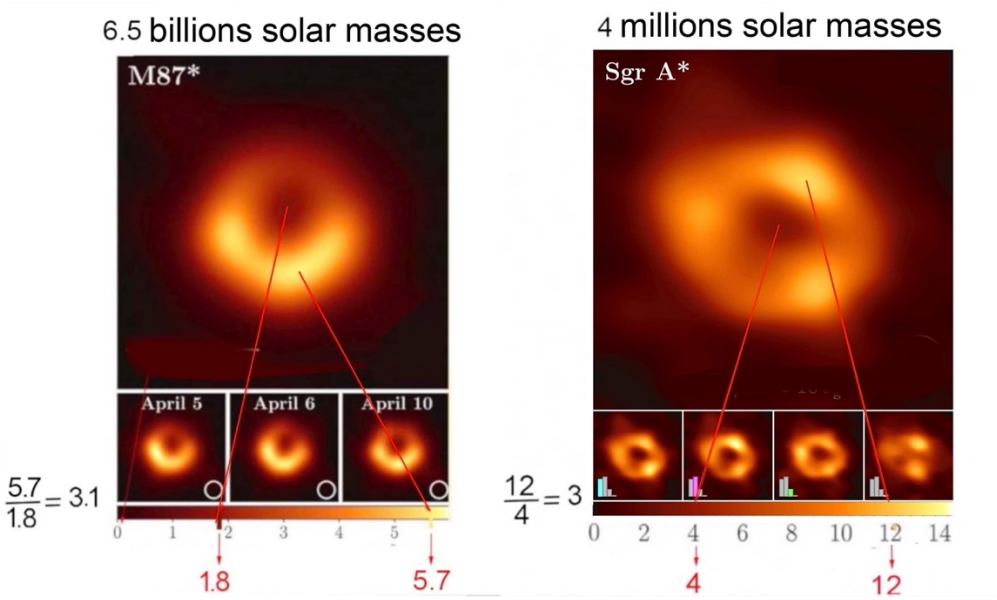


Image of the supermassive objects M87[91] and SgrA*[92]*

The human eye is precise enough to assess color ranges. If we take a small square from these images and compare it to the data from the color bar, we obtain—not a temperature—but the ratio of the maximum to the minimum temperature. And in both cases, these ratios are practically equal to 3.

This in no way means that this light is emitted by a source with that temperature. It means that we are receiving radiation with a bandwidth centered around a wavelength of 1.3 mm, which corresponds to a frequency of 230 GHz. However, the temperature indicated on the bar cannot be derived from:

$$T = \frac{h\,\nu}{k_B} = 11°\,K$$

It is a "luminous temperature" given by the Rayleigh-Jeans law:

---

[91] First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. K. Akiyama, A. Alberdi, W. Alef, K. Asada, R. Azulay, A. K. Bcako, D. Ball, M. Balakovic, J. Barrett. Astrophysical Journal, vol. 875, no. 1, 2019

[92] First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole at the Center of the Milky Way. K. Akiyama, A. Alberdi, W. Alef, J. C. Algaba, E. Anantua, K. Asada, R. Azulay, U. Bach, A. K. Backo. The Astrophysical Journal Letters, vol. 930, no. 2, 2022. DOI 10.3847/2041-8213/ac6674. April 10, 2019.

$$T_b = \frac{c^2 I_\nu}{2\, k_B \nu^2}$$

Thus, the figures shown tell us nothing about the actual temperature of the source, nor about the mechanism of radiation emission. It is a measurement of the light intensity at this frequency of 230 GHz.

That said, commentators point out that this radiation is emitted via the synchrotron effect. However, regardless of how this radiation is emitted, it undergoes a gravitational redshift, and we must therefore conclude that this corresponds to the "Buchdahl effect" at this value of 3. Thus, these objects could be Plugstars.

Proponents of the giant black hole model suggest that the radiation emitted by the central regions results from the presence of a cloud of hot gas located in front of these objects.

But what a happy coincidence! The luminosity temperature of this hot gas cloud yields exactly this value of 3 in both cases, even though these two objects differ in mass by a factor of 1,600!

All of this can be found in this article, published in 2025

J.P. Petit, G. d'Agostini. Alternative to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*. Vol. 16, No. 10. October 2025. 497 views. 3,610 downloads[93]

In France, retired astrophysicist Jean-Pierre Luminet—who has not produced a single scientific contribution in the form of a peer-reviewed journal article in 20 years, but has authored numerous popular science books, particularly on black holes—has just published an article[94] on arXiv in which he attempts to play the role of "cosmology's firefighter," doing his best to discredit our work on the Janus model.

Since his article was posted in a section (gr-qc): general relativity and quantum cosmology—which is heavily visited by theorists (but where I have been blacklisted)—I feel compelled to offer a few comments.

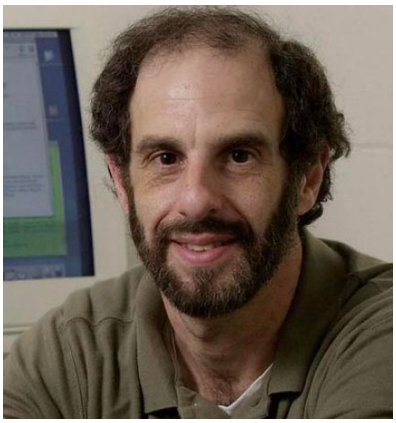
Censorship on arXiv

The website www.arxiv.org

The founder: Paul Guinsparg, a theoretical physicist at the Los Alamos National Laboratory.

[93] http://www.jp-petit.org/papers/cosmo/2025-10-14-JMP-Alternative-to-Black-Holes-Gravastars-and-Plugstars.pdf

[94] http://www.jp-petit.org/papers/cosmo/2026-Luminet-bimetric-arxiv.pdf



Paul Guinsparg, theoretical physicist
Los Alamos National Laboratory.

In 1991, he created:

xxx.lanl.gov.

The domain is a .gov (meaning it was created with government approval).

Goal: to accelerate the dissemination of scientific information before it is published in journals.

Ginsparg had anticipated about a hundred manuscripts, which would remain available for three months.

But the value of the platform became apparent immediately. Ginsparg received 400 submissions during the first six months, and the authors insisted that these articles remain available indefinitely.

The name **arXiv** was adopted in late 1998**. In 2001,** the server was transferred to **Cornell University**, where Ginsparg had been hired.

To prevent an influx of "inappropriate" submissions, an **endorsement**[95] system was introduced in **2004,** and the name **arXiv** was retained.

On July 1, 2026, arXiv became an **independent nonprofit organization**, after twenty-five years under Cornell's stewardship.

By June 2026, the archive had surpassed three million manuscripts, with approximately 27,500 new submissions per month since the beginning of the year

[95] I was "endorsed" in the early 2000s, in the (gr-qc) section, by the Italian Christian Corda, who subsequently lost his ability to sponsor others for having allowed me to set foot in this "sanctuary."

In 2023, arXiv added this clause to its bylaws:

arXiv reserves the right to reject manuscripts at its sole discretion

The official policy states: “arXiv, in its sole discretion, may decline to post works submitted to the platform.” This provision explicitly grants arXiv the discretionary authority to reject submitted manuscripts.

Publication in a peer-reviewed journal does not guarantee acceptance by arXiv

The policy further states:

“Some works that have been published or accepted by a journal may still be declined at arXiv’s discretion.”

Consequently, a manuscript that has successfully undergone peer review and has been accepted or published in an established scientific journal may nevertheless be rejected by arXiv.

The platform therefore does not consider acceptance by a scientific journal to be sufficient grounds for inclusion in its repository.

arXiv moderation is not scientific peer review

arXiv explicitly distinguishes its moderation procedures from conventional peer review.

Its moderators evaluate whether submissions meet the platform’s requirements, including relevance, scientific presentation, originality, and suitability for the selected subject category.

However, moderation does not constitute a formal scientific review process.

This distinction is important because an arXiv rejection should not automatically be interpreted as a scientific refutation of the manuscript’s arguments or conclusions.

Grounds for Rejection

The moderation policy lists several possible grounds for rejecting or reclassifying manuscripts, including:

- Failure to comply with accepted standards of scientific presentation.
- Insufficient originality, novelty, or scientific interest.
- Manuscripts deemed to require substantial revision.
- Submissions lacking substantial original research.
- Manuscripts deemed inappropriate for the selected subject category.

- Repeated submissions or copyright-related issues.

This policy does not present these grounds as an exhaustive list.

We are allowed two submissions per month. The following two:

Article Submissions — Update | Delete | Unsubmit

Identifier	Type	Title	Status	Actions	Expires
submit/8357021	New	Is the Real World, as a Part of Complex Reality	on hold		
submit/8357190	New	The True Schwarzschild Solution and its Subsequent Geometry	on hold		

START NEW SUBMISSION — You have reached your monthly limit. You may start and work on a new submission but cannot submit until next month.

On arXiv, papers immediately blocked and canceled

Met the same fate immediately. Here is the standard rejection message for papers published in peer-reviewed journals:

MOD-113982 arXiv - important notification regarding submit/8308519

(Alternative to Black Hole: Gravastar and Plugstars)

J — jira@arxiv-org.atlassian.net
À : moi · mar. 10 nov. à 16:00

Dear author,

Thank you for submitting your work to arXiv. We regret to inform you that arXiv's moderators have determined that your submission will not be accepted and made public on http://arxiv.org[arXiv.org |http://arxiv.org].

While we acknowledge that this article has been published, our moderators determined it is not of plausible interest for inclusion within arXiv. As a result, this submission has been declined.

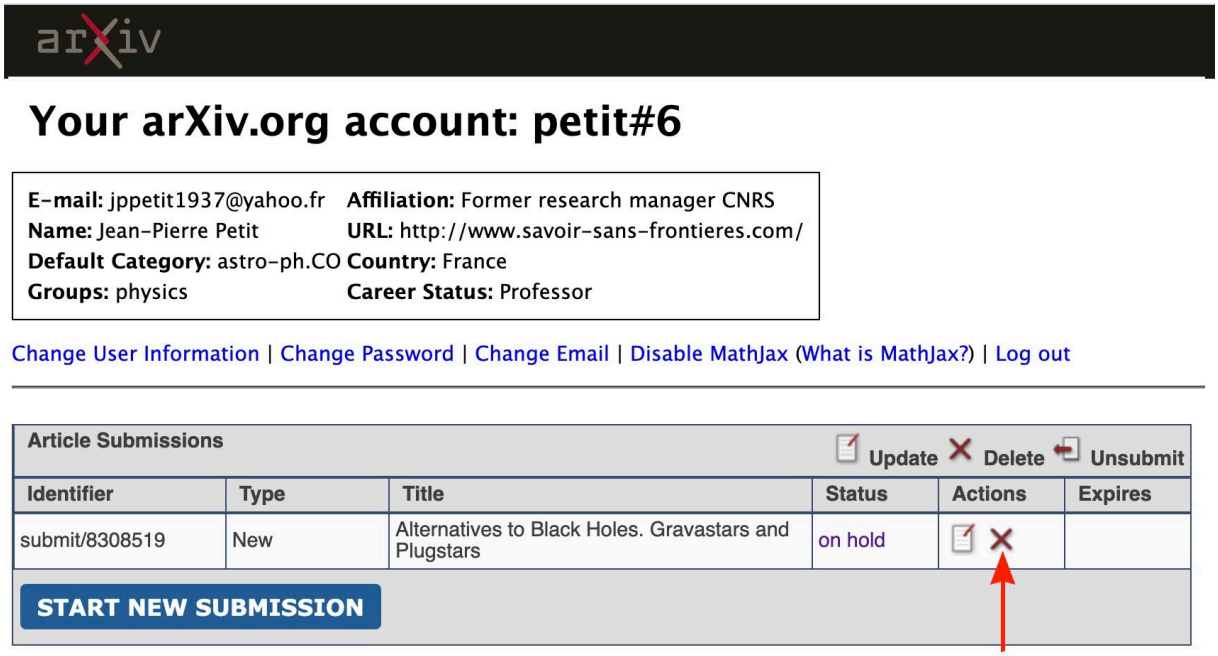
arXiv, rejection message

For French speakers, highlighted in red:

Although we acknowledge the publication of this article, **our moderators determined that it did not present a public interest sufficient to justify its inclusion in arXiv.** Consequently, the submission was rejected.

November 13, update: two new articles—submitted, blocked, rejected, and deleted.

Quite expeditious….

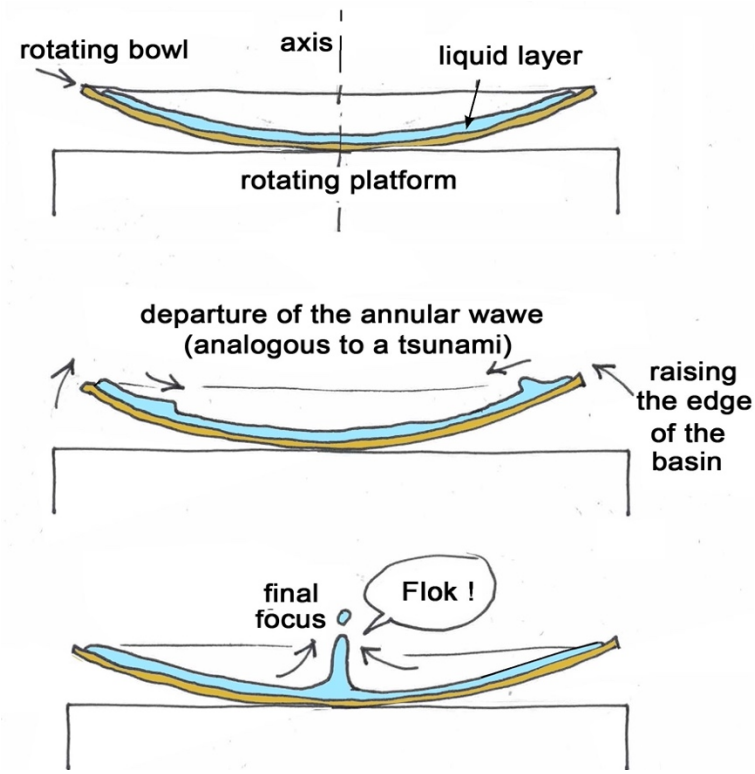


207 *arXiv completely erases all traces of the submission*

The article, submitted on October 31, 2026, to the arXiv platform, was deleted by the "moderators"
Regarding our Plugstars model, Luminet criticizes the use of a spherically symmetric solution, noting that this object—formed by accretion through successive mergers of increasingly massive black holes—should rotate on its own axis.

As mentioned in our article, we believe that these objects are the result of joint metric fluctuations, a phenomenon we are attempting to model based on the Janus equations. In doing so, we draw on the idea—present in the Ummo texts (we'll need to locate the exact source)—that the expansion of the universe underwent a highly turbulent early phase.

From this perspective, the formation of such objects would have occurred very long ago and would be the result of the focusing of a density wave. The illustrative image is as follows:



Simulation of the Quasar Phenomenon

Imagine a rotating basin where the slope creates a gravitational force that, by counterbalancing the centrifugal force at every point, allows a layer of water of nearly constant depth to settle in contact with the basin. This tends to represent the nearly constant density distribution of interstellar gas within a galaxy's gravitational field—a field created primarily by the surrounding negative mass.

Two scenarios arise.

- Either this gravitational field is weakened. The rim of the basin is lowered, and the water spills out. This simulates an irregular galaxy.

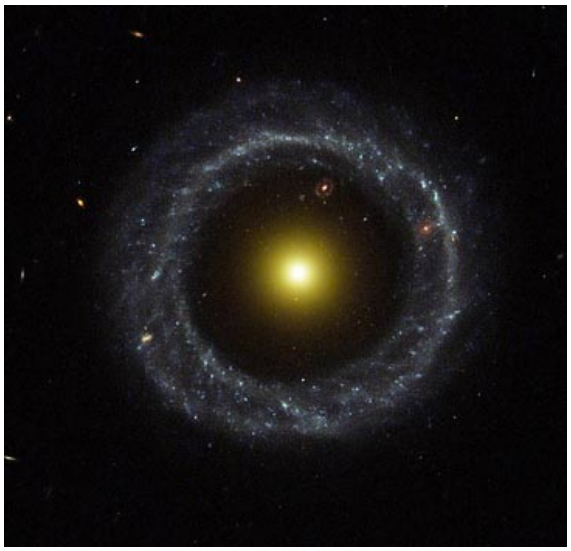


Irregular Galaxy

In the Ummo letters, the authors state that, upon coming across Halton[96] Aarp's systematic observations of irregular galaxies, they sent him a report indicating that he had identified a key observation for understanding the phenomena at work in the cosmos and that this revealed the interaction between our universe and its twin. They were taken aback when he simply tossed the report into the trash.

- In the second scenario, it is the confinement field that is strengthened. This is simulated by raising the rim of the basin. This creates a circular liquid wave—in every respect similar to a tsunami—which then propagates toward the galactic center.

In the article on plugstars, we suggested that this might correspond to Hoag's Galaxy:



Hoag's[97] Galaxy

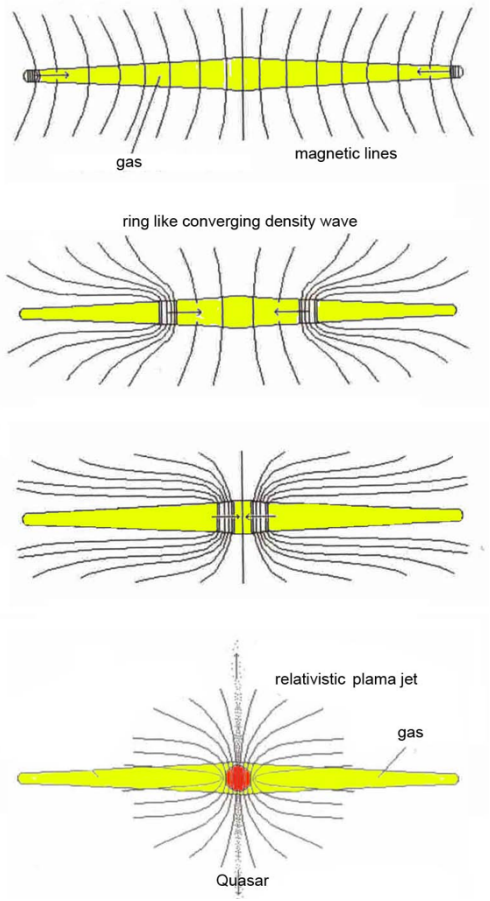
[96]Arp, H. C. (1966). Atlas of Peculiar Galaxies. The Astrophysical Journal Supplement Series, vol. 14, no. 123, pp. 1–20. DOI: https://doi.org/10.1086/190147

[97] Hoag, A. A. (1950). A Peculiar Object in Serpens. The Astronomical Journal, vol. 55, p. 170. DOI: https://doi.org/10.1086/106427

When the wave reaches the center, it causes a significant spike in density. The temperature also rises, and Lawson's conditions (for fusion) are met throughout the entire mass, leading to a large release of energy through thermonuclear reactions.

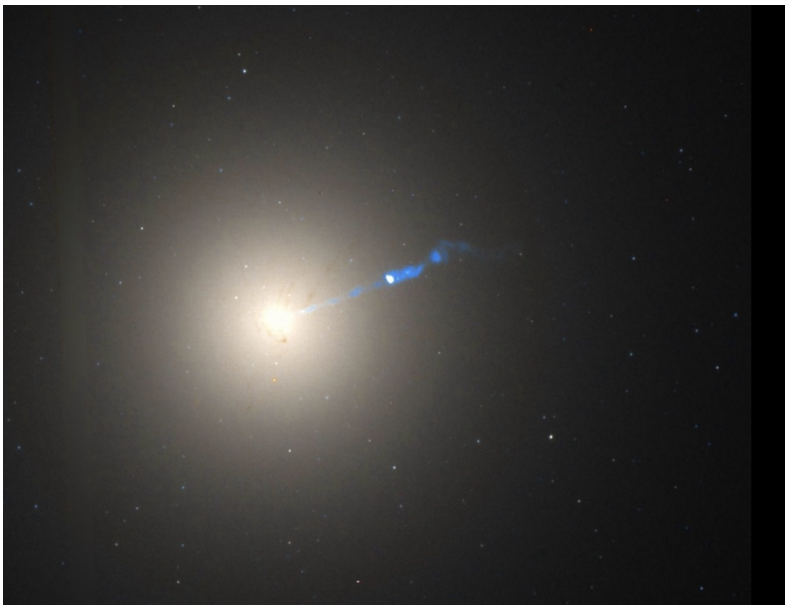
As the density wave travels through the galaxy's gas, it triggers the formation of new stars, which, by emitting in the UV, ionize the gas (as in the spiral arms). This results in conditions of high magnetic Reynolds number. The field lines of the weak magnetic field present in the galaxy are trapped within the plasma (*frozen in*) and gathered together much as a harvester would gather ears of wheat. The conservation of magnetic flux converts this very weak pre-existing field (one microgauss) into an extremely high dipole field.

This dipole field channels the plasma emissions into two jets, resulting in a quasar.



Creation of a powerful dipole magnetic field.

This emission phenomenon can cease. The supermassive object at the center of the Milky Way can thus be described as an "extinct quasar." But these bursts can continue periodically, producing a "dotted" jet. An example is the phenomenon produced by the galaxy M87:



The fragmented jet of the M87 galaxy

This jet undergoes a “blue shift” because it is directed toward the observer. The second jet is not visible because it is “red-shifted.” Below is an artist’s rendering:



M87, artist’s rendering

These quasars are located at the centers of galaxies. The fact that, statistically, their jets are oriented perpendicular to the plane of the galaxies that host them contradicts the theory of their formation by accretion (in which case the jets would have random directions). On the contrary, this aspect supports the idea of formation via a density wave.

When the density wave reaches the center of the galaxy, it causes a sudden increase in density but does not imply the conservation of angular momentum. This is why the metric solution in 0(3) symmetry is well-suited to describing the phenomenon.

This makes the plusgstar model falsifiable in the sense that it predicts that the ratio z = 2 will be present in all future images, regardless of the mass of the objects.

The conclusion is that in the article we reproduce below, the second sentence received by Dominguez in 1988 supports the idea that this interlocutor was indeed an extraterrestrial; the probability that a prankster—or even an Earth-based scientist—could have made such a statement seems highly unlikely.

Scientific Research Publishing

Journal of Modern Physics, 2025, 16(10), 1479-1490
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Alternatives to Black Holes: Gravastars and Plugstars

Jean-Pierre Petit, Gilles D'Agostini

Manaty Laboratory, Dijon, France
Email: jean-pierre.petit@manaty.net, gilles.dagostini@manaty.net

How to cite this paper: Petit, J.-P. and D'Agostini, G. (2025) Alternatives to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*, **16**, 1479-1490.
https://doi.org/10.4236/jmp.2025.1610072

Received: July 23, 2025
Accepted: October 21, 2025
Published: October 24, 2025

Copyright © 2025 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

The images collected by the EHT of objects M87* and SgrA* show darkened, but not black areas. So that the immediate identification with supermassive black holes remains questionable. Attention is thus focused on the main alternative candidates: gravastars, which have attracted a great deal of attention for decades. However, none of these models provides any evidence that can successfully confront these observational data. We present an interpretation derived directly from Schwarzschild's inner geometry, in the subcritical case where the intense pressure gradient counterbalances the force of gravity. The object then exhibits a gravitational redshift effect leading to a wavelength ratio of 3, which is remarkably consistent with image analysis. A mechanism is proposed to ensure the self-stability of such objects, which are then referred to as plugstars.

Keywords

Black Hole, Quasar, Janus Cosmological Model

1. Introduction: Alternative to Black Holes

Recent observational data from EHT ref. [1] [2], do not coincide with the expected images of hypermassive black holes. Their central parts, while dark, are not black. Attention was then turned to alternative models, mainly the various models of gravastar, objects hypothesized in a 2001 paper by Pawel O. Mazur and Emil Mottola ref. [3]-[5] as an alternative to the black hole theory. It has the usual black hole metric outside of the horizon, but de Sitter metric inside. On the horizon there is a thin shell of matter. This solution to the Einstein equations is stable and has no singularities. In their original formulation, a gravastar is composed of three regions, differentiated by the relationship between pressure p and energy

density ρ. The central region consists of false vacuum or "dark energy", and in this region $p=-\rho$. Surrounding it is a thin shell of perfect fluid f where $p=\rho$. On the exterior is true vacuum, where $p=\rho=0$. The dark-energy-like behavior of the inner region prevents collapse to a singularity, and the presence of the thin shell prevents the formation of an event horizon, avoiding the infinite blue shift. The inner region has thermodynamically no entropy and may be thought of as a gravitational Bose-Einstein condensate. Severe red-shifting of photons as they climb out of the gravity well would make the fluid shell also seem very cold, almost absolute zero. In addition to the original thin-shell formulation, gravastars with continuous pressure have been proposed. These objects must contain anisotropic stress ref. [6]. Externally, a gravastar appears similar to a black hole: it is visible by the high-energy radiation it emits while consuming matter, and by the Hawking radiation it creates. Astronomers search the sky for X-rays emitted by infalling matter to detect black holes. A gravastar would produce an identical signature. It is also possible, if the thin shell is transparent to radiation, that gravastars may be distinguished from ordinary black holes by different gravitational lensing properties, as null geodesics may pass through ref. [7]. Mazur and Mottola suggest that the violent creation of a gravastar might be an explanation for the origin of our universe and many other universes because all the matter from a collapsing star would implode "through" the central hole and explode into a new dimension and expand forever, which would be consistent with the current theories regarding the Big Bang ref. [8]. This "new dimension" exerts an outward pressure on the Bose-Einstein condensate layer and prevents it from collapsing further. Gravastars also could provide a mechanism for describing how dark energy accelerates the expansion of the universe. One possible hypothesis uses Hawking radiation as a means to exchange energy between the "parent" universe and the "child" universe, and so cause the rate of expansion to accelerate, but this area is under much speculation. Anyway gravastars have been considered as a serious alternative to black holes in the past decades. Stable models of gravastar have been constructed in many of the alternate gravity modelsbesides standard General Relativity. The Randall-Sundrum braneworld model ref. [9] has been apopular alternative to GR, specially in the cosmological and astrophysical context. In ref. [10] gravastar is considered in RS brane gravity. In ref. [11] the model includes the impact of Kuchowicz metric function. In ref. [12] an extension to Gauss-Bonnet gravity. In ref. [13] the collapase of gravastar is driven by a scalar field, taking account of back-reaction effects. In ref. [14] the authors suggest how to detect the gravastar shadow and how one can distinguish from the black hole event horizon while in ref. [15] various electrically charge gravastars configurations are explored. For recent works, see also references [16] [17].

However, to date there has been no successful comparison of data from a gravastar model with the only observational data available for the supermassive objects M87* and SgrA*. In what follows, we present a model, based on Schwazrschild's inner metric solution, which offers remarkable agreement with these data

as they emerge from the current data processing work carried out by the Event Horizon Telescope teams ref. [1] [2].

2. The Schwarzschild model for an Incompressible Fluid Star

In his article from February 1916 ref. [18] Karl Schwarzschild constructed a metric describing the geometry inside a sphere filled with an incompressible fluid. We reproduce this expression exactly as it appears in Equation (35) of the original German edition:

$$\mathrm{d}s^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 \mathrm{d}t^2 - \frac{3}{\kappa\rho_0}\left(\mathrm{d}\chi^2 + \sin^2\chi \mathrm{d}\vartheta^2 + \sin^2\chi \sin^2\vartheta \mathrm{d}\phi^2\right) \quad (1)$$

Schwarzschild takes the value c of the speed of light in vacuum to be equal to 1. ρ_0 is the density, which is constant inside the sphere. We have:

$$\kappa = 8\pi G \quad (2)$$

Angles (χ, ϑ, ϕ) are classical spherical coordinates. The characteristic length is given by expression:

$$\hat{R}^2 = \frac{3c^2}{8\pi G\rho_0} \quad (3)$$

We can pass from the angular coordinate to the radial coordinate r, and Euclidean norm of the space vector (x, y, z) as

$$r = \hat{R}\sin\chi \quad (4)$$

The radius of the star is given by:

$$R^* = \hat{R}\sin\chi_a \quad (5)$$

Schwarzschild denotes by f_4 the term g_{tt} of the metric

$$f_4 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 \quad (6)$$

The time factor is an expression corresponding to a stationary observer located inside the sphere.

$$f = \frac{3\cos\chi_a - \cos\chi}{2} \quad (7)$$

Then we have

$$\mathrm{d}s = f\mathrm{d}t \quad (8)$$

Schwarzschild then obtained (Equation (10)) as the way in which the pressure inside the sphere varies.

$$(\rho_0 + p)\sqrt{f_4} = konst. = \gamma \quad (9)$$

In this parenthesis, Schwarzschild sums the constant energy density $\rho_0 c^2$ (with $c = 1$) and the pressure. We can consider that the matter at the centre of a star, when the energy density becomes very high, is in the form of radiation, with a radiation pressure:

$$p = \frac{\rho_0 c^2}{3} \tag{10}$$

The modern form of this internal metric ref. [19] is:

$$\mathrm{d}s^2 = \left(\frac{3}{2}\sqrt{1-\frac{R^{*2}}{\hat{R}^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{\hat{R}^2}} \right)^2 \mathrm{d}t^2 - \left(1-\frac{r^2}{\hat{R}^2}\right)^{-1} \mathrm{d}r^2 - r^2 \mathrm{d}\Omega^2 \tag{11}$$

The term $f_4 = g_{tt}$ becomes zero when $\cos\chi_a = 1/3$, *i.e.* when

$$R^* = R_{crit,phys} = \sqrt{\frac{8}{9}}\hat{R} = \frac{c}{3\pi G\rho_0} \tag{12}$$

According to relation (9), as Schwarzschild notes, the pressure at the centre of the star becomes infinite. We are therefore faced with a first physical singularity. As the star's mass increases, at constant density (which could describe a neutron star where the centrifugal force linked to rotation could be neglected), this physical singularity occurs before a geometric singularity. This last occurs when the star's radius identifies with its Schwarzschild radius, *i.e.* when:

$$R^* = R_{crit,geom} = \sqrt{\frac{3c^2}{8\pi G\rho_0}} \tag{13}$$

The two critical radii are linked by the relation:

$$R_{crit,phys} = \sqrt{\frac{8}{9}} R_{crit,geom} \tag{14}$$

We can consider that this situation does not belong to the physical world ref.[20], which amounts to considering that this exact solution to Einstein's equation, independent of time, does not belong to physics, by invoking a "peed of sound"? varying as the square root of the pressure gradient. But if we consider that the medium can be assimilated to radiation, then this speed of propagation becomes that of light. This leads us to consider, as Schwarzschild was the first to do, that the speed of light varies, and can reach an infinite value at the centre of the star. He gives the law of variation of this speed of light in his equation (44):

Die Lichtgeschwindigeit in unserer Kugel

(The speed of light in our sphere)

$$V = \frac{2}{3\cos\chi_a - \cos\chi} \tag{15}$$

This fraction must be multiplied by the value c of the speed of light in a vacuum:

$$V = \frac{2c}{3\cos\chi_a - \cos\chi} \tag{16}$$

This variation in the speed of light when matter reaches conditions of extreme density is considered in ref. [21] [22]. Unlike other attempts at modeling c variable, this model, based on a generalized gauge relation, preserves Lorentz invariance and the fine structure constant. If we assume that the speed of light can

take on a value, if not infinite, then at least very high, at the centre of the object, we obtain a situation where the force of pressure balances the force of gravity. All that remains is to examine the stability of such an object.

What happens when it benefits from an influx of matter, which would be the case for a subcritical neutron star receiving matter from a nearby standard star?

If we rely on the mathematical solution, the time factor f then becomes negative, but along a geodesic it is impossible to turn back:

$$\mathrm{d}s > 0 \Rightarrow f\mathrm{d}t > 0 \tag{17}$$

In this central part of the object, the time coordinate t reverses. Following the dynamic group theory of mathematician J.M.Souriau (ref. [23] [24]), this inversion of the time coordinate implies the inversion of energy and mass.

General relativity opposes the presence of negative mass in the universe ref. [25], because of the unmanageable runaway effect. To go a step further, we need to place ourselves in the geometric framework of the Janus model ref. [26].

In this new scheme, masses of opposite signs repel each other. We also conjecture that a gravitational and quantum phenomenon occurs under these conditions of criticality, in which the excess masses are reversed.

The object in criticality, having received mass from the outside, contains an equivalent quantity in a small sphere filled with negative mass and initially centered on the origin.

The presence of negative mass at the star's center only makes a very weak contribution to the gravitational field mainly created by the positive mass. Thus negative mass is expelled from the object and then expelled from the galaxy into the negative-mass environment between galaxies.

We thus obtain a mechanism that guarantees the stability of such subcritical objects, which we propose to call "plugstars"? like an overflow valve that evacuates excess water.

3. Calculation of the Gravitational Redshift

Such objects will exhibit a gravitational redshift effect with respect to a distant observer. This corresponds to:

$$1+z = \frac{\lambda(\text{observer})}{\lambda(\text{emitter})} = \frac{\sqrt{g_{tt}(\text{observer})}}{\sqrt{g_{tt}(R^*)}} = \frac{1}{f(R^*)} \tag{18}$$

In a situation close to criticality:

$$f = \frac{3}{2}\sqrt{1-\frac{8}{9}} - \frac{1}{2}\sqrt{1-\frac{8}{9}} = \frac{1}{3} \tag{19}$$

An observer will therefore see an object whose central part has an attenuated luminosity, such that the ratio of wavelengths resulting from a gravitational redshift effect is

$$\frac{\lambda(\text{observer})}{\lambda(\text{emitter})} = 3 \tag{20}$$

4. Observational Data

Let us refer to the images of supermassive objects obtained using the EHT Collaboration (ref. [1] [2]). The chromatic bars provide equivalent temperature values, which are in fact indications of luminosity (Figure 1 and Figure 2). This light, in any case, is emitted by photons of energy $h\nu$. By forming the ratio of maximum and minimum luminosity, we obtain the wavelength ratio.

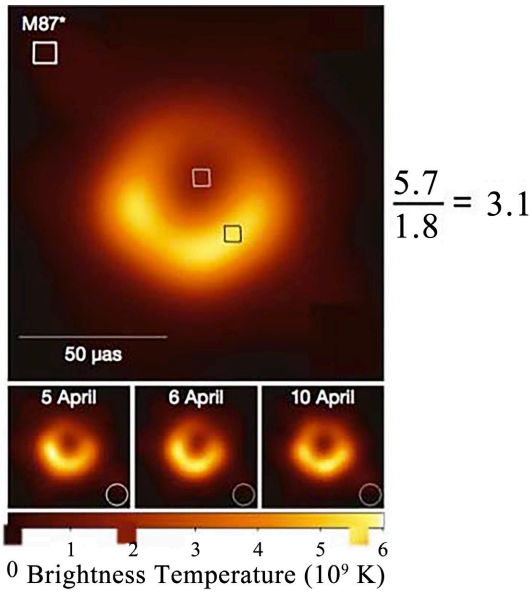


Figure 1. Images of M87*.

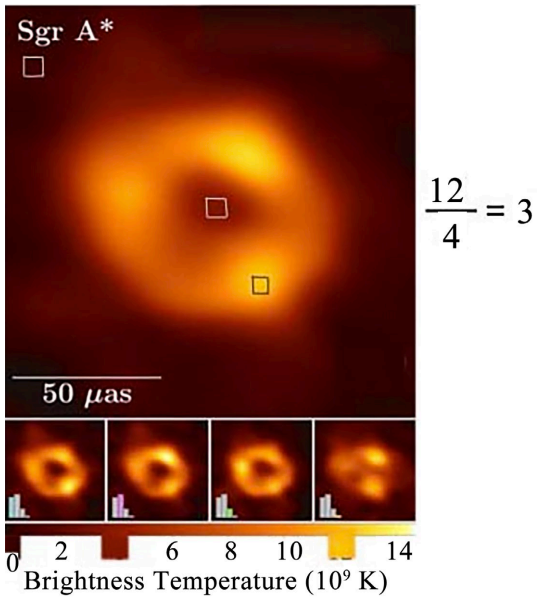


Figure 2. Images of SgrA*.

In the image of the hypermassive object at the centre of M87, the colour bar shows that:

- The maximum equivalent temperature is 5.7×10^9 K.
- The minimum equivalent temperature is 1.8×10^9 K.

The corresponding wavelength ratio is:

$$\frac{\lambda(\text{observer})}{\lambda(\text{emitter})} \approx 3.2 \pm 0.3 \tag{21}$$

On the image of the hypermassive object at the centre of SgrA* the colour bar shows that:

- The maximum equivalent temperature, approximately, is 12×10^9 K.
- The minimum equivalent temperature is 4×10^9 K.

The corresponding wavelength ratio is:

$$\frac{\lambda(\text{observer})}{\lambda(\text{emitter})} \approx 3.0 \pm 0.3 \tag{22}$$

This result is in very agreement with observational data.

Of course, these central objects are not giant neutron stars. But the very high pressure inside them, due to the increase in the speed of light, allows the radiation pressure to counterbalance the force of gravity.

5. Physical Implications of Plugstars

Unlike black holes, these objects:

- have a finite gravitationnal redshift, explaining the darkening of their central part.
- have no horizon or internal singularity.
- are stabilized by the masse inversion processes, in the case of the Janus model, preventing collapse.

When the axes of symmetry of the objects are accessed through the axes of symmetry of the jets, they often correspond to the axes of symmetry of the galaxies ref.[27]. If this phenomenon is general, then these objects would not be formed by mass accretion.

6. An Alternative Interpretation of the Quasar Phenomenon

They could be the result of the convergence of a centripetal density wave, like the one suggested in the Hoag galaxy (**Figure 3**).

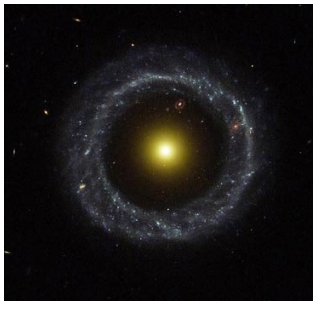


Figure 3. Hoag galaxy.

This galaxy, located in the constellation Serpens Caput, was discovered by Arthur Hoag in 1950. The absence of neighbouring galaxy and debris seems to rule out the possibility that it formed as a result of a collision. Its structure resembles that of a density wave, whose propagation would then be radial.

If a density wave propagates in a galaxy, whether it is a spiral wave or this type of circular, converging wave, its speed of propagation is not that of the matter itself.

This is the classic difference between group velocity and phase velocity. The wave from the Hoag galaxy, if its propagation is centripetal, would then be comparable to a circular tsunami. The rise in density in the wave gives rise to young stars which, emitting in the UV, ionise the medium, making the formation visible, as is the case for spiral waves. In this plasma, if the magnetic Reynolds number is high, this wave will drag along the field lines of the very weak magnetic field (one microgauss) that already exists in the galaxies (see Figure 4).

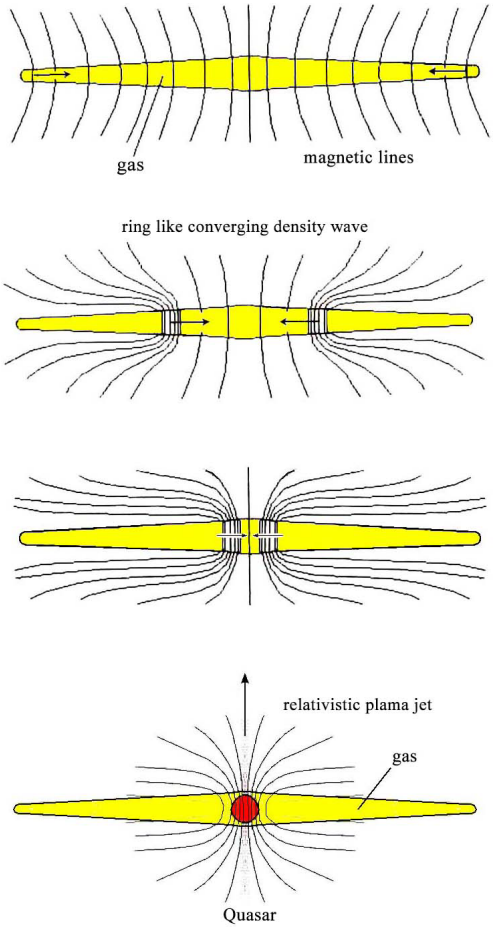


Figure 4. Model for a quasar formation.

Such a rise in the field in the ring could be demonstrated by polarisation measurements.

When the wave would reaches the centre of the galaxy, there would be a sudden

rise in density and temperature[1]. Lawson's conditions would be then realised in a considerable mass, triggering a massive production of energy by fusion. Such model would explain the quasar phenomenon.

The convergence of the magnetic field lines would give to the object a very high dipolar magnetic field. The fusion energy would be then focused on the two jets with which M87* is equipped.

This field has a gradient that accelerates charged particles over very large distances, giving them very high energies. This is how cosmic rays would be formed.

The M87* jet is discontinuous, which seems to indicate that the phenomenon is sporadic. The absence of jets for SgrA* would qualify it as an "extinct quasar"? This model is based on the hypothesis of the repetitive birth of spiral density waves.

We still need a theory to explain their appearance. This is what we are currently trying to construct, within the framework of the Janus model, in terms of joint fluctuations in the metrics of positive and negative masses.

We conjecture that future images of hypermassive objects located at the center of galaxies will all present the same wavelength ratio 3.

7. Discussion

To moderate our conclusions, it should be noted that our model is consistent with the images of the objects M87* and SgrA*, which are produced by model-dependent decoding. In any case, it seems difficult to envisage a decoding based on a model in which the central part is completely non-emissive, unless this model is a priori that of a black hole. There will therefore remain a finite gravitational redshift and a maximum/minimum wavelength ratio. The future will tell whether this value close to three can be considered reliable, and above all whether such a ratio will appear in future images of other supermassive objects. It is hoped that images obtained from orbital radio telescopes will provide finer images, which can then be compared with the brightness profiles from the model, which will be produced in a future article. Nevertheless, a few ideas can be outlined. The supermassive objects M87* and Sgr* are obviously not "giant neutron stars". They still need to be modeled. This attempt to describe them as objects with constant density, entirely contained within a sphere, can only be indicative. The fact remains that these objects emit radiation and that a gravitational redshift effect $z=2$ is present in their central part. Their radius (schematic) is therefore $\sqrt{9/8}R_s = 1.06066R_s$. **Figure 5** shows the schematic evolution of z as a function of distance from the center. If the object is surrounded by a vacuum, the gravitational redshift tends toward zero at infinity.

[1]When the wave causes a rise in density, the angular momentum of the mass concerned is conserved, and the speed of rotation increases. But this phenomenon generates a centrifugal force that cannot be compared with the forces of gravity and pressure. This explains the excellent agreement with the data derived from the Schwarzchild solution.

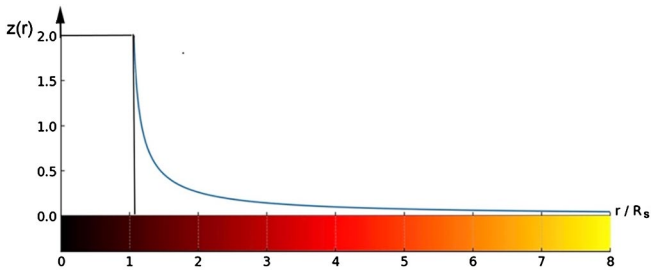


Figure 5. Schematic gravitational redshift as a function of distance.

However, there must be a source of radiation at a distance. The chromatic bar gives us the darkening towards the central part (which is not black). We can assume that the object is surrounded by plasma, which is emissive but whose temperature gradually decreases. This gives us another cause of reddening: the fact that the radiation temperature decreases. And we then have a change in wavelength according to:

$$\lambda_{obs} = \frac{b}{T(r)} \frac{1}{\sqrt{1-\frac{R_s}{r}}} \tag{23}$$

By combining the two, we can then, very schematically, produce an image that matches the observational data (see **Figure 6**).

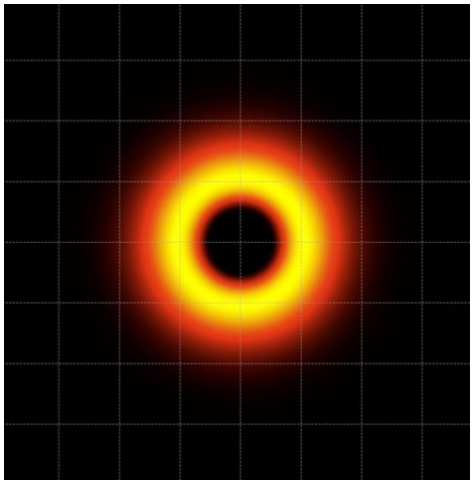


Figure 6. Schematic representation of a plugstar.

But until we have a coherent physical vision of the object, with density decreasing from the center to the edge and where magnetohydrodynamics undoubtedly plays a role, all this cannot be considered a model. It merely combines two phenomena that alter the wavelength: a gravitational redshift effect toward the center and a drop in radiation temperature toward the periphery, without specifying the mechanism of radiation emission. All this is, we admit, only a rough draft. Others will develop it further, younger people. We are old, and there is so much work to be done in this immense project that is the Janus Cosmological Model.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Event Horizon Telescope Collaboration (2019) First M87 Event Horizon Telescope Results, Volume 875. The Astrophysical Journal Letters.

[2] Event Horizon Telescope Collaboration (2022) First Sagittarius an Event Horizon Results, Volume 930. The Astrophysical Journal Letters.

[3] Mazur, P.O. and Mottola, E. (2001) Gravitational Condensate Stars: An Alternative to Black Holes.

[4] Mazur, P.O. and Mottola, E. (2004) Gravitational Vacuum Condensate Stars: An Alternative to Black Holes. *Proceedings of the National Academy of Sciences*, **101**, 9545-9550. https://doi.org/10.1073/pnas.0402717101

[5] Mazur, P.O. and Mottola, E. (2023) Gravitational Condensate Stars: An Alternative to Black Holes. *Universe*, **9**, Article No. 88. https://doi.org/10.3390/universe9020088

[6] Cattoen, C., Faber, T. and Visser, M. (2005) Gravastars Must Have Anisotropic Pressures. *Classical and Quantum Gravity*, **22**, 4189-4202. https://doi.org/10.1088/0264-9381/22/20/002

[7] Sakai, N., Saida, H. and Tamaki, T. (2014) Gravastar Shadows. *Physical Review D*, **90**, Article ID: 104013. https://doi.org/10.1103/physrevd.90.104013

[8] Chown, M. (2006) Is Space a Superfluid. New Scientist.

[9] Randall, L. and Sundrum, R. (1999) Large Mass Hierarchy from a Small Extra Dimension. *Physical Review Letters*, **83**, 3370-3373. https://doi.org/10.1103/physrevlett.83.3370

[10] Sengupta, R., Ghosh, S., Ray, S., Mishra, B. and Tripathy, S.K. (2020) Gravastar in the Framework of Braneworld Gravity. *Physical Review D*, **102**, Article ID: 024037. https://doi.org/10.1103/physrevd.102.024037

[11] Rikpatrick, S., Saibal, R. and Shounak, G. (2020) Gravastar in the Framework of Braneworld Gravity II: Effect of the Kuchowicz Metric Function. *Modern Physics Letters A*, **37**.

[12] Farasat, S. and Mushtaq, A. (2018) Gravastars in f(g,t) Gravity. *Physical Review D*, **97**. https://doi.org/10.1103/PhysRevD.97.104031

[13] Bellini, M. (2024) Gravastars Formation with Back-Reaction Effects from Extended General Relativity. *The European Physical Journal C*, **84**, Article No. 1097. https://doi.org/10.1140/epjc/s10052-024-13381-6

[14] Sahoo, P.K., Mohanty, D. and Gosh, S. (2024) Study of Charged Gravastars in f(q) Gravity. *Annals of Physics*, **461**.

[15] Horvat, D., Ilijić, S. and Marunović, A. (2008) Electrically Charged Gravastar Configurations. *Classical and Quantum Gravity*, **26**, Article ID: 025003. https://doi.org/10.1088/0264-9381/26/2/025003

[16] Mohanty, D., Ghosh, S. and Sahoo, P.K. (2025) Impact of Chaplygin Gas Model on the Characteristics of Gravastar in F(q,t) Gravity. *International Journal of Modern Physics D*, **34**, Article ID: 2550041. https://doi.org/10.1142/s0218271825500415

[17] Ghosh, S., Pradhan, S. and Sahoo, P.K. (2025) Maximum Mass Limit of Generalized MIT Compact Star: A Theoretical Study of Thin Shell Dynamics and Gravitational Lensing. *Fortschritte der Physik*, **73**, e70020. https://doi.org/10.1002/prop.70020

[18] Schwarzschild, K. (1916) Über das gravitationsfeld einer kugel aus inkompressibler flus-sigkeit nach der einsteinschen theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin.

[19] Oppenheimer, J.R. and Volkoff, G.M. (1939) On Massive Neutron Cores. *Physical Review*, **55**, 374-381. https://doi.org/10.1103/physrev.55.374

[20] Wheeler, J.A., Misner, C.W. and Thorne, K.S. (2017) Gravitation. Princeton University Press.

[21] Petit, J. (1995) Twin Universes Cosmology. *Astrophysics and Space Science*, **226**, 273-307. https://doi.org/10.1007/bf00627375

[22] D'Agostini, G. and Petit, J.P. (2024) Variable Speed of Light Cosmological Model with Lorentz and Fine Structure Constant Invariance Maintained.

[23] Souriau, J.-M. (1970) Structure des systèmes dynamiques: Maîtrises de mathématiques. Collection Dunod Université.

[24] Souriau, J.-M. (1997) Structure of Dynamical Systems (Structure des systèmes dynamiques): A Symplectic View of Physics. Progress in Mathematics. Birkhäuser.

[25] Bondi, H. (1957) Negative Mass in General Relativity. *Reviews of Modern Physics*, **29**, 423-428. https://doi.org/10.1103/revmodphys.29.423

[26] Petit, J., Margnat, F. and Zejli, H. (2024) A Bimetric Cosmological Model Based on Andreï Sakharov's Twin Universe Approach. *The European Physical Journal C*, **84**, Article No. 1226. https://doi.org/10.1140/epjc/s10052-024-13569-w

[27] Fernández Gil, D., Hodgson, J.A., L'Huillier, B., Asorey, J., Saulder, C., Finner, K., *et al.* (2024) Detection of an Orthogonal Alignment between Parsec-Scale AGN Jets and Their Host Galaxies. *Nature Astronomy*, **9**, 302-313. https://doi.org/10.1038/s41550-024-02407-4

We have compiled, in sequence, the main articles demonstrating the physical and geometric inconsistencies of the black hole model.

They are:

J-P. Petit, G. D'Agostini. "Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I." Journal of Modern Physics, 2026, Vol. 17, No. 2. 3,223 views. 423 downloads
Cited 4 times.

J-P Petit, G. D'Agostini, J-M Pechinot: The True Schwarzschild Solution and Its Subsequent Geometry. Journal of Modern Physics. 2026, 17(8), 997–1027.
https://content.scirp.org/pdf/jmp_7506149.pdfAugust 2026 31 downloads 329 views cited 2 times.

J-P Petit. Kruskal coordinates, semi-complex extension of spacetime. Journal of Modern Physics, 2026. 17(9), 1197–1128 39 downloads 197 views

J-P. Petit: Physical and Geometrical Inconsistency of the Janus Cosmological Model. Part II. Journal of Modern Physics

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(2), 199-239
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I

Jean-Pierre Petit, Gilles D'Agostini

Manaty Research Group, Glanon, France
Email: Jean-pierre.petit@manaty.net, Gilles.d'agostini@manaty.net

How to cite this paper: Petit, J.-P. and D'Agostini, G. (2026) Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I. *Journal of Modern Physics*, **17**, 199-239.
https://doi.org/10.4236/jmp.2026.172014

Received: August 26, 2025
Accepted: February 25, 2026
Published: February 28, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

We show that the model of the black hole is a complete mathematical and geometrical chimera, born from wrong interpretations of the Schwarzschild external metric solution. The first misconception stems from Hilbert's confusion of the Schwarzschild intermediate magnitude R with a radial coordinate r. The second arises from the omission of the cross term in drdt, which, by yielding a finite free-fall time, allowed Oppenheimer and Snyder in 1939 to suggest using a stationary solution to describe a highly unsteady phenomenon. The third is a consequence of the assumption of space contractibility, which creates the myth of the existence of the interior of black holes and a central singularity, with real spiral trajectories endowed with an imaginary length, thus located outside the hypersurface. This translates in the literature into the surreal idea that space and time coordinates exchange roles. Starting from the only accessible observational quantities, the images of the objects M87* and SgrA*, that is, a ratio of maximum to minimum brightness temperatures of 3, we show that this gravitational redshift effect then fits within the Schwarzschild inner metric solution when physical criticality is approached. The criticism formulated by Einstein in 1939 regarding this solution must be reconsidered, as the physical conditions in such criticality are such that the distances between baryons become on the order of their Compton length. Thus, the precise physical description of such states, and the suggested mass inversion process in objects described as plugstars, will require the completion of the unification between general relativity and quantum mechanics. As it stands, this reinterpretation of the Schwarzschild outer and inner solutions then fits into the Janus Cosmological Model. It is predicted that all future objects of this type will exhibit this same ratio of 3 and that the masses of neutron stars are less than 2.5 solar masses, therefore, in general, black holes of any size simply do not exist. It is suggested that the supermassive objects M87* and Sgr A* result from the implosion of density waves similar to those in Hoag's Galaxy.

Keywords

Black Hole, M87*, SgrA*, Birkhoff Theorem, Plugstar, Hilbert, Two Folds Cover Manifold, Bounded Manifold, PT-Symmetry, Geometrical Criticity, Physical Criticity, Mass Inversion, Negative Mass, Time Factor, Throat Sphere, Hoag galaxy, Density Wave, Supermassive Objects, Neutron Stars, Janus Cosmological Model

1. Foreword

In 1902 [1], it became increasingly clear that time could no longer be treated as something entirely independent of the phenomena it was supposed to describe. In 1905, Einstein published [2] his special relativity, breaking this centuries-old independence. The very nature of this fourth coordinate remained to be defined. In 1902, the mathematician Henri Poincaré suggested that it could be purely imaginary, and thus came very close to anticipating Einstein's discovery. In 1909, the mathematician Minkowski [3] united the four dimensions in a single space, but in his initial formulation, he retained the purely imaginary nature of the time coordinate. It was only later that time ct began to be considered as measured in meters, modulo the factor c, which is the speed of light. After the invariance of the speed of light, the second problem Einstein tackled was the variation in the orbital period of the planet Mercury. The astronomer Le Verrier had attempted to interpret these variations as the consequence of the perturbation of the orbit by a new planet, located closer to the Sun, which he named Vulcan. But observation did not confirm his calculations. At the time, no one dared to call it a dark planet. At the beginning of the century, Einstein was far from isolated in this quest for a new paradigm. Thus, when he published the equation of what then became the key to the new geometric paradigm of general relativity, the mathematician and astronomer Karl Schwarzschild immediately produced exact solutions to his equation, in stationary and spherically symmetric motion, equations without a forcing term [4] and with a forcing term [5].

Before this, Einstein had numerous exchanges with the mathematician Hilbert. Prior to this meeting with Einstein, Hilbert was convinced that the world of high mathematics, to which he was making significant contributions, was unrelated to the world of physics. Having been persuaded by Einstein, Hilbert tackled the problem and, through variational calculus, constructed the first version of the field equation [6] and submitted his publication a few days before Einstein. After a moment of confusion, the two decided to prioritize their relationship of esteem and friendship, acknowledging that Einstein would be the true discoverer. To understand the problems faced by today's theorists, it is essential to delve into the writings of the time, and in particular Hilbert's two articles [6] [7], which are difficult and perplexing to read. Both articles are titled "The Foundations of Physics". In its time, it was the first "theory of everything". Indeed, in 1915, only two of the

four forces were known: the electromagnetic force and the force of gravity. Hilbert's ambition was therefore to integrate both of them into his variational formalism. Another essential aspect: for Hilbert (as for Poincaré), the time coordinate, denoted by the letter *l*, is purely imaginary. The solutions to the field equation he envisioned are not fundamentally lengths, but bilinear forms.

A few months after the publication of his first paper by Hilbert, Schwarzschild published two articles in January and February 1916. The first [4] gave the solution to Einstein's equation in a steady state and spherically symmetric, and the second [5] gave the solution to the equation with a right-hand side. He then perfectly described the geometry inside an object consisting of a sphere of constant density, and outside of it. For the solution in a vacuum, Schwarzschild did not express it in terms of the variables (t,x,y,z) he initially defined. He introduced a coordinate $r=\sqrt{x^2+y^2+z^2}$, but using an "intermediate variable *R*" defined as follows:

$$\mathrm{d}s^2=\left(1-\frac{R}{\alpha}\right)c^2\mathrm{d}t^2-\frac{\mathrm{d}R^2}{1-\frac{R}{\alpha}}-R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\quad R=\left(r^3+\alpha^3\right)^{1/3} \tag{0}$$

Hilbert then made a mistake by deciding to identify this coordinate *R* with *r*, which he presented as a simplification. This is described in the following article, in sections 3 to 6, indicating its implications. The error was first pointed out in 2001, eighty-five years later (…) by the Canadian mathematician Abrams [8], in an article that went completely unnoticed. Today, scientific work is disseminated rapidly and efficiently. But at that time, the primary means of dissemination was the book, in English, which was becoming the standard scientific language. In works that immediately became references, Eddington in 1924 [9] and Tolman [10] in 1934 echoed "Hilbert's approximation", that is, the identification of the intermediate quantity *R* with a "radial coordinate" *r*. But no one paid any attention. In 1939, Einstein referred to Schwarzschild's second article, describing the geometry inside a mass, where the pressure p and a density ρ are linked by an equation of state. Schwarzschild then highlighted the conditions for physical criticality, occurring before the star's radius is caught by its Schwarzschild radius, such that the pressure soars towards infinity.

In the few lines Einstein devotes to this solution, he concludes that under such conditions the speed of sound would exceed the speed of light and that, consequently, this solution is not physical. Difficult to read, the article [5] was not available in English until…1999. In 1939, Oppenheimer and Snyder [11], based on what is now considered "the standard form of the Schwarzschild solution", pointed out that the free-fall and escape times of a reference mass, measured in the time frame of a distant observer, are infinite. This article then became the birth of the black hole model, which uses a stationary solution to describe an implosion process that lasts only a few days. However, as shown in 2021 by the mathematician Koiran [12], this stems from the absence of a cross term in drdt, also historically considered a "simplification". Its presence, depending on its sign, completely alters the scenario, one of the two times becoming finite. Thus, the foundational

hypothesis of the black hole model disappears. Furthermore, while this geometry had been perfectly understood and described in 1916 by the mathematician Flamm [13], the authors repeated Hilbert's second error, which consisted of taking as the Lagrange function, in the action that determines the geodesics, not the length, that is, the square root of the bilinear form, but the form itself. It turns out that when the derivatives $\dot{x}^i$ are calculated with respect to the length s, the Lagrange equations obtained are the same. They therefore do indeed provide the geodesics. But they then provide what can be called virtual geodesics, real curves, equipped with a purely imaginary length, which do not belong to the hypersurface. Thus, the curves "spiraling towards the central singularity, inside the black hole" exist only in the imagination of the theorists who study them. In a future article, I will show, as an example, how this error allows us to equip the torus with these "virtual geodesics", spiraling towards its center and towards infinity, and how we can demonstrate that the "virtual geodesics of the sphere" are ellipses!

As a conclusion, the black hole model is a mathematical and geometrical chimera.

So, what happens when a massive star collapses? What about the supermassive objects identified at the centers of galaxies? At the end of the article, we present another scenario, which has the advantage of aligning with observational data (brightness temperature ratio 3). Regarding Einstein's critique, it's worth remembering that it's based on a constant density. However, for example, in neutron stars, when the criticality he invokes is reached, the distances between baryons tend to become smaller than their Compton length. A purely geometric description thus proves inadequate. The physics of such a medium can only be described through a complete and real unification of gravitation and quantum mechanics.

2. Introduction: On What Basis Did the Black Hole Model Emerge?

a) The founding article is that of Oppenheimer and Snyder [11], in 1939. It is based on the fact that the free-fall time of a test mass toward the Schwarzschild sphere, when measured with the coordinate t, is assumed to represent the proper time experienced by a distant observer, and is infinite.

b) Moreover, the vanishing of the g_{tt} term on the Schwarzschild sphere implies that any light emitted from this surface undergoes an infinite gravitational redshift and thus behaves like a cosmological horizon.

c) The hypothesis is that the local topology is $\mathbb{R}_+ \times \mathbb{R}^3$. Thus, the universe is supposed to be locally contractile.

d) It is then assumed that in the bilinear form—solution of Einstein's field equation without source term, invariant under time translation, known as the "Schwarzschild solution":

$$G_{ext}\left(t,r,\theta,\varphi,\dot{t},\dot{r},\dot{\theta},\dot{\varphi}\right)=\left(1-\frac{r}{\alpha}\right)\dot{t}^2-\frac{\dot{r}^2}{1-\frac{r}{\alpha}}-r^2\left(\dot{\theta}^2+\sin^2\theta,\dot{\varphi}^2\right) \tag{1}$$

the letter r is supposed to represent a radial variable.

e) It is assumed that the solution curves derive from the Lagrangian:

$$L = G_{ext}\left(t, r, \theta, \varphi, \dot{t}, \dot{r}, \dot{\theta}, \dot{\varphi}\right) \tag{2}$$

involved in the construction of the action:

$$J = \int L \mathrm{d}p \tag{3}$$

It is assumed that these curves describe the geodesics of the entire contractile spacetime hypersurface, with topology $\mathbb{R}_+ \times \mathbb{R}^3$.

3. The So-Called "Schwarzschild Solution" Is Not the Original Solution

In his first article [4] from 1916, Schwarzschild clearly defines his initial coordinates $\{t, x, y, z\}$ as well as a coordinate r defined by:

$$r = \sqrt{x^2 + y^2 + z^2} \tag{4}$$

although he does not state it explicitly $\{t, x, y, z\} \in \mathbb{R}_+ \times \mathbb{R}^3$. Thus $r \geq O$

By expressing that the solution is spherically symmetric, which introduces a constant of integration α (later called the "Schwarzschild radius"), he effectively chooses to express his solution not with the coordinates $\{t, r, \theta, \varphi\}$ but with the coordinates $\{t, R, \theta, \varphi\}$ where R is an intermediate variable (called "*Hilfsgröße*") defined by:

$$R = \left(r^3 + \alpha^3\right)^{1/3} \geq \alpha \tag{5}$$

Which gives:

$$\mathrm{d}s^2 = \left(1 - \frac{R}{\alpha}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}R^2}{1 - \frac{R}{\alpha}} - R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right) \qquad R = \left(r^3 + \alpha^3\right)^{1/3} \tag{6}$$

The true Schwarzschild solution, from January 1916, expressed using his coordinates $\{t, r, \theta, \varphi\}$, is therefore:

$$\mathrm{d}s^2 = \frac{\left(r^3 + \alpha^3\right)^{1/3} - \alpha}{\left(r^3 + \alpha^3\right)^{1/3}}c^2\mathrm{d}t^2 - \frac{r^4\mathrm{d}r^2}{\left(r^3 + \alpha^3\right)\left[\left(r^3 + \alpha^3\right)^{1/3} - \alpha\right]} - \left(r^3 + \alpha^3\right)^{2/3}\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right) \tag{7}$$

If we take $\mathrm{d}t = \mathrm{d}r = 0$ what remains is the spatial part of the metric:

$$\mathrm{d}\sigma^2 = \left(r^3 + \alpha^3\right)^{2/3}\left(\mathrm{d}\theta^2 + \sin^2\theta, \mathrm{d}\varphi^2\right) \tag{8}$$

This is the metric of a family of spheres with radius: $\left(r^3 + \alpha^3\right)^{1/3}$, with a minimal area value:

$$A = 4\pi\left(r^3 + \alpha^3\right)^{2/3} \tag{9}$$

The geometric object is therefore non-contractile. In terms of the spatial coordinates $\{r, \theta, \varphi\}$ the topology of the geometric object is $\mathbb{R}_+ \times \mathbb{R}^3$ but, based on

the system $\{R,\theta,\varphi\}$ it a manifold with boundary, the boundary being the Schwarzschild sphere.

What happens to the metric coefficients as $r \to 0$?

$$g_{tt} = \frac{\left(r^3+\alpha^3\right)^{1/3} - \alpha}{\left(r^3+\alpha^3\right)^{1/3}} c^2 \to +0 \tag{10}$$

This vanishing implies that on the Schwarzschild sphere, one cannot define a volume form, and thus no orientation.

For the g_{rr} we must perform a Taylor expansion:

$$g_{rr} \approx -\frac{r^4}{\alpha^3\left[\left(1+\frac{r^3}{\alpha^3}\right)^{1/3} - 1\right]} \approx -\frac{r^4}{\alpha^3} \to -0 \tag{11}$$

At $r=0$ or $R=\alpha$, spacetime is locally non-orientable. The regions of space corresponding to values $R<\alpha$ lie outside the realm of real numbers. In this space, Flamm [13] constructs the meridional section in the form of a lying half-parabola in **Figure 1**, with equation:

$$z = \pm 2\sqrt{\alpha\left(r-\alpha\right)} \tag{12}$$

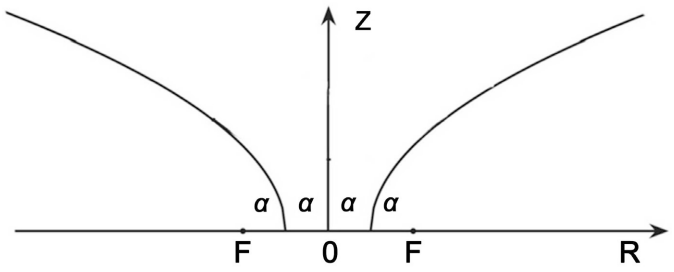


Figure 1. Half-Meridian of Flamm's surface [13].

The external metric is being considered in isolation. Only a part of this meridian refers to the exterior of the star. The complete meridian therefore has no physical significance and its path is given only as an expression of a part ($z>0$) of the solution to the differential Equation (12) above.

Who initiated this choice? It was David Hilbert [7]. He treats the solutions to the field equation—of which he had published his own version [6] a few days before Einstein [14]—not as metrics but as bilinear differential forms:

$$G\left(x^i, \frac{\mathrm{d}x^i}{\mathrm{d}p}\right) \tag{13}$$

His field equation [6] [14] is:

$$K_{\mu\nu} - \frac{1}{2} K g_{\mu\nu} = \frac{1}{\sqrt{g}} \frac{\partial \sqrt{g} L}{\partial g^{\mu\nu}} \tag{14}$$

where $K_{\mu\nu}$ is the Ricci tensor and K the corresponding Ricci scalar.

He places his solution in a space $\{x_1, x_2, x_3, x_4\}$, with the fourth coordinate referring to time. He considers the space to be quasi-Euclidean, writing (his Equation (37)):

$$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu} \tag{15}$$

He then studies a stationary solution where the $g_{\mu\nu}$ are independent of x_4. He assumes the $g_{\mu\nu}$ are centrally symmetric with respect to the origin of the coordinates ("*zentrisch symmetrisch*"). On page 67, he chooses spherical coordinates:

$$x_1 = r\cos\theta \tag{16}$$

$$x_2 = r\sin\theta \tag{17}$$

$$x_3 = r\sin\theta\sin\varphi \tag{18}$$

$$x_4 = l \tag{19}$$

Taking symmetry into account, he writes the bilinear form (Equation (42)):

$$F(r)\mathrm{d}r^2 + G(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) + H(r)\mathrm{d}l^2 \tag{20}$$

At this stage, he decides to set:

$$r^* = \sqrt{G(r)} \tag{21}$$

Then he writes:

Züren wir in (42) *r anstatt r ein und lassen dann wieder das Zeichen * weg** (*Let us insert r* into* (42) *instead of t and then drop the asterisk again*)

In doing so, without realizing it, he expresses his solution not in terms of the coordinate

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2} \tag{22}$$

but according to Schwarzschild's intermediate quantity ("*Hilfsgrobe*"):

$$R = \left(r^3 + \alpha^3\right)^{1/3} \geq \alpha \tag{23}$$

The first to point out this discrepancy was the Canadian mathematician Abrams [8]. Thus, Hilbert's bilinear form becomes, in his Equation (43):

$$M(r) + r^2\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2 + W(r)\mathrm{d}l^2 \tag{24}$$

4. Hilbert's Legacy: The Survival of a Purely Imaginary Time

After determining the form of his functions $M(r)$ and $W(r)$, he writes (Equation (45), p. 70)

$$G(\mathrm{d}r, \mathrm{d}\theta, \mathrm{d}\varphi, \mathrm{d}l) = \frac{r}{r-\alpha}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2 + \frac{r-\alpha}{r}\mathrm{d}l^2 \tag{25}$$

Then, after writing:

"*…so ergibt sich aus* (43) *für l = it die gesuchte Maßbestimmung in der von Schwarzschild gefundenen Gestalt.*"

"*Thus, from Equation* (43), *for l = it, one obtains the sought-after determina-*

tion of the metric in the form found by Schwarzschild."

He writes:

$$G\left(\mathrm{d}r,\mathrm{d}\theta,\mathrm{d}\varphi,\mathrm{d}t\right)=\frac{r}{r-\alpha}\mathrm{d}r^2+r^2\mathrm{d}\theta^2+r^2\sin^2\theta\mathrm{d}\varphi^2-\frac{r-\alpha}{r}\mathrm{d}t^2 \tag{26}$$

One must recall that before Einstein presented his formulation of special relativity in 1905 [2], the majority of scientists adhered to the interpretation of the French mathematician Henri Poincaré, who in 1902 proposed the idea that the time variable was purely imaginary [1]. Even when Hermann Minkowski developed what would become spacetime in special relativity [3], he retained the idea of imaginary time—though he would soon abandon it [15], adopting Einstein's view that time is ultimately measured in meters.

In 1915, in his article on the advance of Mercury's perihelion [16], Einstein explicitly presents his Gram matrix, on page 832:

…der unspünglichen Relativitätstheorie entsprechende Schama gegeben:

…given the original theory of relativity corresponding to the shape:

$$\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix} \tag{27}$$

5. Everything Depends on the Meaning Given to the Word "Geodesic"

One may say that at the time, before World War II, most authors (Einstein, Schwarzschild, Droste, Weyl, and others) explicitly opted for the signature (+ - - -), equipping the solution hypersurface with a metric where the length element s, identified with the proper time t through the relation $s = ct$, is real. One might consider that it was Hilbert's influence that suggested the shift (or return) to a signature inheriting the interpretation of time dt as a purely imaginary quantity. By contrast, with Schwarzschild everything is clear from the start, when he writes at the beginning of his article [4]:

$$\mathrm{d}s=\sqrt{\sum g_{\mu\nu}\mathrm{d}x_\mu\mathrm{d}x_\nu},\ \ \mu=1,2,3,4 \tag{28}$$

The terms are set out very clearly. s is a length, essentially real. As for the constructed curves—the geodesics—they are the shortest paths, materialized by the variational equation:

$$\delta\int\mathrm{d}s=\delta\int\sqrt{\sum g_{\mu\nu}\mathrm{d}x_\mu\mathrm{d}x_\nu}=0 \tag{29}$$

Thus, Schwarzschild only considers curves that have a real length. But with Hilbert, things change completely when he writes [7], page 68:

Der erste Schritt hierzu ist die Aufstellung der Differentialgleichungen der geodätischen Linien durch Variation des Integrals (*The first step is to formulate the differential equations of the geodesic lines by varying the integral*).

$$\int M\left(\frac{\mathrm{d}r}{\mathrm{d}p}\right)^2 + r^2\left(\frac{\mathrm{d}\theta}{\mathrm{d}p}\right)^2 + r^2\sin^2\theta\left(\frac{\mathrm{d}\varphi}{\mathrm{d}p}\right)^2 + H\left(\frac{\mathrm{d}l}{\mathrm{d}p}\right)^2 \tag{30}$$

This integral is Hilbert's action J. The variational calculus is applied to it $\delta J = 0$, leading to the Lagrange equations. But then, what does one call a geodesic?

For Schwarzschild, Einstein, Droste, and Weyl, they are curves along which the real length s is minimized.

For Hilbert, these curves minimize the *square* of that length. The resulting Lagrange equations are the same, but in one case there is a constraint on the domain of validity of the solution, while in the other this constraint disappears.

Later in his article, the fact that his bilinear form $G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)$ may be positive or negative does not bother Hilbert much, since he defines *two real* lengths:

- In the region where the bilinear form $G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)$ is positive, he defines a real length referring to curve segments he calls *segments*.

$$\lambda = \int\sqrt{G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)}\,\mathrm{d}p \tag{31}$$

- In the region where the bilinear form $G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)$ is negative, he defines a second real length referring to curve portions he calls *time lines*.

$$\tau = \int\sqrt{-G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)}\,\mathrm{d}p \tag{32}$$

- Finally, when the bilinear form $G\left(\frac{\mathrm{d}x_s}{\mathrm{d}p}\right)$ is null, it corresponds to "null lines" (*Nulllinien*).

6. When the Choice of Lagrangian Determines the Topology of the Object

This identification of the bilinear form with the Lagrangian appearing in the action will become generalized. One may cite, as an example, the mention in the relatively recent (1992) book by S. Chandrasekhar, *The Mathematical Theory of Black Holes* [17].

After presenting on page 92, in his Equation (60), the Schwarzschild metric in the form:

$$\mathrm{d}s^2 = \left(1-\frac{2M}{r}\right)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{33}$$

with signature (+ - - -), he defines on page 96, in Equation (80), what he calls "the Lagrangian of the Schwarzschild spacetime":

$$L = \frac{1}{2}\left[\left(1-\frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1-\frac{2M}{r}} - r^2\dot{\theta}^2 - r^2\sin^2\theta\,\dot{\varphi}^2\right] \tag{34}$$

This allows him to trace the geodesic curves located "*inside the Schwarzschild sphere*" in Figure 2, that is, with a purely imaginary ds:

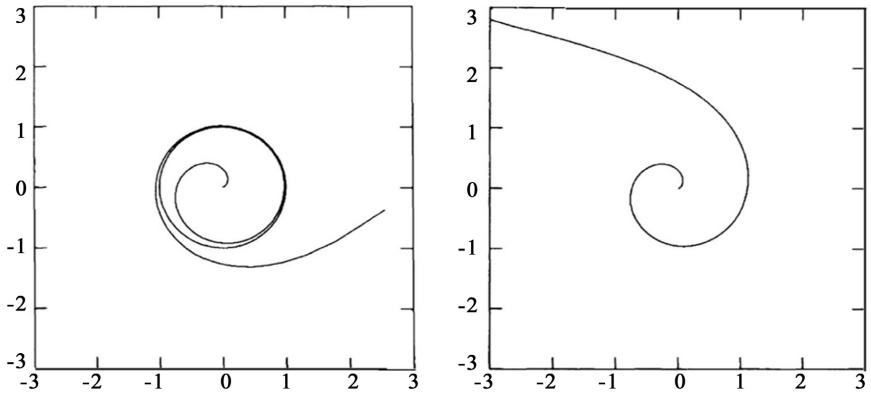


Figure 2. Geodesics spiraling "*inside the Schwarzschild sphere*" according to S. Chandrasekhar, from his book [17], p. 121.

It is clear that this choice of Lagrangian, differing from that of Schwarzschild, "*extends the expression of the solution to the entire spacetime*", which goes hand in hand with the analytic continuation constructed by Kruskal [18]. It is no longer a manifold with boundary, but a space whose topology is $\mathbb{R}_+ \times \mathbb{R}^3$.

It turns out that when quantities are calculated using the length parameter s, the variation of an action based on the length or its square produces the same system of Lagrange equations. Therefore, when the length is real, when we are within the hypersurface, this system of equations does indeed yield its geodesics. However, it provides what could be called "virtual geodesics", that is, real curves equipped with a purely imaginary length, which are thus outside the hypersurface. In a future article, we will show how, by constructing the Lagrange equations by basing the action on the square of the length, we equip the torus with pseudo-geodesics spiraling "inside" towards its "center" and outwards towards infinity. As a "geometry of the imaginary", we thus demonstrate that the virtual geodesics of the sphere, extending to infinity, are ellipses!

7. Everything Depends on What One Considers the Physical World to Be

This choice of Lagrangian, a "modern" choice, is at the root of the black hole model. Since it is accompanied by an alteration of the metric signature when crossing the Schwarzschild sphere, the interpretation accepted by all specialists is that in this region t becomes a spatial coordinate and r becomes a time coordinate.

As an indication only, let us now examine the various interpretations of the two-dimensional geometric object defined by the bilinear form:

$$G = \frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2 \tag{35}$$

If, like Chandrasekhar, one chooses the Lagrangian:

$$L = G = \frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2 \tag{36}$$

And the action:

$$J = \int L\left(r, \dot{r}, \varphi, \dot{\varphi}\right) \mathrm{d}s \tag{37}$$

then the Lagrange equations yield the differential equation:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}r} = \pm\frac{h}{r^2}\frac{1}{\sqrt{\left(1-\frac{\alpha}{r}\right)\left(1-\frac{h^2}{r^2}\right)}} \tag{38}$$

Let us assign the constant α the value 1. Following Figure 3 shows different solution curves:

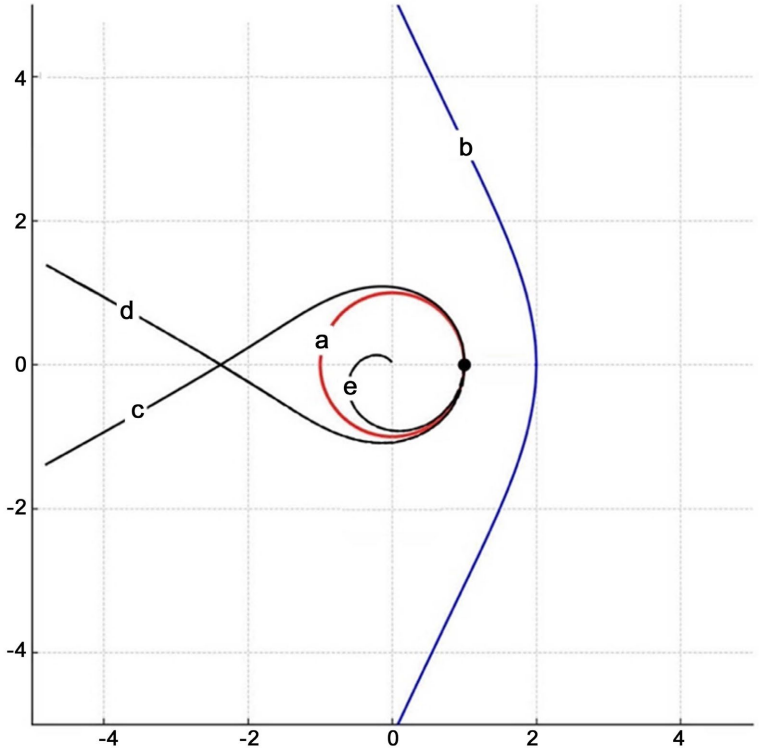


Figure 3. Solution curves. (b): ($r > a$; $h < a$). (c): ($r > a$; $h = a$). (e): ($r < a$; $h < a$).

This choice of Lagrangian amounts to choosing an object whose topology is homotopic to $\mathbb{R}^2$. If we define the Lagrangian as:

$$L = \sqrt{G} = \sqrt{\frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2} \tag{39}$$

Then we have two possible topologies, since the drawing of curves inside the circle is now forbidden, as shown in Figure 4.

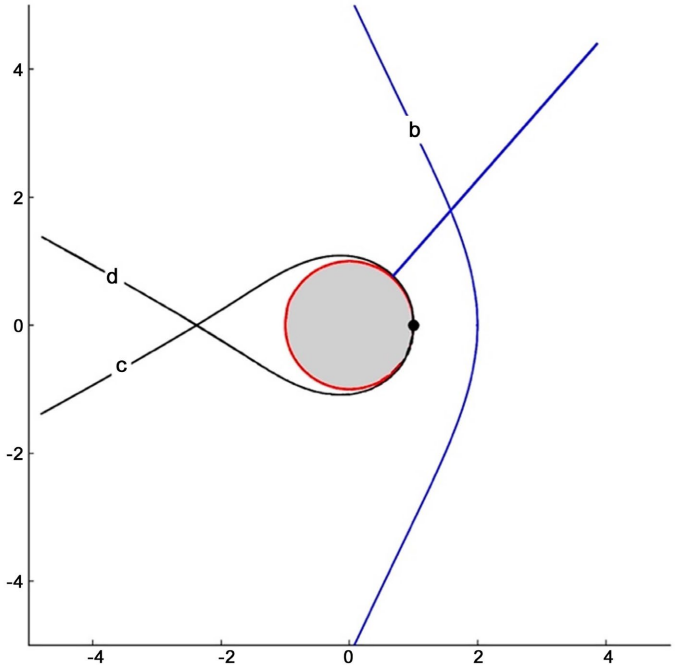


Figure 4. Non-Contractible: The object cannot be homotopic to $\mathbb{R}^2$.

Either we consider that the geometry is that of a manifold with boundary. It turns out that the object can be embedded in $\mathbb{R}^3$. By adding a third dimension z, and setting:

$$\mathrm{d}r^2 + \mathrm{d}z^2 = \frac{\mathrm{d}r^2}{1-\frac{\alpha}{r}} \tag{40}$$

This gives the meridian curve:

$$z = \pm 2\sqrt{\alpha(r-\alpha)} \tag{41}$$

But we have a third option: that of the two-sheeted covering of this manifold with boundary as shown in Figure 5. This covering is then regular, and the meridian becomes:

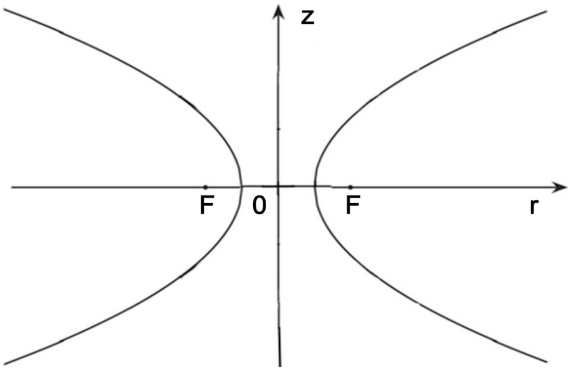


Figure 5. Meridian of the object viewed as a two-sheeted covering of a manifold with boundary.

Since this object has the particularity of being embeddable in $\mathbb{R}^3$, we can provide in Figure 6 a perspective view of it in this three-dimensional representation space:

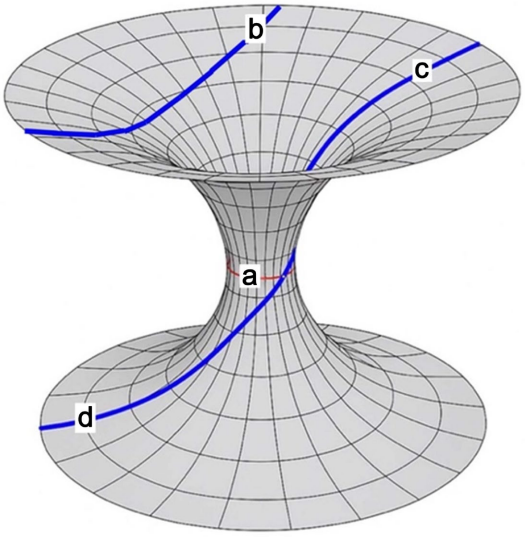


Figure 6. Perspective view of the Flamm surface, embedded in $\mathbb{R}^3$.

It is the surface generated by the rotation of a lying parabola around its axis—a Flamm surface. This allows us to consider that curves (c) and (d) each lie on one of the two sheets joined along a throat circle.

It is immediately evident that this reasoning can be extended to the bilinear form corresponding to the spatial 3D part of the modern form of the Schwarzschild solution.

$$G = \frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\theta}^2 + \sin^2\theta\,\dot{\varphi}^2 \tag{42}$$

When Schwarzschild presents his solution, the object is then the union of two manifolds joined along their common boundary, which is not the Schwarzschild sphere but the surface of the star. Flamm presents in Figure 7 its meridian [13]:

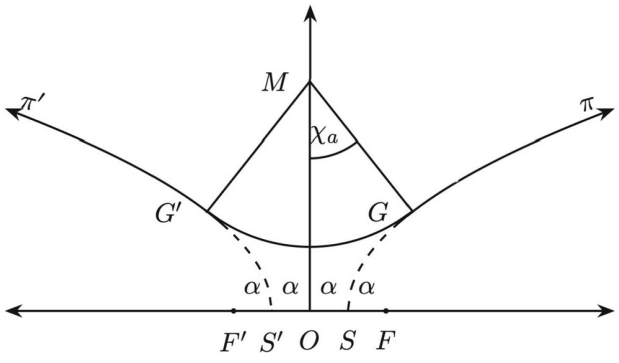


Figure 7. Meridian of the Schwarzschild hypersurface [13].

Flamm then uses only part of the lying parabolas, which connect via a circular arc. The interior geometry is that of a portion of S3 sphere, with metric:

$$\mathrm{d}\sigma^2 = \hat{R}^2\left(\mathrm{d}\chi^2 + \sin^2\chi\,\mathrm{d}\theta^2 + \sin^2\chi\sin^2\theta\,\mathrm{d}\varphi^2\right) \tag{43}$$

With:

$$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}} \tag{44}$$

The star's surface corresponds to $\chi = \chi_a$. Schwarzschild and Flamm obviously had no problem with topology. The 3D hypersurface is contractile and unfolds by translation in the time dimension.

The black hole model consists in considering, as a full-fledged geometric object, the one associated with the bilinear form:

$$\left(1-\frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1-\frac{2M}{r}} - r^2\dot{\theta}^2 - r^2\sin^2\theta\,\dot{\varphi}^2 \tag{45}$$

with the a priori hypothesis of contractibility, that is, of topology $\mathbb{R}\times\mathbb{R}^3$, which amounts to asserting that what corresponds to values $0 \le r \le \alpha$ is part of physics and gives rise to a true singularity at $r = 0$.

8. The Birkhoff Theorem [19]

When a theoretical physicist constructs a mathematical solution to an equation and wants that solution to correspond to a phenomenon situated within the domain of physics, it then appears essential to him that this solution be unique. The Birkhoff theorem is therefore, above all, a uniqueness theorem which immediately prohibits the presence of a cross term in drdt. Put differently, it requires that the solution be not only invariant under time translation—that is, stationary—but also invariant under the change $t \to -t$, that is, static.

In 1934, in his book [10], the mathematician Richard Tolman was the first to note that, mathematically, the general solution to Einstein's equation without source term, in spherical symmetry, includes a cross term in drdt. What is the consequence of this term? It is easy to reveal it by applying the time coordinate change proposed by A. Eddington in 1924 [20]:

$$t = t_E - \frac{\varepsilon\alpha}{c}\ln\left|\frac{r}{\alpha}-1\right| \quad \text{with} \quad \varepsilon = \pm 1 \tag{46}$$

Then the bilinear form becomes:

$$G = \left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t_E^2 - \left(1+\frac{\alpha}{r}\right)\mathrm{d}r^2 - \frac{2\varepsilon\alpha c}{r}\mathrm{d}r\,\mathrm{d}t_E - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right) \tag{47}$$

We note in passing that this transformation eliminates the coordinate singularity, which was Eddington's goal. Focusing on radial geodesics, P. Koiran [12] analyzed free-fall and escape times. Without a cross term, the null-length geodesics lead to the same velocity evolution as perceived by a distant observer:

$$V_\varphi = \pm c\left(1-\frac{\alpha}{r}\right) \tag{48}$$

The value is identical for emerging and plunging light rays. This velocity cancels out on the Schwarzschild sphere. In the form resulting from the Eddington transformation, this velocity depends on the direction and sign of ε.

First case: $\varepsilon = -1$ the case of photons in free fall, they have a constant radial velocity $V_\varphi = -c$.

Escaping photons have radial velocity:

$$V_\varphi = c\frac{r-\alpha}{r+\alpha} \tag{49}$$

Then this time becomes infinite.

If we choose $\varepsilon = +1$ he values are reversed. This time it is the photons on the escape trajectory that have a constant velocity c. If we refer to the Kerr metric, which also has a cross term in $\mathrm{d}r\mathrm{d}\varphi$ it also has two different speeds, depending on whether or not the azimuthal limine ray accompanies the rotational movement. This is known as "frame dragging", which is reminiscent of Mach's principle, as opposed to the covariance principle. In this way, the solution mentioned above could be associated with "radial frame-dragging".

Free-fall time calculation ($\nu = -1$), or escape time ($\nu = +1$), of a mass with zero velocity at infinity gives:

$$\mathrm{d}t_E = \nu\frac{r+\alpha\varepsilon\nu\sqrt{\frac{\alpha}{r}}}{c\left(r-\alpha\right)}\sqrt{\frac{\alpha}{r}}\,\mathrm{d}r \tag{50}$$

Whether this time is finite or infinite depends on how the equation behaves in the vicinity of $r = \alpha$:

$$\mathrm{d}t_E \approx \nu\frac{r+\alpha\varepsilon\nu}{c\left(r-\alpha\right)}\mathrm{d}r \tag{51}$$

We see that if $\varepsilon = +1$ this time is finite for plunging radial trajectories, infinite for escape trajectories. Inverse conclusion if $\varepsilon = -1$.

To manage this non-uniqueness of the solution, we are forced to abandon the topology $\mathbb{R}\times\mathbb{R}^3$ to that of a two-sheet covering of a manifold with a (spherical) edge. The first person to consider a two-sheet space-time covering structure was H. Weyl, in 1917 [21] with the aim of attempting a purely topological description of the masses, an approach which was taken up in 1935 by Einstein and Rosen [22]. This concept was also the basis of the wormhole object, supposed to represent a bridge, either between two universes, or between two different regions of space-time as shown in **Figure 8**. In this case, the two identical metrics represent an abstraction. Free-fall times, expressed as proper times, are finite and brief. It has been deduced that these unstable objects must reform immediately, but this adverb is only relative, since to a distant observer this time would be infinite.

If we try to interpret the reformulation of the solution with a cross term and impose continuity in speed, we are led to opt for $\varepsilon = -1$ in our own fold and for

$\varepsilon = +1$ in the adjacent fold.

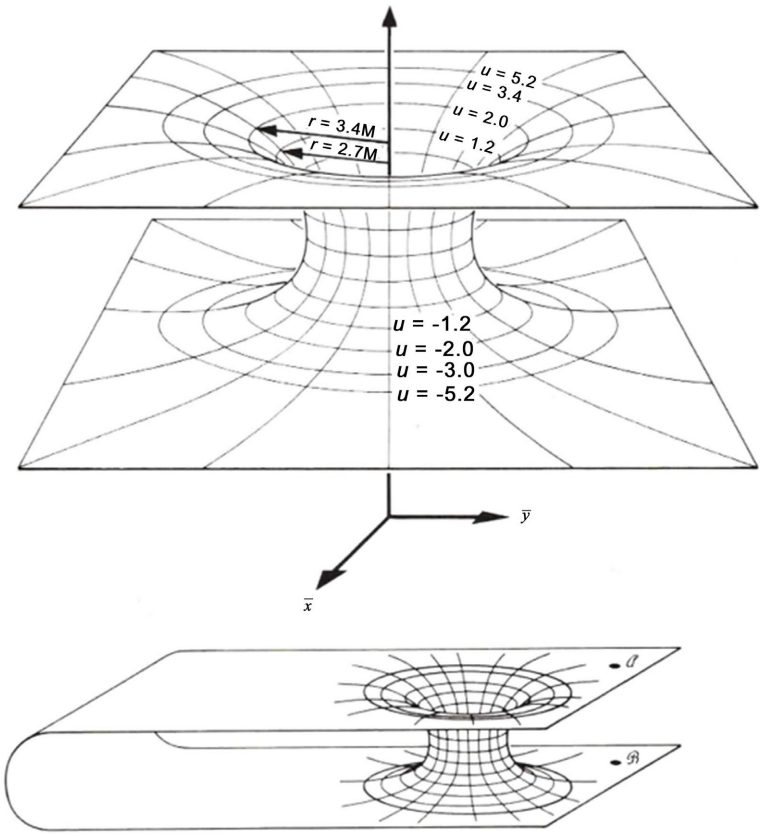


Figure 8. Wormholes [18].

If we opt for an "old-fashioned" definition of length and proper time, we have in our fold:

$$ds^2 = \left(1-\frac{\alpha}{r}\right)c^2 dt_E^2 - \left(1+\frac{\alpha}{r}\right)dr^2 - \frac{2\alpha c}{r}dr dt_E - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (52)$$

In the adjacent fold:

$$ds^2 = \left(1-\frac{\alpha}{r}\right)c^2 dt_E^2 - \left(1+\frac{\alpha}{r}\right)dr^2 + \frac{2\alpha c}{r}dr dt_E - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (53)$$

This results in a very short one-way transit time, which would invalidate Oppenheimer and Snyder's hypothesis [11]. Another view [23] is to attribute the inversion of the sign of the cross term to the inversion of the time coordinate in the second sheet, which becomes T-symmetrical to ours. With the help of an extremely simple diagram, it is also shown in [23] that the passage of masses through the throat sphere also generates a PT-symmetry on **Figure 9**.

Figure 9 may be confusing for readers who do not have good 3D vision. The 2D version in **Figure 10** is easier to visualize.

In 2D, an oriented object can be represented by a triangle with arrows, indicating an arbitrary direction of rotation. In fact, sliding this triangle according to the

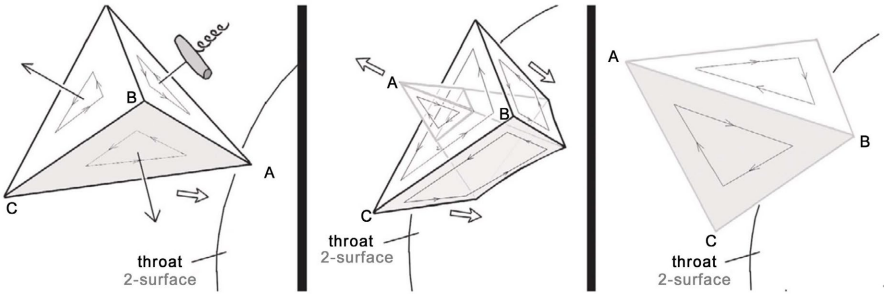


Figure 9. P-symmetry when crossing the throat sphere [23].

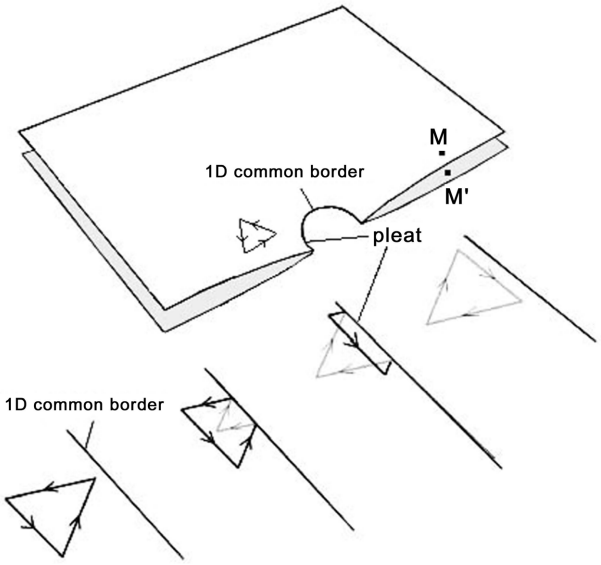


Figure 10. 2D-P-symmetry when passing the throat circle connecting two 2D spaces.

Figure 9 above, it passes over the other layer. The gray parts in Figure 9 show that, with respect to an observer living in the first layer, its orientation has been reversed. It crosses a sphere with a throat S1, that is, a circle. Note that any point M of the first layer is associated with a corresponding point M4, belonging to the second layer. This amounts to saying that these two points have the same polar coordinates. Here, the thing is clear because this 2D surface can be imbedded in our familiar Euclidean 3D representation space. By moving to three dimensions, we cannot produce a drawing and it is difficult to mentally represent two 3D spaces constituting a two-layer covering. What we can keep in mind is that any point M, with coordinates (x, y, z) of the first 3D sheet will have its equivalent M', identified with the same coordinate values. If we were to consider a film, taking place over a time t, the four vertices of the tetrahedron would evolve over time according to coordinates $\left(x^i_{(t)}, y^i_{(t)}, z^i_{(t)}\right)$. The reader can then imagine that the four vertices of the tetrahedron are steel balls sent onto the throat sphere, also made of steel. They bounce off this sphere and, see Figure 9, ball A being the first to touch the surface of the sphere, will also be the first to bounce, which will cause

the tetrahedron to turn over. Thanks to this, the reader can then admit that crossing a throat sphere causes the adjacent sheets to be P-symmetric.

This PT-symmetry refers to the extension of the Poincaré group described in [24] and to the idea that this inversion of the time coordinate then leads to the inversion of the mass in the later publication [25], leading to a reformulation of the pair of metrics, with:

$$\alpha = \frac{2GM}{c^2} \tag{54}$$

In our own old (attractive field):

$$ds^2 = \left(1-\frac{\alpha}{r}\right)c^2 dt_E^2 - \left(1+\frac{\alpha}{r}\right)dr^2 - \frac{2\alpha c}{r}dr dt_E - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{55}$$

In the adjacent fold (repulsive field):

$$ds^2 = \left(1+\frac{\alpha}{r}\right)c^2 dt_E^2 - \left(1-\frac{\alpha}{r}\right)dr^2 + \frac{2\alpha c}{r}dr dt_E - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{56}$$

9. Observational Data. The Giant Black Holes Hypothesis

Recently, the combination of powerful observing resources has enabled us to obtain images on **Figure 11** of the hypermassive objects M87* [26] and SgrA* [27].

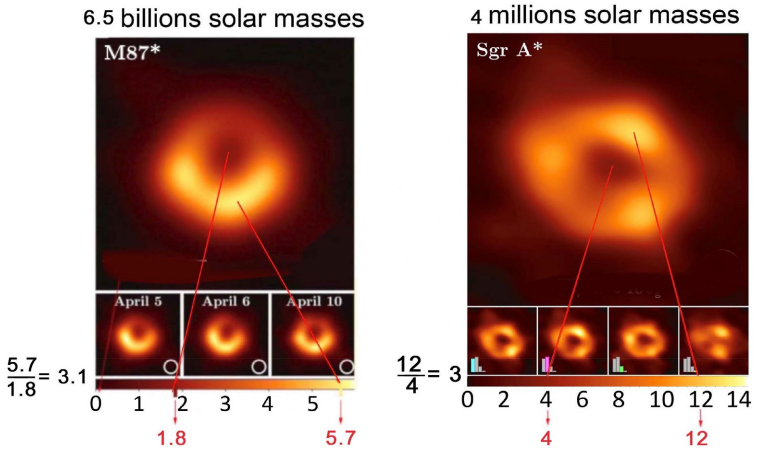


Figure 11. Images of hypermassive objects M87* and SgrA*.

One may wonder what meaning to give to the "equivalent temperature" values appearing on the two color bars. The radio telescopes forming the EHT system capture electromagnetic energy whose wavelength is between 0.3 mm and 1.3 mm. This is therefore a simple measurement of the energy emitted by the different parts of the object and, without prejudging the phenomenon which is at the origin of this emission of radiation, these values correspond to a brightness temperature. The figures obtained are surprisingly close to 3. Strictly speaking, therefore, we can't immediately identify them as giant black holes, for which the gravitational redshift effect would then lead to infinite values, unless the luminosity of the central part can be attributed to a mass of hot gas in the foreground. However, given

the vast differences in the masses and brightness temperatures of these objects, this hypothesis is not very credible. There's another aspect to the formation of such giant black holes, by successive accretions. If this were the case, the axis of symmetry of these objects would have a direction uncorrelated with that of the galaxy that hosts it. However, observers agree with the generalization of this correlation between the two axes. If these objects are Giant Black Holes, the emitted radiation does not emanate from the objects themselves, but from their accretion disks and corresponds to the synchrotron phenomenon, that is to say, the radiation emitted by electrons, orbiting in a powerful magnetic field. What we observe is that the central part is less emissive and that there is a region where this power is maximum. Using the color bars of the images we can form the ratio Maximum Brightness Temperature/Minimum Brightness Temperature and starting from these two images we find a value very close to 3. Is this value significant? This is a central question and we cannot affirm it. Indeed, in 2024 new images on **Figure 12** of M87* were obtained [28]. If the displayed value of the minimum brightness temperature has not changed, the same is not true of the maximum value:

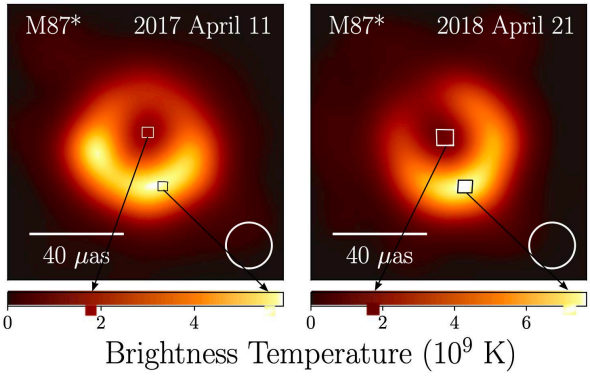


Figure 12. No variations in the display of extreme brightness temperatures of M87* [28] [29].

The colors are very similar. The article focuses on determining the diameter of the central dark spot. We will therefore take this assessment of the similarity of the ratios of these maximum brightness temperatures as the minimum value, with all due reservations. Let us therefore assume that the proximity of these two values is reliable and significant. If this is the case, and if these objects are Giant Black Holes, then it is necessary to explain why their [30]. Central regions do not include completely black areas, with an infinite gravitational redshift. Proponents of Giant Black Holes respond that this emission is due to the presence of hot gas in the foreground. Under these conditions (again, if we can trust these ratio values!), we might find it surprising that this is the case, given that the mass of M87* is 1925 times greater than that of SgrA* while its brightness temperature is, on the contrary, 2.1 times lower. Let us now see what happens with the simulations carried out, assuming that these objects are giant black holes [30]. The image obtained from M87* corresponds to **Figure 13** below.

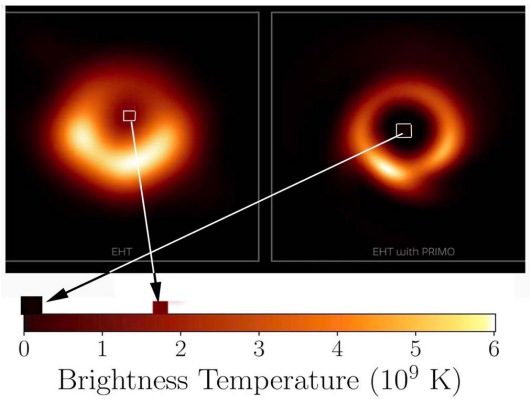


Figure 13. Attempt to reconstruct the image of M87 [30].

It should be noted that the authors did not consider the brightness value in the central region, as if this meant "this is the Giant Black Hole, as you would see it if we removed the mass of hot gas located in the foreground". Everything that follows depends on the assumption that this number 3, representing the ratio of the maximum brightness temperature to the minimum brightness temperature, is reliable and meaningful. We will only have possible confirmation of this in the relatively distant future. Indeed, these images could only be obtained for two very specific objects. The first is located at the center of our galaxy, and therefore the closest. The second is distant, but M87* is a monster. All the radio telescopes located on Earth, forming the EHT system, do not seem capable of producing images of other objects, which are too distant. For these future images to reach us, we will need to position orbital radio telescopes, a technically complex and costly operation.

Note that if these new images produce Maximum Brightness Temperature/ Minimum Brightness Temperature ratios significantly different from 3, then everything that follows will be null and void!

10. A Look Back at Karl Schwarzschild's Interior Metric Solution

The alternative is to assume that these supermassive objects have a central part darkened by a non-infinite gravitational redshift effect, which would then correspond to the value $z = 2$. This gravitational redshift then depends on the value of the g_{tt} coefficient of the interior metric, published by Karl Schwarzschild in his second article [5], which was not available in English until 1999 [31]. In its original form:

$$\mathrm{d}s^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 c^2\mathrm{d}t^2 - \hat{R}^2\left(\mathrm{d}\chi^2 + \sin^2\chi\mathrm{d}\theta^2 + \sin^2\chi\sin^2\theta\mathrm{d}\varphi^2\right) \quad (57)$$

The quantity $\hat{R}$ is the characteristic length:

$$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}} \quad (58)$$

The angle χ allows us to locate the position within the mass. The relationship with the radial variable r is:

$$r = \hat{R}\sin\chi \tag{59}$$

The wall of the object $r = r_a$ corresponds to the value χ_a. We then find the more familiar expression:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{1-\frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1-\frac{8\pi G\rho r^2}{3c^2}}\right]^2 c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{8\pi G\rho r^2}{3c^2}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right) \tag{60}$$

Which connects with the external metric:

$$\mathrm{d}s^2 = \left(1-\frac{8\pi G\rho}{3c^2 r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{8\pi G\rho}{3c^2 r}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right) \tag{61}$$

The time factor is then:

$$f = \frac{3}{2}\sqrt{1-\frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1-\frac{8\pi G\rho r^2}{3c^2}} \tag{62}$$

The gravitational redshift effect we want to consider corresponds to $f = \frac{1}{3}$

The ratio $\frac{r_a}{\hat{R}}$ is then equal to $\sqrt{8/9}$. And we have:

$$f = \frac{3}{2}\sqrt{1-\frac{8}{9}} - \frac{1}{2}\sqrt{1-\frac{8}{9}} = \frac{1}{3} \tag{63}$$

Thus we have a gravitational redshift:

$$1+z = \frac{\lambda_{\text{observer}}}{\lambda_{\text{emitter}}} = \frac{\sqrt{g_{tt_{\text{observer}}}}}{\sqrt{g_{tt_{\text{emitter}}}}} = \frac{1}{f} = 3 \tag{64}$$

Note that the Classical criticality, arising from the exterior metric, when the Schwarzschild radius catches up with the star's radius r_a, at constant, corresponds to:

$$1+z = \frac{\lambda_{\text{observer}}}{\lambda_{\text{emitter}}} = \frac{\sqrt{g_{tt_{\text{observer}}}}}{\sqrt{g_{tt_{\text{emitter}}}}} = \frac{1}{f} = 3 \tag{65}$$

We would describe this configuration (linked to the external metric, considered as describing the object entirely and in isolation) as:

Geometric criticality

Let us now return to this situation where the gravitational redshift effect $z = 2$ leads to a brightness temperature ratio of 3. Let us search the literature for curves referring to this situation as shown in **Figure 14**:

We have three curves before us. **Figure 14(b)**, top left, gives the time factor. The physical criticality situation corresponds to:

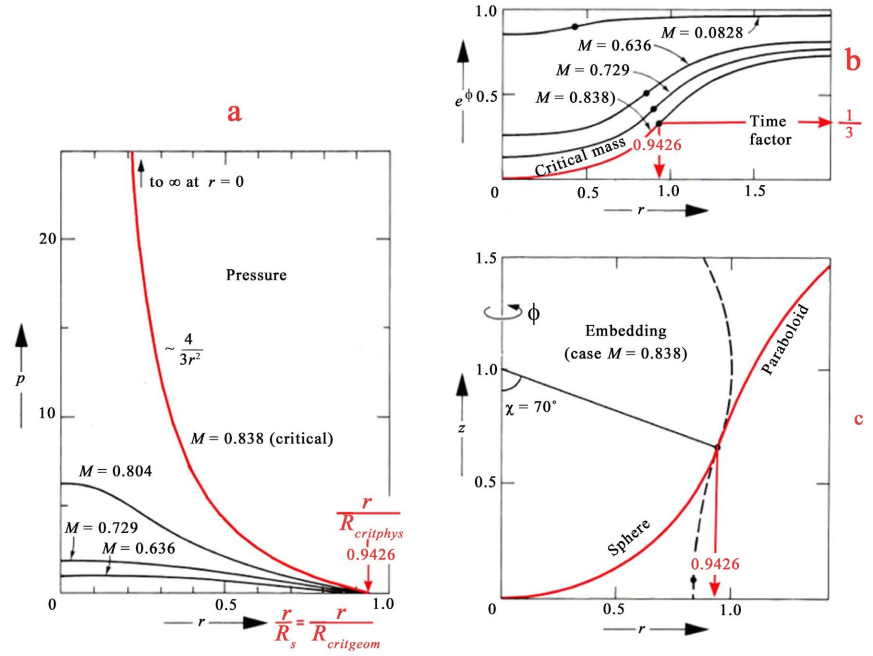


Figure 14. Physical criticality [32], page 611.

$$R_{\text{critphys}} = \sqrt{\frac{8}{9}}\hat{R} = \sqrt{\frac{8}{9}}R_{\text{critgeom}} = \sqrt{\frac{c^2}{3\pi G\rho}} = 0.9426\hat{R} \tag{66}$$

What we will call "geometric critical mass" M_{critgeom} will be, for a given density ρ :

$$M_{\text{critgeom}} = \frac{4\pi\left(R_{\text{critgeom}}\right)^3\rho}{3} = \frac{4\pi\hat{R}^3\rho}{3} \tag{67}$$

This is the "mass of the black hole". It is also the reference mass corresponding to the **Figure 14**, noted *M*. The curves indicated in red correspond to a mass:

$$M_{\text{critphys}} = \frac{4\pi\left(R_{\text{critphys}}\right)^3\rho}{3} = 0.838M_{\text{critgeom}} \tag{68}$$

Let's consider the diagram of the formation of a black hole by the capture of matter emitted by a companion star by a neutron star, as shown in **Figure 15**. Considering a scenario with constant density ρ , the mass will regularly increase.

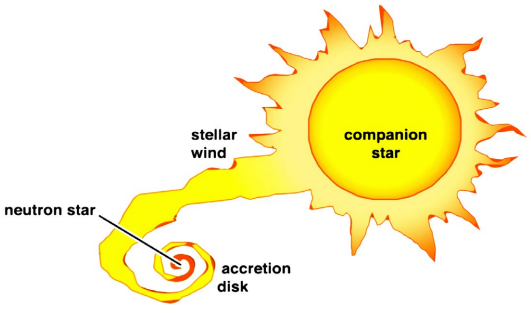


Figure 15. Capture of matter by a neutron star.

We then classically consider that the neutron star transforms into a black hole when its mass reaches the geometric critical mass M_{critgeom}, close to 3 solar masses. However, in this scenario, we ignore physical criticality, for $M_{\text{critphys}} < M_{\text{critgeom}}$, which is necessarily crossed, with a lower physical critical mass, close to 2.5 solar masses. Note that a similar situation would occur in the more brutal scenario envisaged in 1939 by Oppenheimer and Snyder [11]. **Figure 14(c)** shows the meridian of the object, that is, the particular case of the Flamm meridian, shown in **Figure 7**, in a situation of physical criticality, which corresponds to:

Figure 14(c) gives the meridian of the object, that is to say, the particular case of the Flamm meridian, of **Figure 7**, in a situation of physical criticality, which corresponds to:

$$\cos \chi_a = \frac{1}{3} \to \chi_a \cong 70^\circ \tag{69}$$

This follows from the original expression given by Schwarzschild in 1916, Equation (55) where the time factor is written:

$$f = \frac{3\cos \chi_a - \cos \chi}{2} \tag{70}$$

The center of the object corresponds to the value $\chi = 0$. The corresponding meridian curve will be more readable in **Figure 16** below:

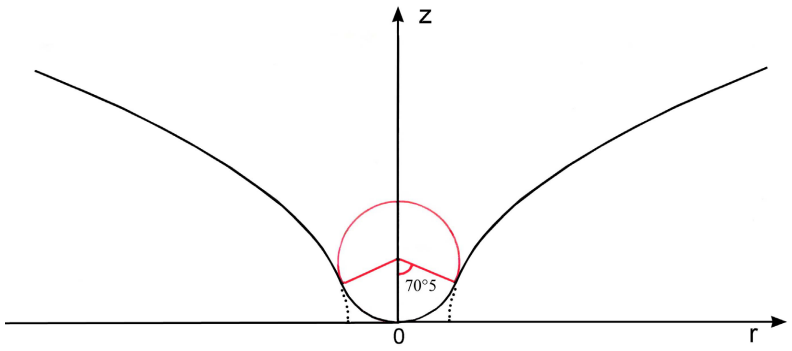


Figure 16. Meridian in a situation of physical criticality.

The curves giving the time factor are those in **Figure 14(b)**. On the red curve, the large black dot locates the object's wall. The time factor then has the value 1/3 at this point, which is consistent with this attempt to interpret the brightness temperature ratio as a gravitational redshift effect. However, we note that this coefficient is then zero at the center of the star.

For comparison, consider geometric criticality, when the object's radius equals the corresponding Schwarzschild length. The angle χ_a is then π/2, as shown in **Figure 17**.

Before the situation of geometric criticality is reached, a prelude to the formation of the black hole, the situation of geometric criticality cannot be avoided, simply by continuity. What happens then in terms of physics? This was known as early as 1916. As we have just seen, it is accompanied by the fact that the coeffi-

cient g_{tt} of the metric becomes zero. Let us quote these very important extracts from Karl Schwarzschild's February 1916 article in Figure 18.

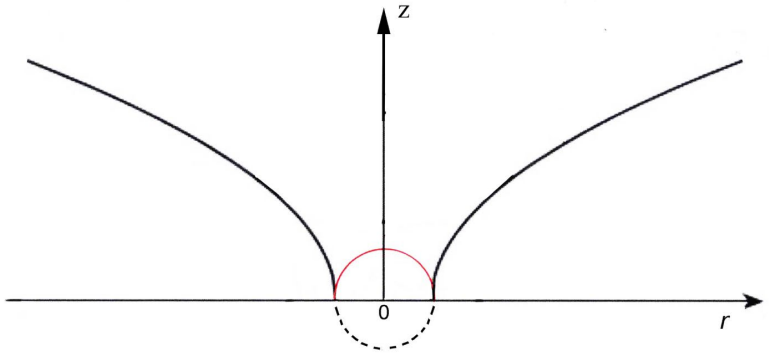


Figure 17. Meridian in a situation of geometric criticality.

This can be immediatly integrated, and gives:

$$(\rho_0 + p)\sqrt{f_4} = \text{konst.} = \gamma . \qquad (10)$$

Figure 18. Schwarzschild 1916. Relationship between pressure and metric potential g_{tt}.

ρ_0 is the density. To ensure homogeneity, we must read $\rho_0 c_0^2$ where c_0 is the value of the speed of light in a vacuum, to which Schwarzschild gives the value unity. f_4 is the metric potential g_{tt}, so $\sqrt{f_4}$ is the time factor. A little further on, we read extract in Figure 19.

through an elementary calculation the equations (13), (26), (10), (24), (25) transform themselves into the following:

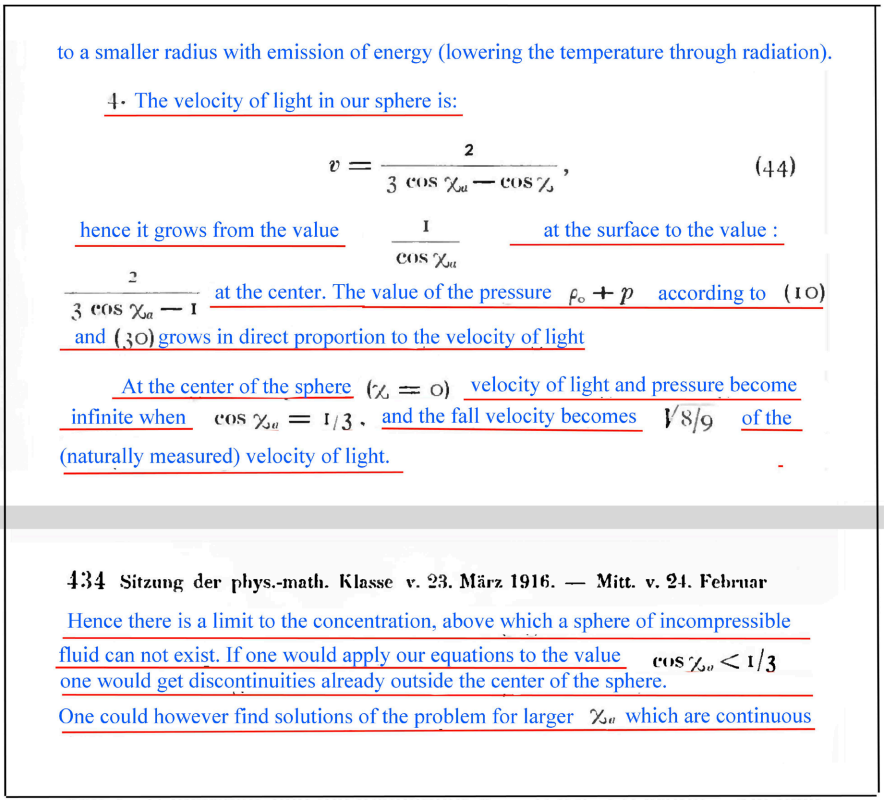
$$f_2 = \frac{3}{\varkappa \rho_0} \sin^2 \chi , \quad f_4 = \left(\frac{3 \cos \chi_a - \cos \chi}{2} \right)^2 , \quad f_1 f_2^2 f_4 = 1 . \qquad (29)$$

$$\rho_0 + p = \rho_0 \frac{2 \cos \chi_a}{3 \cos \chi_a - \cos \chi} \qquad (30)$$

Figure 19. Schwarzschild: Evolution of pressure inside the mass.

And Schwarzschild concludes with extract in Figure 20.

We see therefore that as early as 1916 Karl Schwarzschild had proposed a law of variation of the speed of light within masses, indicating the existence of a configuration of physical criticality such that the pressure and the speed of light became infinite. This remained ignored from 1916 to 1999, for 83 years, until this second article became available in the form of its English translation [31]. Until this date those who called themselves cosmologists had access to Schwarzschild's



to a smaller radius with emission of energy (lowering the temperature through radiation).

4. The velocity of light in our sphere is:

$$v = \frac{2}{3\cos\chi_a - \cos\chi}, \qquad (44)$$

hence it grows from the value $\frac{1}{\cos\chi_a}$ at the surface to the value : $\frac{2}{3\cos\chi_a - 1}$ at the center. The value of the pressure $\rho_0 + p$ according to (10) and (30) grows in direct proportion to the velocity of light

At the center of the sphere ($\chi = 0$) velocity of light and pressure become infinite when $\cos\chi_a = 1/3$, and the fall velocity becomes $\sqrt{8/9}$ of the (naturally measured) velocity of light.

434 Sitzung der phys.-math. Klasse v. 23. März 1916. — Mitt. v. 24. Februar

Hence there is a limit to the concentration, above which a sphere of incompressible fluid can not exist. If one would apply our equations to the value $\cos\chi_a < 1/3$ one would get discontinuities already outside the center of the sphere.

One could however find solutions of the problem for larger χ_a which are continuous

Figure 20. In 1916 Karl Schwarzschild obtained a variable speed of light within masses [5].

work only through second-hand texts, noting that neither Hilbert [7], nor Eddington, who first echoed this general relativity in a book published in 1924, reissued in 1960 [9], mentioned it. No mention of this physical criticality in Tolman's work, 1934 [10]. In 1939 Tolman [33], Oppenheimer and Volkoff [34] joined forces to publish this equation of state in a differential form, which would become the famous TOV equation [35]:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left(\rho + \frac{p}{c^2}\right)\left(m + \frac{4\pi G p r^3}{c^4}\right)c^2}{r\left(r - 2m\right)} \qquad (71)$$

About thirty years ago, out of curiosity, we had numerically constructed the solution to this equation, at constant ρ (unpublished as shown in **Figure 21**), and thus put our finger on this physical criticality, while ignoring that Karl Schwarzschild had already written it down, providing the exact solution:

It was this observation that triggered all the work of examining the foundations of black hole theory, spread over decades; of which this article represents the conclusion. Returning to the TOV equation, the authors' idea was obviously to try to extend Schwarzschild's work by considering configurations with non-constant density. But, in this form, physical criticality was no longer evident and its existence simply remained ignored by the entire scientific community for decades. It was only in later editions that the curves in **Figure 14(a)** appeared in [32]. If this

last reference represents the state of the art in the matter, what do we read about this surge in pressure? Here is this passage:

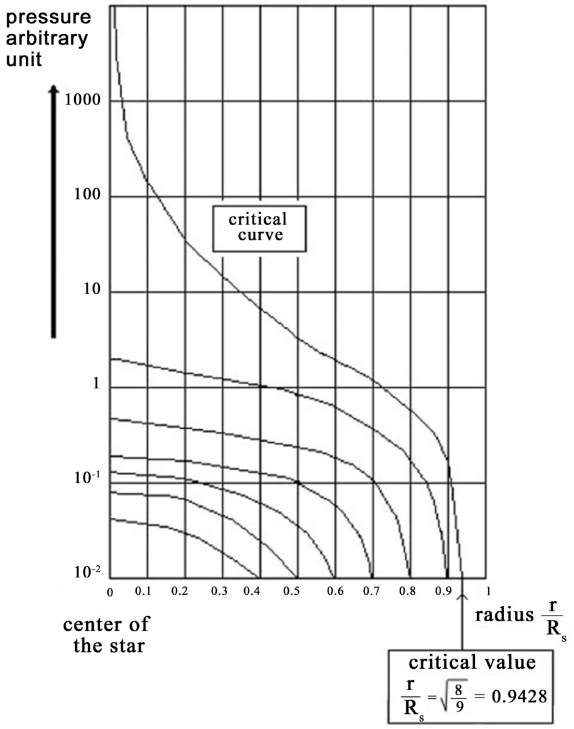


Figure 21. Physical criticity evidence (1988, unpublished).

"*Incompressibility would imply a speed of sound* $v=\left(\mathrm{d}p/\mathrm{d}\rho\right)^{1/2}$, *of unlimited magnitude, therefore in excess of the speed of light, and therefore in contradiction with a central principle of special relativity* ("*principle of causality*") *that no physical effect can propagate at a speed* $v>c$."

This property of a limitation of the speed of light applies to vacuum. There is no conceptual tool, nor experimental result, which imposes such a principle when electromagnetic waves propagate in matter at very high density. In an ideal gas the pressure is the volume density of kinetic energy of thermal agitation and is based on the value of the root mean square value $\left\langle V^2\right\rangle$ associated with a thermal agitation speed *v*.

$$p_m=\frac{\rho\left\langle V^2\right\rangle}{3} \tag{72}$$

When considering the state of such a Subcritical Schwarzschild Body (SSB) it can only be that of a fully ionized plasma where the dominant component of pressure is radiation pressure:

$$p_r=\frac{\rho c^2}{3} \tag{73}$$

In a relatively cold (non-relativistic) gas, sound corresponds to the propagation of acoustic waves, from near to far, resulting from collisions between particles

with mass. This propagation speed is then of the same order of magnitude as the thermal agitation speed of the elements considered. In the plasma composing an SSB, the density and temperature are then comparable to that which governs in the primitive state of the universe, where the thermal agitation speed then reaches the speed of light. S. Weinberg, in his famous book "The first three minutes", summarized this by writing that at that time the universe "was filled with all kinds of radiation", in the plural. Thus the speed of light, as the speed of propagation of information, replaces the thermal agitation speed of the components. This leads us to focus our attention on this primitive state, knowing that at constant c we are confronted with the paradox of the extreme homogeneity of the medium. Two solutions then present themselves. The mainstream thesis is that of inflation. Another is to envisage an era of "variable constants" [36] such that the cosmological horizon increases as the spatial scale factor a. In 1916, Karl Schwarzschild was the first to envisage a variation in the speed of light within mass, resulting from an exact solution to Einstein's equation with a right-hand side. In the absence of credible ontological arguments, as in [31], one cannot make a choice by retaining only part of an exact solution, considering the other part as incredulous.

But one can just as well consider that this variation in the speed of light within masses, at very high density, as it emerges from this exact solution to Einstein's equation with a right-hand side, belongs to the world of physics. It therefore seems perfectly legitimate to us to conclude that this ratio Maximum brightness temperature/Minimum brightness temperature = 3 (if this data can be considered reliable) fits with a model based on this Schwarzschild Subcritical Body (SSB) model where, at the center, the pressure and the speed of light reach considerable, if not infinite, values, which then allows the pressure forces to counterbalance the force of gravity.

11. About the Stability of Such Schwarzschild Subcritical Bodies

To address this question of the stability of these SSBs, it would be necessary to have a perturbational, time-dependent solution of the equation, which is not the case today and will have to be done. Returning to the passage from Schwarzschild's article and its translation into English, we see that he considers that if these conditions were exceeded, a singularity could appear, breaking the topology of the solution, that is to say, the total contractibility of space, at all points. Using the word from modern cosmology, we would use the term "wormhole", a concept that suggests a connection between two distinct geometric structures, two sheets of space-time. Before Einstein and Rosen published their own article in 1935 [22], the first person to suggest the existence of such a geometric structure was H. Weyl in 1917 [21]. Of course, with respect to what we are going to consider, his reflection, like that of Einstein and Rosen, only concerns the exterior metric solution, but the general idea is the same, and we will return to this part of his article in more detail. Like Einstein and Rosen, he therefore envisages a description of

masses in the form of a topological singularity. We note that he endows his coordinates and his length with the same properties that Einstein and Schwarzschild attributed to him at the time:

$$ds^2 = g_{ik}dx_i dx_k > 0 \tag{74}$$

The same lack of ambiguity concerns its variable r:

$$r = \sqrt{x_1^2 + x_2^2 + x_3^2} \tag{75}$$

By qualifying the length a as the "gravitational radius", which will later be called the "Schwarzschild radius", he produces, like Flamm, the expression for the meridian:

$$z = \sqrt{8a(r-2a)} \tag{76}$$

And then he writes:

"This metric characterizes the geometry of a paraboloid of revolution whose equation is written, in Euclidean space with orthogonal coordinates x_1, x_2, z. Its projection on the plane $z = 0$, *whose points are identified by polar coordinates* r, ϑ, *covers twice the area outside the circle of radius* 2a. *But the surface does not project inside this circle*.

Analytically and continuously, the set of points in real space, identified by the coordinates x_i, which is used to represent the object, *will cover twice the area corresponding to* $r \geq 2a$. These two coverings are separated by the sphere $r = 2a$, on which the mass is located, and which is a singular place."

Above, we have indicated in italics two key passages from Weyl's article [21]. The first illustrates the non-contractibility of the object, which goes hand in hand with this dreamlike structure that Weyl was the first to report in physics, but which has its source in Riemann's doctoral thesis, 1851 [37], which for the first time introduces the concept of a two-sheeted covering of a manifold, which he later took up again in 1856 [38]. In 1882 the mathematician Felix Klein [39] systematized this work of Riemann, emphasizing the geometry of covered surfaces and automorphic transformations. In [40] Klein presents an extension to automorphic functions, that is to say functions invariant by discrete groups of isometries (Fuchsian, Kleinian groups), directly linked to the coverings of the hyperbolic plane. Weyl is therefore the first to use this concept by applying it to physics. In this perspective, the Janus Cosmological Model [24] extends the use of this geometry and mathematics tool, created more than a century ago, to the entire cosmological model. The elements presented here therefore do not simply reflect a physical interpretation, devoid of foundations. In the rest of the article, the reader will find what is today presented as a representation of this exterior Schwarzschild solution using "isotropic coordinates", but without mentioning the motivation for the approach.

$$d\sigma^2 = \left(1 + \frac{a}{2r}\right)^4 \left(dx_1^2 + dx_2^2 + dx_3^2\right), \quad f = \left(\frac{r - a/2}{r + a/2}\right)^2.$$

Revisiting Weyl's presentation:

In the new coordinates, this metric becomes consistent with a Euclidean metric. The elongation factor being:

$$\left(1+\frac{a}{2r}\right)^2.$$

$\mathrm{d}\sigma^2$ is regular for all values $r>0$ f is positive and becomes zero for:

$$r>0$$

The circumference of the circle $x_1^2+x_2^2=r^2$ is given below. The circle is nothing but the equator of the throat sphere of such new geometric structure.

$$2\pi r\left(1+\frac{a}{2r}\right)^2;$$

this function, if we pass r in a decreasing way through the values starting from $+\infty$, decreases monotonically up to the value $4\pi a$, which is reached for

$$r=\frac{a}{2},$$

but then starts to increase again as r approaches zero. This geometric structure is therefore non-contractile. This circle is nothing but the equator of a family of circles whose area becomes minimal on the throat sphere. This ensures the passage between two space-times, described by Weyl as "exterior" and "interior" of this geometric object which he identifies with a mass. According to the view above, here the area

$$r>\frac{a}{2}$$

would correspond to the exterior of the mass point, and

$$r<\frac{a}{2}$$

inside the mass-point.

If we continue analytically, in this region $r<\frac{a}{2}$ the term:

$$\sqrt{f}=\frac{r-a/2}{r+a/2}$$

becomes negative, in this region of space-time, so that the time coordinate (x_4) and proper time are opposed to it for a stationary observer.

Here we recognize the first introduction of isotropic coordinates and the "time factor" conceived as the square root $\sqrt{g_{44}}=\sqrt{g_{tt}}$. A quantity that we have called f in our article, as is customary in the literature. We will not make the confusion: in Weyl's article we have $f=g_{tt}$. The same observation applies to the inversion of the time coordinate (x_4 in Weyl), which arises from the simple fact that we cannot make a "half-turn" along a geodesic, that is to say, invert ds. The formulation of Schwarzschild's exterior metric solution, using isotropic coordinates, appears in all books presenting the theory of general relativity. But none of them presents what originally motivated its author, Hermann Weyl, in 1917: an attempt at a

purely geometric model of masses, as topological singularities. What we will remember is that this approach leads to a second space-time sheet, where the time coordinate is reversed. The passage through a throat sphere means that this second world sheet is also P-symmetric (see Figure 9). Thus this second world sheet is PT-symmetrical with respect to ours ([23]-[25]). A detour through Weyl's article shows us that considering the appearance of a topological singularity (suggested by Schwarzschild [5]) leading to a PT-symmetrical space-time sheet is therefore neither artificial nor arbitrary. Furthermore, this reaction of the system would translate, locally, a connection of the two symmetrical PT sheets of the Janus Cosmological Model. In the absence of this time-dependent perturbational solution, we will seek the "response" that the stationary solution would provide when we go beyond the subcritical situation. This leads us to plot the curve giving the time factor in such conditions.

12. The Suggested Plugstar Mechanism [41]

In the present article we take the same approach as Weyl by exploring what happens to this interior metric, a solution, let us recall, stationary to Einstein's equation with right-hand side. When we exceed what we have called "physical criticality" the time factor becomes negative as shown in Figure 22.

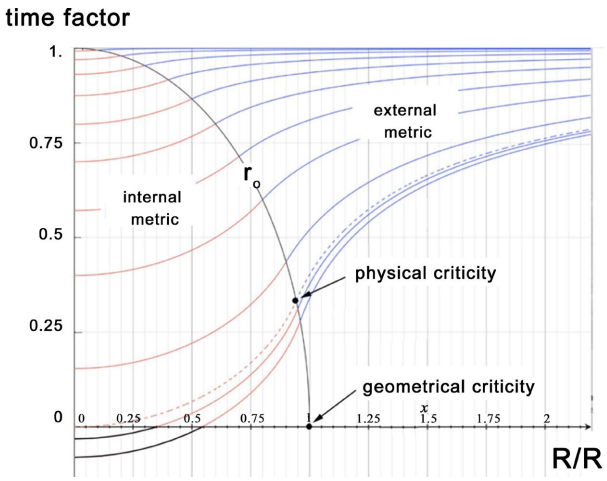


Figure 22. Time factor.

Beyond physical criticality, the time factor f becomes negative:

$$d\tau = \frac{ds}{c} f dt < 0 \tag{77}$$

Doing as Weyl did in 1917 we deduce that this leads to the inversion of the time coordinate t (we can also say that maintaining the sign of ds has the physical meaning that we cannot turn around on a timeline). Like this inversion of the time coordinate (improperly called the "arrow of time") this brings the image of the inversion of the light cone, which turns over like an umbrella in a gust of wind. In its median position the cone transforms into a flat disk as shown in Figure 23:

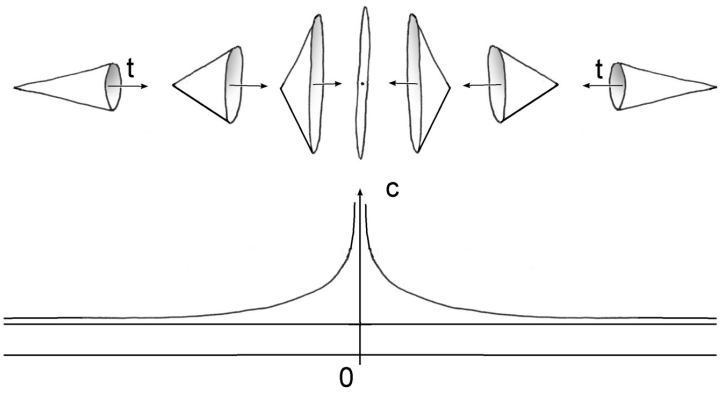


Figure 23. The image of the light cone turning over.

We must then appeal to dynamic group theory [42] to obtain the physical meaning of this inversion of the time coordinate. All this is detailed in the article [24]. To summarize in a few words: Physicists, classically, only use the restricted Poincaré group, limited to its orthochronous components, a subgroup of the isometry group of Minkowski space. The complete Poincaré group then contains antichronous components, inverting the time coordinate. A first action of the group consists of making its components act on the points of space-time. In symplectic geometry we introduce a second action on the dual of the Lie algebra of the group. This brings up momentum space, which has the structure of a vector space and whose dimension is the same as that of the group: 10. Among its components, we see a scalar appear: energy, and two 3-vectors, momentum and spin, quantities which then emerge as objects of pure geometry. We can also make them appear as an application of Noether's theorem. Energy is then the invariant linked to the subgroup of time translations, momentum to the subgroup of space translations and spin to the subgroup of four-dimensional rotations, arising from the Lorentz group. The action of symplectic geometry adds that of the group on these different quantities. Thus the action of the antichronous components leads to that of energy E, and therefore of mass. This property constitutes the group basis of the Janus model, as a two-sheet covering of a compact and inorientable 4-manifold, this creating the PT-symmetry of the two sheets [24]. The second sheet therefore contains negative masses and negative energy photons which travel according to a second metric $\overline{g}_{\mu\nu}$, while the positive energy masses and photons follow the geodesics resulting from the metric $g_{\mu\nu}$. We therefore have a new geometry, bimetric. The unique equation of general relativity then becomes one of the two equations of a system of two coupled field equations, constructed from an action and satisfying the Bianchi conditions, which is mathematically consistent [24]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi \left(T_{\mu\nu} + K_{\mu\nu} \right) \tag{78}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \overline{g}_{\mu\nu} = -\chi \left(\overline{T}_{\mu\nu} + \overline{K}_{\mu\nu} \right) \tag{79}$$

χ is the classical Einstein constant. Thus we can say that the Einstein constant

of the second sheet is equal and opposite to the first:

$$\bar{\chi} = -\chi \tag{80}$$

$R_{\mu\nu}$, R, $\bar{R}_{\mu\nu}$, $\bar{R}$, are the Ricci tensors and scalars from the two metrics. $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ are the field tensors of the positive and negative masses. $K_{\mu\nu}$ and $\bar{K}_{\mu\nu}$ are interaction tensors,

Not to be confused with field tensors!

While the attempt to introduce negative masses into general relativity (by requiring them to follow the geodesics from the single metric $g_{\mu\nu}$) led to the forces as shown in Figure 24:

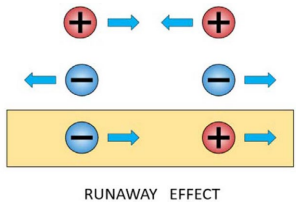


Figure 24. Force laws from general relativity.

That is to say, to the runaway phenomenon, contradicting the principle of action reaction, the Janus Model provides a different scheme as shown in Figure 25 (derived from the relation $\bar{\chi} = -\chi$)

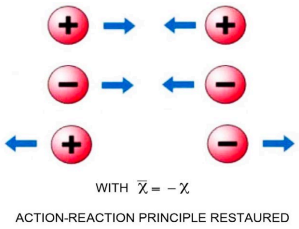


Figure 25. Force laws from the Janus Cosmological Model [24].

In the positive layer the metric $g_{\mu\nu}$ corresponds, inside, then outside the object, before geometric criticality is reached. These metrics are given by Equations (60) and (61). In an approximation the geometry in the adjacent portion of the second layer, in the Janus Cosmological Model (limiting ourselves to the case we can apply the Newtonian approximation) then corresponds to the two metrics $\bar{g}_{\mu\nu}$, interior and exterior:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{1+\frac{8\pi G\rho r_a^2}{3c^2}} - \frac{1}{2}\sqrt{1+\frac{8\pi G\rho r^2}{3c^2}}\right]^2 c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1+\frac{8\pi G\rho r^2}{3c^2}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right) \tag{81}$$

$$\mathrm{d}s^2 = \left(1+\frac{8\pi G\rho}{3c^2 r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1+\frac{8\pi G\rho}{3c^2 r}} - r^2\left(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right) \tag{82}$$

So we have the answer to the question: "what happens when we exceed the physical criticality and a small amount of mass is inverted at the center of the object?" This mass, sensitive to the dominant gravitational field, created by the positive mass, is violently ejected. As the Janus model is based on the hypothesis that masses of opposite signs only interact antigravitationally, these negative masses pass through the object freely. Figure 26 below is a didactic image evoking the repulsion experienced by inverted masses.

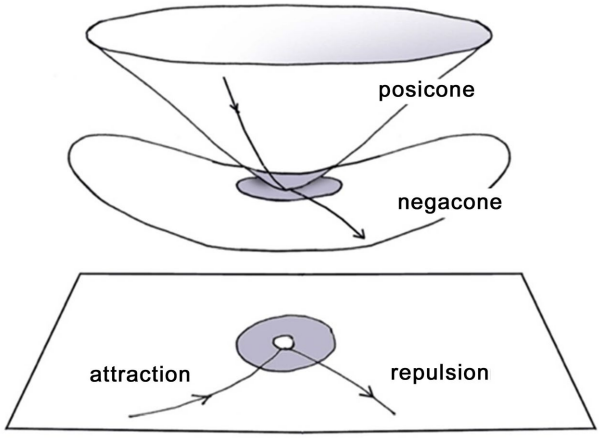


Figure 26. Didactic 2D image showing the repulsive effect on negative masses.

In the positive sheet, the portion of space containing the positive mass is represented by a spherical cap, which is consistent with the Flamm meridian. Its image, in the negative sheet, is then a portion of a horse saddle. These negative masses are then taken over by the gravitational field of the galaxy and continue their journey to intergalactic space. This is also populated by negative masses, but this time it is primordial antimatter [24] (the Janus model is built, no longer only on a PT-symmetry but on the basis of a CPT-symmetry). There are therefore annihilations between the particles of matter of negative mass ejected from the Supermassive Shwarzschild Body and the particles of antimatter of negative mass populating intergalactic space (and ensuring the confinement of galaxies) [24], these annihilations producing negative energy photons, escaping all observation. We thus obtain a phenomenon that would ensure the self-stability of these SSBs to which we propose to give the name of Plugstars.

In the case of neutron stars, this negative feedback would then limit their mass to around 2.5 solar masses. Figure 27 illustrates the phenomenon:

But supermassive objects at the centers of galaxies are obviously not giant neutron stars.

Note 1: Little attention has been paid to Schwarzschild's interior solution [5], for two reasons. The first is that this article was only available in English in 1999. The second is the criticism Einstein made of it in 1939 [43], in a short commentary. When the object evolves toward physical criticality, at constant density, Schwarzschild finds that the pressure tends toward infinity. But the constancy of density

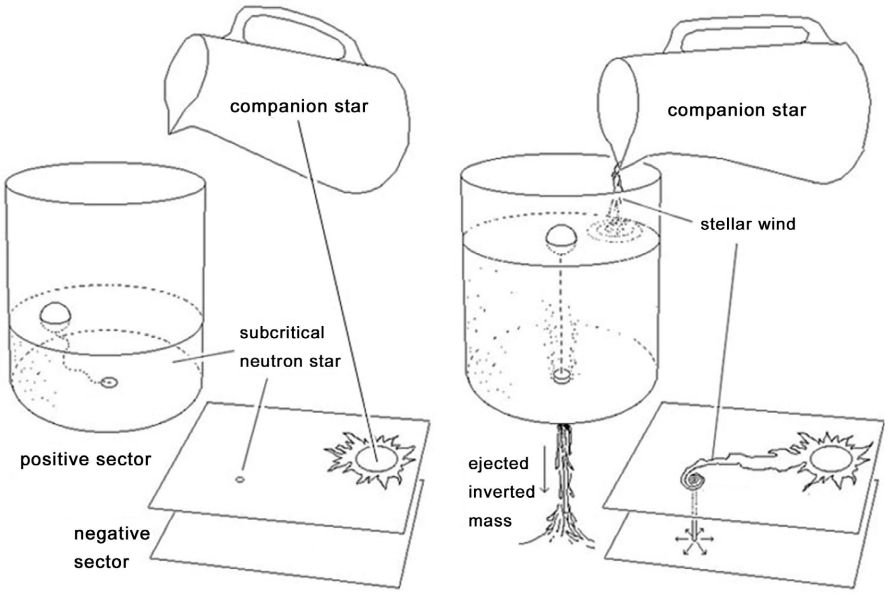


Figure 27. 2D educational image evoking the Plugstar phenomenon and the self-stability of subcritical neutron stars.

within subcritical objects is not physical. In the core of neutron stars, for example, the density becomes so high that the distance between baryons becomes on the order of their Compton length. Under such conditions, general relativity is no longer sufficient to describe the state of matter. Mastering all the phenomena, including the suggested mass inversion, will only be possible once the cosmological model has been unified, incorporating quantum mechanics, a process whose beginnings are laid by the work of N. Debergh [44]-[47]. The phenomenon of excess mass inversion, in the case of object mergers, would be accompanied by a powerful emission of gravitational waves. Integrating this phenomenon into the decoding of signals from the LIGO and Virgo devices could lead to a downward revision of the estimated masses involved, which are immediately identified as black holes.

Note 2: This plugstar model was initially developed based on the steady-state solution, an approach that should be reconsidered using an unsteady, possibly perturbational, solution. However, we can observe, purely as an example, how this central region of the object would evolve when the physical criticality is exceeded, where the inversion of the time factor leads to the inversion of the mass.

We then find that the diameter of this region grows extremely rapidly as shown in **Figure 28**. We can then imagine that when a subcritical neutron star receives an influx of matter from a companion star, the topological singularity that ensures the removal of an equivalent amount of matter to the center opens and closes extremely rapidly, like a "3D camera diaphragm". This perspective leads us to reconsider the merger of two neutron stars, when the sum of their masses exceeds the critical value and the inversion of the excess mass is accompanied by powerful dissipation in the form of gravitational waves. Finally, a third scenario emerges, associated with the implosion of a massive star. Here again, we would have a mass

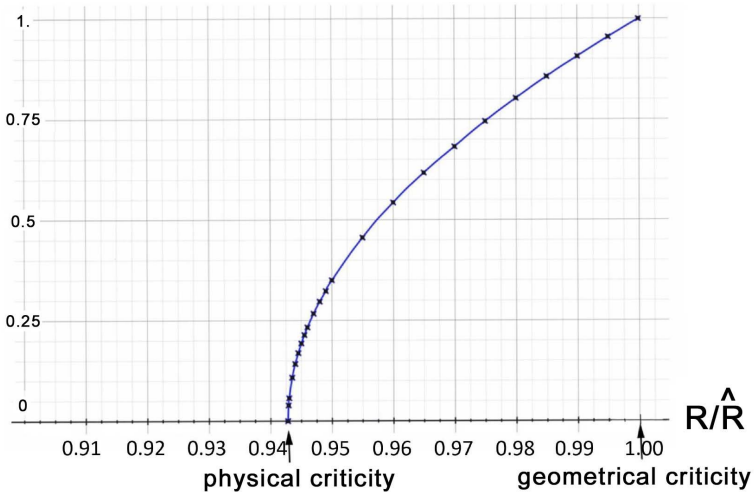


Figure 28. Suggestion for the window opening speed.

inversion effect. But what would happen if the inverted, self-attractive mass exceeded the remaining positive mass? This negative mass would not disperse, but would constitute the nucleus of a new object made up of an invisible, spheroidal negative mass, surrounded by a remnant of positive mass in the form of a halo, which would be observable. It is conceivable that this relatively diluted positive mass would lose all energy through radiative dissipation and adapt to the ambient background radiation temperature. A new field of investigation to explore.

13. About the Origin of Such Hypermassive Objects and the Nature of Quasars

Seyfert galaxies, modulo the difficulty of evaluating the distance, seem to produce a Hubble constant significantly lower than the standard value. If we use **Figure 29** for the Hoag galaxy with $D = 56$ MPc and $z = 0.004$ we obtain a value of H_o of 21.4 km/s/Mpc. We conjecture that very irregular galaxies should give a value of H_o higher than the standard value. The idea is to consider a phenomenon of expansion turbulence, resulting from joint fluctuations of the metrics, which would have the effect of varying the confinement of the galaxies. In the case of weakening of this field, this could go as far as the dislocation of the structure. In the opposite case where there is a strengthening of the gravitational field, this could cause the start of a centripetal density wave. This, destabilizing the gas, would generate the birth of new stars which, during their early youth, would reveal their presence, as in the spiral structure, by illuminating the gas through fluorescence. This is the interpretation that we propose to give to the circular formation present in the Hoag galaxy, as a density wave.

Young stars are emissive for times between 5 and 20 million years. Measurements relating to the year give a rotation speed of 250 km/s, but no radial speed. If the ring is a density wave, this is normal, because it is the speed of the wave (comparable to a tsunami with spherical symmetry) and not that of the wave of matter that reacts to its passage. The width of the ring, being Hoag's 7.4 kpc,

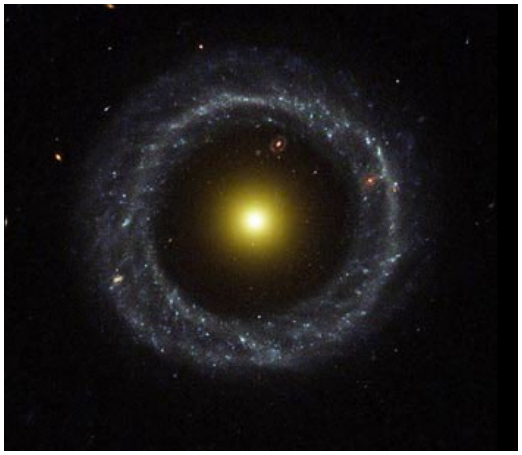


Figure 29. Hoag galaxy ([48]).

this allows an evaluation of the radial speed of the wave between 730 and 1470 km/s. It would then reach the center of the galaxy in a time between thirteen and fifty million years. The magnetic Reynolds number being large compared to unity, the wave then brings together the lines of force of the pre-existing magnetic field in the galaxy, which is evaluated at one microgauss. By bringing these lines together, an intense dipole magnetic field is formed in a direction that coincides with the axis of symmetry of the galaxy. The object that forms when the wave is focused at the galactic center is small. Its angular rotation speed, as a solid body, is that which animated the galactic center before the impact, therefore moderate. There is a sudden excursion of density and temperature such that the Lawson conditions are reached in a very large mass. Hence a powerful plasma emission according to the two poles of the magnetic field. This is our vision of the quasar phenomenon. Moreover, this system automatically behaves like a natural accelerator of charged particles, capable of giving them considerable energies and we see there the origin of cosmic rays. From this angle, the object located at the center of our galaxy could be considered as an extinct quasar. This mechanism of joint fluctuations of metrics, which we are currently trying to model, could periodically restart the emission, stimulated by the convergence of new density waves, of very low amplitude, but each time causing an emissive burst. Hence the dashed structure of the M87 jet. We conjecture that these joint fluctuations of metrics would have been very important at the very moment of the formation of galaxies. This is the reason why we observe the presence of these hyperdense objects in the oldest galaxies.

14. Conclusions

The model of black holes, of all sizes—mini, stellar, giant—has gradually taken its place in the scientific community over the past half-century, to the point of becoming the deus ex machina of astrophysics. Are Quasars fantastic sources of radiation emission? This is due to matter falling into a black hole. A question stirs the community of specialists: "What happens to the information when it falls in-

side black holes?" Gravitational waves are detected? The signals are immediately interpreted as the result of black hole mergers. Very massive objects are identified at the centers of galaxies? As soon as the first images reach us, the journals don't headline: "Images of supermassive objects", but "Images of giant black holes". The argument: we can't see what else they could be.

However, a necessary return to the ontological foundations of this model is required. And it is here that we realize it is merely a chimera, mathematically and geometrically speaking. In 1939, advances in observations raised the question of the fate of massive stars. What happens when these stars collapse onto their iron cores? In 1939, Oppenheimer and Snyder provided the answer. Considering that, according to the standard form of the solution to Einstein's equation without a second member, the free-fall and escape times of a test mass are infinite, as perceived by a distant observer, this allows us to envision describing an eminently unsteady phenomenon using a steady-state solution. The concept of the cosmological horizon then emerges. For this solution to be meaningful, since it refers to the vacuum, all the mass must be contained within a central singularity. We then calculate the spiral trajectories towards it, within what is considered the interior of this Schwarzschild sphere. In this domain, are the signs of the terms in the description of the metric reversed? The explanation is found: it's because the space and time coordinates are reversed. All this because the idea that this Schwarzschild geometry could escape contractibility is not considered by anyone, even though it was perfectly described as early as 1916 by the young mathematician Flamm.

We show how all of this relies both on a topological assumption and on a singular definition of the Lagrange function used for calculating geodesics. Indeed, when, in this Lagrange function, the derivatives $\dot{x}$ are calculated with respect to the parameter s, the resulting Lagrange equations are the same, whether the action is evaluated based on a length or any power of this quantity, in this case, its square. The equations then provide real curves, but equipped with a purely imaginary length, and no one notices that this is simply because these curves lie outside the considered hypersurface. In addition, in 2021, the mathematician Koiran further showed that taking into account a cross term in $\mathrm{d}r\mathrm{d}t$, depending on its sign, gives either the free-fall time or the escape time a finite value, which invalidates the idea of using this stationary solution. Birkhoff's theorem is then invoked. However, this theorem does not impose the absence of this term; it indicates that it can be reintroduced using a simple change of variable, which is entirely different. If treated as an existence theorem, it then prescribes the coexistence of two solutions (where these terms happen to have opposite signs), which aligns with the reinterpretation of the solution through the Janus Cosmological Model. In this representation of the universe, based on a two-sheet covering, there must indeed be two PT-symmetric solutions for each set of four coordinates.

But then, what is the result of massive stars collapsing in on themselves? What happens to neutron stars whose mass inevitably grows due to the capture of stellar wind emitted by a companion star? What are the supermassive objects M87* and

SgrA*? We then focus on the main piece of data leading to observations: the brightness temperature ratio, which turns out to be significantly close to 3. We then observe that this is the value of the gravitational redshift phenomenon that an object described by the second Schwarzschild metric solution would exhibit when it is close to a subcritical situation where the pressure at the center reaches a value that is, if not infinite, at least considerable, thus counterbalancing the force of gravity. Immediately, the objection raised by Einstein as early as 1939 resurfaces. But this objection remained based on an object of constant density. And he concludes, we quote, "It would be necessary, therefore, to introduce a compressible liquid whose question of states excludes the possibility of signals with a speed in excess of the velocity of light." If we consider the core of neutron stars, the concentration of baryons is such that the distance separating them becomes on the order of their Compton length. Here we touch upon a domain where giving a physical description of matter itself becomes problematic. The question can only be clarified when a common formalism has unified gravitation and quantum mechanics. A mechanism of self-stability is then suggested for objects henceforth designated as plugstars. The slightest criticality exceedance then leads to a local inversion of the term g_{tt} at the center of the object. By using the concepts developed through the Janus Cosmological Model, where the universe is the two-folds cover of a P4 projective, which generates the PT-symmetry of the two, this inversion of the time factor is then interpreted as a local inversion of mass, thanks to the dynamic groups theory [24] with its immediate evacuation from the object. Incidentally, such a mechanism would limit the mass of neutron stars to a value close to 2.5 solar masses. In the event of a neutron star merger, if the sum of their masses exceeds this value, the excess would be immediately reversed, resulting in a strong emission of gravitational waves. According to this new model, the reinterpretation of the detected signals could then lead to a downward revision of the estimated masses involved in these mergers.

The ratio of maximum to minimum brightness temperatures for the two objects M87* and SrgA* is significantly close to 3, while their masses differ by more than three orders of magnitude, which seems to rule out mere chance. If the proposed model is valid, then all future images of such objects will exhibit this same value of 3. Recent studies have shown that the rotational axes of such objects, considered as their emission lobes when they possess them, coincide with the rotational axis of galaxies, which seems to rule out their formation by accretion. The result of the implosion of a centripetal density wave would seem more likely. It is worth noting that Hoag's Galaxy exhibits such an appearance. At this stage, these are obviously only conjectures, but we suggest that the formation of supermassive objects results from joint fluctuations of metrics in the early universe. The periodic recurrence of this phenomenon would reactivate these objects, transforming them into quasars. The object at the center of the Milky Way thus appears as an extinct quasar.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Poincaré, H. (1902) La mesure du temps. *Revue de Métaphysique et de Morale*, **10**, 1-13. http://www.jstor.org/stable/40892291

[2] Einstein, A. (1905) Zur Elektrodynamik bewegter Körpner. *Annalen der Physik*, **17**, 89-92.

[3] Minkowski, H. (1909) Das Relativitätsprinzip. *Physikalische Zeitschrift*, **10**, 104-111. https://de.wikisource.org/wiki/Das_Relativit%C3%A4tsprinzip_(Minkowski)

[4] Schwarzschild, K. (1916) Über das Gravitationsfeld Massenpunktes nach der Einsteinschen Theorie. Sitzungsberichte der Deutschen Akademie der Wissenschaften.

[5] Schwarzschild, K. (1916) Über das Gravitionalsfeld einer Kugel Aus incompressibler Flüssigkeit nach der Einsteinschen Theorie. Sitzung der Physikalisch-Mathematischen Klasse, Vol. 23.

[6] Hilbert, D. (1915) Die Grundlagen der Physik (Erste Mitteilung). Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse. 395-407.

[7] Hilbert, D. (1916) Die Grundlagen der Physik (Erste Mitteilung). Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse.

[8] Abrams, L.S. (1989) Black Hole: The Legacy of Hilbert's Error. *Canadian Journal of Physics*, **67**, 919.

[9] Eddington, A. (1960) The Mathematical Theory of Relativity. 2nd Edition, Cambridge University Press.

[10] Tolman, R. (1934) Relativity, Thermodynamics and Cosmology. The International Series of Monographs on Physics. The Clarendon Press.

[11] Oppenheimer, J.R. and Snyder, H. (1939) On Continued Gravitational Contraction. *Physical Review*, **56**, 455-459. https://doi.org/10.1103/physrev.56.455

[12] Koiran, P. (2021) Infall Time in the Eddington-Finkelstein Metric, with Application to Einstein-Rosen Bridges. *International Journal of Modern Physics D*, **30**.

[13] Flam, L. (1916) Beiträge zur Einsteinschen Gravitationstheorie. *Physikalische Zeitschrift*, **17**, 448-454.

[14] Einstein, A. (1915) Die feldgleichungen der gravitation Sitzungsberichte der Königlich Preuishen 24 November.

[15] Minkowski, H. (1909) Raum und Zeit. *Physikalische Zeitschrift*, **10**, 75-88.

[16] Einstein, A. (1915) Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie. *Sitzungsberichte der Preussischen Akademie der Wissenschaften*, **47**, 831-839. https://ui.adsabs.harvard.edu/abs/1915SPAW...47..831E

[17] Chandrasekhar, S. (1992) The Mathematical Theory of Black Holes. Oxford University Press. https://ui.adsabs.harvard.edu/abs/1992mtbh.book.....C/

[18] Kruskal, M.D. (1960) Maximal Extension of Schwarzschild Metric. *Physical Review*, **119**, 1743-1745. https://doi.org/10.1103/physrev.119.1743

[19] Birkhoff, G.D. (1923) Relativity and Modern Physics. Cambridge Harvard University Press.

[20] Eddington, A.S. (1924) A Comparison of Whitehead's and Einstein's Formulæ. *Nature*, **113**, 192-192. https://doi.org/10.1038/113192a0

[21] Weyl, H. (1917) Zur Gravitationstheorie. *Annalen der Physik*, **359**, 117-145. https://doi.org/10.1002/andp.19173591804

[22] Einstein, A. and Rosen, N. (1935) The Particle Problem in the General Theory of Relativity. *Physical Review*, **48**, 73-77. https://doi.org/10.1103/physrev.48.73

[23] Koiran, P., Zejli, H., Levy, J., Margnat, F., Duval, M. and Zejli, H. (2024) PT-Symmetry in One-Way Wormholes. *Annals of Physics*, **470**, Article ID: 169765. https://doi.org/10.1016/j.aop.2024.169765

[24] Petit, J. and Zejli, H. (2024) Study of Symmetries through the Action on Torsors of the Janus Symplectic Group. *Reviews in Mathematical Physics*, **37**, n001. https://doi.org/10.1142/s0129055x24500545

[25] Petit, J., Margnat, F. and Zejli, H. (2024) A Bimetric Cosmological Model Based on Andreï Sakharov's Twin Universe Approach. *The European Physical Journal C*, **84**, Article No. 1126. https://doi.org/10.1140/epjc/s10052-024-13569-w

[26] The Event Horizon Telescope Collaboration (2024) The Persistent Shadow of the Supermassive Black Hole of M87? I. Observations, Calibration, Imaging and Analysis. *Astronomy and Astrophysics*, **681**, A79.

[27] The Event Horizon Telescope Collaboration (2019) First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole. *Astrophysical Journal*, **875**, L1.

[28] Event Horizon Telescope Collaboration (2023) First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way. *The Astrophysical Journal Letters*, **930**, L12.

[29] Dokuchaev, V.I. and Nazarova, N.O. (2020) Visible Shapes of Black Holes M87 and SgrA. *Universe*, **6**, Article No. 154. https://doi.org/10.3390/universe6090154

[30] Medeiros, L., Psaltis, D., Lauer, T.R. and Özel, F. (2023) Principal-Component Interferometric Modeling (PRIMO), an Algorithm for EHT Data. I. Reconstructing Images from Simulated EHT Observations. *The Astrophysical Journal*, **943**, Article No. 144. https://doi.org/10.3847/1538-4357/acaa9a

[31] Antoci, F. (1999) English Translation of the Second Schwarzschild's Paper: On the Gravitational Field of a Sphere of Incompressible Fluid According to Einstein Theory. https://arxiv.org/abs/physics/9905030

[32] Misner, C.W., Thorne, K.S. and Wheeler, J.A. (2017) Gravitation. Princeton University Press. https://de.wikisource.org/wiki/Gravitationsfeld_einer_Kugel_aus_inkompressibler_Fl%C3%BCssigkeit

[33] Tolman, R.C. (1939) Static Solutions of Einstein's Field Equations for Spheres of Fluid. *Physical Review*, **55**, 364-373. https://doi.org/10.1103/physrev.55.364

[34] Oppenheimer, J.R. and Volkoff, G.M. (1939) On Massive Neutron Cores. *Physical Review*, **55**, 374-381. https://doi.org/10.1103/physrev.55.374

[35] Adler, R., Bazin, M. and Schiffer, M. (1965) Introduction to General Relativity. McGraw Hill Books. https://catalog.hathitrust.org/Record/001484221

[36] Petit, J. (1995) Twin Universes Cosmology. *Astrophysics and Space Science*, **226**, 273-307. https://doi.org/10.1007/bf00627375

[37] Riemann, B. (1851) Grundlagen für eine allgemeine Theorie der Functionen einer veränderlichen complexen Grösse. Mémoire de doctorat, Göttingen. https://www.maths.tcd.ie/pub/HistMath/People/Riemann/Grund/Grund.pdf

[38] Riemann, B. (1857) Theorie der Abel'schen Functionen. *Journal für die Reine und Angewandte Mathematik* (*Crelle's Journal*), **54**, 115-155.

[39] Klein, F. (1882) Über Riemann's Theorie der algebraischen Funktionen und ihrer Integrale. Teubner. https://www.gutenberg.org/ebooks/36959

[40] Klein, F. (1890) Collected Papers in Mathematische Annalen et ses Gesammelte Mathematische Abhandlungen.

[41] Petit, J. and D'Agostini, G. (2025) Alternatives to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*, **16**, 1479-1490. https://doi.org/10.4236/jmp.2025.1610072

[42] Souriau, J.M. (1970) Structure of Dynamical Systems. Dunod.

[43] Einstein, A. (1939) On a Stationary System with Spherical Symmetry Consisting of Many Gravitating Masses. *The Annals of Mathematics*, **40**, Article No. 922. https://doi.org/10.2307/1968902

[44] Debergh, N., Petit, J. and D'Agostini, G. (2018) On Evidence for Negative Energies and Masses in the Dirac Equation through a Unitary Time-Reversal Operator. *Journal of Physics Communications*, **2**, Article ID: 115012. https://doi.org/10.1088/2399-6528/aaedcc

[45] Debergh, N. and Petit, J.P. (2022) On Some Reasons Why the Time Reversal Operator Could Be Unitary. *Physics Letters A*, **442**, Article ID: 127791.

[46] Debergh, N. and Petit, J.P. (2023) On Spacetime Algebra and Its Relation with Negative Mass. *Reports on Mathematical Physics*, **91**, 165-181.

[47] Debergh, N., D'Agostini, G. and Petit, J. (2021) On the Poincaré Algebra in a Complex Space-Time Manifold. *Journal of Modern Physics*, **12**, 218-228. https://doi.org/10.4236/jmp.2021.123017

[48] Schweizer, F., Ford, W.K.J., Jederzejewski, R. and Giovanelli, R. (1987) The Structure and Evolution of Hoag's Object. *The Astrophysical Journal*, **320**, 454-463. https://doi.org/10.1086/165562

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(8), 997-1027
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

The True Schwarzschild Solution and Its Subsequent Geometry

Jean-Pierre Petit, Gilles d'Agostini, Jean-Marc Pechinot

Manaty Research Group, Glanon, France
Email: Jean-pierre.petit@manaty.net

How to cite this paper: Petit, J.-P., d'Agostini, G. and Pechinot, J.-M. (2026) The True Schwarzschild Solution and Its Subsequent Geometry. *Journal of Modern Physics*, **17**, 997-1027.
https://doi.org/10.4236/jmp.2026.178045

Received: May 14, 2026
Accepted: August 25, 2026
Published: August 28, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

We present the solution to Einstein's stationary, spherically symmetric field equation, expressed using variables initially chosen by Schwarzschild. We show that the geodesics of the Flamm surface are not the geodesics of the hypersurface solution and how the Flamm's 2D cross-section allows us to understand the topology of the 4D object.

Keywords

Schwarzschild Metric, Schwarzschild Geometry, Throat Sphere

1. Introduction

This article will be particularly long. We will begin by revisiting how Karl Schwarzschild constructed the first exact solution to Einstein's equation in 1916—a stationary, spherically symmetric solution [1].

Very few people have followed his calculations in detail, as they require close attention. Indeed, Schwarzschild performs several successive changes of variable, making it quite difficult to follow his work. We shall therefore go through this again in detail, specifying his assumptions and variables at each step. This yields the line element shown in Equation (3), which is what Schwarzschild should have written when expressing his solution metric in terms of his coordinates (t, r, θ, φ).

The reason why he did not do so is then explained. Quite simply, his primary goal was to recover the approximate solution published a few months earlier by Einstein, which accounted for the advance of Mercury's perihelion [2]. To achieve this, he needed to calculate the geodesic trajectories of a test mass (Mercury) around the mass (the Sun). He chose to do this not using the radial coordinate r, but rather using what he termed an intermediate quantity R (*Hilfsgröße*), a choice he explained in a letter sent to Einstein as early as December 1915 [3] (see **Figure**

8), even before the article had been published by the journal.

It is absolutely essential to understand that the real formulation of this metric solution, whose construction is presented in details in the present paper, is nothing other than the true form of his original solution—expressed in the coordinates (t,r,θ,φ) he had chosen, and that this calculation involves no extraneous input or additional geometric or physical assumptions. The invariant remains the length *s*. What is truly extraordinary is that no one has revisited his calculation in the way we are about to do together.

Naturally, this analysis is inextricably linked to the black hole model. The present work stems from an article published in 1989 by the Canadian mathematician L.S. Abrams [4]—a paper that went entirely unnoticed at the time. Abrams pointed out what he regarded as an error in D. Hilbert's work [5]. However, the issue is more complex; it is not, strictly speaking, an error. In fact, Hilbert based his geometry not on length measurement but on a bilinear form—an approach that appeared more general to him—which entails the extension of geodesics along autoparallel curves. This aspect will be analyzed in a future article, as will the reinterpretation of the solution by Kruskal and matters relating to Birkhoff's theorem. The present article is confined to a precise examination of the solution constructed by Karl Schwarzschild in February 1916 [1] and a similarly precise description of Flamm's surface [6].

2. Detailed Re-Examination of Karl Schwarzschild's Calculation

Figure 1 shows how Schwarzschild presents his solution [1]. To convince the reader of the importance of this examination, we deemed it appropriate to also provide the German versions of the passages from the articles.

We felt it important to present an excerpt from the original article, published in German, noting that the English translation of this document was only made available in 1999, 83 years after its publication. The corresponding excerpt, in English translation, follows on Figure 1.

where the auxiliary quantity

$$\longrightarrow R = (3x_1 + \rho)^{1/3} = (r^3 + \alpha^3)^{1/3}$$

has been introduced.

When one introduces these values of the functions f in the expression (9) of the line element and goes back to the usual polar co-ordinates one gets the line element that forms the exact solution of Einstein's problem:

$$ds^2 = (1 - \alpha/R)dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2(d\vartheta^2 + \sin^2\vartheta d\phi^2), \quad R = (r^3 + \alpha^3)^{1/3}. \qquad (14)$$

The latter contains only the constant α that depends on the value of the mass at the origin.

Figure 1. The way in which Karl Schwarzschild presented his exterior metric solution in January 1916 [1].

It seems essential to draw attention to the fact that this form of the Schwarzschild metric, described in the literature as "standard", is expressed using a varia-

ble *R*, auxiliary quantity (*Hilfsgröbe*), and not the variable *r* [1], explicitly designated by Schwarzschild as a radial variable. See **Figure 2**.

§3. If one calls t the time, x, y, z, the rectangular co-ordinates, the most general line element that satisfies the conditions 1-3 is clearly the following:

$$ds^2 = Fdt^2 - G(dx^2 + dy^2 + dz^2) - H(xdx + ydy + zdz)^2$$

where F, G, H are functions of $r = \sqrt{x^2 + y^2 + z^2}$

Figure 2. The true Schwarzschild's radial coordinate [1].

In his article, Schwarzschild presents the above expression as, in his view, the most general form of the solution to Einstein's spherically symmetric and time-translation invariant equation. He then performs a series of changes of variable, for the sake of computational convenience, which means that, in this complex process, we lose the true final expression of his solution, expressed in its initial coordinate system, with its variable *r* and not this intermediate quantity *R*. We will reconstruct it precisely. He goes from its coordinates (t, r, θ, ϕ) to a set (x_4, x_1, x_2, x_3) according to **Figure 3**.

One puts:

$$x_1 = \frac{r^3}{3}, \quad x_2 = -\cos\vartheta, \quad x_3 = \phi. \tag{7}$$

Figure 3. The first Schwarzschild's coordinate change [1].

Which gives, see **Figure 4**:

In the new polar co-ordinates the line element reads:

$$ds^2 = Fdx_4^2 - \left(\frac{G}{r^4} + \frac{H}{r^2}\right)dx_1^2 - Gr^2\left[\frac{dx_2^2}{1-x_2^2} + dx_3^2(1-x_2^2)\right], \tag{8}$$

for which we write:

$$ds^2 = f_4 dx_4^2 - f_1 dx_1^2 - f_2\frac{dx_2^2}{1-x_2^2} - f_3 dx_3^2(1-x_2^2) \tag{9}$$

Then f_1 , $f_2 = f_3$, f_4 are three functions of x_1

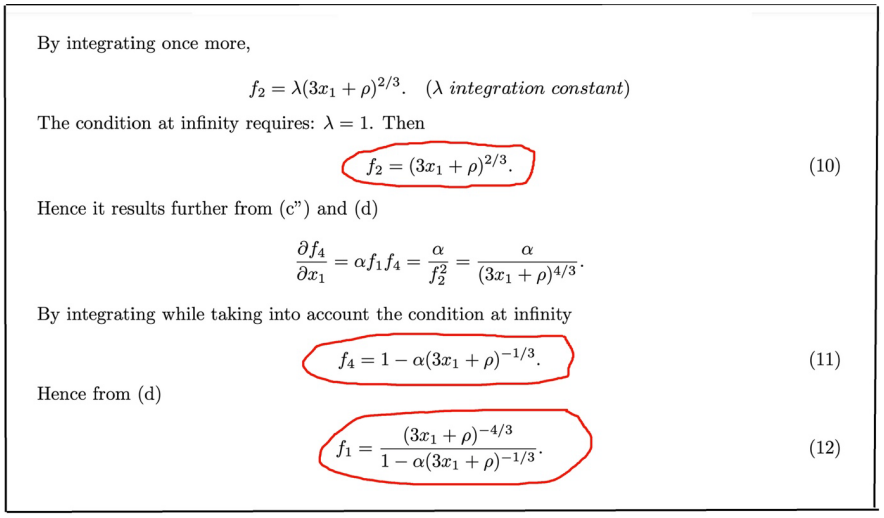
Figure 4. Schwarzschild's line element with his new space coordinates [1].

In the following pages, he calculates the Christoffel symbols, introduces all of this into the field equation, solves the equations, and obtains the expressions of his functions f_1, f_2, f_3, f_4. See **Figure 5**.

By exploiting this result, and taking into account de $f_3 = f_2, x_4 = t$, we obtain:

$$\begin{aligned} \mathrm{d}s^2 = &\left[1-\alpha\left(3x_1+\rho\right)^{-1/3}\right]\mathrm{d}t^2 - \frac{\left(3x_1+\rho\right)^{-4/3}}{1-\alpha\left(3x_1+\rho\right)^{-1/3}}\mathrm{d}x_1^2 \\ &-\left(3x_1+\rho\right)^{2/3}\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right) \end{aligned} \tag{1}$$

Using the change mentioned on **Figure 6**:



By integrating once more,

$$f_2 = \lambda(3x_1 + \rho)^{2/3}. \quad (\lambda \text{ integration constant})$$

The condition at infinity requires: $\lambda = 1$. Then

$$f_2 = (3x_1 + \rho)^{2/3}. \tag{10}$$

Hence it results further from (c") and (d)

$$\frac{\partial f_4}{\partial x_1} = \alpha f_1 f_4 = \frac{\alpha}{f_2^2} = \frac{\alpha}{(3x_1 + \rho)^{4/3}}.$$

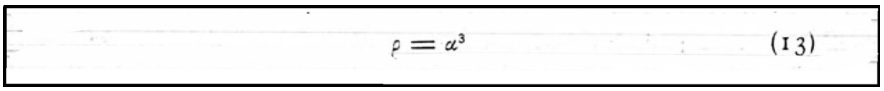
By integrating while taking into account the condition at infinity

$$f_4 = 1 - \alpha(3x_1 + \rho)^{-1/3}. \tag{11}$$

Hence from (d)

$$f_1 = \frac{(3x_1 + \rho)^{-4/3}}{1 - \alpha(3x_1 + \rho)^{-1/3}}. \tag{12}$$

Figure 5. The *f* functions [1].



$$\rho = \alpha^3 \tag{13}$$

Figure 6. From original document [1].

We give for the first time, presented rigorously in a peer-reviewed scientific journal, 110 years after the publication of the original article, using its variables.

$$\left(t, r = \sqrt{x^2 + y^2 + z^2} \geq 0, \theta, \phi\right) \tag{2}$$

the true solution, given in 1916 by Karl Schwarzschild, of Einstein's homogeneous equation, in spherical symmetry and invariant under time translation.

$$\begin{pmatrix} \mathrm{d}s^2 = \dfrac{\left(r^3 + \alpha^3\right)^{1/3} - \alpha}{\left(r^3 + \alpha^3\right)^{1/3}} c^2 \mathrm{d}t^2 - \dfrac{r^4 \mathrm{d}r^2}{\left(r^3 + \alpha^3\right)\left[\left(r^3 + \alpha^3\right)^{1/3} - \alpha\right]} \\ -\left(r^3 + \alpha^3\right)^{2/3} \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right) \end{pmatrix} \tag{3}$$

Following Schwarzschild, Einstein's homogeneous equation—assuming spherical symmetry and invariance under time translation—is presented here in its author's original coordinates. This is the first time in 110 years that this form—despite its essential nature—has been presented in an article.

The invariant remains the length *s*. It is simply a reformulation of the exact solution provided by Schwarzschild, expressed not in terms of his intermediate quantity *R*, but in terms of his true radial coordinate:

$$r = \sqrt{x^2 + y^2 + z^2} \geq 0\,. \tag{4}$$

We have represented and highlighted it in red in the hope that the many specialists will notice this expression, which is being presented to them for the first time. At this point, a question immediately arises: Why didn't Schwarzschild present it in this form, with its variable $r = \sqrt{x^2 + y^2 + z^2}$, in his article, instead of using this intermediate variable *R*? The answer is found in a letter

Schwarzschild sent to Einstein on December 22, 1915 [3]. He wrote, see **Figure 7**:

As follows, it had beside your α yet a second term, and the problem was physically undetermined. From this I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result: It gave only a line element, which fulfills your conditions 1) to 4), as well as field- and determinant equations, and at the null point and only in the null point is singular.

If:

$$x_1 = r\cos\phi\cos\theta, \quad x_2 = r\sin\phi\cos\theta, \quad x_3 = r\sin\theta$$

$$R = (r^3 + \alpha^3)^{1/3} = r\left(1 + \frac{1}{3}\frac{\alpha^3}{r^3} + \ldots\right)$$

then the line element becomes:

$$ds^2 = \left(1 - \frac{\gamma}{R}\right)dt^2 - \frac{dR^2}{\left(1 - \frac{\gamma}{R}\right)} - R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

R, θ, ϕ are not "allowed" coordinates, with which one must build the field equations, because they do not have the determinant = 1, however the line element expresses itself as the best.

Figure 7. Letter from Schwarzschild to Einstein, December 2015 [3].

Einstein and Schwarzschild were in contact and exchanged letters regularly. In his letter of December 22, 1915, Schwarzschild responded to the article Einstein had just published on November 25, in which he calculated the perihelion advance of Mercury using an approximate solution [2]. Schwarzschild was preparing to send what was then the exact solution. But as he specified in his article, he knew that the radius, for the Sun, was limited to 3 km. He therefore decided to present his result using an intermediate variable R by performing a series expansion, as indicated in the article. We thus find here the origin of what would later be called the "standard form of the Schwarzschild solution".

3. December 1916: David Hilbert's Impact on Geometric Cosmology [5]

During 1915, for seven months, through numerous exchanges—first in person, then through copious correspondence—a very close relationship developed between Einstein and Hilbert, in an atmosphere of great mutual respect. Through Einstein's words, Hilbert discovered the link between advanced mathematics and the development of a new physics, toward which he had always been skeptical. Einstein spoke of the project in which he had become completely invested: a geometric translation, within a framework of covariance of the force of gravity. Intrigued, Hilbert had access to the most advanced mathematical tools of his time. It was a world in which Einstein, first and foremost a fantastic physicist endowed with exceptional intuition, always admitted to feeling uneasy. In this context, Hilbert conceived the project of integrating the two forces known at the time—gravita-

tion and electromagnetism—into a single formalism. Today we would call this a "theory of everything". For him, cosmic geometry is that of a four-dimensional space, designated by the letters (w_1, w_2, w_3, w_4), where the indices 1 to 3 refer to space and where w_4 is the time coordinate. Space is quasi-Euclidean and stationary (invariant under time translation): the phenomenon of cosmic expansion had not yet been discovered by Edwin Hubble. The geometry of the cosmos is determined by a number of functions, the set forming a tensor $g_{\mu\nu}$, from which one can calculate the components of the Ricci tensor, which Hilbert denotes by $K_{\mu\nu}$. The contraction of this tensor gives the Ricci scalar, designated by the letter *K*.

Hilbert then published a long article on November 20, 1915 [7] entitled "Foundations of Physics". While Einstein struggled as best he could with tensors, aided by his friend the mathematician Grassmann, Hilbert took a further step into abstraction by deriving a field equation from a variational calculus, centered on an action constructed around a "world function" *H*:

$$\int H\sqrt{|g|}\,\mathrm{d}\omega \tag{5}$$

$$|g| = |\det g_{\mu\nu}| \tag{6}$$

See **Figure 8**.

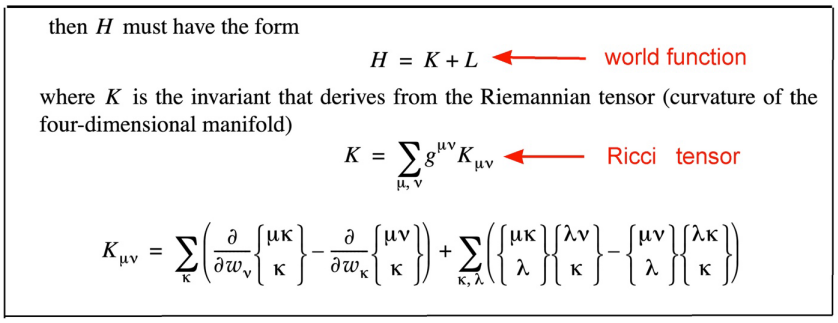


Figure 8. The Hilbert's world function [7].

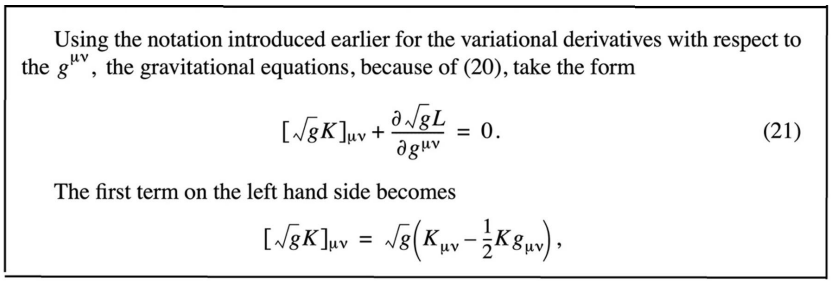
His stroke of genius was to identify this world function as the sum of two terms, the first being the Ricci scalar and the second the Lagrangian of matter, defining a field where, in principle, gravitation and electromagnetism are ambitiously intertwined. The publication of this article was a real shock for Einstein, who, when he sent his own five days later, on November 25, 1915 [8], presenting the goal of his quest—the famous field equation of this general relativity, which completes his special relativity—discovered that Hilbert had beaten him to it. See **Figure 9**.

If we replace the letter *K* with the letter *R* (the Ricci tensor and scalar *r*), and setting:

$$\frac{1}{\sqrt{g}}\frac{\partial\sqrt{g}L}{\partial g^{\mu\nu}} = \chi T_{\mu\nu} \tag{7}$$

The reader will recognize the field equation of general relativity with which they are familiar. Einstein then wrote Hilbert a letter of protest, and the following weeks reflected a certain amount of confusion. However, by mutual agreement, to preserve the feelings of esteem that bound them, the decision was made that this

equation would bear only Einstein's name. In the literature, the reader will now find references to the Einstein-Hilbert collaboration.



Using the notation introduced earlier for the variational derivatives with respect to the $g^{\mu\nu}$, the gravitational equations, because of (20), take the form

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial\sqrt{g}L}{\partial g^{\mu\nu}} = 0. \quad (21)$$

The first term on the left hand side becomes

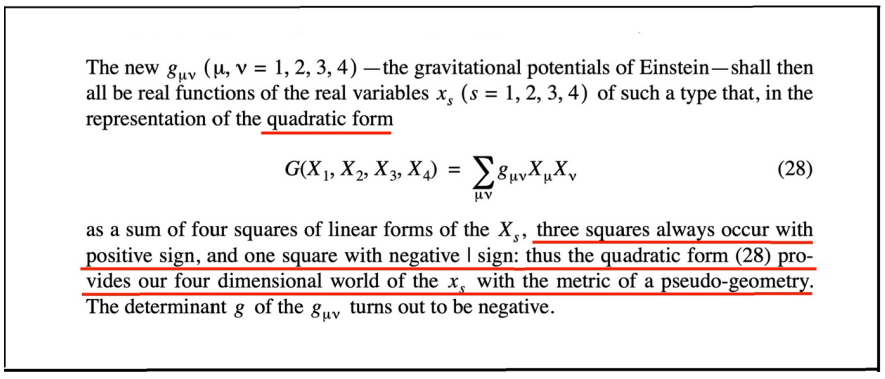
$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\left(K_{\mu\nu} - \frac{1}{2}Kg_{\mu\nu}\right),$$

Figure 9. The Hilbert's field equation [7].

History makes its choices. It is, indeed essential to note that at no point in Hilbert's article does the word "metric" appear, nor expression beginning with "ds^2 =".

Schwarzschild then published, in quick succession, in January and February 1916, his two solutions to this field equation [1] [9]. Hilbert felt compelled to urgently incorporate this into a second version of his paper, bearing the same title, which he published on December 23, 1916 [5]. Only the solution to the homogeneous equation [1] held his attention. He made no mention of the second.

For Hilbert, a solution to the field equation was not a metric but a bilinear form which, in his view, determined the geometry. This is a crucial point that must be emphasized. Hilbert is, first and foremost, a mathematician. It therefore seems natural to him to consider the most general formulation from a purely mathematical perspective, specifically, a geometry derived from a bilinear form that could have any sign. He operates, in other words, like someone living in the world of mathematics. Einstein, by contrast, inhabits the world of physics. That is where the entire difference lies. See **Figure 10**.



The new $g_{\mu\nu}$ ($\mu, \nu = 1, 2, 3, 4$) —the gravitational potentials of Einstein—shall then all be real functions of the real variables x_s ($s = 1, 2, 3, 4$) of such a type that, in the representation of the quadratic form

$$G(X_1, X_2, X_3, X_4) = \sum_{\mu\nu} g_{\mu\nu}X_\mu X_\nu \quad (28)$$

as a sum of four squares of linear forms of the X_s, three squares always occur with positive sign, and one square with negative | sign: thus the quadratic form (28) provides our four dimensional world of the x_s with the metric of a pseudo-geometry. The determinant g of the $g_{\mu\nu}$ turns out to be negative.

Figure 10. Hilbert's Quadratic form [5].

The tedious presentation of these numerous plates is intended to show how Hilbert's highly personal conception of this "geometric cosmology", very different from that of Einstein [8], Schwarzschild [1], Droste [10], Flamm [6], Weyl [11], and others, profoundly influenced the development of this discipline. We will

show later how the main proponents of these ideas echoed, without citing their source, the choices made by Hilbert.

Thus, in the plate above, we find the origin of the change in signature. Among the authors cited, this signature was ubiquitous:

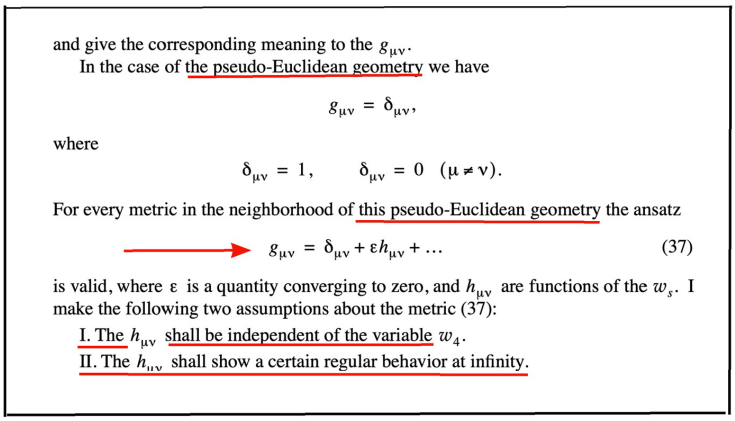
$$\text{The « old » signature: } (+---) \tag{8}$$

In the sentences underlined in red in the excerpt from Hilbert's December 1916 article, he explicitly introduces what will become the "modern" signature:

$$(+++-) \text{ or } (-+++) \tag{9}$$

This change will occur gradually and will eventually become widespread in the postwar period, even though, at least in cosmology, there is no article that provides an argument in favor of such a change.

But what does Hilbert mean by pseudo-geometry? The answer is in the following excerpt. See **Figure 11**.



and give the corresponding meaning to the $g_{\mu\nu}$.
In the case of the pseudo-Euclidean geometry we have

$$g_{\mu\nu} = \delta_{\mu\nu},$$

where

$$\delta_{\mu\nu} = 1, \qquad \delta_{\mu\nu} = 0 \quad (\mu \neq \nu).$$

For every metric in the neighborhood of this pseudo-Euclidean geometry the ansatz

$$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu} + \ldots \tag{37}$$

is valid, where ε is a quantity converging to zero, and $h_{\mu\nu}$ are functions of the w_s. I make the following two assumptions about the metric (37):

I. The $h_{\mu\nu}$ shall be independent of the variable w_4.

II. The $h_{\mu\nu}$ shall show a certain regular behavior at infinity.

Figure 11. Hilbert's pseudo (Euclidean) geometry [5].

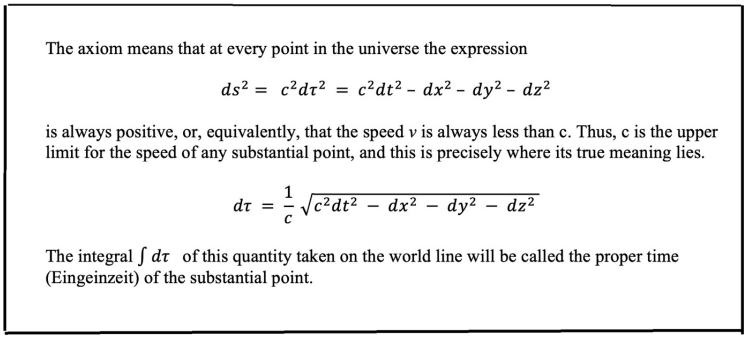
In equation (37) of the extract, it is clear that for Hilbert the universe is not only stationary (independent of its time coordinate w_4) but practically Euclidean. All this relativistic physics introduced by Einstein manifests itself only as a tiny perturbation, through the presence of a term $g_{44} = \varepsilon h_{44}$ [5]. The conception of spacetime introduced by Hermann Minkowski [12] in 1909[1], with its metric, is completely neglected. See Minkowski's statement on **Figure 12**.

These elements were very well assimilated by people like Schwarzschild [1]. See **Figure 13**.

In Schwarzschild's work, as with Schwarzschild and all others at that time, the assumptions are unambiguous:

- The coordinates are defined in $\mathbb{R}^4$.
- The element of length ds, associated with the hypersurface, is fundamentally real.
- We are looking for curves that minimize the length.

[1]Minkowski died a few months later from appendicitis that degenerated into a generalized infection. Antibiotics had not yet been invented. A detail of crucial importance.



The axiom means that at every point in the universe the expression

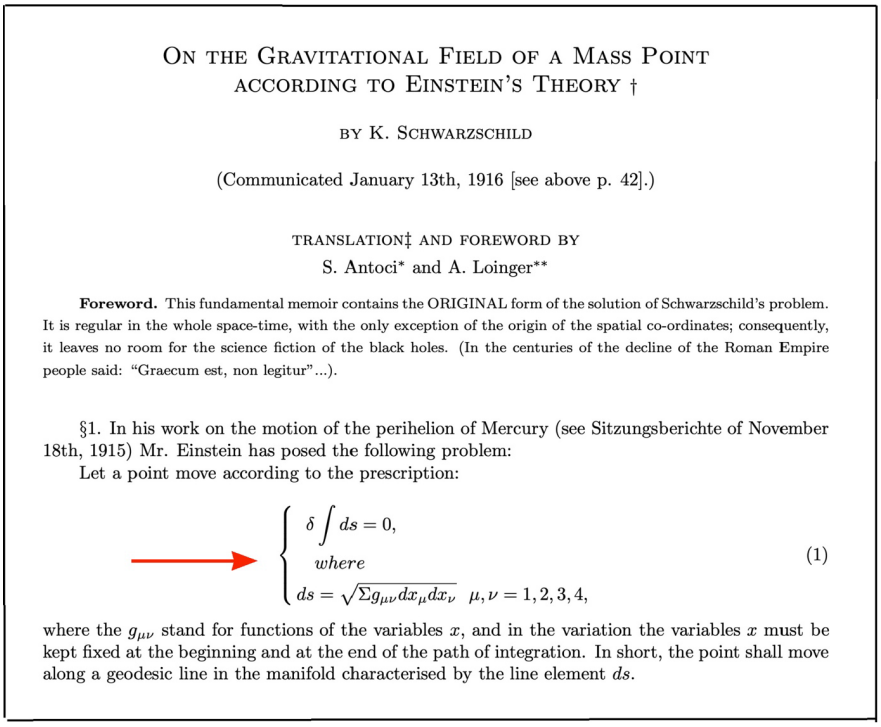
$$ds^2 = c^2d\tau^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

is always positive, or, equivalently, that the speed v is always less than c. Thus, c is the upper limit for the speed of any substantial point, and this is precisely where its true meaning lies.

$$d\tau = \frac{1}{c}\sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}$$

The integral $\int d\tau$ of this quantity taken on the world line will be called the proper time (Eingeinzeit) of the substantial point.

Figure 12. Minkowski's axiomatic definition [12] of spacetime (1909).



ON THE GRAVITATIONAL FIELD OF A MASS POINT
ACCORDING TO EINSTEIN'S THEORY †

BY K. SCHWARZSCHILD

(Communicated January 13th, 1916 [see above p. 42].)

TRANSLATION‡ AND FOREWORD BY
S. Antoci* and A. Loinger**

Foreword. This fundamental memoir contains the ORIGINAL form of the solution of Schwarzschild's problem. It is regular in the whole space-time, with the only exception of the origin of the spatial co-ordinates; consequently, it leaves no room for the science fiction of the black holes. (In the centuries of the decline of the Roman Empire people said: "Graecum est, non legitur"...).

§1. In his work on the motion of the perihelion of Mercury (see Sitzungsberichte of November 18th, 1915) Mr. Einstein has posed the following problem:

Let a point move according to the prescription:

$$\begin{cases} \delta \int ds = 0, \\ \text{where} \\ ds = \sqrt{\Sigma g_{\mu\nu} dx_\mu dx_\nu} \quad \mu, \nu = 1, 2, 3, 4, \end{cases} \tag{1}$$

where the $g_{\mu\nu}$ stand for functions of the variables x, and in the variation the variables x must be kept fixed at the beginning and at the end of the path of integration. In short, the point shall move along a geodesic line in the manifold characterised by the line element ds.

Figure 13. Schwarzschild foreword [1].

In short, it's very clear and simple:

When the element ds ceases to be real, we are outside the hypersurface.

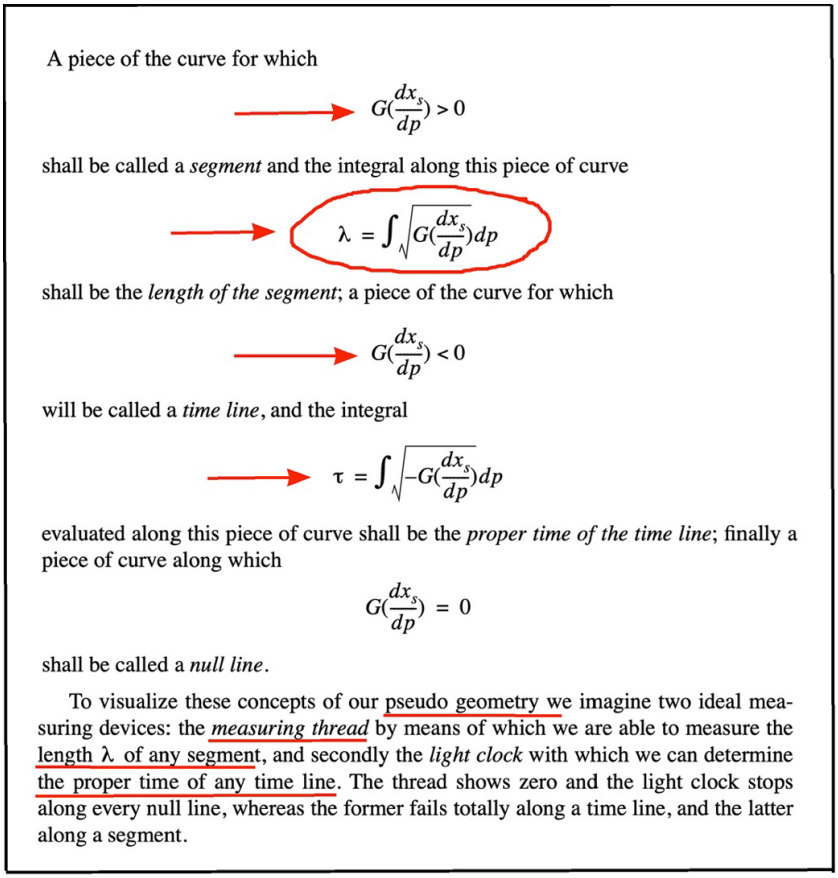
In doing so, he is merely conforming to the prestigious geometrization of physics associated with the lecture Minkowski gave in 1909 [12], just months before his death.

Since Hilbert defines his geometry not in terms of an elementary length d*s*, but rather based on a bilinear form, he arrives at what appears to him to be the most general geometry. It therefore seems logical to him to define two lengths rather than one, a step he takes by introducing an arbitrary sign change. See **Figure 14**.

For Hilbert, it is possible to define two real lengths from different expressions, depending on the sign of the quadratic form *G*. When this form is negative, this length becomes the proper time τ and a real value, measured along timelines, time-like trajectories, and is then obtained by inserting a negative sign in front of

it. In the domain of definition, when this form is positive, the scalar obtained is then called the length of a segment, along curves described today as being "space-like". Hilbert is therefore the first to achieve an extension of the solution, now called an "extension to the whole of spacetime". These are thus two different interpretations of the solution. In Schwarzschild's work, the result of the variational calculus is limited to the determination of authentic geodesics where the length is minimized:

$$\delta\int \mathrm{d}s = \delta\int \sqrt{g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}} = 0 \tag{10}$$


A piece of the curve for which

$$G\left(\frac{dx_s}{dp}\right) > 0$$

shall be called a *segment* and the integral along this piece of curve

$$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}dp$$

shall be the *length of the segment*; a piece of the curve for which

$$G\left(\frac{dx_s}{dp}\right) < 0$$

will be called a *time line*, and the integral

$$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}dp$$

evaluated along this piece of curve shall be the *proper time of the time line*; finally a piece of curve along which

$$G\left(\frac{dx_s}{dp}\right) = 0$$

shall be called a *null line*.

To visualize these concepts of our pseudo geometry we imagine two ideal measuring devices: the *measuring thread* by means of which we are able to measure the length λ of any segment, and secondly the *light clock* with which we can determine the proper time of any time line. The thread shows zero and the light clock stops along every null line, whereas the former fails totally along a time line, and the latter along a segment.

Figure 14. Hilbert's two different lengths [5].

Hilbert constructs the set of curves directly from the quadratic form, without worrying about whether it is positive or negative.

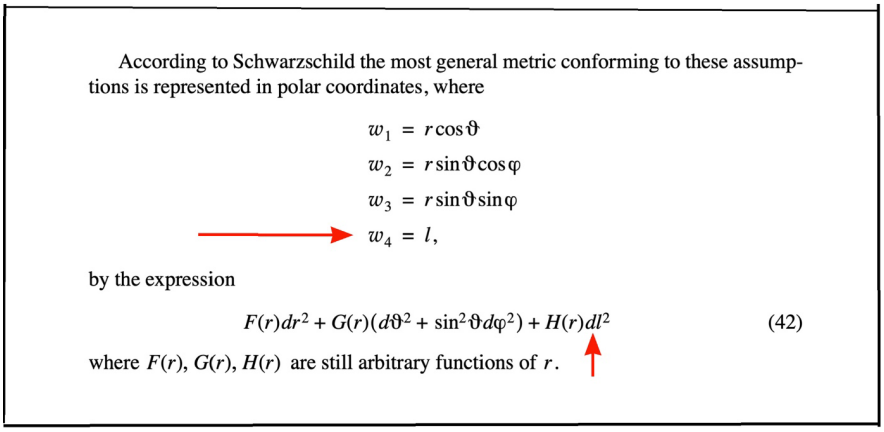
$$\delta\int\left(g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu}\right) = 0 \tag{11}$$

Hilbert was also the first to introduce the concept of the light cone, which henceforth has an interior and an exterior, where curves of a spatial nature then unfold.

4. Hilbert's Construction of the Solution to the Field Equation, Stationary and Spherically Symmetric [5]

For Hilbert, a solution to the field equation is a quadratic form, which can then

have any sign. Using polar coordinates, he writes, see Figure 15.



According to Schwarzschild the most general metric conforming to these assumptions is represented in polar coordinates, where

$$w_1 = r\cos\vartheta$$
$$w_2 = r\sin\vartheta\cos\varphi$$
$$w_3 = r\sin\vartheta\sin\varphi$$
$$w_4 = l,$$

by the expression

$$F(r)dr^2 + G(r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + H(r)dl^2 \qquad (42)$$

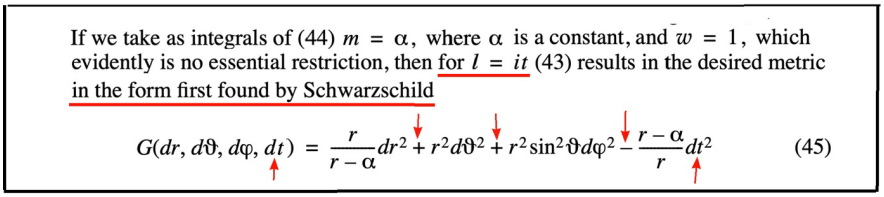
where $F(r)$, $G(r)$, $H(r)$ are still arbitrary functions of r.

Figure 15. Hilbert's coordinate choice [5].

He denotes his time coordinate by the letter l. This coordinate does not intervene at any point in his calculation, since the solution was assumed to be independent of it. For Hilbert, what determines the geometry is a quadratic form. It is therefore in this form that he presents his result (the typographical errors of the original manuscript, reproduced in the English translation, have been corrected):

$$G\left(\mathrm{d}r,\mathrm{d}\theta,\mathrm{d}\vartheta,\mathrm{d}l\right) = \frac{r}{r-\alpha}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\vartheta^2 + \frac{r-\alpha}{r}\mathrm{d}l^2 \qquad (12)$$

And it is only at this stage that Hilbert adopts the interpretation of relativity suggested in 1902 by the mathematician Henri Poincaré [13]. For Poincaré, as or Hilbert, the time coordinate is purely imaginary ($l = it$). See Figure 16.

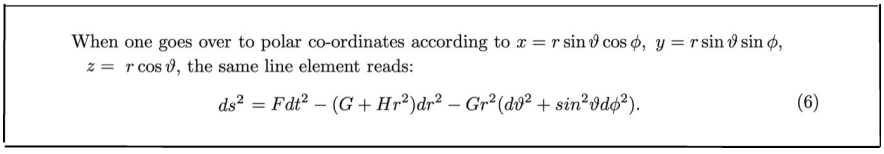


If we take as integrals of (44) $m = \alpha$, where α is a constant, and $w = 1$, which evidently is no essential restriction, then for $l = it$ (43) results in the desired metric in the form first found by Schwarzschild

$$G(dr, d\vartheta, d\varphi, dt) = \frac{r}{r-\alpha}dr^2 + r^2d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 - \frac{r-\alpha}{r}dt^2 \qquad (45)$$

Figure 16. When Hilbert identifies [5] its own result to the Schwarzschild's one.

Like Hilbert, Schwarzschild adopts what appears to him to be the most general form of the solution to Einstein's equation without second member, stationary and spherically symmetric. See Figure 17.



When one goes over to polar co-ordinates according to $x = r\sin\vartheta\cos\phi$, $y = r\sin\vartheta\sin\phi$, $z = r\cos\vartheta$, the same line element reads:

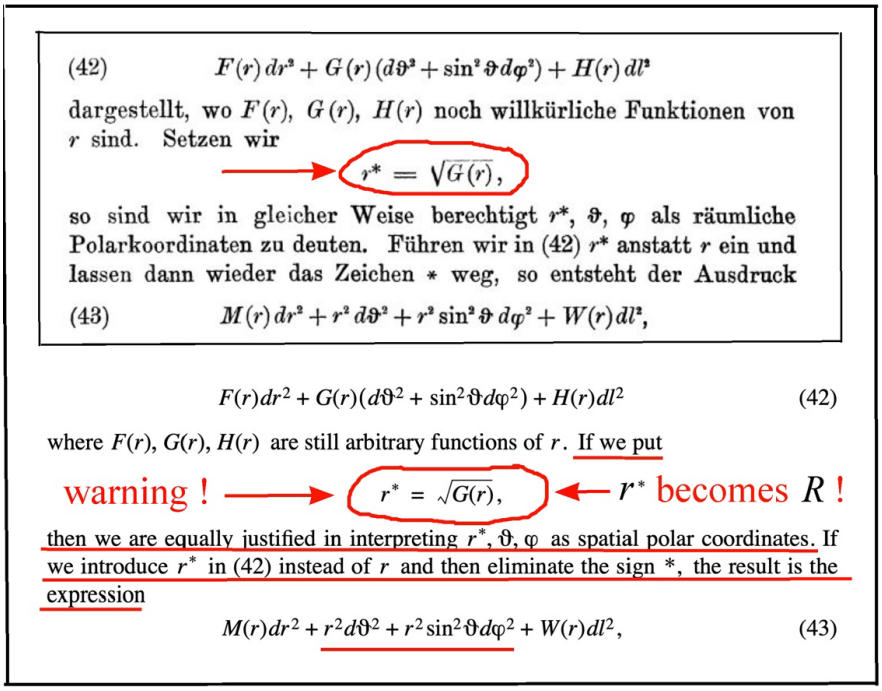
$$ds^2 = Fdt^2 - (G + Hr^2)dr^2 - Gr^2(d\vartheta^2 + sin^2\vartheta d\phi^2). \qquad (6)$$

Figure 17. Schwarzschild's line element [1].

As can be seen, in Schwarzschild coordinates (t,r,θ,ϑ), this corresponds to

$$Gr^2 = \left(r^3+\alpha^3\right)^{2/3} \neq r^2 \tag{13}$$

In his calculation, Hilbert introduces a simplification concerning the coefficient of $\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\vartheta^2\right)$ which seemed to him likely to converge more rapidly toward what he identified as Schwarzschild's result in (t,r,θ,ϑ). See Figure 18.



(42) $F(r)\,dr^2 + G(r)\,(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) + H(r)\,dl^2$

dargestellt, wo $F(r)$, $G(r)$, $H(r)$ noch willkürliche Funktionen von r sind. Setzen wir

$r^* = \sqrt{G(r)}$,

so sind wir in gleicher Weise berechtigt r^*, ϑ, φ als räumliche Polarkoordinaten zu deuten. Führen wir in (42) r^* anstatt r ein und lassen dann wieder das Zeichen $*$ weg, so entsteht der Ausdruck

(43) $M(r)\,dr^2 + r^2\,d\vartheta^2 + r^2\sin^2\vartheta\,d\varphi^2 + W(r)\,dl^2$,

$$F(r)dr^2 + G(r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + H(r)dl^2 \tag{42}$$

where $F(r)$, $G(r)$, $H(r)$ are still arbitrary functions of r. If we put

warning ! $r^* = \sqrt{G(r)}$, r^* becomes R !

then we are equally justified in interpreting r^*, ϑ, φ as spatial polar coordinates. If we introduce r^* in (42) instead of r and then eliminate the sign $*$, the result is the expression

$$M(r)dr^2 + r^2d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 + W(r)dl^2, \tag{43}$$

Figure 18. Hilbert's approximation [5].

He then identifies his variable r^* with r. At a time when no one could have imagined for a single moment the existence of astronomical objects whose characteristic scale would be anything other than an infinitesimal perturbation, this error had little importance. But in reality, this amounts to identifying $r=\sqrt{x^2+y^2+z^2}\geq 0$ with $R=\left(r^3+\alpha^3\right)^{1/3}\geq\alpha$ (the intermediate Schwarzschild variable). This point was only pointed out in 1989 by L.S. Hence, the expression of Hilbert solution, as presented in its Equation (45) is not strictly identical to the Schwarzschild's one since it is expressed with the intermediary variable *R*, as noticed in 1989 by the Canadian mathematician Abrams [4], 73 years after the publication of Hilbert's paper.

5. Impact on Physicists

The first person to widely publicize these different works on this cosmology was Arthur Eddington [14], as early as 1919. His work testifies to his extraordinary openness to the science of his time. Representing a considerable undertaking, the book presents the tools of differential geometry upon which the progress represented by general relativity is based. However, on page 139, in Chapter III, he re-

produces the manner in which Hilbert treats this solution of Einstein's equation. See Figure 19.

CH. III THE FIELD OF AN ISOLATED PARTICLE 139

In flat space-time the interval, referred to spherical polar coordinates and time, is

$$ds^2 = -dr^2 - r^2\,d\theta^2 - r^2\sin^2\theta\,d\phi^2 + dt^2. \qquad (38.11)$$

If we consider what modifications of this can be made without destroying the spherical symmetry in space, the symmetry as regards past and future time, or the static condition, the most general possible form appears to be

$$ds^2 = -U(r)\,dr^2 - V(r)\,(r^2\,d\theta^2 + r^2\sin^2\theta\,d\phi^2) + W(r)\,dt^2, \qquad (38.12)$$

where U, V, W are arbitrary functions of r. Let

$$r_1^2 = r^2 V(r).$$

Then (38.12) becomes of the form

$$ds^2 = -U_1(r_1)\,dr_1^2 - r_1^2\,d\theta^2 - r_1^2\sin^2\theta\,d\phi^2 + W_1(r_1)\,dt^2, \qquad (38.13)$$

where U_1 and W_1 are arbitrary functions of r_1. There is no reason to regard r in (38.12) as more immediately the counterpart of r in (38.11) than r_1 is. If the functions U, V, W differ only slightly from unity, both r and r_1 will have approximately the properties of the radius-vector in Euclidean geometry; but no length in non-Euclidean space can have exactly the properties of a Euclidean radius-vector, and it is arbitrary whether we choose r or r_1 as its closest representative. We shall here choose r_1, and accordingly drop the suffix, writing (38.13) in the form

$$ds^2 = -e^{\lambda}\,dr^2 - r^2\,d\theta^2 - r^2\sin^2\theta\,d\phi^2 + e^{\nu}\,dt^2, \qquad (38.2)$$

where λ and ν are functions of r only.

Figure 19. From Eddington's book [14], 1919.

One should note the old signature $(+---)$ as well as the use of exponentials e^{λ} and e^{ν} to ensure invariance when coordinates and functions belong to the real domain.

The same formulation appears identically in the book published in 1934 by the mathematician Richard Tolman [15], see on this book, Equations (94.2) and (95.1).

6. Geometric Interpretation of the Schwarzschild Solution

The young German mathematician Ludwig Flamm provided [6], immediately after publication of Schwarzschild's papers, the geometric interpretation of the two solutions [1] [9]. Since these are invariant under time translation, the study re-

duces to three-dimensional geometric structures undergoing translation in time. The 3D structure results from the joining of two manifolds along their common boundary, which is the spherical surface of the star. The author then considers the section of these two objects at constant θ, which yields 3D objects. See Figure 20.

§2. Now we can, in an analogous way, examine the first case which was treated by S c h w a r z s c h i l d, the gravitational field of a point mass. The line-element in its simplest form has the same structure as above and reads

$$d s^2 = \left(1 - \frac{\alpha}{R}\right) d t^2 - \frac{d R^2}{1 - \frac{\alpha}{R}} - R^2 \left(d \vartheta^2 + \sin^2 \vartheta \, d \varphi^2\right)$$

Figure 20. L.Flamm's choice for the external metric representation [6].

Let us note in passing that Flamm adopts the signature $(+---)$ and identifies the trajectories of test masses with geodesics minimizing the length s, identified with the proper time τ. See Figure 21.

Therefore, if the elementary rulers and clocks are influenced by the gravitational field, then the propagation of light must be influenced in exactly the same way, so that when compared to each other they do not show any differences. The same must hold for all the other physical phenomena. This might be why one can describe the motion of a point particle by the simple variational equation

$$\delta \int_{P_1}^{P_2} d s = 0, \tag{5}$$

Figure 21. Flamm's variational procedure [6].

He then embeds the object into $\mathbb{R}^3$. See Figure 22.

The spatial part of the line-element is again of a non-Euclidean nature. Once more, due to the spherical symmetry it seems to be enough to restrict the consideration to the metric properties in an arbitrary planar section through the origin, *i.e.*, to $\vartheta = \frac{\pi}{2}$. Now one obtains

$$d {\sigma_e}^2 = \frac{d R^2}{1 - \frac{\alpha}{R}} + R^2 \, d \varphi^2. \tag{4}$$

This line-element is also of the form (2'), because one can put

$$\cos^2 \chi = 1 - \frac{\alpha}{R},$$

and it is also identical with a line-element on a surface of rotation. Denoting by z the coordinate in the direction of the rotational axis, the equation for the meridional curve follows from

$$\frac{d z}{d R} = \operatorname{tg} \chi = \sqrt{\frac{\alpha}{R - \alpha}}$$

as

$$z^2 = 4\alpha(R - \alpha).$$

This describes a parabola with parameter

$$p = 2\alpha,$$

Figure 22. L. Flamm's embedment operation [6].

He then obtains the meridian (r,z) of this manifold with boundary in the form of a sideways parabola. See Figure 23.

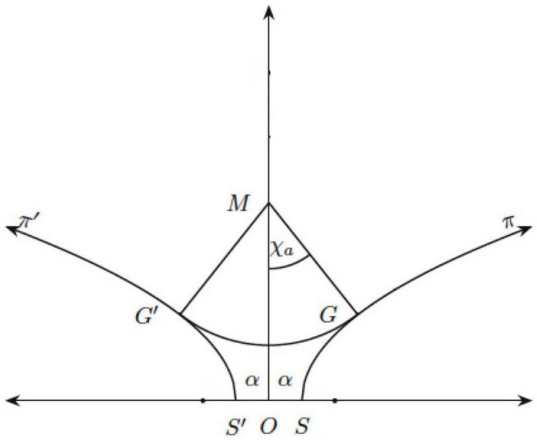


Figure 23. Flamm's medidian [6].

This Schwarzschild geometry is thus perfectly defined. The points G and G' represent the ends of the segment corresponding to the diameter of the sphere along which the two 3D surface elements are joined. The circular arc GG' completes the parabolic portions Gp and G'p'.

The Flamm surface is often described in the literature as a pedagogical artifact intended to illustrate how masses curve the surrounding space. When the spacetime geodesics arising from the 4D hypersurface corresponding to the exterior metric are projected, one obtains curves suggestive of an attractive effect, with the classical perihelion precession, as shown on Figure 24.

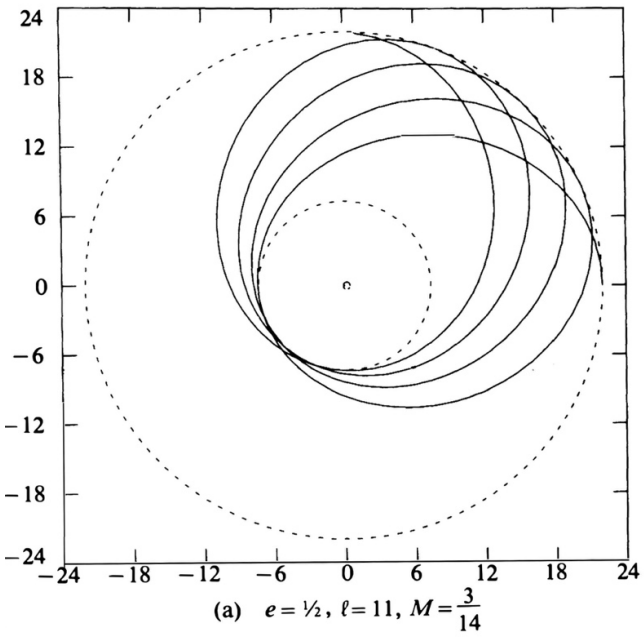


Figure 24. 2D (R,φ) projection of 4D Schwarzschild geodesic [16].

The system of geodesics of the Flamm surface was first studied only very recently [17], 113 years after its first presentation by its author. Later (2019!) this study was extended [18]. In this last paper, the geodesics are grouped into two families, depending on whether or not they cross the throat circle. The authors used a color code to distinguish portions of curves suggesting attraction, and at greater distance, repulsion. Consequently, they cannot constitute a pedagogical image intended to illustrate the way masses modify the geometry of the vacuum surrounding them. See **Figure 25**.

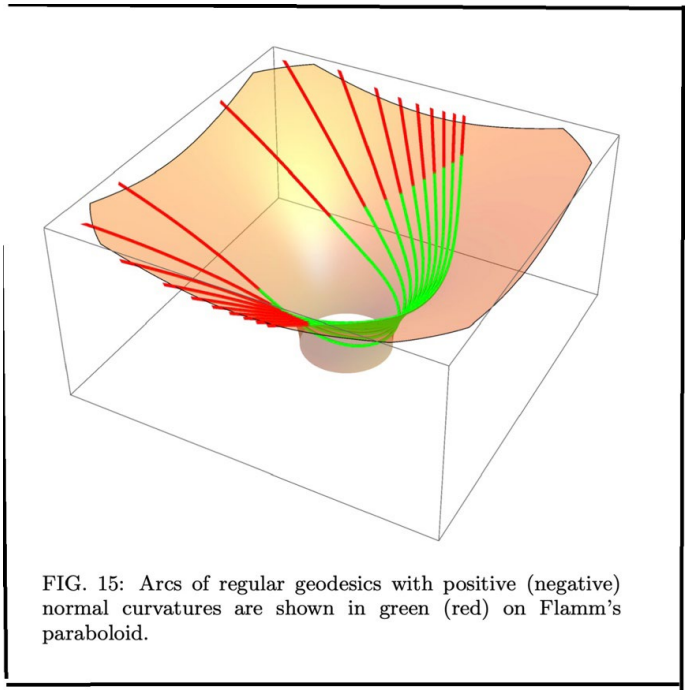


FIG. 15: Arcs of regular geodesics with positive (negative) normal curvatures are shown in green (red) on Flamm's paraboloid.

Figure 25. Flamm's surface geodesics. first family [18].

In **Figure 15** of the cited reference, the authors represented in green the portions of geodesics that do not cross the throat sphere, whose concavity is directed toward the symmetry axis (suggesting attraction). At larger distances, this concavity reverses (red color: repulsion).

This shows that the purely spatial geodesics of the Flamm surface must not be identified with the back-projections onto that surface of the 2D projections, in (R,φ), of spacetime geodesics. Under these conditions, what information does the Flamm surface provide? It is of a topological nature. Since Flamm's analysis shows that the 4D geodesics necessarily project onto this surface, conversely there cannot exist real geodesics crossing the throat sphere that correspond to the region $0<R<\alpha$ in the representation (t,R,θ,φ).

Schwarzschild's approach is perfectly consistent from the astrophysical point of view. He sought to construct two stationary, spherically symmetric solutions of Einstein's equation. He constructed the interior metric solution [9] in the coordinate system (t,χ,θ,φ).

$$ds^2 = \left(\frac{3\cos\chi_a - \cos\chi}{2}\right)^2 c^2 dt^2 - \hat{R}^2\left(d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi \sin^2\theta d\varphi^2\right) \quad (14)$$

With:

$$R = R_a \sin\chi \quad (15)$$

R_a being the ray of the star. With:

$$\hat{R}^2 = \frac{3c^2}{8\pi G\rho} \quad (16)$$

By going back to the system (t, R, θ, φ) it comes:

$$ds^2 = \left[\frac{3}{2}\sqrt{1+\frac{8\pi G\rho R_a^2}{3c^2}} - \frac{1}{2}\sqrt{1+\frac{8\pi G\rho R^2}{3c^2}}\right]^2 c^2 dt^2 + \frac{dR^2}{1-\frac{8\pi G\rho R^2}{3c^2}} - R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (17)$$

The geodesics of the two solutions join along the sphere representing the boundary of the star, assimilated to a sphere of radius R_a, filled with an incompressible material of constant density ρ. The geometric solution presented by Schwarzschild is illustrated in the following figure, expressing the union of a portion of the Flamm surface with a spherical cap. The global result is a contractible object, as shown on Figure 26.

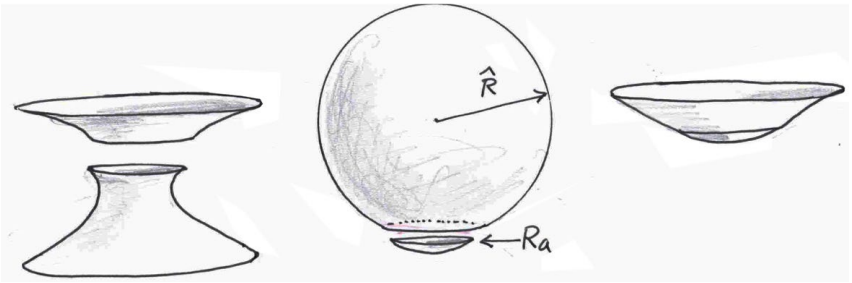


Figure 26. The global geometry of the Schwarzschild's solution.

This constitutes the true geometry of "Schwarzschild spacetime".

7. The Flamm Surface as a Whole

However, some authors considered the isolated exterior metric as a possible geometric description of masses: first H. Weyl in 1917[2] [11], then Einstein and Rosen in 1935 [19]. In the postwar period, the isolated exterior metric solution was considered either as describing a "frozen snapshot" of an object undergoing rapid implosion [20], or as a wormhole [21]. See the Kruskal's wormhole on Figure 27.

[2]Using the isotropic coordinate system he introduced, and noting the possible inversion of the time factor on a second sheet, Weyl was the first to suggest a two-sheeted covering structure involving an inversion of the time coordinate x^4. This will be analyzed in a future article; detailing it here would make the exposition overly cumbersome.

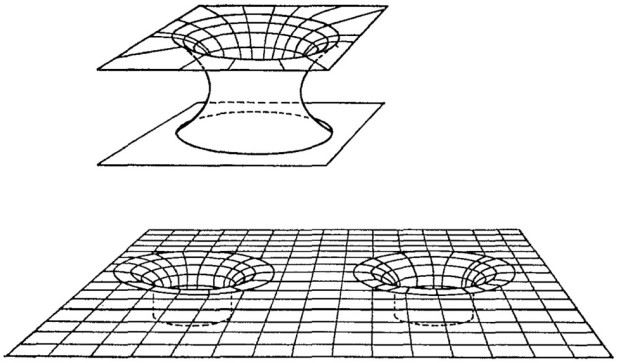


FIG. 1. Two interpretations of the 3-dimensional "maximally extended Schwarzschild metric" at the time $T=0$. Above: A connection or bridge in the sense of Einstein and Rosen between *two* otherwise Euclidean spaces. Below: A wormhole in the sense of Wheeler connecting two regions in *one* Euclidean space, in the limiting case where these regions are extremely far apart compared to the dimensions of the throat of the wormhole.

Figure 27. Kruskal wormhole [21].

In both cases, this leads us to consider the Flamm surface in its entirety. Below are its geodesics when they cross the throat circle, as shown on **Figure 28**.

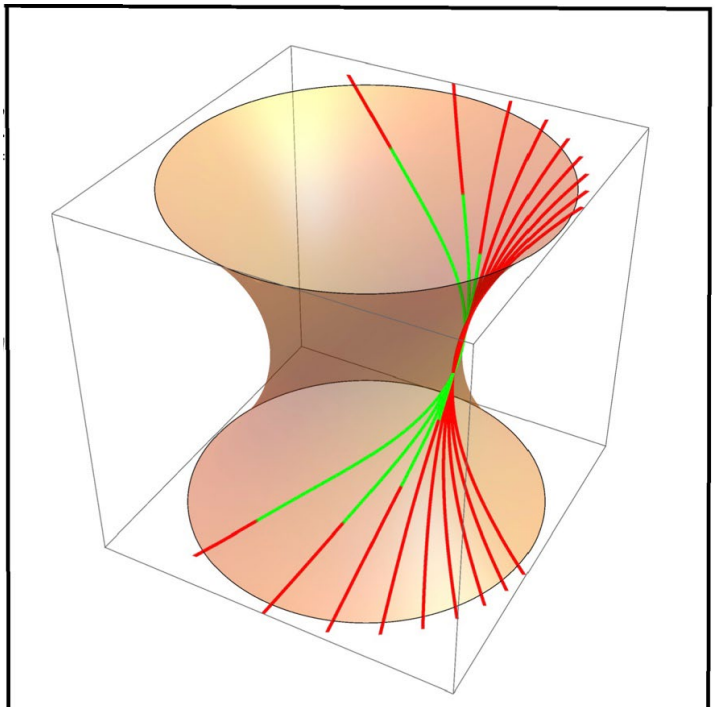


FIG. 16: Arcs of s-geodesics with positive (negative) normal curvatures are shown in green (red) on Flamm's paraboloid. The (normal) curvature of radial ($L = 0$) geodesics, $\kappa_n = \kappa = -\frac{1}{2}\sqrt{\frac{r_S}{r^3}}$, is always negative, pointing away from the hole region, although vanishing asymptotically.

Figure 28. Flamm's surface geodesics. Second family [18].

The same color coding is found again: green when these geodesics direct their concavity toward the axis (attraction), and red when this concavity reverses (repulsion). This is also visible when these Flamm surface geodesics are projected. See **Figure 29**.

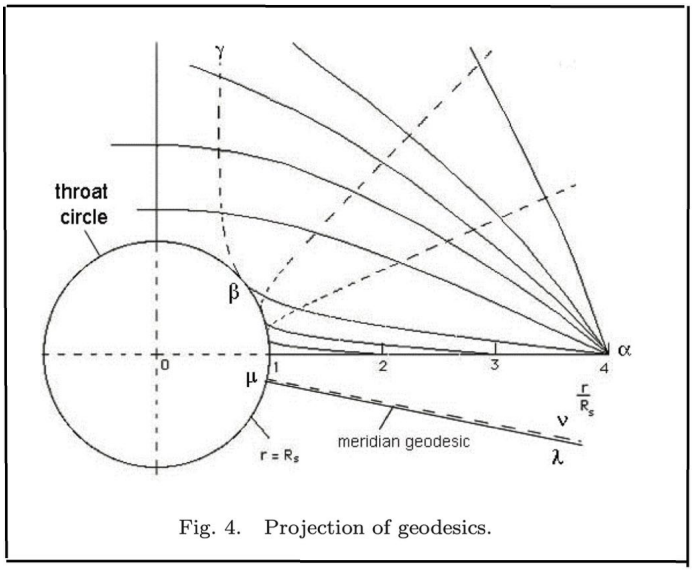


Figure 29. Geodesic projection of Flamm surface [17].

In these figures the purely spatial geodesics of the Flamm surface must not be identified with the back-projections onto that surface of the 2D projections, in (R,φ), of spacetime geodesics. This shows that since the 4D geodesics necessarily project onto this surface, there cannot exist real geodesics crossing the throat sphere that correspond to the region $R<\alpha$ in the representation (t,R,θ,φ) with $(0<R<\alpha)$.

When one constructs the curves solving the Lagrange equations, one finds in the projection (R,φ) real curve segments that give the illusion that test masses penetrate into the region $0<R<\alpha$. This illusion disappears when one constructs 3D images of these geodesics in a coordinate system (t,R,φ), as shown on **Figure 30**.

In **Figure 30(A)**, the trajectory of a test mass converging toward the boundary and then emerging from it has been represented. Both operations occur, with this coordinate *t*, over an infinite time interval. In **Figure 30(B)**, one has the impression that the test mass "bounces" off this boundary. In **Figure 30(C)**, this projection **Figure 30(B)** of the geodesic has been back-projected onto the Flamm surface along the green curve, which therefore appears distorted. It should be kept in mind that this is not a geodesic of the Flamm surface.

If one considers separating the two sheets of the hypersurface associated with this isolated exterior metric solution, it then becomes appropriate to introduce a new variable [17]:

$$R=\propto\left(1+\ln ch\rho\right) \tag{18}$$

$$\mathrm{d}s^2=\frac{\ln ch\rho}{1+\ln ch\rho}c^2\mathrm{d}t^2-\alpha^2\left[\frac{1+\ln ch\rho}{\ln ch\rho}th^2\rho\mathrm{d}\rho^2+\left(1+\ln ch\rho\right)^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right)\right] \tag{19}$$

The throat sphere corresponds to the value $\rho=0$. The two sheets are distinct and correspond to $\rho>0$ and to $\rho<0$. At the throat:

$$G_{tt} \propto \frac{\rho^2}{2} \to +0 \qquad g_{\rho\rho} \to 2 \tag{20}$$

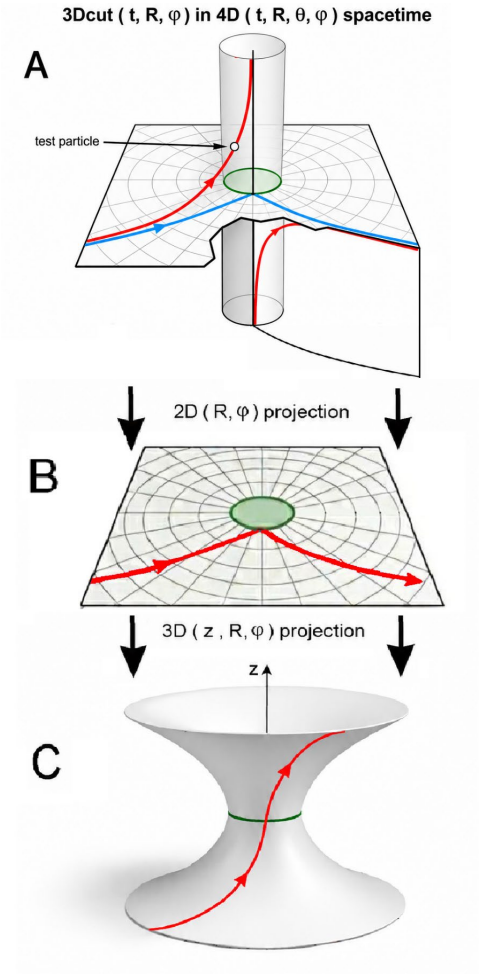


Figure 30. 4D geodesics and projections.

If one restricts oneself to the spatial part described by the metric:

$$\mathrm{d}\sigma^2 = \alpha^2 \left[\frac{1+\ln ch\rho}{\ln ch\rho} th^2\rho \mathrm{d}\rho^2 + \left(1+\ln ch\rho\right)^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right) \right] \tag{21}$$

it is regular everywhere. Let us now consider the projections in (ρ,φ) of spacetime geodesics with $\theta = \pi/2$. In a representation (t,R,θ,φ), the projections (R,φ) are solutions of the classical equation:

$$\mathrm{d}\varphi = \pm \frac{\mathrm{d}R}{R^2\sqrt{\frac{c^2l^2}{h^2} + \frac{\alpha}{h^2R} - \frac{1}{R^2} + \frac{\alpha}{R^3}}} \tag{22}$$

where l and h are parameters of the solution curve. Let ϕ denote the angle formed by the tangent to the curve and the radius vector. At the throat:

$$tg\,\phi = \alpha \left|\frac{\mathrm{d}\phi}{\mathrm{d}R}\right|_{R=\alpha} = \frac{h}{\alpha c l} \tag{23}$$

This quantity is finite. This is what gives the impression, when projecting the geodesic lying on the other sheet, of a discontinuity. It seems to bounce on the throat sphere. But this is only an artifact resulting from the embedding into $\mathbb{R}^3$. If one instead adopts a representation (t,ρ,θ,φ) and projects the geodesics with $\theta=\pi/2$ onto the plane (ρ,φ), this angular discontinuity disappears. Let β denote the angle formed by the tangent to the curve and the radius vector in this representation.

$$(tg\,\beta)_{\rho\to 0}=\frac{\rho \mathrm{d}\varphi}{\mathrm{d}\rho}=\frac{R\mathrm{d}\varphi}{\mathrm{d}R}\frac{\rho}{R}\frac{\mathrm{d}R}{\mathrm{d}\rho}=\frac{h}{\alpha^2 cl}\rho^2\to 0 \tag{24}$$

One can therefore draw the corresponding projection of the geodesic. See Figure 31.

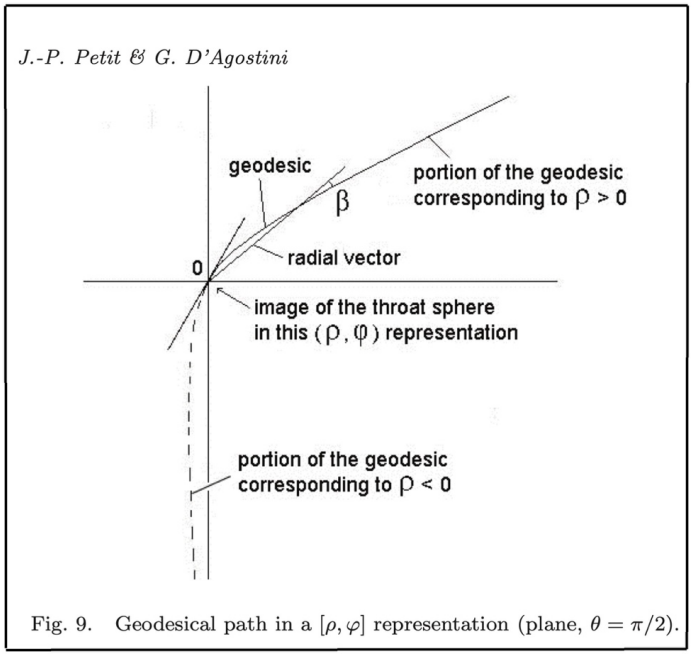


Figure 31. Projection of spatiotemporal geodesics in the representation (ρ,φ) [17].

This illustrates the interpretation of the Schwarzschild exterior metric solution as a two-sheeted covering of a manifold. Such a situation could be visualized by drawing the geodesics $(\rho>0)$ on the front side of a sheet of paper and extending them with the curves $(\rho<0)$ on the back side (dashed curve). The throat sphere is then reduced to a point. What opposes our intuition is that this sphere reduced to a point nevertheless possesses a nonzero area $4\pi\alpha^2$, so that the manifold is locally non-contractible even though it visually appears contractible. A similar representation would be obtained by exploiting the true expression of the Schwarzschild exterior metric solution in coordinates (t,r,θ,φ). See Figure 10. In a projection (r,φ), using the true variable $r=\sqrt{x^2+y^2+z^2}\geq 0$ the interior of the sphere of area $4\pi\alpha^2$ would likewise reduce to a point.

These elements show that artifacts may arise when one attributes physical real-

ity to a representation obtained through embedding in $\mathbb{R}^3$. That being said, can the Schwarzschild exterior metric solution be considered in isolation? The first response was given in 1939 by Oppenheimer and Snyder [20]. In article [22], a second interpretation is proposed, outlining a model for a neutron star reaching Schwarzschild criticality [9], leading to the establishment of a passage toward a second sheet. Whatever physical meaning one assigns to this model, the following figure illustrates the P-symmetry of the two sheets. In three-dimensional space, four points form a tetrahedron whose volumetric orientation is linked to the orientation of its faces. When this tetrahedron crosses the throat sphere, its orientation becomes inverted, showing that the two sheets are related by an enantiomorphic relationship, as shown on **Figure 32**.

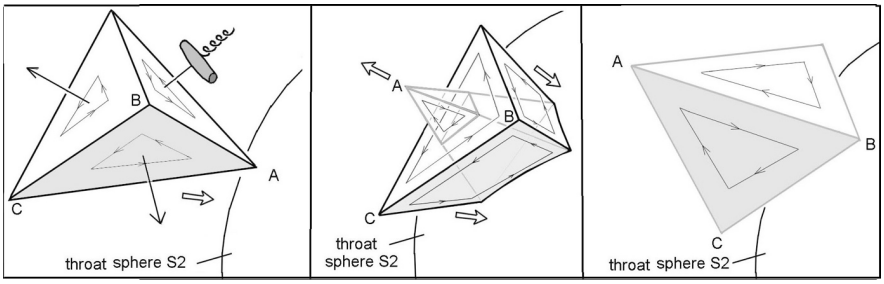


Figure 32. Crossing the throat sphere produces space-inversion [23].

In general, a choice of coordinates leads to a representation of the solution-object that is both geometric and physical. Each such choice corresponds to a particular way of conceiving spacetime, both geometrically and physically. When the stationary, spherically symmetric exterior solution to Einstein's equations is viewed in isolation as a physical object, a black hole, this choice of a geometry that allows for contraction implies a swapping of the roles of space and time coordinates within the object; this issue does not arise in interpretations derived from Flamm's, in which the object is non-contractile and possesses no interior.

8. Conclusion

We must now draw conclusions from this review of the various works in question. This could be summarized by saying that "everything depends on what one calls spacetime". As defined in 1909 by H. Minkowski, it is a hypersurface defined by a real elementary line element: $ds^2 \geq 0$. This was also the position held by Schwarzschild, Einstein, Droste, Weyl, Flamm, and many others at the time. In December 1916, the mathematician D. Hilbert proposed that the solution hypersurface be defined directly from the bilinear form—representing what is today referred to as the "extension of the solution to the whole of spacetime". Consequently, the signature ceases to act as a rigid constraint and can take either the form $(+---)$ or $(-+++)$. This corresponds to the standard view of spacetime expressed through the geometric interpretation of the Schwarzschild solution known as the black hole model. This article demonstrates that this interpretation differs from the reading

of the geometry provided by Flamm. Achieving this "extension to the whole of spacetime" requires that geodesics penetrating "into the interior"—beyond the Schwarzschild sphere—connect differently. That is, instead of extending onto the second sheet of Flamm's surface (organized around a throat sphere and corresponding to a wormhole), they extend into a region of spacetime "where time and space coordinates exchange roles". These correspond to two different topological options. Mathematically, Flamm's physical interpretation corresponds to a non-contractile geometric object, as can be seen directly from the line element in **Figure 10**, whereas the black hole model reflects the preservation of the object's contractibility, at the cost of swapping the roles of the coordinates. This article represents only the beginning of a broader line of reasoning linked to the black hole model. The ultimate goal is to clarify the nature of the geometric extension associated with this model. A future article will address the Kruskal extension and aspects related to Birkhoff's theorem.

Author Contributions

J.-P. Petit is the author of the article. G. D'Agostini verified the calculations, and J.-M. Pechinot contributed to the production of some of the figures.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Schwarzschild, K. (1916) Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin.

[2] Einstein, A. (1915) Erlklärung der Perihelbewegung des Merkur aus allgemeinen Relarivitätstheorie. Königlish Preubische Akademie der Wissenschaften (Berlin) Sitzungberichet, 831-838.

[3] Schwarzschild, K. (1915) Letter to A. Einstein. Collected Papers of Albert Einstein, The Berlin Years, Correspondence 1914-1918, Document n°169. https://einsteinpapers.press.princeton.edu/vol6-doc

[4] Abrams, L.S. (1989) Black Holes: The Legacy of Hilbert's Error. *Canadian Journal of Physics*, **67**, 919-926.

[5] Hilbert, D. (1916) Die Grundlagen der Physik (Esrte Mitteilung). Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse.

[6] Flamm, L. (1916) Beiträge zur Einsteinschen Gravitationstheorie. *Physikalische Zestschrift*, **17**, 448-454.

[7] Hilbert, D. (1915) Die Grundlagen der Physik (Esrte Mitteilung). Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse, 20 November 1915, 395-407.

[8] Einstein, A. (1915) Die Feldgleichungen des Gravitation (The Field Equation of Gravitation). *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, Berlin, 844-847.

[9] Schwarzschild, K. (1916) Über das Gravitionalsfeld einer Kugel Aus incompressibler Flü-

ssigkeit nach der Einsteinschen Theorie. Sitzung der phys. Math. Klasse v.23 märz.

[10] Droste, J. (1917) The Field of a Single Centre in Einstein's Theory of Gravitation and the Motion of a Particle in That Field. *Proceedings of the Koninklijke Nederlandsche Akademie van Wetenschappen*, **19**, 197-215.

[11] Weyl, H. (1917) Zur Gravitationstheorie. *Annalen der Physik*, **359**, 117-145. https://doi.org/10.1002/andp.19173591804

[12] Minkowski, H. (1909) Raum und Zeit. *Physikalische Zeitschrift*, **10**, 75-88.

[13] Poincaré, H. (1902) La mesure du temps. *Revue de métaphysique et de morale*, **10**, 1-13.

[14] Eddington, A. (1919) The Mathematical Theory of Relativity. Cambridge University Press.

[15] Tolman, R.C. (1946) Relativity, Thermodynamics and Cosmology. The Clarendon Press.

[16] Chandrasekhar, S. (1992) The Mathematical Theory of Black Holes. Oxford University Press.

[17] Petit, J.P. and D'Agostini, G. (2015) Cancellation of the Singularity of the Schwarzschild Solution with Natural Mass Inversion Process. *Modern Physics Letters A*, **30**, Article ID: 1550051.

[18] Eufrasio, R.T., Mecholsky, N.A. and Resca, L. (2018) Curved Space, Curved Time, and Curved Space-Time in Schwarzschild Geodetic Geometry. *General Relativity and Gravitation*, **50**, Article No. 159. https://doi.org/10.1007/s10714-018-2481-2

[19] Einstein, A. and Rosen, N. (1935) The Particle Problem in the General Theory of Relativity. *Physical Review*, **48**, 73-77. https://doi.org/10.1103/physrev.48.73

[20] Oppenheimer, J.R. and Snyder, H. (1939) On Continued Gravitational Contraction. *Physical Review*, **56**, 455-459. https://doi.org/10.1103/physrev.56.455

[21] Kruskal, M.D. (1960) Maximal Extension of Schwarzschild Metric. *Physical Review*, **119**, 1743-1745. https://doi.org/10.1103/physrev.119.1743

[22] Petit, J. and D'Agostini, G. (2025) Alternatives to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*, **16**, 1479-1490. https://doi.org/10.4236/jmp.2025.1610072

[23] Koiran, P., Zejli, H., Levy, J., Margnat, F., Duval, M. and Zejli, H. (2024) PT-Symmetry in One-Way Wormholes. *Annals of Physics*, **470**, Article ID: 169765. https://doi.org/10.1016/j.aop.2024.169765

Supplementary Information

wobei die Hilfsgröße

$$R = (3x_1 + \rho)^{1/3} = (r^3 + \alpha^3)^{1/3}$$

eingeführt ist.

Setzt man diese Werte der Funktionen f im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des Einsteinschen Problems bildet:

$$ds^2 = (1 - \alpha/R)dt^2 - \frac{dR^2}{1 - \alpha/R} - R^2(d\vartheta^2 + \sin^2\vartheta\, d\phi^2),\ R = (r^3 + \alpha^3)^{1/3}. \quad (14)$$

Dasselbe enthält die eine Konstante α, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

Figure S1. German version of **Figure 1** (the line element expressed with the intermediate quantity R).

§ 3. Nennt man die Zeit t, die rechtwinkligen Koordinaten x, y, z, so ist das allgemeinste Linienelement, welches die Forderungen 1—3 erfüllt, offenbar das folgende:

$$ds^2 = Fdt^2 - G(dx^2 + dy^2 + dz^2) - H(xdx + ydy + zdz)^2$$

wobei F, G, H Funktionen von $r = \sqrt{x^2 + y^2 + z^2}$ sind.

Figure S2. German version of **Figure 2** (the true Schwarzschild's radial coordinate r).

Man setze

$$x_1 = \frac{r^3}{3}, \quad x_2 = -\cos\vartheta, \quad x_3 = \phi. \quad (7)$$

Figure S3. German version of **Figure 3** (Schwarzschild's first coordinate change).

In den neuen Polarkoordinaten lautet das Linienelement:

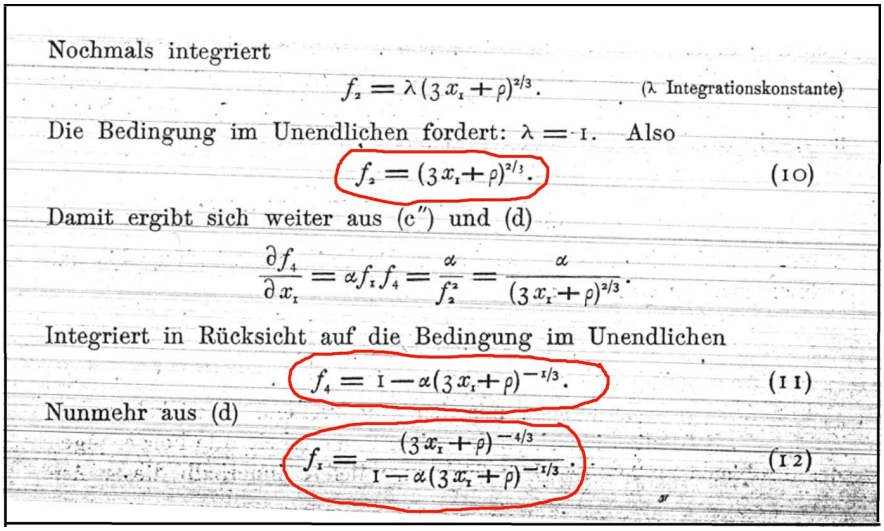
$$ds^2 = Fdx_4^2 - \left(\frac{G}{r^4} + \frac{H}{r^2}\right)dx_1^2 - Gr^2\left[\frac{dx_2^2}{1 - x_2^2} + dx_3^2(1 - x_2^2)\right], \quad (8)$$

wofür wir schreiben wollen:

$$ds^2 = f_4 dx_4^2 - f_1 dx_1^2 - f_2\frac{dx_2^2}{1 - x_2^2} - f_3 dx_3^2(1 - x_2^2). \quad (9)$$

Dann sind f_1, $f_2 = f_3$, f_4 drei Funktionen von x_1

Figure S4. German version of **Figure 4** (the three functions f).



Nochmals integriert

$$f_2 = \lambda(3x_1+\rho)^{2/3}. \qquad (\lambda \text{ Integrationskonstante})$$

Die Bedingung im Unendlichen fordert: $\lambda = 1$. Also

$$f_2 = (3x_1+\rho)^{2/3}. \qquad (10)$$

Damit ergibt sich weiter aus (c″) und (d)

$$\frac{\partial f_4}{\partial x_1} = \alpha f_1 f_4 = \frac{\alpha}{f_2^2} = \frac{\alpha}{(3x_1+\rho)^{2/3}}.$$

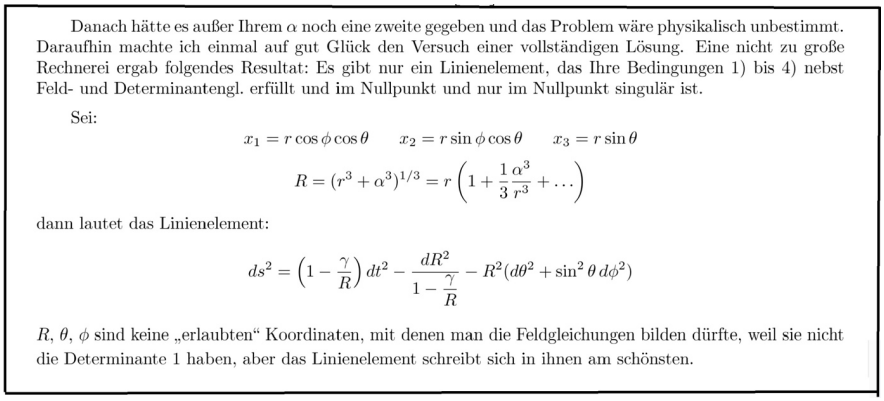
Integriert in Rücksicht auf die Bedingung im Unendlichen

$$f_4 = 1-\alpha(3x_1+\rho)^{-1/3}. \qquad (11)$$

Nunmehr aus (d)

$$f_1 = \frac{(3x_1+\rho)^{-4/3}}{1-\alpha(3x_1+\rho)^{-1/3}}. \qquad (12)$$

Figure S5. German version of **Figure 4** (expressions of the three functions *f*) [1].



Danach hätte es außer Ihrem α noch eine zweite gegeben und das Problem wäre physikalisch unbestimmt. Daraufhin machte ich einmal auf gut Glück den Versuch einer vollständigen Lösung. Eine nicht zu große Rechnerei ergab folgendes Resultat: Es gibt nur ein Linienelement, das Ihre Bedingungen 1) bis 4) nebst Feld- und Determinantengl. erfüllt und im Nullpunkt und nur im Nullpunkt singulär ist.

Sei:

$$x_1 = r\cos\phi\cos\theta \qquad x_2 = r\sin\phi\cos\theta \qquad x_3 = r\sin\theta$$

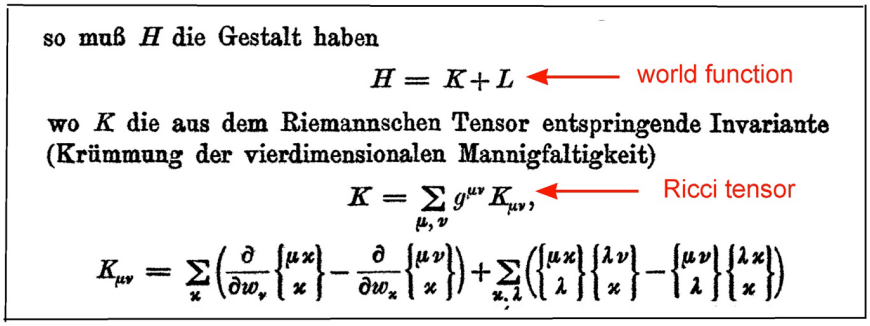
$$R = (r^3+\alpha^3)^{1/3} = r\left(1+\frac{1}{3}\frac{\alpha^3}{r^3}+\dots\right)$$

dann lautet das Linienelement:

$$ds^2 = \left(1-\frac{\gamma}{R}\right)dt^2 - \frac{dR^2}{1-\frac{\gamma}{R}} - R^2(d\theta^2+\sin^2\theta\,d\phi^2)$$

R, θ, ϕ sind keine „erlaubten" Koordinaten, mit denen man die Feldgleichungen bilden dürfte, weil sie nicht die Determinante 1 haben, aber das Linienelement schreibt sich in ihnen am schönsten.

Figure S6. German version of **Figure 8** (the letter to Einstein).



so muß H die Gestalt haben

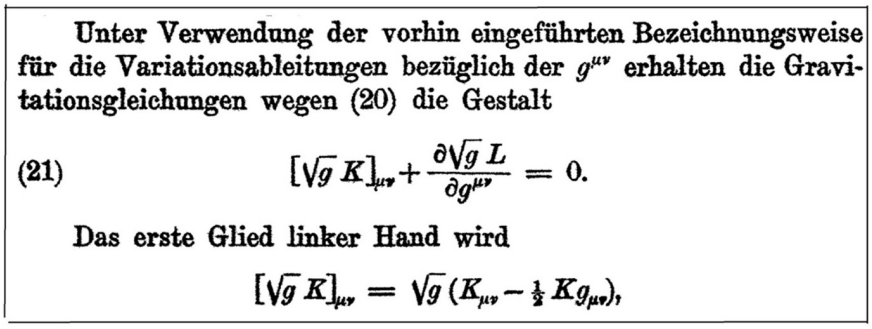
$$H = K+L$$ ← world function

wo K die aus dem Riemannschen Tensor entspringende Invariante (Krümmung der vierdimensionalen Mannigfaltigkeit)

$$K = \sum_{\mu,\nu} g^{\mu\nu}K_{\mu\nu},$$ ← Ricci tensor

$$K_{\mu\nu} = \sum_{\varkappa}\left(\frac{\partial}{\partial w_\nu}\left\{\begin{matrix}\mu\varkappa\\ \varkappa\end{matrix}\right\} - \frac{\partial}{\partial w_\varkappa}\left\{\begin{matrix}\mu\nu\\ \varkappa\end{matrix}\right\}\right) + \sum_{\varkappa,\lambda}\left(\left\{\begin{matrix}\mu\varkappa\\ \lambda\end{matrix}\right\}\left\{\begin{matrix}\lambda\nu\\ \varkappa\end{matrix}\right\} - \left\{\begin{matrix}\mu\nu\\ \lambda\end{matrix}\right\}\left\{\begin{matrix}\lambda\varkappa\\ \varkappa\end{matrix}\right\}\right)$$

Figure S7. German version of **Figure 9** (the Hilbert's world function).



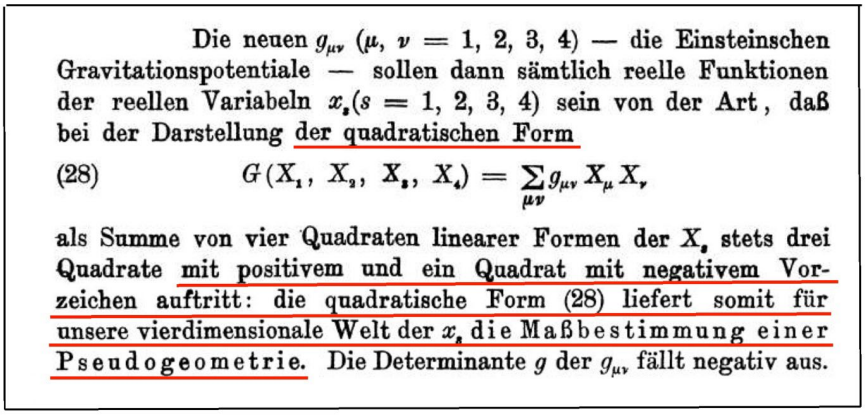
Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (20) die Gestalt

$$[\sqrt{g}\,K]_{\mu\nu} + \frac{\partial \sqrt{g}\,L}{\partial g^{\mu\nu}} = 0. \tag{21}$$

Das erste Glied linker Hand wird

$$[\sqrt{g}\,K]_{\mu\nu} = \sqrt{g}\,(K_{\mu\nu} - \tfrac{1}{2} K g_{\mu\nu}),$$

Figure S8. German version of Figure 10 (The Hilbert's Field Equation).

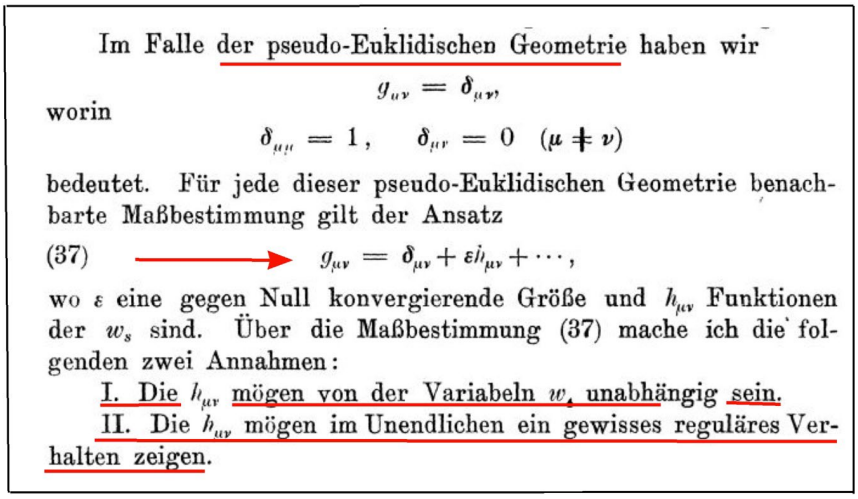


Die neuen $g_{\mu\nu}$ ($\mu, \nu = 1, 2, 3, 4$) — die Einsteinschen Gravitationspotentiale — sollen dann sämtlich reelle Funktionen der reellen Variabeln $x_s (s = 1, 2, 3, 4)$ sein von der Art, daß bei der Darstellung der quadratischen Form

$$G(X_1, X_2, X_3, X_4) = \sum_{\mu\nu} g_{\mu\nu} X_\mu X_\nu \tag{28}$$

als Summe von vier Quadraten linearer Formen der X_s stets drei Quadrate mit positivem und ein Quadrat mit negativem Vorzeichen auftritt: die quadratische Form (28) liefert somit für unsere vierdimensionale Welt der x_s die Maßbestimmung einer Pseudogeometrie. Die Determinante g der $g_{\mu\nu}$ fällt negativ aus.

Figure S9. German version of Figure 11 (Hilbert's quadratic form).



Im Falle der pseudo-Euklidischen Geometrie haben wir

$$g_{\mu\nu} = \delta_{\mu\nu},$$

worin

$$\delta_{\mu\mu} = 1, \qquad \delta_{\mu\nu} = 0 \quad (\mu \neq \nu)$$

bedeutet. Für jede dieser pseudo-Euklidischen Geometrie benachbarte Maßbestimmung gilt der Ansatz

$$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu} + \cdots, \tag{37}$$

wo ε eine gegen Null konvergierende Größe und $h_{\mu\nu}$ Funktionen der w_s sind. Über die Maßbestimmung (37) mache ich die folgenden zwei Annahmen:

I. Die $h_{\mu\nu}$ mögen von der Variabeln w_s unabhängig sein.

II. Die $h_{\mu\nu}$ mögen im Unendlichen ein gewisses reguläres Verhalten zeigen.

Figure S10. German version of Figure 12 (Hilbert's pseudo Euclidean space).

Das Axiom bedeutet, dass an jedem Punkt im Universum der Ausdruck

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

ist immer positiv, oder, gleichbedeutend damit, dass die Geschwindigkeit v immer kleiner als c ist. Somit ist c die obere Grenze für die Geschwindigkeit eines beliebigen wesentlichen Punktes, und genau darin liegt seine wahre Bedeutung.

$$d\tau = \frac{1}{c}\sqrt{c^2 dt^2 - dx^2 - dy^2 - dz^2}$$

Das Integral $\int d\tau$ dieser Größe entlang der Weltgeraden wird als Eigenzeit des substanziellen Punktes bezeichnet.

Figure S11. German version of **Figure 13** (Minkowski's axiomatic definition of spacetime).

Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie.

Von K. Schwarzschild.

(Vorgelegt am 13. Januar 1916 [s. oben S. 42].)

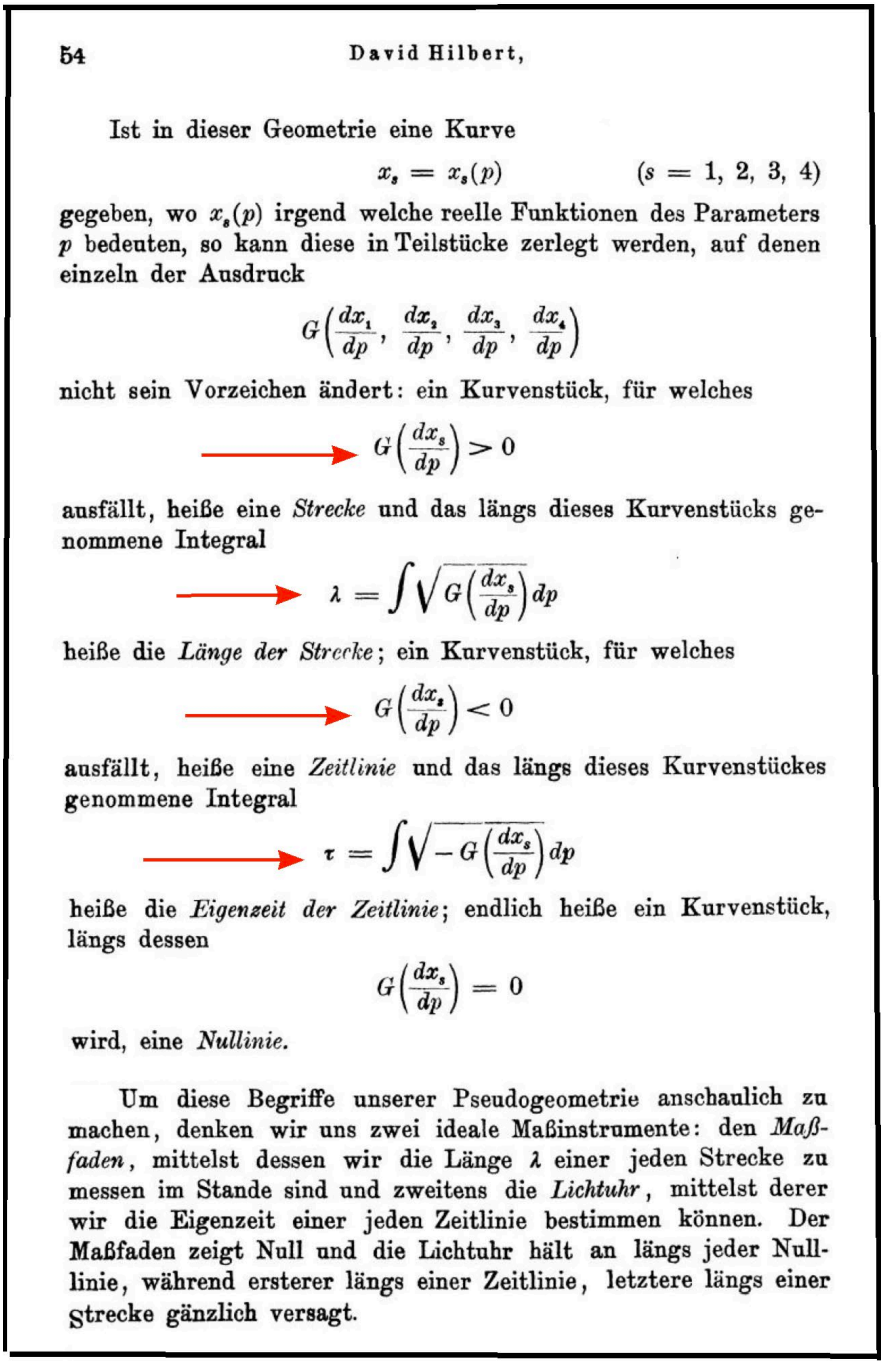
§ 1. Hr. Einstein hat in seiner Arbeit über die Perihelbewegung des Merkur (s. Sitzungsberichte vom 18. November 1915) folgendes Problem gestellt:

Ein Punkt bewege sich gemäß der Forderung

$$\text{wobei} \quad \left.\begin{aligned} \delta\int ds = 0, \\ ds = \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \qquad \mu,\nu = 1,2,3,4 \end{aligned}\right\} \quad (1)$$

ist, $g_{\mu\nu}$ Funktionen der Variabeln x bedeuten und bei der Variation am Anfang und Ende des Integrationswegs die Variablen x festzuhalten sind. Der Punkt bewege sich also, kurz gesagt, auf einer geodätischen Linie in der durch das Linienelement ds charakterisierten Mannigfaltigkeit.

Figure S12. German version of **Figure 14** (foreword of the Schwarzschild's paper).



54 David Hilbert,

Ist in dieser Geometrie eine Kurve

$$x_s = x_s(p) \qquad (s = 1, 2, 3, 4)$$

gegeben, wo $x_s(p)$ irgend welche reelle Funktionen des Parameters p bedeuten, so kann diese in Teilstücke zerlegt werden, auf denen einzeln der Ausdruck

$$G\left(\frac{dx_1}{dp}, \frac{dx_2}{dp}, \frac{dx_3}{dp}, \frac{dx_4}{dp}\right)$$

nicht sein Vorzeichen ändert: ein Kurvenstück, für welches

$$G\left(\frac{dx_s}{dp}\right) > 0$$

ausfällt, heiße eine *Strecke* und das längs dieses Kurvenstücks genommene Integral

$$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\, dp$$

heiße die *Länge der Strecke*; ein Kurvenstück, für welches

$$G\left(\frac{dx_s}{dp}\right) < 0$$

ausfällt, heiße eine *Zeitlinie* und das längs dieses Kurvenstückes genommene Integral

$$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}\, dp$$

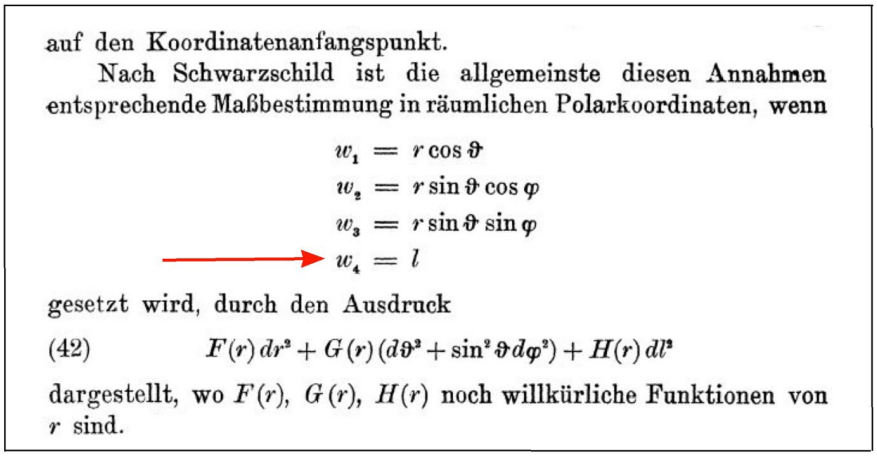
heiße die *Eigenzeit der Zeitlinie*; endlich heiße ein Kurvenstück, längs dessen

$$G\left(\frac{dx_s}{dp}\right) = 0$$

wird, eine *Nullinie*.

Um diese Begriffe unserer Pseudogeometrie anschaulich zu machen, denken wir uns zwei ideale Maßinstrumente: den *Maßfaden*, mittelst dessen wir die Länge λ einer jeden Strecke zu messen im Stande sind und zweitens die *Lichtuhr*, mittelst derer wir die Eigenzeit einer jeden Zeitlinie bestimmen können. Der Maßfaden zeigt Null und die Lichtuhr hält an längs jeder Nullinie, während ersterer längs einer Zeitlinie, letztere längs einer Strecke gänzlich versagt.

Figure S13. German version of Figure 15 (Hilbert's definition of lengths).



auf den Koordinatenanfangspunkt.

Nach Schwarzschild ist die allgemeinste diesen Annahmen entsprechende Maßbestimmung in räumlichen Polarkoordinaten, wenn

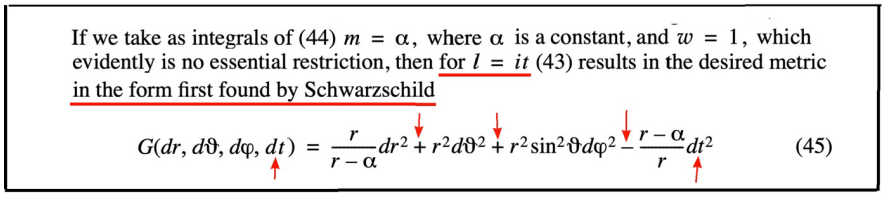
$$w_1 = r\cos\vartheta$$
$$w_2 = r\sin\vartheta\cos\varphi$$
$$w_3 = r\sin\vartheta\sin\varphi$$
$$w_4 = l$$

gesetzt wird, durch den Ausdruck

$$F(r)\,dr^2 + G(r)\,(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) + H(r)\,dl^2 \tag{42}$$

dargestellt, wo $F(r)$, $G(r)$, $H(r)$ noch willkürliche Funktionen von r sind.

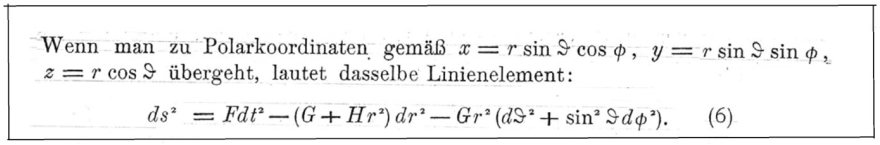
Figure S14. German version of Figure 16 (Hilbert's coordinates' choice).



If we take as integrals of (44) $m = \alpha$, where α is a constant, and $w = 1$, which evidently is no essential restriction, then for $l = it$ (43) results in the desired metric in the form first found by Schwarzschild

$$G(dr, d\vartheta, d\varphi, dt) = \frac{r}{r-\alpha}dr^2 + r^2 d\vartheta^2 + r^2\sin^2\vartheta\,d\varphi^2 - \frac{r-\alpha}{r}dt^2 \tag{45}$$

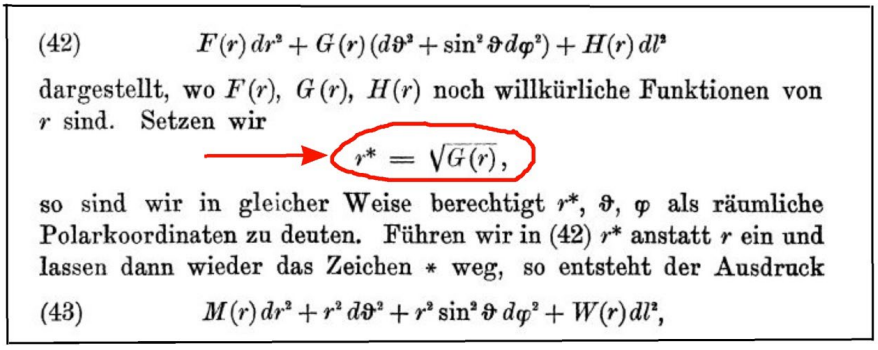
Figure S15. German version of Figure 17-German version of Figure 17 (Hilbert's identification of his solution to Schwarzschild's).



Wenn man zu Polarkoordinaten gemäß $x = r\sin\vartheta\cos\phi$, $y = r\sin\vartheta\sin\phi$, $z = r\cos\vartheta$ übergeht, lautet dasselbe Linienelement:

$$ds^2 = Fdt^2 - (G + Hr^2)\,dr^2 - Gr^2(d\vartheta^2 + \sin^2\vartheta\,d\phi^2). \tag{6}$$

Figure S16. German version of Figure 18 (Schwarzschild's line element).



$$F(r)\,dr^2 + G(r)\,(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) + H(r)\,dl^2 \tag{42}$$

dargestellt, wo $F(r)$, $G(r)$, $H(r)$ noch willkürliche Funktionen von r sind. Setzen wir

$$r^* = \sqrt{G(r)},$$

so sind wir in gleicher Weise berechtigt r^*, ϑ, φ als räumliche Polarkoordinaten zu deuten. Führen wir in (42) r^* anstatt r ein und lassen dann wieder das Zeichen $*$ weg, so entsteht der Ausdruck

$$M(r)\,dr^2 + r^2\,d\vartheta^2 + r^2\sin^2\vartheta\,d\varphi^2 + W(r)\,dl^2, \tag{43}$$

Figure S17. German version of Figure 19 (Hilbert's approximation).

§2. Nun können wir in analoger Weise den ersten Fall untersuchen, der von Schwarzschild behandelt wurde, nämlich das Gravitationsfeld einer Punktmasse. Das Linienelement hat in seiner einfachsten Form dieselbe Struktur wie oben und lautet:

$$d s^2 = \left(1 - \frac{\alpha}{R}\right) d t^2 - \frac{d R^2}{1 - \frac{\alpha}{R}} - R^2 \left(d \vartheta^2 + \sin^2 \vartheta \, d \varphi^2\right) .$$

Figure S18. German version of Figure 21 (Flamm's choice for the metric representation).

Wenn also elementare Lineale und Uhren vom Gravitationsfeld beeinflusst werden, muss die Ausbreitung des Lichts in exakt derselben Weise beeinflusst werden, sodass sie im Vergleich keine Unterschiede aufweisen. Dasselbe gilt für alle anderen physikalischen Phänomene. Dies könnte erklären, warum sich die Bewegung eines Punktteilchens durch die einfache Variationsgleichung beschreiben lässt.

$$\delta \int_{P_1}^{P_2} d s = 0 \quad , \tag{5}$$

Figure S19. German version of Figure 22 (Flamm's variational procedure).

Der räumliche Anteil des Linienelements ist wiederum nichteuklidischer Natur. Aufgrund der sphärischen Symmetrie scheint es wiederum ausreichend zu sein, die Betrachtung auf die metrischen Eigenschaften in einem beliebigen ebenen Schnitt durch den Ursprung zu beschränken, $\vartheta = \pi/2$. un erhält man

$$d \sigma_e{}^2 = \frac{d R^2}{1 - \frac{\alpha}{R}} + R^2 \, d \varphi^2 \tag{4}$$

Bezeichnet man mit z die Koordinate in Richtung der Rotationsachse, so ergibt sich die Gleichung für die Meridionalkurve aus

$$\frac{d z}{d R} = \operatorname{tg} \chi = \sqrt{\frac{\alpha}{R - \alpha}}$$

$$z^2 = 4\alpha(R - \alpha).$$

Dies beschreibt eine Parabol mit Parameter

$$p = 2\alpha,$$

Figure S20. German version of Figure 23 (Flamm's embedment).

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(9), 1197-1228
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Kruskal Coordinates, Semi-Complex Extension of Spacetime

Jean-Pierre Petit

Manaty Research Group, Glanon, France
Email: jppetit1937@yahoo.fr

How to cite this paper: Petit, J.-P. (2026) Kruskal Coordinates, Semi-Complex Extension of Spacetime. *Journal of Modern Physics*, **17**, 1197-1228.
https://doi.org/10.4236/jmp.2026.179053

Received: May 26, 2026
Accepted: September 26, 2026
Published: September 29, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

In this article, we re-examine the geometric nature of the extension of the Schwarzschild solution into the domain $0<r<\alpha$. A preliminary study of the sphere, the torus, and Flamm's surface demonstrates that real geometries can admit semi-complex extensions beyond the limits of their real embedding domains; in such cases, certain coordinates become purely imaginary, while their projections onto real subspaces remain observable as *ghost images*. Kruskal's standard construction is then repeated line by line, separately in the outer $r>\alpha$ and the inner region $0<r<\alpha$. It appears that the usual actual definition of the tortoise coordinate, $r^{*}=r+\alpha\ln\left|\frac{r}{\alpha}-1\right|$ encompasses two distinct changes of variables on either side of $r=\alpha$, supplemented by different sign choices for the exponential coordinates that allow for the construction of the four sectors of the Kruskal diagram. We then consider another continuation, obtained by maintaining a single analytic change of variables across $r=\alpha$, without introducing the absolute value. In the domain $0<r<\alpha$, the Kruskal coordinates thus extended naturally become purely imaginary. The standard real representation can be recovered via an imaginary translation of the time coordinate, which simultaneously produces the swapping of hyperbolic functions characteristic of the interior regions. A three-dimensional representation $r=H(u,v)$ allows for the visualization of the connection between the real and semi-complex sectors. This analysis leads to an alternative interpretation of the Schwarzschild extension: rather than attributing swapped causal roles to the coordinates r and t in the interior region, the latter can be viewed as belonging to a semi-complex extension of spacetime geometry. Finally, the physical consequences of this interpretation are discussed, along with the possibility of alternative compact states, such as *plugstars*, that could potentially lead to discriminating observational tests.

Keywords

Semi-Complex Extension, Kruskal Coordinates, Kruskal Diagram, Schwarzschild Geometry, Autoparallel Curves, Ghost Images, Ghost Geometry, Ghost Symmetry, Pseudo-Real Coordinates

1. Real and Semi-Complex Geometric Extension: The Ghost Phenomenon

Let us consider the sphere S^2, defined by its metric, expressed in terms of the two angular coordinates θ and φ:

$$ds^2 = R^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{1}$$

Introduce the change of coordinate:

$$\theta = \arcsin\frac{r}{R} \tag{2}$$

The line element becomes:

$$ds^2 = r^2 d\varphi^2 + \frac{R^2 dr^2}{R^2 - r^2} \tag{3}$$

We thus have a Riemannian metric, in the sense that the signs of the signature (+ +) are identical. One can construct the geodesics of this object as length-minimizing curves:

$$J = \int ds = \int\sqrt{r^2 d\varphi^2 + \frac{R^2 dr^2}{R^2 - r^2}} \tag{4}$$

By locating the position of the point using a parameter λ:

$$J = \int\sqrt{r^2\left(\frac{d\varphi}{d\lambda}\right)^2 + \frac{R^2}{R^2 - r^2}\left(\frac{dr}{d\lambda}\right)^2} \tag{5}$$

By posing:

$$\dot{\varphi} = \frac{d\varphi}{d\lambda} \qquad \dot{r} = \frac{dr}{d\lambda} \tag{6}$$

We get:

$$J = \int\sqrt{r^2\dot{\varphi}^2 + \frac{R^2}{R^2 - r^2}\dot{r}^2}\, d\lambda = \int F\left(r, \varphi, \dot{r}, \dot{\varphi}\right) d\lambda \tag{7}$$

The quantity $F\left(r, \varphi, \dot{r}, \dot{\varphi}\right)$ ...being the corresponding Lagrange function. Minimizing the length would lead us to the Lagrange equations:

$$\frac{d}{d\lambda}\left(\frac{\partial F}{\partial \dot{r}}\right) = \frac{\partial F}{\partial r} \tag{8}$$

$$\frac{d}{d\lambda}\left(\frac{\partial F}{\partial \dot{\varphi}}\right) = \frac{\partial F}{\partial \varphi} \tag{9}$$

What happens if we replace the Lagrange function F by its square F^2?

$$J=\int\left(r^2\dot{\varphi}^2+\frac{R^2}{R^2-r^2}\dot{r}^2\right)\mathrm{d}\lambda=\int F^2\left(r,\varphi,\dot{r},\dot{\varphi}\right)\mathrm{d}\lambda=\int\Phi\mathrm{d}\lambda \tag{10}$$

The Lagrange equations are:

$$\frac{\mathrm{d}F}{\mathrm{d}\lambda}\left(\frac{\partial F}{\partial\dot{r}}\right)+F\left(\frac{\partial F}{\partial\dot{r}}\right)=F\frac{\partial F}{\partial r} \tag{11}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\lambda}\left(\frac{\partial F}{\partial\dot{\varphi}}\right)+F\left(\frac{\partial F}{\partial\dot{\varphi}}\right)=F\frac{\partial F}{\partial\varphi} \tag{12}$$

To obtain the same equations, a specific parameterization is required, such as:

$$\frac{\mathrm{d}F}{\mathrm{d}\lambda}=0 \tag{13}$$

But we have:

$$\left(\frac{\mathrm{d}s}{\mathrm{d}\lambda}\right)^2=r^2\dot{\varphi}^2+\frac{R^2}{R^2-r^2}\dot{r}^2=F^2=\Phi \tag{14}$$

The condition $\frac{\mathrm{d}F}{\mathrm{d}\lambda}=0$ means that λ is an affine parameter along the geodesic. Since $F=\frac{\mathrm{d}s}{\mathrm{d}\lambda}$, this condition imposes a constant rate of variation of arc length with respect to λ. Choosing $F=1$ simply amounts to taking $\lambda=s$.

We now choose an affine parameterization, and more precisely the normalization $F=1$, which, in the Riemannian sector, amounts to choosing $\lambda=s$. Thus:

$$1=r^2\dot{\varphi}^2+\frac{R^2}{R^2-r^2}\dot{r}^2 \tag{15}$$

Since we have a cyclical nature:

$$\frac{\partial\Phi}{\partial\varphi}=0 \tag{16}$$

Choosing Φ/2 as Lagrangian:

$$r^2\dot{\varphi}^2=\mathrm{Cst}\rightarrow\dot{\varphi}=\frac{h}{r} \tag{17}$$

By combining these two equations, one immediately obtains the differential equation yielding the solution curves:

$$\mathrm{d}\varphi=\frac{hR}{r}\frac{\mathrm{d}r}{\sqrt{\left(R^2-r^2\right)\left(r^2-h^2\right)}}) \tag{18}$$

From which we can construct two families of curves:

- For $r<R$ with $|h|<R$
- For $r>R$ with $|h|>R$

For $r<R$, these are ellipses—planar projections of the geodesics of the sphere S^2, which are its great circles. For $r<R$, these elliptical projections can be viewed as the planar projections of the great circles of the sphere S^2 embedded in $\mathbb{R}^3$.

But what about the curves corresponding to $r>R$, and what meaning should be assigned to them? Let us first show that they are ellipses. Let us set:

$$r = \frac{1}{u} \tag{19}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}u} = -\frac{hRu}{\sqrt{\left(R^2u^2-1\right)\left(1-h^2u^2\right)}} \tag{20}$$

$$v = u^2 = \frac{1}{r^2} \tag{21}$$

$$\left(\frac{\mathrm{d}v}{\mathrm{d}u}\right)^2 = \frac{4}{h^2R^2}\left(R^2v-1\right)\left(1-h^2v\right) \tag{22}$$

$$\frac{\mathrm{d}^2v}{\mathrm{d}\varphi^2} + 4v = 2\left(\frac{1}{h^2}+\frac{1}{R^2}\right) \tag{23}$$

$$v = \frac{1}{2}\left(\frac{1}{h^2}+\frac{1}{R^2}\right) + A\cos 2\left(\varphi-\varphi_0\right) \tag{24}$$

The turning points are

$$r = R \text{ et } r = |h| \tag{25}$$

What fixes the origin and provides, after the angular origin has been chosen:

$$\frac{1}{r^2} = \frac{\cos^2\left(\varphi-\varphi_0\right)}{h^2} + \frac{\sin^2\left(\varphi-\varphi_0\right)}{R^2} \tag{26}$$

With:

$$x = r\cos\left(\varphi-\varphi_0\right) \tag{27}$$

$$y = r\sin\left(\varphi-\varphi_0\right) \tag{28}$$

$$\frac{x^2}{h^2} + \frac{y^2}{R^2} = 1 \tag{29}$$

All these curves are ellipses, as shown in Figure 1.

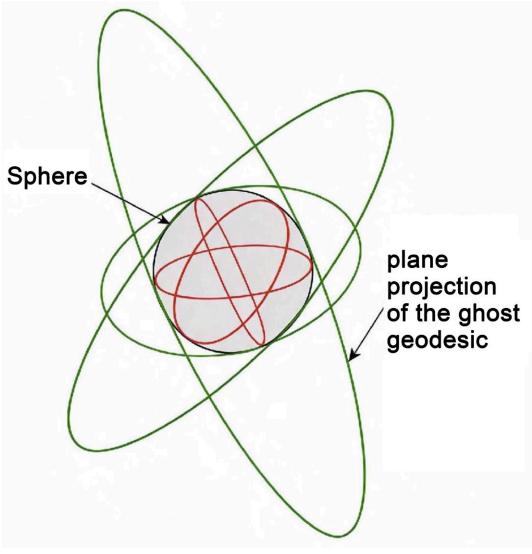


Figure 1. Real plane projections of geodesics and ghost images of antiparallels inscribed on the semi-complex extension of the sphere for $r > \alpha$.

The curves lying outside the projection of the sphere's apparent contour are therefore also ellipses. However, these real curves are 2D projections; they cannot be regarded as the projections of actual curves capable of being embedded in $\mathbb{R}^3$. (See Figure 2.)

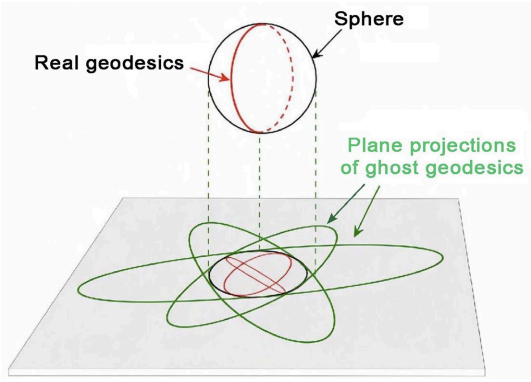


Figure 2. Sphere and projection of the sphere, plus geodesics, and ghost images of antiparallel curves that belong to the semi-complex geometric extension.

Let us show that they can be embedded in a three-dimensional semi-complex space with coordinates (r,φ,η), where

$$(r,\varphi)\in\mathbb{R}^2 \qquad \eta\in\mathbb{C} \tag{30}$$

That is to say, where the first two are real and the third is purely imaginary.

Regarding the region $r < R$ the sphere S^2, embedded in the space described by the real coordinates (r,φ,z) and projected onto the subspace—exhibits an apparent contour in the form of the circle:

$$r^2+z^2=R^2 \tag{31}$$

For $r > R$, if we attempt to embed the geometric object defined by its metric into $\mathbb{R}^3$, using coordinates (r,φ,z), we obtain

$$\mathrm{d}z=\frac{r\mathrm{d}r}{\sqrt{R^2-r^2}} \tag{32}$$

z becomes purely imaginary. If we then move to an embedding in a semi-complex space described by the coordinates (r,φ,η) where

$$(r,\varphi)\in\mathbb{R}^2 \qquad \eta\in\mathbb{C} \tag{33}$$

We would have an apparent contour corresponding to the equation:

$$r^2+\eta^2=R^2 \tag{34}$$

The curves we have obtained thus lie within a space formed by joining, along the sphere S^2, which serves as a common boundary—the portion of the space $(r,\varphi,z)\in\mathbb{R}^3$ contained inside ($r < R$) that sphere and the portion of a semi-complex space (r,φ,η) corresponding to ($r > R$). The embeddings are merely systems of representation. Let us set:

$$\eta = i\zeta \qquad \zeta \in \mathbb{R} \tag{35}$$

The semi-complex geometric object upon which the curves corresponding to $r > R$ are traced then becomes a hyperboloid of one sheet with the equation:

$$r^2 - \zeta^2 = R^2 \tag{36}$$

For $r < R$, the metric of the sphere $\mathcal{S}^2$ is Riemannian, with signature (+, +). For $r > R$, in this representation (r, φ, ζ), the (induced) metric becomes Lorentzian, with signature (+, −).

$$\mathrm{d}s^2 = r^2 \mathrm{d}\varphi^2 - \frac{R^2}{r^2 - R^2}\mathrm{d}r^2 \tag{37}$$

In the figure below, we have the extension of the meridian constructed in this way. In **Figure 3**, this extension corresponds to the blue curve:

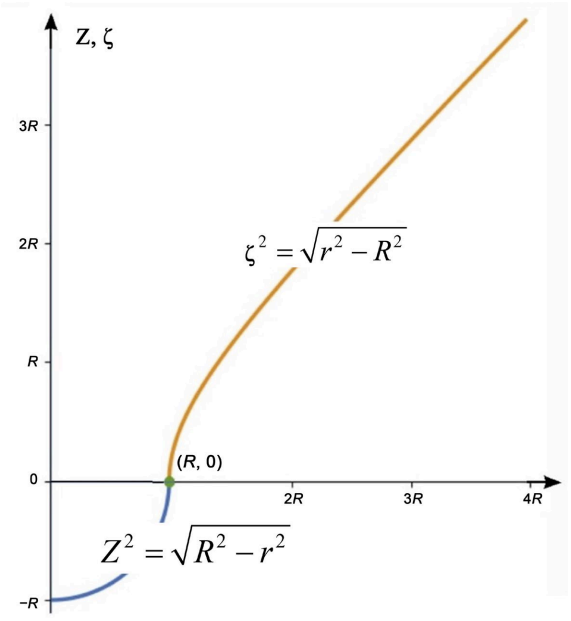


Figure 3. Extended apparent outline.

And its representation as a double 3D embedding. In **Figure 4** below, we have only the half part of the global semi-complex extension.

Of course, this needs to be completed by the reflection of this figure, including the symmetrical part:

$$z \to -z \qquad \zeta \to -\zeta \tag{38}$$

Note the analogy with the same operation applied later to Flamm's surface. These figures serve merely to illustrate the extension of a real geodesic arc on the sphere $\mathcal{S}^2$ by a branch lying within the semi-complex spatial region. We shall not pursue this concept of extending real geodesics from the sphere $\mathcal{S}^2$ into a semi-complex extension of space any further. Nevertheless, this sheds light on the nature of the curves shown in the figure, which arise from Lagrange's equations. These curves fall into two families.

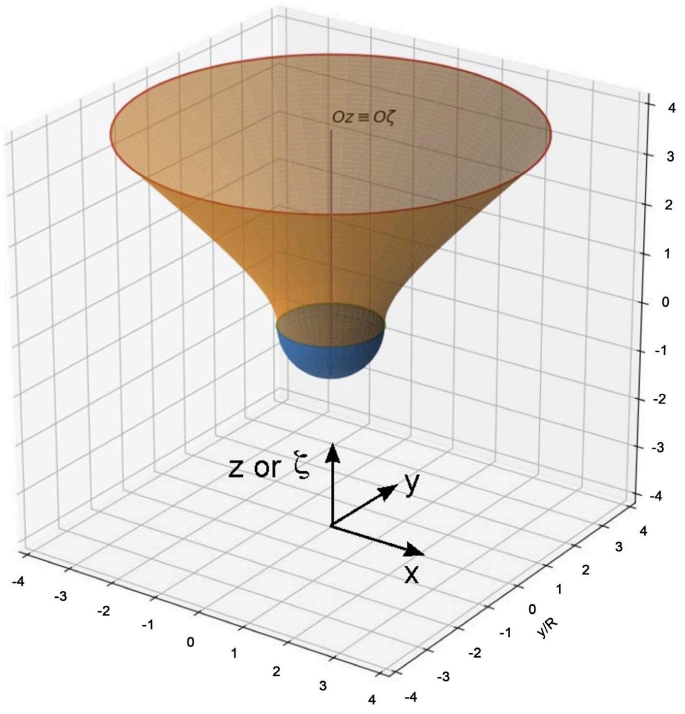


Figure 4. Part of the semi-complex extension of the S^2 sphere.

- Real curves
- Curves embedded in a semi-complex space

We have immediate access to the projection of these paths onto the real component of these possible projections. For $r < R$, these curves are geodesics of the sphere. But what name should be given to the semi-complex extensions corresponding to $r > R$? We might consider classical geodesics as belonging to the "here and now," whereas those lying within the semi-complex extension inhabit a sort of "beyond." Yet, we have access to their projection via the real coordinates of the semi-complex extension; they thus behave like ghosts. Our existence in the "here and now" unfolds within a real four-dimensional space (t, x, y, z). Let us suppose that death transports us to a semi-complex space (τ, x, y, z), where the "beyond" is simply a space in which the time coordinate becomes purely imaginary (which would, of course, make communication problematic).

2. The Ghost Symmetry

However, if an observer in the "here-and-now" were to glimpse a ghost, they would gain access to the projection—onto the real (x, y, z) component of the semi-complex space—of the realm where that ghost exists. By analogy, we propose terming these curves (where $r > R$) "ghost images."

Conversely, if the observer were situated within the semi-complex world, the great circles of the sphere would appear to them as ghosts. Under this hypothesis, death would correspond to a shift from the real world to a semi-complex world—a phenomenon comparable to a 90˚ rotation of the time vector. This brings to

mind individuals in extreme situations who report seeing their entire life's events flash before them in rapid succession; this could be likened to the onset of a time-vector rotation that was subsequently aborted.

3. Autoparallel Curves of the Torus

Let us leave aside these digressions. What do Lagrange's equations represent? They yield autoparallel trajectories, the existence of which in no way requires length and coordinates to be real. We could repeat this exercise with the torus, defined by its metric:

$$\mathrm{d}s^2 = r_0^2 \mathrm{d}\theta^2 + \left(R + r_0 \cos\theta\right)^2 \mathrm{d}\varphi^2 \tag{39}$$

The topology $S^1 \oplus S^1$ then appears. The corresponding signature is (+ +).

By considering the paths at $\theta = \mathrm{Cst}$ and then at $\varphi = \mathrm{cst}$, we immediately highlight the non-contractility of this surface. Let's now introduce the change of variable:

$$\theta = \arccos\frac{r-R}{r_0} \tag{40}$$

The line element becomes:

$$\mathrm{d}s^2 = r^2 \mathrm{d}\varphi^2 + \frac{\mathrm{d}r^2}{-R^2 + r_0^2 - r^2 + 2rR} \tag{41}$$

It is immediately apparent that the signature (+ +) will only be maintained if:

$$R - r \le r \le R + r_0 \tag{42}$$

Otherwise, we are outside the torus. The length s is expressed as follows:

$$s = \int \sqrt{r^2\dot{\varphi}^2 + \frac{\dot{r}^2}{-R^2 + r_0^2 - r^2 + 2rR}}\,\mathrm{d}s \tag{43}$$

But we know that we will obtain the same system of Lagrange equations by doing:

$$\delta \int \left(r^2\dot{\varphi}^2 + \frac{\dot{r}^2}{-R^2 + r_0^2 - r^2 + 2rR} \right) \mathrm{d}s = 0 \tag{44}$$

Lagrange's equations give us the projection of the curves resulting from this variation calculation onto the plane (r, φ):

$$\mathrm{d}\varphi = \frac{\pm h \mathrm{d}r}{r^2 \sqrt{\left(-R^2 + r_0^2 - r^2 + 2rR\right)\left(1 - \frac{h}{r^2}\right)}} \tag{45}$$

This allows us to plot the projection of a geodesic of the torus. See Figure 5.

But this solution also generates real curves that are the projection of curves equipped with an imaginary length. Below are the planar projections of these virtual geodesics, "inside and outside the torus", as shown in Figure 6.

Things become clearer when we group these results into a perspective view. See Figure 7.

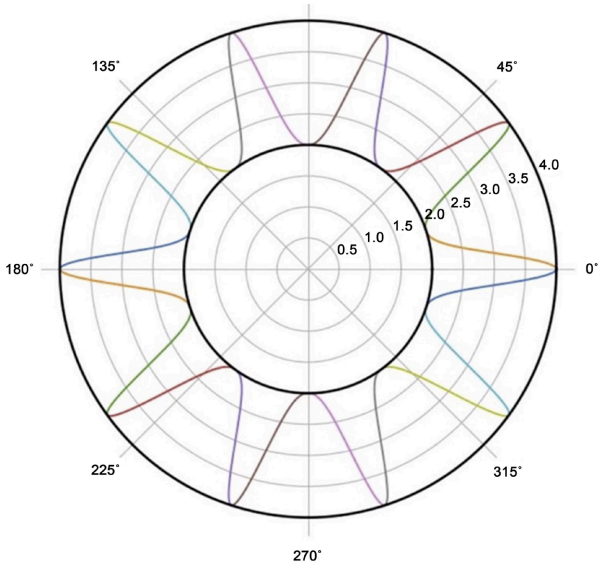


Figure 5. Plane projection of the real geodesics of the torus.

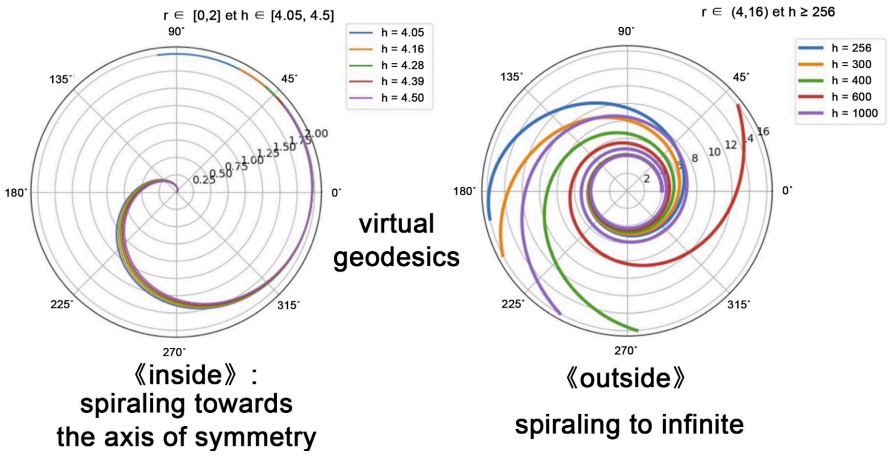


Figure 6. Projections of ghost geodesics of the torus.

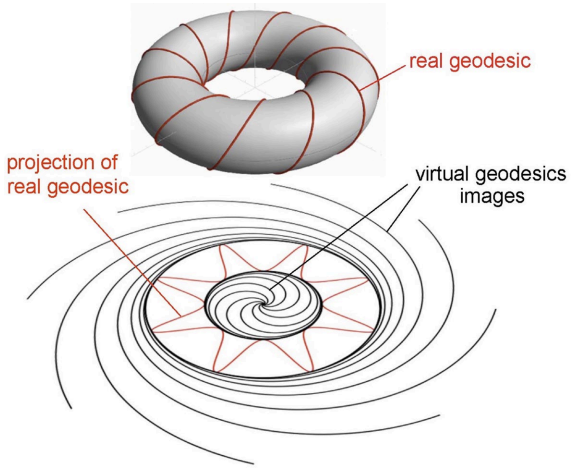


Figure 7. The torus. Projections of real and ghost images of autoparallel curves inscribed on the semi-complex extension of the torus.

By extending the meridian curves, one could visualize the double semi-complex extension of the torus—"inside its throat circle" and "outside its rim circle."

We shall not dwell on this, moving on instead to another, far more interesting 2D geometric object: the surface presented by L. Flamm as early as 1916 [1].

4. Semi-Complex Extension of the Flamm Surface

Its line elements is:

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{1-\frac{\alpha}{r}} + r^2\mathrm{d}\varphi^2 \tag{46}$$

For $r > \alpha$ the two coefficients are positive. The metric is Riemannian and the signature is (++). Let us embed this geometric object in 3D Euclidean space. In the 3D Euclidean space (x, y, z):

$$x = r\cos\varphi \tag{47}$$

$$y = r\sin\varphi \tag{48}$$

The corresponding line element is:

$$\mathrm{d}l^2 = \mathrm{d}r^2 + r^2\mathrm{d}\varphi^2 + \mathrm{d}z^2 \tag{49}$$

We have:

$$1+\left(\frac{\mathrm{d}z}{\mathrm{d}r}\right)^2 = \frac{r}{r-\alpha} \tag{50}$$

$$\frac{\mathrm{d}z}{\mathrm{d}r} = \pm\sqrt{\frac{\alpha}{r-\alpha}} \tag{51}$$

$$z = \pm 2\sqrt{\alpha(r-\alpha)} \tag{52}$$

It is the horizontal parabola:

$$r = \alpha - \frac{z^2}{4\alpha} \tag{53}$$

We therefore have two layers that meet at the throat circle in

$$r = \alpha z = 0 \tag{54}$$

For $r < \alpha$, we can consider an embedding into the semi-complex space (r, φ, η) with:

$$(r, \varphi) \in \mathbb{R}^2 \qquad \eta \in \mathbb{C} \tag{55}$$

Posing:

$$\eta = i\zeta \tag{56}$$

$$\mathrm{d}s^2 = r^2\mathrm{d}\varphi^2 - \frac{r}{r-\alpha}\mathrm{d}r^2 \tag{57}$$

The induced metric becomes Lorentzian, with signature:

$$(+-) \tag{58}$$

Two embeddings can be considered. The autoparallel curves for $r > \alpha$, which

lie on Flamm's surface—and thus serve as its geodesics—can be embedded in a 3D Euclidean space (r,φ,z), and the meridian of this surface of revolution is a sideways-oriented parabola. The surface generated by rotating this sideways parabola consists of two sheets located on either side of the throat circle; we will designate these as regions I and III. The autoparallel curves corresponding to $r<\alpha$ lie on a 2D object that constitutes the semi-complex extension of Flamm's surface. This object can be embedded in a 3D Euclidean space $(r,\varphi,\eta=-\zeta)$. A representation can be created by identifying ζ with *z*. This yields two surface elements—portions of sideways parabolas—that meet at the throat circle, each featuring a conical point on the Oz axis. The real meridian and the meridian of the semi-complex extension are shown in Figure 8 below.

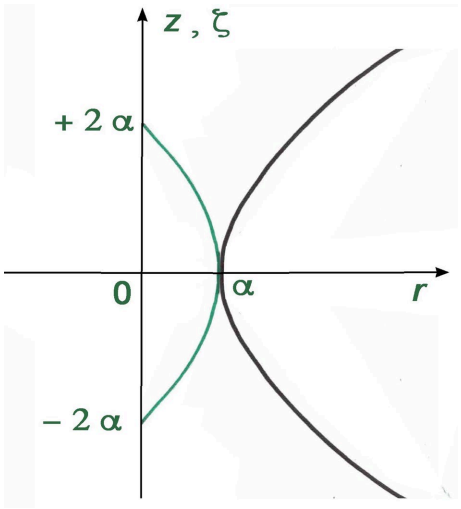


Figure 8. Meridians. In black, from the Flamm surface. In green, of its semi-complex extension.

In Figure 9, 3D-views.

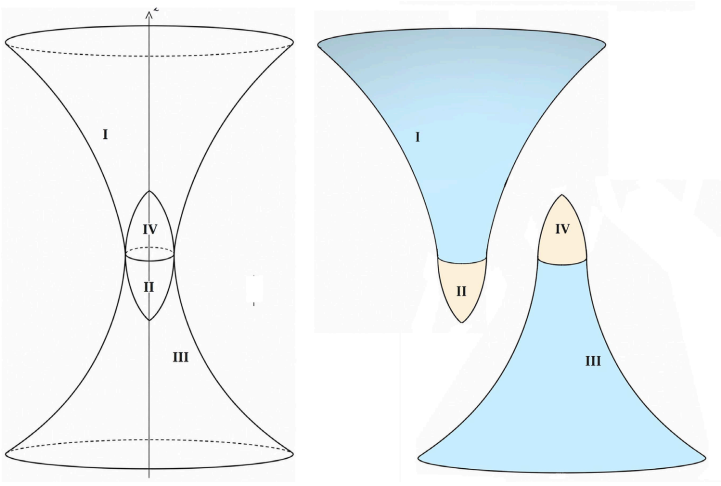


Figure 9. Semi complex extension of the Flamm's surface.

Let's deal with the calculation of the autoparallel curves.

$$L = \frac{1}{2}\left[\frac{r}{r-\alpha}\dot{r}^2 + r^2\dot{\varphi}^2\right] \tag{59}$$

$$\frac{\partial L}{\partial \dot{\varphi}} = r^2\dot{\varphi} = h \tag{60}$$

$$\dot{\varphi} = \frac{h}{r^2} \tag{61}$$

$$1 = \frac{r}{r-\alpha}\dot{r}^2 + r^2\dot{\varphi}^2 \tag{62}$$

$$r > \alpha \to r > |h| \tag{63}$$

$$r < \alpha \to r < |h| \tag{64}$$

In the semi-complex part:

$$|h| > \alpha \tag{65}$$

In the vicinity of the throat, when r tends to α :

$$\frac{\mathrm{d}\varphi}{\mathrm{d}r} \sim \frac{\mathrm{cst}}{\sqrt{\alpha - r}} \tag{66}$$

$$\frac{\mathrm{d}r}{\mathrm{d}\varphi} \to 0 \tag{67}$$

In its embedding, the parallel curve is tangent to the circle of striction, the junction between the two surfaces.

Behaviour in $r = 0$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}r} \sim \frac{1}{\sqrt{\alpha r}} \tag{68}$$

$$\Delta\varphi \cong 2\sqrt{\frac{r}{\alpha}} \tag{69}$$

The curve does not spiral. In its semi-complex part, the meridian is the section of a horizontal parabola:

$$\zeta^2 = 4\alpha(\alpha - r) \tag{70}$$

In $r = 0$, we have $\zeta = \pm 2\alpha$ and:

$$g_{\varphi\varphi} = r^2 \to 0 \tag{71}$$

$$g_{rr} = -\frac{r}{\alpha - r} \to 0 \tag{72}$$

The induced metric is degenerate. In **Figure 10**, below, a perspective view of the Flamm's surface, imbedded in à 3D Euclidean space. Joined, the plane projections of real and ghost geodesics.

5. Where Is the Physical World?

In 1909, H. Minkowski invented the concept of spacetime [2] [3] and proposed an initial set of rules of the game. Hereafter, in **Figure 11**, the English translation

of his axiom, presented [3] in 1909.

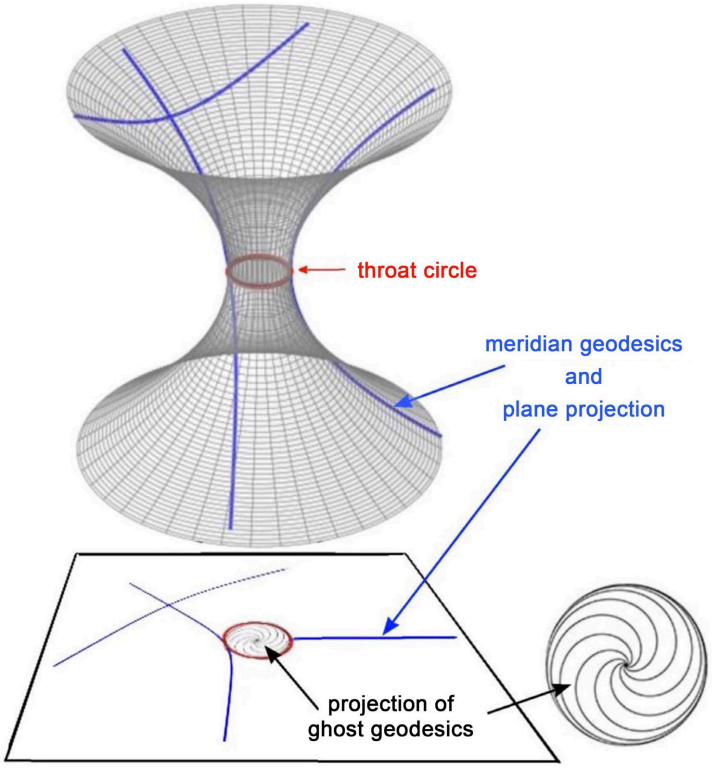


Figure 10. Flamm's surface and the ghost images of the autoparallels of its semi-complex extension.

The axiom means that at every point in the universe the expression

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

is always positive, or, equivalently, that the speed v is always less than c. Thus, c is the upper limit for the speed of any substantial point, and this is precisely where its true meaning lies.

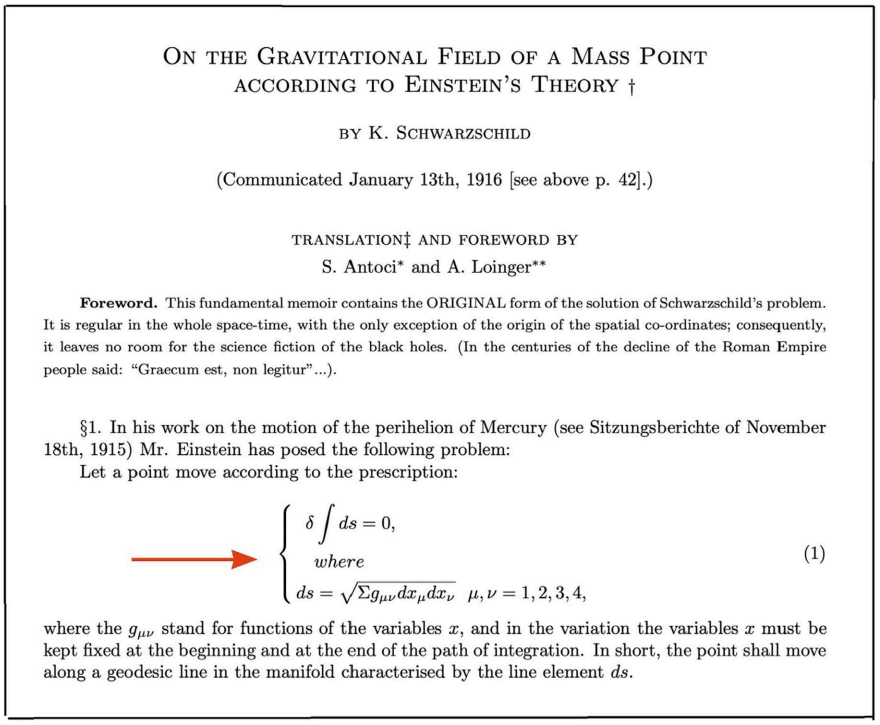
$$d\tau = \frac{1}{c}\sqrt{c^2 dt^2 - dx^2 - dy^2 - dz^2}$$

The integral $\int d\tau$ of this quantity taken on the world line will be called the proper time (Eingeinzeit) of the substantial point.

Figure 11. Minkowski's axiomatic definition [3] of spacetime (1909).

According to its prescriptions, material particles follow geodesics defined by a length equated to a real proper time. Photons follow geodesics of zero length within the manifold. This approach would subsequently be adopted—either explicitly or implicitly—by Einstein [4] and Schwarzschild [5] [6]. In **Figure 12**, the English translation of the very beginning of his paper [5].

Droste [7], Flamm [1], and Weyl [8] adopt the same view of physics. In **Figure 13**, the English translation of Weyl's article.



ON THE GRAVITATIONAL FIELD OF A MASS POINT
ACCORDING TO EINSTEIN'S THEORY †

BY K. SCHWARZSCHILD

(Communicated January 13th, 1916 [see above p. 42].)

TRANSLATION‡ AND FOREWORD BY
S. Antoci* and A. Loinger**

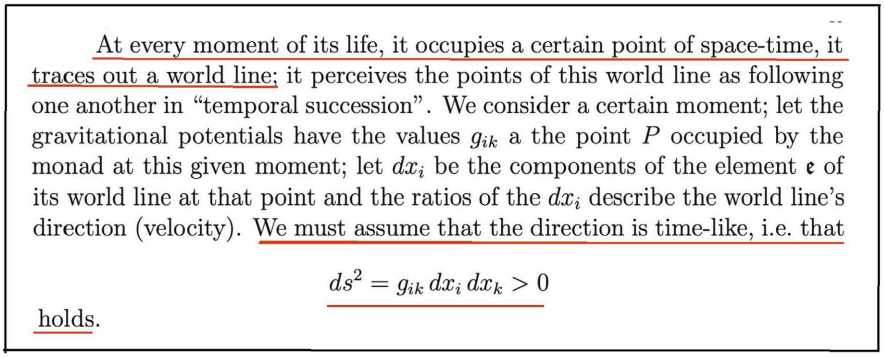
Foreword. This fundamental memoir contains the ORIGINAL form of the solution of Schwarzschild's problem. It is regular in the whole space-time, with the only exception of the origin of the spatial co-ordinates; consequently, it leaves no room for the science fiction of the black holes. (In the centuries of the decline of the Roman Empire people said: "Graecum est, non legitur"...).

§1. In his work on the motion of the perihelion of Mercury (see Sitzungsberichte of November 18th, 1915) Mr. Einstein has posed the following problem:
Let a point move according to the prescription:

$$\begin{cases} \delta \int ds = 0, \\ where \\ ds = \sqrt{\Sigma g_{\mu\nu} dx_\mu dx_\nu} \quad \mu,\nu = 1,2,3,4, \end{cases} \tag{1}$$

where the $g_{\mu\nu}$ stand for functions of the variables x, and in the variation the variables x must be kept fixed at the beginning and at the end of the path of integration. In short, the point shall move along a geodesic line in the manifold characterised by the line element ds.

Figure 12. Schwarzschild foreword [5].



At every moment of its life, it occupies a certain point of space-time, it traces out a world line; it perceives the points of this world line as following one another in "temporal succession". We consider a certain moment; let the gravitational potentials have the values g_{ik} a the point P occupied by the monad at this given moment; let dx_i be the components of the element $\mathfrak{e}$ of its world line at that point and the ratios of the dx_i describe the world line's direction (velocity). We must assume that the direction is time-like, i.e. that

$$ds^2 = g_{ik}\, dx_i\, dx_k > 0$$

holds.

Figure 13. Weyl restricts himself to real proper time [8].

For H. Weyl, the path of a material particle lies along a trajectory endowed with a necessarily real length [8]. In this context, any solution curve possessing an imaginary length element does not belong to the physical world and lies outside the real hypersurface. All this is consistent with a Lorentzian signature $(+---)$.

6. In Search of the Geometry of the Hypersurface

In 1916, L. Flamm provided a complete geometric interpretation of the two solutions presented by Schwarzschild. In the exterior region, where $r > R_a$, the cross-section of the solution hypersurface at constant t and θ is a portion of one of the two sheets of the surface generated by rotating a sideways-oriented parabola. In the interior region, where $0 < r < R_a$, the cross-section of the solution hyper-

surface at constant t and θ is a portion of a sphere of radius. In Figure 14, building the final cut of the Schwarzschild global surface at t and θ constants.

$$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}} \tag{73}$$

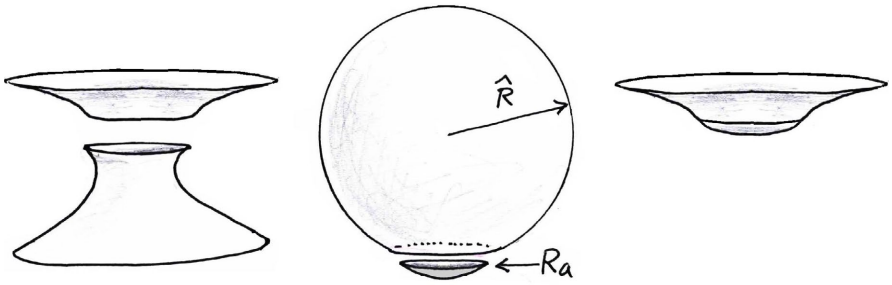


Figure 14. The Flamm's construction [9].

However, in January 1916, Karl Schwarzschild did not present the true expression of his exterior metric solution [5], expressed in terms of his coordinates (t,r,θ,φ) which would not be published in that form until 2026 [9].

$$\begin{aligned} \mathrm{d}s^2 = &\frac{\left(r^3+\alpha^3\right)^{1/3}-\alpha}{\left(r^3+\alpha^3\right)^{1/3}}c^2\mathrm{d}t^2 - \frac{r^4\mathrm{d}r^2}{\left(r^3+\alpha^3\right)\left[\left(r^3+\alpha^3\right)^{1/3}-\alpha\right]} \\ &-\left(r^3+\alpha^3\right)^{2/3}\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right) \end{aligned} \tag{74}$$

With:

$$r = \sqrt{x^2+y^2+z^2} \geq 0 \qquad x,y,z,r \in \mathbb{R} \tag{75}$$

In this context (with real variables x, y, z, and r), the topology of the solution hypersurface is clearly revealed. The variable r—a simple spatial marker—is not a "radius" in the Euclidean sense. The surface can be foliated by a family of S^2 spheres with area:

$$A = 4\pi\left(r^3+\alpha^3\right)^{2/3} \tag{76}$$

With a minimum value in $r = 0$

$$A_{\min} = 4\pi\alpha^2 \tag{77}$$

The hypersurface associated with the exterior metric solution is therefore non-contractible, a fact confirmed by its cross-sections at constant t and θ (Flamm's paraboloid). However, Schwarzschild published his result in this form:

$$\mathrm{d}s^2 = \left(1-\frac{\alpha}{R}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}R^2}{1-\frac{\alpha}{R}} - R^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right) \qquad R = \left(r^3+\alpha^3\right)^{1/3} \tag{78}$$

By explicitly stating, in a letter sent to Einstein [10], that this quantity R is merely an "intermediate quantity" (*Hilfsgrobe*).

David Hilbert modified his ambitious 1915 article, "The Foundations of Physics"

[11], and set out to incorporate Schwarzschild's exterior metric solution into a new paper published in December 1916 [12]—a paper in which he made no mention whatsoever of the existence of an interior metric solution. He defined geometry as deriving not from the expression of an elementary length (the term "ds" is absent from his article) but from a bilinear form that could take on either sign; depending on that sign, this approach led to the definition of two lengths associated with two sets of solution curves, described either as timelike trajectories or as "segments" (spacelike). He made no mention of the interior metric. As explained in detail in [9], in his reconstruction of the solution to the Einstein equation, he "de facto" conflated Schwarzschild's coordinates "r" and "R" while making what appeared to him to be a logical simplification to reach the solution more quickly—a confusion first pointed out in 1989, 73 years later, by the Canadian mathematician Abrams [13].

This confusion is implicitly reflected in the works published by A. Eddington in 1918 [14] and R. C. Tolman in 1934 [15]—extensively documented works that subsequently popularized general relativity worldwide.

In 1939, Albert Einstein published a paper [16] containing a few lines in which—without relying on substantive arguments—he expressed skepticism regarding the description of geometry within masses as presented by Schwarzschild in his second paper of 1916. This may well have deterred many scientists from focusing on that work, particularly given that Schwarzschild's paper was difficult to read, was written in German, and was not translated into English until 1999.

These two factors focus the scientific community's interest on the exterior metric. Many propose changes of variables aimed at eliminating the singularity at $r=\alpha$. The challenge is, in particular, to ensure that the metric coincides with the Lorentz metric at infinity. In 1935, A. Einstein and N. Rosen proposed [17] the change of variable:

$$u^2 = r - 2\alpha \tag{79}$$

And get:

$$\mathrm{d}s^2 = \frac{u^2}{u^2+\alpha}\mathrm{d}t^2 - 4\left(u^2+\alpha\right)\mathrm{d}u^2 - \left(u^2+\alpha\right)^2\left(\mathrm{d}\theta^2+\sin^2\theta\right) \tag{80}$$

Their goal was to propose a geometric description of masses. They introduce the concept of wormhole, writing:

We *call such a connection between two sheets a "bridge"*.

However, this metric is not Lorentzian at infinity. In 1924, however, A. Eddington [18] proposed a change of variable involving the time coordinate that resolves the issue. Denoting "Schwarzschild time" by t_S and Eddington time by t_E, we have:

$$t_s = t_E + \varepsilon\frac{\alpha}{c}\ln\left(\frac{r}{\alpha}-1\right) \qquad \varepsilon=\pm 1 \tag{81}$$

$$\mathrm{d}s^2 = \left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t_E^2 + \varepsilon c\alpha\frac{\mathrm{d}t_E\mathrm{d}r}{r} - \left(1+\frac{\alpha}{r}\right)\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right) \tag{82}$$

The coordinate singularity is eliminated, but a cross-term in $\mathrm{d}r\mathrm{d}t_E$ appears.

In 1923, G.D. Birkhoff [19] stated a theorem:

Any spherically symmetric solution to Einstein's vacuum field equations is locally isometric to the Schwarzschild metric.

This result does not rule out the presence of a cross-term in the metric, provided that it arises from a simple coordinate change and the new formulation remains isometric to the Schwarzschild solution. This is precisely the case for the Eddington-Finkelstein metric, which is derived from the Schwarzschild metric via a transformation of the time coordinate and introduces a *dtdr* cross-term without altering the underlying geometry. A metric lacking a cross-term and admitting a hypersurface-orthogonal timelike Killing field is termed static. The Eddington-Finkelstein solution, by contrast, is stationary but not static, while remaining isometric to Schwarzschild.

In 1939, Oppenheimer and Snyder [20]—drawing on the fact that, in the Schwarzschild solution to the vacuum Einstein field equations, an external observer assigns an infinite Schwarzschild time to the crossing of the surface $r=\alpha$ (an infinite value for the free-fall time of a test particle, corresponding to an infinite escape time)—proposed [19] that this solution describes the highly non-stationary phenomenon of the implosion of a massive star at the end of its life.

D. Finkelstein [21] in 1958 and H. A. Buchdahl [22] in 1959—recalling that non-static solutions containing a cross-term in *drdt* are solutions to the vacuum Einstein equations—begin by presenting the most general form of the exterior metric, as given by Tolman [15]:

$$\mathrm{d}s^2=\left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t^2+q\frac{\mathrm{d}t\mathrm{d}r}{r}-\frac{\mathrm{d}r^2}{1-\dfrac{\alpha}{Rr}}-r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right) \tag{83}$$

However, they do not associate this form with a physical solution and hasten to point out that the cross term can be eliminated by a change of variable involving the time coordinate (the inverse of the Eddington coordinate transformation [18]). Nevertheless, Finkelstein, focusing on the idea of a past-future asymmetry, writes:

The surface is thus a true unidirectional membrane: causal influences can pass through it only in one sens. This again demonstrates the asymmetry between past and future. Moreover it demonstrates the existence of two distinct completions of the Schwarzschild solution is asserted. One has the structure:

$$\mathrm{d}s^2=\left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t^2+\frac{2\alpha c\mathrm{d}t\mathrm{d}r}{r}-\frac{dr^2}{1-\dfrac{\alpha}{Rr}}-r^2\Omega^2 \tag{84}$$

and the other, obtained by time-reversal, has a negative coefficient for drdt. The completions must be distinct even if the resulting manifolds are isomorphic under t, because a particular geodesic segment reaches the center and in the other completion does not. A transformation of the variable t that cancels the cross term

shows that the "core" of the manifold is that of the interior solution[1]. *The only information, in this work, accordingly, is the connection between the interior and exterior solutions to form a single manifold.*

The community is thus still seeking a geometric interpretation of this Schwarzschild solution. It is worth noting that Finkelstein suggests a two-sheeted, T-symmetric extension.

Incidentally, in the post-war period—and without any supporting paper—the cosmology community gradually opted for a change of signature:

$$(+---)\rightarrow(-+++) \tag{85}$$

Thus, what is henceforth considered the standard form of the Schwarzschild metric becomes:

$$\mathrm{d}s^2=-\left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t^2+\frac{\mathrm{d}r^2}{1-\dfrac{\alpha}{Rr}}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2\right) \tag{86}$$

In 1959, during a private meeting, Martin Kruskal presented to John Archibald Wheeler a coordinate transformation he had devised to eliminate the coordinate singularity at $r=\alpha$. Wheeler immediately grasped its significance and presented the construction a few weeks later at a colloquium in Royaumont, France. He was so enthusiastic and eager to share this with the scientific community that he wrote the paper [23] himself and proposed that Kruskal put his name to it—a proposal Kruskal accepted[2]. It should be noted that the change of variable is presented without details regarding its construction.

We now arrive at the core of this article's subject. We must therefore present a version that can be considered standard, and we opt for the one found in Wald's work [24].

7. The Standard Construction of the Kruskal Representation [24]

Région I	**Région II**
$r>\alpha \qquad A(r)>0$ (87.I)	$r<\alpha \qquad A(r)<0$ (87.II)
	→ *We then decide to define the length, and the proper time, according to*:
$\mathrm{d}s^2=A(r)\mathrm{d}t^2-\dfrac{1}{A(r)}\mathrm{d}r^2$ (88.I)	$\mathrm{d}s^2=\dfrac{1}{\lvert A(r)\rvert}\mathrm{d}r^2-\lvert A(r)\rvert\mathrm{d}t^2$ (88.II)
t is timelike. r is spacelike.	→ r *becomes timelike and* t *is spacelike*, in other

[1] Finkelstein thus discusses Schwarzschild's second solution—describing the geometry inside the mass—which he implicitly cites by referring to Tolman's writings.

[2] It would be of considerable historical interest to determine to what extent the global four-region interpretation already existed in Kruskal's original work or whether it emerged through Wheeler's physical interpretation of the new coordinate system. The proceedings of the Royaumont conference (June 1959), where Wheeler first presented Kruskal's construction before its publication, could shed light on this question.

We perform the change of variable:

$$r^* = r + \alpha \ln\left(\frac{r}{\alpha} - 1\right) \quad (89.I)$$

$$\mathrm{d}r^* = \mathrm{d}r\left(1 + \frac{1}{\frac{r}{\alpha} - 1}\right) = \mathrm{d}r\left(\frac{1}{1 - \frac{r}{\alpha}}\right) \quad (90.I)$$

$$\mathrm{d}r^* = \frac{\mathrm{d}r}{A(r)} \quad (91.I)$$

The line element becomes:

$$\mathrm{d}s^2 = A(r)\left(\mathrm{d}t^2 - \mathrm{d}r^{*2}\right) \quad (92.I)$$

We introduce null coordinates:

$$x^- = t - r^* \quad (93a.I)$$

$$x^+ = t + r^* \quad (93b.I)$$

Whence:

$$\mathrm{d}t = \frac{1}{2}\left(\mathrm{d}x^+ + \mathrm{d}x^-\right) \quad (94.I)$$

$$\mathrm{d}r^* = \frac{1}{2}\left(\mathrm{d}x^+ - \mathrm{d}x^-\right) \quad (95.I)$$

$$x^- = t - r^* \quad (96.I)$$

$$x^+ = t + r^* \quad (97.I)$$

$$\mathrm{d}t = \frac{1}{2}\left(\mathrm{d}x^+ + \mathrm{d}x^-\right) \quad (98.I)$$

$$\mathrm{d}r^* = \frac{1}{2}\left(\mathrm{d}x^+ - \mathrm{d}x^-\right) \quad (99.I)$$

$$\mathrm{d}s^2 = A(r)\left(\mathrm{d}t^2 - \mathrm{d}r^{*2}\right) = A(r)\mathrm{d}x^+\mathrm{d}x^- \quad (100.I)$$

In the outer region $r > \alpha$ we set up:

$$U = -\exp\left(-\frac{x^-}{2\alpha}\right) \quad (101.I)$$

$$V = \exp\left(\frac{x^+}{2\alpha}\right) \quad (102.I)$$

$$\mathrm{d}U = \frac{1}{2\alpha}\exp\left(-\frac{x^-}{2\alpha}\right)\mathrm{d}x^- - \frac{U}{2\alpha}\mathrm{d}x^- \quad (103.I)$$

$$\mathrm{d}V = \frac{V}{2\alpha}\mathrm{d}x^+ \quad (104.I)$$

words, the causal and temporal nature of the directions $\frac{\partial}{\partial t}$ *et* $\frac{\partial}{\partial r}$ are swapped.

→ We use a *different* change of variable:

$$r^* = r + \alpha \ln\left(1 - \frac{r}{\alpha}\right) \quad (89.II)$$

$$\mathrm{d}r^* = \mathrm{d}r\left(1 - \frac{1}{1 - \frac{r}{\alpha}}\right) = \mathrm{d}r\left(-\frac{\alpha}{\alpha - r}\right) \quad (90.II)$$

$$\mathrm{d}r^* = \frac{\mathrm{d}r}{A(r)} \quad (91.II)$$

The line element becomes:

$$\mathrm{d}s^2 = A(r)\left(\mathrm{d}t^2 - \mathrm{d}r^{*2}\right) \quad (92.II)$$

We introduce null coordinates:

$$x^- = t - r^* \quad (93a.II)$$

$$x^+ = t + r^* \quad (93b.II)$$

Whence:

$$\mathrm{d}t = \frac{1}{2}\left(\mathrm{d}x^+ + \mathrm{d}x^-\right) \quad (94.II)$$

$$\mathrm{d}r^* = \frac{1}{2}\left(\mathrm{d}x^+ - \mathrm{d}x^-\right) \quad (95.II)$$

$$x^- = t - r^* \quad (96.II)$$

$$x^+ = t + r^* \quad (97.II)$$

$$\mathrm{d}t = \frac{1}{2}\left(\mathrm{d}x^+ + \mathrm{d}x^-\right) \quad (98.II)$$

$$\mathrm{d}r^* = \frac{1}{2}\left(\mathrm{d}x^+ - \mathrm{d}x^-\right) \quad (99.II)$$

$$\mathrm{d}s^2 = A(r)\left(\mathrm{d}t^2 - \mathrm{d}r^{*2}\right) = A(r)\mathrm{d}x^+\mathrm{d}x^- \quad (100.II)$$

→ In the inner region $r < \alpha$ we set up:
(This is the second act of this mathematical artifice).

$$U = +\exp\left(-\frac{x^-}{2\alpha}\right) \quad (101.II)$$

$$V = \exp\left(\frac{x^+}{2\alpha}\right) \quad (102.II)$$

$$\mathrm{d}U = \frac{1}{2\alpha}\exp\left(-\frac{x^-}{2\alpha}\right)\mathrm{d}x^- - \frac{U}{2\alpha}\mathrm{d}x^- \quad (103.II)$$

$$\mathrm{d}V = \frac{V}{2\alpha}\mathrm{d}x^+ \quad (104.II)$$

$$\mathrm{d}U\mathrm{d}V = -\frac{UV}{4\alpha^2}\mathrm{d}x^+\mathrm{d}x^- \quad (105.\mathrm{I})$$

$$\mathrm{d}x^+\mathrm{d}x^- = -\frac{4\alpha^2}{UV}\mathrm{d}U\mathrm{d}V \quad (106.\mathrm{I})$$

Moreover:

$$UV = -\exp\left(\frac{x^+ - x^-}{2\alpha}\right) \quad (107.\mathrm{I})$$

$$x^+ - x^- = 2r^* \quad (108.\mathrm{I})$$

$$\frac{r^*}{\alpha} = \frac{r}{\alpha} + \ln\left(\frac{r}{\alpha} - 1\right) \quad (109.\mathrm{I})$$

$$\mathrm{e}^{r^*/\alpha} = \left(\frac{r}{\alpha} - 1\right)\mathrm{e}^{r/\alpha} \quad (110.\mathrm{I})$$

$$UV = -\left(\frac{r}{\alpha} - 1\right)\mathrm{e}^{r/\alpha} \quad (111.\mathrm{I})$$

The line element becomes:

$$ds^2 = -4\alpha^2\frac{A(r)}{UV}dUdV \quad (112.\mathrm{I})$$

That's to say:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\mathrm{d}U\mathrm{d}V \quad (113.\mathrm{I})$$

In four dimensions:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\mathrm{d}U\mathrm{d}V - r^2\mathrm{d}\Omega^2 \quad (114.\mathrm{I})$$

$$U = v - u \qquad V = v + u \quad (115.\mathrm{I})$$

$$\mathrm{d}U\mathrm{d}V = \mathrm{d}v^2 - \mathrm{d}u^2 \quad (116.\mathrm{I})$$

Whence:

$$\boxed{\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-2\alpha}\left(\mathrm{d}v^2 - \mathrm{d}u^2\right) - r^2\mathrm{d}\Omega^2} \quad (117.\mathrm{I})$$

$$UV = (v-u)(v+u) = v^2 - u^2 \quad (118.\mathrm{I})$$

In this region I: $U < 0$ $V > 0$ we set up:

$$q_{ext} = \mathrm{e}^{r/2\alpha}\sqrt{\frac{r}{\alpha} - 1} \quad (119.\mathrm{I})$$

$$U = -q_{ext}\mathrm{e}^{-t/2\alpha} \quad (120.\mathrm{I})$$

$$V = q_{ext}\mathrm{e}^{t/2\alpha} \quad (121.\mathrm{I})$$

$$u = \frac{V-U}{2}q_{ext}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right) \quad (122.\mathrm{I})$$

$$\mathrm{d}U\mathrm{d}V = -\frac{UV}{4\alpha^2}\mathrm{d}x^+\mathrm{d}x^- \quad (105.\mathrm{II})$$

$$\mathrm{d}x^+\mathrm{d}x^- = -\frac{4\alpha^2}{UV}\mathrm{d}U\mathrm{d}V \quad (106.\mathrm{II})$$

Moreover:

$$UV = -\exp\left(\frac{x^+ - x^-}{2\alpha}\right) \quad (107.\mathrm{II})$$

$$x^+ - x^- = 2r^* \quad (108.\mathrm{II})$$

$$\frac{r^*}{\alpha} = \frac{r}{\alpha} + \ln\left(1 - \frac{r}{\alpha}\right) \quad (109.\mathrm{II})$$

$$\mathrm{e}^{r^*/\alpha} = \left(1 - \frac{r}{\alpha}\right)\mathrm{e}^{r/\alpha} \quad (110.\mathrm{II})$$

$$UV = +\left(1 - \frac{r}{\alpha}\right)\mathrm{e}^{r/\alpha} \quad (111.\mathrm{II})$$

The line element becomes:

$$\mathrm{d}s^2 = -4\alpha^2\frac{A(r)}{UV}\mathrm{d}U\mathrm{d}V \quad (112.\mathrm{II})$$

That's to say:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\mathrm{d}U\mathrm{d}V \quad (113.\mathrm{II})$$

In four dimensions:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\mathrm{d}U\mathrm{d}V - r^2\mathrm{d}\Omega^2 \quad (114.\mathrm{II})$$

$$U = v - u \qquad V = v + u \quad (115.\mathrm{II})$$

$$\mathrm{d}U\mathrm{d}V = \mathrm{d}v^2 - \mathrm{d}u^2 \quad (116.\mathrm{II})$$

→ We get the *same* line element:

$$\boxed{\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-2\alpha}\left(\mathrm{d}v^2 - \mathrm{d}u^2\right) - r^2\mathrm{d}\Omega^2} \quad (117.\mathrm{II})$$

$$UV = (v-u)(v+u) = v^2 - u^2 \quad (118.\mathrm{II})$$

In this region II: $U > 0$ $V > 0$ we set up:

$$q_{int} = \mathrm{e}^{r/2\alpha}\sqrt{1 - \frac{r}{\alpha}} \quad (119.\mathrm{II})$$

$$U = +q_{int}\mathrm{e}^{-t/2\alpha} \quad (120.\mathrm{II})$$

$$V = q_{int}\mathrm{e}^{t/2\alpha} \quad (121.\mathrm{II})$$

→ And that is where the *permutation* $\mathrm{sh} \leftrightarrows \mathrm{ch}$ appears.

$$u = \frac{V-U}{2}q_{int}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right) \quad (122.\mathrm{II})$$

$$v=\frac{V+U}{2}q_{ext}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right) \quad (123.I)$$

Hence the change of variables:

$$\boxed{u=\mathrm{e}^{r/2\alpha}\sqrt{\frac{r}{\alpha}-1}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right)} \quad (124.I)$$

$$\boxed{v=\mathrm{e}^{r/2\alpha}\sqrt{\frac{r}{\alpha}-1}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right)} \quad (125.I)$$

$$\boxed{u^2-v^2=\left(\frac{r}{\alpha}-1\right)>0}. \quad (126.I)$$

$$v=\frac{V+U}{2}q_{int}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right) \quad (123.II)$$

Hence the change of variables:

$$\boxed{u=\mathrm{e}^{r/2\alpha}\sqrt{1-\frac{r}{\alpha}}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right)} \quad (124.II)$$

$$\boxed{v=\mathrm{e}^{r/2\alpha}\sqrt{1-\frac{r}{\alpha}}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right)} \quad (125.II)$$

$$\boxed{u^2-v^2=-\left(1-\frac{r}{\alpha}\right)<0} \quad (126.II)$$

We thus see that this result is obtained through the use of two computational devices:

A—We have two changes of variables that differ for $r>\alpha$ and for $r<\alpha$.

$r>\alpha$:

$$r^*=r+\alpha\ln\left(\frac{r}{\alpha}-1\right) \quad (127.I)$$

$r<\alpha$

$$r^*=r+\alpha\ln\left(1-\frac{r}{\alpha}\right) \quad (127.II)$$

B—Different sectors are created through different choices of signs in the definition of the exponential variables U and V.

Sector I

$$U=-\exp\left(-\frac{x^-}{2\alpha}\right) \quad (128.I)$$

$$V=+\exp\left(-\frac{x^+}{2\alpha}\right) \quad (129.I)$$

Sector II

$$U=+\exp\left(-\frac{x^-}{2\alpha}\right) \quad (128.II)$$

$$V=+\exp\left(-\frac{x^+}{2\alpha}\right) \quad (129.II)$$

Additional calculations could be performed based on the choices:

Sector III

$$U=+\exp\left(-\frac{x^-}{2\alpha}\right) \quad (130.I)$$

$$V=-\exp\left(-\frac{x^+}{2\alpha}\right) \quad (131.I)$$

Sector IV

$$U=-\exp\left(-\frac{x^-}{2\alpha}\right) \quad (130.II)$$

$$V=-\exp\left(-\frac{x^+}{2\alpha}\right) \quad (131.II)$$

It is thanks to this that the permutation of hyperbolic lines appears in the various changes of variable. $\boxed{\mathrm{sh}\rightleftarrows\mathrm{ch}}$

These calculations gave rise to the Kruskal diagram. See **Figure 15** below.

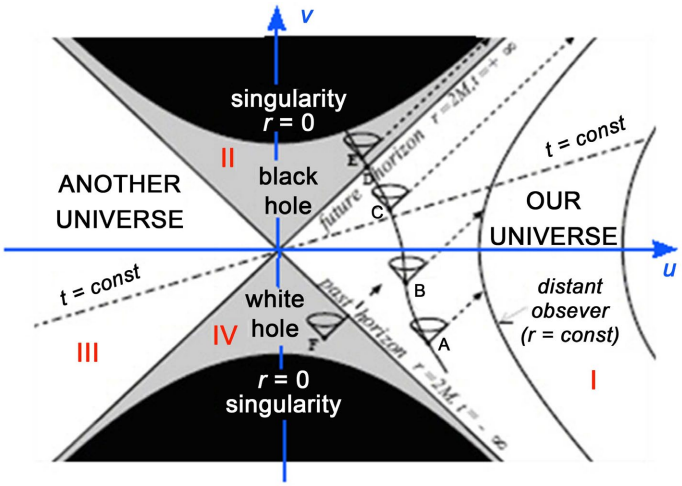


Figure 15. The Kruskal diagram.

8. Interpretation of the Region $r < \alpha$ as a Semi-Complex Extension of the Manifold

We start again from the form of the metric with constant θ and φ:

$$ds^2 = A(r)dt^2 - \frac{dr^2}{A(r)} \tag{132}$$

The 1960 article specifies that the aim is to express it in the form:

$$ds^2 = F^2(r)\left(dv^2 - du^2\right) \tag{133}$$

The curves with $ds = 0$ correspond to:

$$\frac{du}{dv} = \pm 1 \tag{134}$$

The coordinates u and v form a conformally Minkowskian pair; their combinations $v-u$ and $v+u$ are null coordinates in the strict sense.

Let's introduce a change of variable of the form:

$$u = f(r)\,\mathrm{ch}(\eta t) \tag{135}$$

$$v = f(r)\,\mathrm{sh}(\eta t) \tag{136}$$

η being a constant to be determined.

$$du = f'(r)\,\mathrm{ch}(\eta t)dr + \eta f(r)\,\mathrm{sh}(\eta t)dt \tag{137}$$

$$dv = f'(r)\,\mathrm{sh}(\eta t)dr + \eta f(r)\,\mathrm{ch}(\eta t)dt \tag{138}$$

This allows the cross-terms to be eliminated by forming:

$$dv^2 - du^2 = \eta^2 f^2 dt^2 - f'^2 dr^2 \tag{139}$$

By identifying:

$$F^2\eta^2 f^2 = A \tag{140}$$

$$F^2 f'^2 = A^{-1} \tag{141}$$

Whence:

$$\frac{f'}{f} = \pm\frac{\eta}{A} \tag{142}$$

We choose the sign +

$$\frac{\mathrm{d}f}{f} = \eta\left(1+\frac{\alpha}{r-\alpha}\right)\mathrm{d}r \tag{143}$$

By integrating:

$$\ln f = \eta r + \alpha\ln\left(r-\alpha\right) + C \tag{144}$$

As long as $r > \alpha$, we have the logarithm of a positive quantity—that's fine, let's continue. So:

$$f\left(r\right) = C\mathrm{e}^{\eta r}\left(r-\alpha\right)^{\alpha\eta} \tag{145}$$

The preceding relations yield:

$$f^2 = C^2\mathrm{e}^{2\eta r}\left(r-\alpha\right)^{2\alpha\eta} \tag{146}$$

Whence:

$$F^2 = \frac{1}{\eta^2C^2r}\mathrm{e}^{-2\eta r}\left(\frac{r}{\alpha}-1\right)^{1-2\alpha\eta} \tag{147}$$

Which gives:

$$\mathrm{d}s^2 = \frac{1}{\eta^2C^2r}\mathrm{e}^{-2\eta r}\left(\frac{r}{\alpha}-1\right)^{1-2\alpha\eta}\left(\mathrm{d}v^2-\mathrm{d}u^2\right) \tag{148}$$

If $\eta < \dfrac{1}{2\alpha}$ then, when:

$$r\to\alpha \Rightarrow F^2\to 0 \tag{149}$$

If $\eta > \dfrac{1}{2\alpha}$ then, when:

$$r\to\alpha \Rightarrow F^2\to +\infty \tag{150}$$

To avoid this, we choose:

$$\eta = \frac{1}{2\alpha} \quad \text{et} \quad C = 1 \tag{151}$$

Which gives:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\left(\frac{r}{\alpha}-1\right)^{0}\left(\mathrm{d}v^2-\mathrm{d}u^2\right) \tag{152}$$

For $r > \alpha$, this factor is identically equal to unity. Its value at $r = \alpha$ is defined by the continuous extension of the conformal factor, rather than by an independent evaluation of the formal expression 0^0. We thus obtain:

$$\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-r/\alpha}\left(\mathrm{d}v^2-\mathrm{d}u^2\right) \tag{153}$$

And, in four dimensions:

$$ds^2 = \frac{4\alpha^3}{r} e^{-r/\alpha}\left(dv^2 - du^2\right) - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \tag{154}$$

And:

$$u = e^{r/2\alpha}\sqrt{r-\alpha}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right) \tag{155}$$

$$v = e^{r/2\alpha}\sqrt{r-\alpha}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right) \tag{156}$$

Let us revisit this construction for $0 < r < \alpha$. We obtain:

$$u = ie^{r/2\alpha}\sqrt{\alpha-r}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right) \tag{157}$$

$$v = ie^{r/2\alpha}\sqrt{\alpha-r}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right) \tag{158}$$

If t is real, then the coordinates u and v become purely imaginary. Let us introduce a translation by a purely imaginary amount $-i\frac{\pi}{2}$ and make use of the relation:

$$\mathrm{ch}\left(\tau - i\frac{\pi}{2}\right) = -i\,\mathrm{sh}(\tau) \tag{159}$$

$$\mathrm{sh}\left(\tau - i\frac{\pi}{2}\right) = -i\,\mathrm{ch}(\tau) \tag{160}$$

When applied to the time variable t, a purely imaginary translation suffices:

$$t \to t - i\pi\alpha \tag{161}$$

Then, for $0 < r < \alpha$ we get:

$$u = e^{r/2\alpha}\sqrt{r-\alpha}\,\mathrm{sh}\left(\frac{t}{2\alpha}\right) \tag{162}$$

$$v = e^{r/2\alpha}\sqrt{r-\alpha}\,\mathrm{ch}\left(\frac{t}{2\alpha}\right) \tag{163}$$

This refers to Kruskal's change of variables. However, for a purely imaginary translation to be performed on a variable, that variable must be complex.

We are thus dealing with a genuine sleight of hand, enabling the description of a complex geometric extension using coordinates that can be termed pseudo-real.

An alternative construction thus emerges, leading to a semi-complex geometric object. Just as we did earlier with the 2D objects—the sphere and Flamm's surface—we obtain, in **Figure 16**, the drawing below.

The blue folds correspond to the real values of the coordinates u and v; the yellow surfaces, to imaginary folds.

Thus, the four-dimensional solution hypersurface in (t, r, θ, φ) lends itself to two representations based on the form of two intersections:

- At t and θ constants, this gives us a semi-complex representation in (r, φ)

- At θ and t φ constants this gives us a semi-complex representation in (t,r)

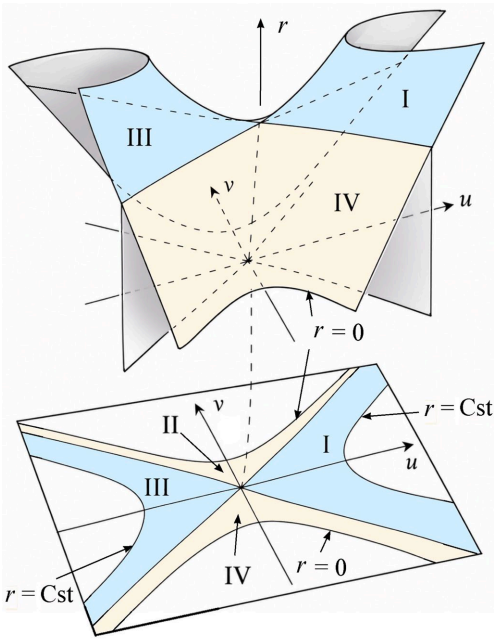


Figure 16. Kruskal complex extension (yellow).

In both cases, one obtains a complex hypersurface comprising four sectors that correspond to one another in pairs. This reflects a fundamental concept in differential geometry: that the nature of a geometric object cannot depend on the choice of coordinates used to describe it.

At the bottom, the Kruskal diagram. At the top, the surface $r(u,v)$. It indeed possesses four sheets. Sheets II and IV correspond to purely imaginary values of the coordinates u and v which a sleight-of-hand maneuver allows to be converted into real quantities. There is thus a one-to-one correspondence with the four sheets of the complex extension of the Flamm surface. This demonstrates that the geometric properties of this object—existing in a complex space—are intrinsic and independent of the chosen mode of representation, a fact that aligns with the principles of differential geometry.

This surface can be constructed using a foliation of hyperbolas defined by the equations:

$$u^2 - v^2 = \frac{e^{r/\alpha}}{r-\alpha} \tag{164}$$

Depending on the values of $r>0$, four branches are obtained. The real branches connect to the complex branches along lines $u=\pm v$ situated in the plane $r=\alpha$. The point $(0,0,\alpha)$ lies on a saddle point of the surface. The surface is bounded by hyperbolas located at $r=0$, which are the image of the central singularity. It has been arbitrarily bounded by the plane $r=2\alpha$, but these elements I and III extend to infinity.

To the best of our knowledge, the literature generally depicts the Kruskal diagram in the (u,v) plane. We have found no study interpreting the implicit relation $r = H(u,v)$ as a three-dimensional surface foliated by hyperbolas of constant r.

Here are some other images—this time, computer-generated. See Figure 17.

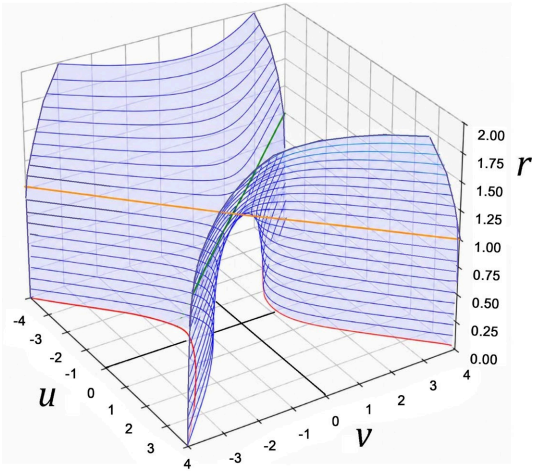


Figure 17. Kruskal complex extension. r > 1 corresponds to real world.

And in Figure 18, another view.

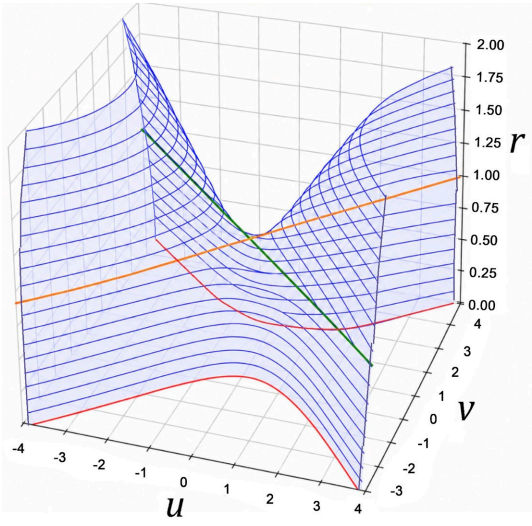


Figure 18. Kruskal complex extension. 0 < r < 1 is the semi-complex world.

We will now consider another way of proceeding.

9. Two Different Representations of the Schwarzschild Geometry

On the left, the Kruskal representation. On the right, construction using a single change of variables.

The Kruskal representation:

It is arbitrarily decided that the following expression will henceforth be called the standard form of the Schwarzschild solution to the source-free Einstein equation.

The line element becomes:

$$\mathrm{d}s^2 = \left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{\alpha}{r}} - r^2\mathrm{d}\Omega^2 \quad (169)$$

In the real field, coordinate singularity for g_{RR} in $r=\alpha$ and $r=0$.

At this stage, the signature is still $(+---)$

But one suddenly ceases to regard the reality of length as a criterion for belonging to physics.

So, arbitrarily and without any argument, they decide to change their signature.

$$(+---)\rightarrow(-+++) \quad (171)$$

It is decided that this spherically symmetric stationary solution describes a physical object and that the "standard Schwarzschild metric" is:

$$\mathrm{d}s^2 = -\left(1-\frac{\alpha}{r}\right)c^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{\alpha}{r}} + r^2\mathrm{d}\Omega^2 \quad (173)$$

It is decided that if we obtain a representation of the geometric object using real coordinates (u,v,θ,φ) this corresponds to a real physical object, known as a black hole. This representation is obtained from the coordinates.

(t,r,θ,φ). But they are the coordinates (t,R,θ,φ) where the intermediate Schwarzschild's quantity R is assimilated to a radial r coordinate.

It is set out to eliminate the singularity at $r=\alpha$ and to obtain a representation using two variables, u and u such that light rays correspond to:

$$\frac{\mathrm{d}u}{\mathrm{d}v} = \pm 1 \quad (177)$$

We consider a cross-section at constant θ and φ. Then, two different changes of variable

Construction using a single change of variables:

We stick to the coordinates chosen by Schwarzschild:

$$(t,x,y,z)\in\mathbb{R}^4 \quad (165)$$

With its polar variable:

$$r=\sqrt{x^2+y^2+z^2}\geq 0 \quad (166)$$

He gets:

$$\mathrm{d}s^2 = \frac{\left(r^3+\alpha^3\right)^{1/3}-\alpha}{\left(r^3+\alpha^3\right)^{1/3}}c^2\mathrm{d}t^2 - \frac{r^4\mathrm{d}r^2}{\left(r^3+\alpha^3\right)\left[\left(r^3+\alpha^3\right)^{1/3}-\alpha\right]} \quad (167)$$

It is noted that this representation is free of singularities and that this portion of spacetime is non-contractible: within a foliation at constant r, the area of this sphere has a minimum value of $4\pi\alpha^7$

Schwarzschild opts for a simpler presentation by introducing an intermediate variable R

$$R=\left(r^3+\alpha^3\right)^{1/3} \Rightarrow R\geq\alpha \quad (168)$$

The line element becomes:

$$\mathrm{d}s^2 = \left(1-\frac{\alpha}{R}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}R^2}{1-\frac{\alpha}{R}} - R^2\mathrm{d}\Omega^2 \quad (170)$$

In the real field, coordinate singularity for g_{RR} in $R=\alpha$

The signature is $(+---)$

Quel que soit l'usage qui soit fait de cette métrique on ne lui attribue un lien avec la physique que si l'élément ds (le temps propre) est réel.

Schwarzschild n'attribue de sens physique à cette métrique que si celle-ci se trouve raccordée à une métrique intérieure, de mêmes symétries.

We keep the signature:

$$(+---) \quad (172)$$

We're staying in shape:

$$\mathrm{d}s^2 = \left(1-\frac{\alpha}{R}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}R^2}{1-\frac{\alpha}{R}} - R^2\mathrm{d}\Omega^2 \quad (174)$$

We use Eddington's time change of variable [14] to eliminate the coordinate singularity in $r=\alpha$

$$t=t_E+\varepsilon\frac{\alpha}{c}\ln\left(\frac{R}{\alpha}-1\right) \quad \varepsilon=\pm 1 \quad (175)$$

are implemented:

For $r>\alpha$

$$r^* = r + \alpha \ln\left(\frac{r}{\alpha} - 1\right) \tag{178}$$

For $r<\alpha$

$$r^* = r + \alpha \ln\left(1 - \frac{r}{\alpha}\right) \tag{179}$$

Exponential coordinates U and V are then introduced:

$$U = \varepsilon_1 \exp\left(\frac{r^* - t}{2\alpha}\right) \tag{180}$$

$$V = \varepsilon_2 \exp\left(\frac{r^* + t}{2\alpha}\right) \tag{181}$$

A description is then obtained using a single metric:

$$\boxed{\mathrm{d}s^2 = \frac{4\alpha^3}{r}\mathrm{e}^{-2\alpha}\left(\mathrm{d}v^2 - \mathrm{d}u^2\right) - r^2\mathrm{d}\Omega^2} \tag{184}$$

We then introduce four possible choices regarding the pairs ($\varepsilon_1 = \pm 1$; $\varepsilon_2 = \pm 1$) that arbitrarily define four sectors.

	ε_1	ε_2
Sector I	−1	+1
Sector II	+1	+1
Sector III	+1	−1
Sector IV	−1	−1

Sectors I and III are then treated as two external regions, situated in two different spacetimes.

Sector II is likened to the interior of a black hole, organized around a central singularity.

Sector IV is identified with the interior of a white fountain, also organized around a central singularity.

From a physical standpoint, one is led to consider that, within these objects, the roles of the time and space coordinates are interchanged.

With $R>\alpha$ still holding in the real domain. The choice $\varepsilon = \pm 1$ introduces two possible solutions (two sheets).

$$\begin{aligned}\mathrm{d}s^2 = &\left(1 - \frac{\alpha}{R}\right)c^2\mathrm{d}t_E^2 + \varepsilon c\alpha\frac{\mathrm{d}t_E\mathrm{d}R}{R} \\ &-\left(1 + \frac{\alpha}{R}\right)\mathrm{d}R^2 - R^2\mathrm{d}\Omega^2\end{aligned} \tag{176}$$

We merge these two sheets into a single one by choosing $\varepsilon = -1$ and introducing the change of variable:

$$R = \alpha\left(1 + \ln \mathrm{ch}\,\rho\right) \tag{182}$$

We get:

$$\begin{aligned}\mathrm{d}s^2 = &\frac{\ln \mathrm{ch}\,\rho}{1 + \ln \mathrm{ch}\,\rho}c^2\mathrm{d}t_E^2 - \frac{c\,\mathrm{th}\,\rho\mathrm{d}t_E\mathrm{d}\rho}{1 + \ln \mathrm{ch}\,\rho} \\ &-\alpha^2\left[\frac{1 + \ln \mathrm{ch}\,\rho}{\ln \mathrm{ch}\,\rho}\mathrm{th}^2\rho\mathrm{d}\rho^2 + \left(1 + \ln \mathrm{ch}\,\rho\right)^2\mathrm{d}\Omega^2\right]\end{aligned} \tag{183}$$

This representation depicts a single *non-contractile* sheet with two leaves, $\rho>0$ and $\rho<0$, connected at a throat sphere $\rho = 0$ with an area of $4\pi\alpha^2$

This object is a bridge between two sheets of Lorentzian spacetime. This metric coincides with the Lorentz metric at infinity on both sheets.

$$\rho \simeq \frac{R}{\alpha} \tag{185}$$

$$g_{t_E t_E} \to c^2 \tag{186}$$

$$g_{\rho t_E} \to 0 \tag{187}$$

$$g_{\rho\rho} \to -\alpha^2 \left(g_{RR} \to -1\right) \tag{188}$$

$$g_{\theta\theta} \to -\alpha^2\rho^2 = -R^2 \tag{189}$$

$$g_{\varphi\varphi} \to -\alpha^2\rho^2\sin^2\theta = -R^2\sin^2\theta \tag{190}$$

All of this (upon grouping terms) is achieved using a single change of variables applied to the initial Schwarzschild variables:

$$r = \alpha\left[1 - \left(1 + \ln \mathrm{ch}\,\rho\right)^3\right]^{1/3} \tag{191}$$

$$t = t_E - \frac{\alpha}{c}\ln\left(\ln\rho\right) \tag{192}$$

This representation and its topology are consistent with the analysis of sections at constant t and θ (Flamm's paraboloid).

10. A Look Back at the Use of a Stationary Solution to Describe a Highly Unsteady Phenomenon

This question was raised by P. Koiran in 2021 [25]. The use of the Schwarzschild exterior metric solution—a specific form of the solution to Einstein's vacuum field equations—was proposed in 1939 by Oppenheimer and Snyder [20]. In this context, the variable t appearing in the line element is intended to correspond to the proper time of a distant observer. Consequently, for such an observer, the time taken for a test mass to fall freely or to escape appears infinite. Based on this, the authors suggest that the solution describes the implosion of a mass—such as a massive star—as a process akin to "freezing the film"; this extremely brief event (lasting a few days) would appear to a distant observer as a virtually static image. However, is this feature an intrinsic property of the solution? The answer is no. Introducing a cross-term involving $drdt$ alters the scenario, as demonstrated in [25].

- If the sign of $drdt$ term is negative, the free-fall time becomes finite and is measured in days, whereas the escape time remains infinite.
- If the sign of $drdt$ term is positive, opposite conclusion

As mentioned in the article, the presence of this term is by no means ruled out by Birkhoff's theorem. The theorem merely requires the form in question to be isometric to the diagonal form, which is indeed the case.

If the validity of this solution is called into question, what alternative scenario might describe the outcome of a massive star's implosion? One proposed model is that of "plugstars" [26]. It predicts that all future images of supermassive objects at galactic centers will exhibit the same 3:1 ratio between maximum and minimum dimensions—a prediction that already appears consistent with data from the first two such objects, M87* and SgrA*. This model relies on the assumption of constant, or quasi-constant, mass density. Regarding subcritical neutron stars—which could also be described by this model, with a mass limit of around 2.5 solar masses (compared to 3 for a black hole)—this description is not unreasonable; beyond a certain threshold of matter compression, treating the matter as a gas is no longer appropriate. Furthermore, nucleons become so closely packed that quantum effects—which are not accounted for in the gas model—must come into play.

In a future article, we will introduce a time-dependent perturbation term into both the interior and exterior metrics—including cross-terms in both—along with a topological discontinuity at the center. If it can then be shown that the core conversion of the excess positive mass into negative mass, and its subsequent expulsion from the celestial body, tend to cancel out the perturbation term, then self-stability will be demonstrated.

11. Conclusions

The aim of this article was to re-examine the geometric significance of the maximal extension of the Schwarzschild solution, starting from a fundamental require-

ment: the geometric nature of an object should not depend on the coordinate system chosen to represent it.

A preliminary study of simple two-dimensional examples has shown that the equations defining autoparallel curves can possess solutions extending beyond the domain where the corresponding metric admits a real Euclidean embedding. In the case of the sphere, the torus, and Flamm's surface, these additional branches can be described using semi-complex extensions of their embedding spaces. Although some coordinates become purely imaginary, their projections onto the real-coordinate subspaces remain real. We have termed these projections "ghost images."

Flamm's surface provides a particularly significant example in this regard. Its real domain $r > \alpha$ and its semi-complex extension ($0 < r < \alpha$) join continuously at the throat $r = \alpha$. These two domains thus appear as two parts of a single, extended geometric construction.

We then revisited the Kruskal construction, detailing separately—line by line—the standard calculation in regions I and II. In its usual physical formulation, the tortoise coordinate is written as follows:

$$r^* = r + \alpha \ln \left| \frac{r}{\alpha} - 1 \right|$$

This compact notation actually encompasses two distinct real expressions on either side of $r = \alpha$. Added to this are different sign choices in the definition of the exponential coordinates U and V, which allow for the construction of the four sectors of the Kruskal diagram and lead to the swapping of hyperbolic functions between the exterior and interior regions.

We therefore pursued a different approach. Instead of requiring the coordinates to remain real on both sides of $r = \alpha$, we retained a single coordinate transformation—derived in the exterior region—and analytically continued it into the domain $0 < r < \alpha$. In this continuation, the coordinates u and v naturally become purely imaginary. The standard real Kruskal representation can then be recovered via a complex translation of the time variable, an operation that results in the swapping of the hyperbolic sine and cosine functions.

From this perspective, regions II and IV can be interpreted as belonging to a semi-complex extension of spacetime geometry, represented in the standard Kruskal construction by coordinates endowed with a pseudo-real character. The three-dimensional representation:

$$r = H(u, v)$$

...foliated by the curves r = const., makes this structure geometrically visible and shows the continuous connection of the four sectors along $r = \alpha$.

Two descriptions of the continuation of the Schwarzschild geometry thus emerge. The conventional description maintains real coordinates throughout the Kruskal manifold and leads, in the domain $0 < r < \alpha$, to an inversion of the causal nature of the directions associated with the Schwarzschild coordinates: r

becomes timelike while t becomes spacelike. By contrast, the description proposed here employs a single analytic change of variables and leads to the interpretation of the interior sectors as belonging to a semi-complex extension.

The question therefore goes beyond the mere elimination of a coordinate singularity at $r = \alpha$. It concerns the geometric nature and, ultimately, the physical status attributed to the domain obtained by extending the solution.

A second difficulty then arises. The Schwarzschild solution is stationary, whereas the gravitational collapse supposed to lead to the formation of a black hole is, by nature, a time-dependent phenomenon. Moreover, Eddington-Finkelstein-type representations and the calculations of free-fall time discussed in this article demonstrate that the divergence of Schwarzschild time at the horizon is not an invariant property of the geometry. The interpretation of an essentially dynamic process based on a stationary solution therefore warrants re-examination.

This situation leaves open the possibility of other descriptions of the final state of gravitational collapse. The previously proposed "plugstar" model constitutes one such alternative. In particular, it leads to observational predictions that could, in principle, allow for a distinction between the two scenarios. A dynamic extension of this model, incorporating time-dependent perturbation terms as well as cross-terms in the interior and exterior metrics, will be the subject of future work.

When it comes to choosing between these two interpretations, the principle of Occam's razor would suggest opting for the simpler one. Furthermore, can Nature accept a physics in which space and time coordinates swap roles? If the black hole model were ever to be abandoned, could the "plugstar" model take its place? If the ratio between the maximum and minimum wavelengths turned out to be significantly close to 3, the question would have to be raised.

Only Nature holds the answer to these questions.

Acknowledgements

We thank J-C Pechinot for his assistance in producing the images.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Flamm, L. (1916) Beiträge zur Einsteinschen Gravitationstheorie. *Physikalische Zestschrift*, **XVII**, 448-454.

[2] Minkowski, H. (1909) Das Relativitätsprinzip. *Physikalische Zeitschrift*, **10**, 104-111.

[3] Minkowski, H. (1909) Raum und Zeit. *Physikalische Zeitschrift*, **10**, 75-88.

[4] Einstein, A. (1916) Die Grundlage der allgemeinen Relativitätstheorie. *Annalen der Physik*, **354**, 769-822. https://doi.org/10.1002/andp.19163540702

[5] Schwarzschild, K. (1916) Über das Gravitationsfeld Messenpunktes nach der Einsteinschen Theorie. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin*, 189-196.

[6] Schwarzschild, K. (1916) Über das Gravitionalsfeld einer Kugel Aus incompressibler

Flü-ssigkeit nach der Einsteinschen Theorie. Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin.

[7] Droste, J. (1917) The Field of a Single Centre in Einstein's Theory of Gravitation and the Motion of a Particle in That Field. *Proceedings of the Koninklijke Nederlandsche Akademie van Wetenschappen*, **19**, 197-215.

[8] Weyl, H. (2012) Republication of: 3. On the Theory of Gravitation. *General Relativity and Gravitation*, **44**, 779-810. https://doi.org/10.1007/s10714-011-1310-7

[9] Petit, J., d'Agostini, G. and Pechinot, J. (2026) The True Schwarzschild Solution and Its Subsequent Geometry. *Journal of Modern Physics*, **17**, 997-1027. https://doi.org/10.4236/jmp.2026.178045

[10] Schwarzschild, K. (1915) Letter to A. Einstein. Collected Papers of Albert Einstein, The Berlin Years, Correspondence 1914-1918, Document n°169.

[11] Hilbert, D. (1915) The Foundations of Physics (First Communication). In: Janssen, M., Norton, J.D., Renn, J., Sauer, T., Stachel, J., Eds., *The Genesis of General Relativity. Boston Studies in the Philosophy of Science*, Vol. 250. Springer, Dordrecht. https://link.springer.com/chapter/10.1007/978-1-4020-4000-9_44

[12] Hilbert, D. (1916) The Foundations of Physics (Second Communication). In: Janssen, M., Norton, J.D., Renn, J., Sauer, T. and Stachel, J., Eds., *The Genesis of General Relativity*, Springer, 1939-1961. https://link.springer.com/chapter/10.1007%2F978-1-4020-4000-9_45

[13] Abrams, L.S. (1989) Black Holes: The Legacy of Hilbert's Error. arXiv: gr-qc/0102055.

[14] Eddington, A. (1919) The mathematical Theory of Relativity. Cambridge University Press.

[15] Tolman, R.C. (1934) Relativity, Thermodynamics and Cosmology. Clarendon Press.

[16] Einstein, A. (1939) On a Stationary System with Spherical Symmetry Consisting of Many Gravitating Masses. *The Annals of Mathematics*, **40**, 922-936. https://doi.org/10.2307/1968902

[17] Einstein, A. and Rosen, N. (1935) The Particle Problem in the General Theory of Relativity. *Physical Review*, **48**, 73-77. https://doi.org/10.1103/physrev.48.73

[18] Eddington, A. (1960) The Mathematical Theory of Relativity. 2nd Edition, Cambridge University Press.

[19] Birkhoff, G.D. (1923) Relativity and Modern Physics. Harvard University Press.

[20] Oppenheimer, J.R. and Snyder, H. (1939) On Continued Gravitational Contraction. *Physical Review*, **56**, 455-459. https://doi.org/10.1103/physrev.56.455

[21] Finkelstein, D. (1958) Past-Future Asymmetry of the Gravitational Field of a Point Particle. *Physical Review*, **110**, 965-967. https://doi.org/10.1103/physrev.110.965

[22] Buchdahl, H.A. (1959) General Relativistic Fluid Spheres. *Physical Review*, **116**, 1027-1034. https://doi.org/10.1103/physrev.116.1027

[23] Kruskal, M.D. (1960) Maximal Extension of Schwarzschild Metric. *Physical Review*, **119**, 1743-1745. https://doi.org/10.1103/physrev.119.1743

[24] Wald, R.M. (1984) General Relativity. University of Chicago Press.

[25] Koiran, P. (2021) Infall Time in the Eddington-Finkelstein Metric, with Application to Einstein-Rosen Bridges. *International Journal of Modern Physics D*, **30**, Article ID: 2150106. https://doi.org/10.1142/s0218271821501066

[26] Petit, J. and D'Agostini, G. (2025) Alternatives to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*, **16**, 1479-1490. https://doi.org/10.4236/jmp.2025.1610072

Mathematical and geometrical Inconsistency of the black hole model. Part ll

Jean-Pierre Petit[1] Gilles d'Agostini

Keywords : virtual geodesics, Lagrangian, Flamm surface, Schwarzschild exterior solution, virtual geodesics, signature.

Abstract : Complementing a previous article, this second part shows that the spiral curves at the center of black holes are merely virtual geodesics, devoid of any physical reality. As an example, these virtual geodesics are constructed for the torus and the sphere. It is suggested that the generalization of the signature (- + + + +), exempt from any ontological justification in cosmology, stems from Hilbert's erroneous interpretation of Schwarzschild's solution in his 1916 article.

1 - **Introduction**.

The evolution of the geometric context of physics, astrophysics, and cosmology was driven by the changing understanding of the nature of light among scientists. In 1690, the Dutchman Christiaan Huygens was the first to attempt to identify light as a wave, without relying on experimental confirmation. He suggested that this wave propagated in a kind of universal, rigid medium: the ether, which would fill all of space and remain constant. In 1704, the Englishman Isaac Newton [2] introduced the idea of a corpuscular nature of light, which dominated for a century, until 1802, when his compatriot Thomas Young, with his slit experiment [3], demonstrated an interference phenomenon, which immediately revived a wave interpretation. In 1814, the German Joseph Fraunhofer demonstrated [4]. The fact that light emitted by heated bodies passing through a prism is distributed according to a spectrum made up of lines. In 1819, the Frenchman Augustin Fresnel published a complete theory of interference and diffraction [5]. In 1838, using the parallax method, the German Friedrich Bessel obtained the first estimate of the distance to a star [6]. The measurement of the distance to light was then added to that of light sources. In 1842, the Austrian Christian Doppler introduced [7] the idea that the color of light could depend on the radial component of the source's velocity relative to the observer. In 1845, the Dutchman Christophorus Buys Ballot set up an experiment to confirm Doppler's idea in air [8]. Musicians were placed aboard a train traveling at 40 km/h. The pitch of the sound from their instrument is compared to that of a tuning fork, manipulated by stationary observers. When the train moves away, the sound is lower. When it approaches, the sound is higher. In 1848, the Frenchman Hippolyte Fizeau gave substance to this idea in a theoretical form [9]. Subsequently, in 1865, the Scotsman James Clerk Maxwell identified light as an electromagnetic wave [8]. In 1868, the Englishman William Huggins demonstrated the relative motion of stars by comparing two spectra of the same type [10]. From this, he deduced the value of the difference in the relative velocities of these stars with respect to the Earth. Astronomers thus acquired, after photometric and distance measurements, a third element of measurement: that of the relative radial velocity.

[1] Jean-pierre.petit@manaty.net

At this stage, it doesn't occur to anyone that the universe could evolve. Light is therefore a wave that affects the ether, fundamentally underlying, invisible, assumed to be homogeneous, isotropic, but inaccessible to observation. Measurements are taken from the Earth, which moves around the Sun. As for the speed of light, it was first estimated in 1676 by the Dane Ole Rømer, then refined in 1728 by the Englishman James Bradley, in 1849 by the Frenchman Hippolyte Fizeau, and finally in 1862 by the Frenchman Léon Foucault. In 1881, the American Abraham Michelson conceived an experiment [12] intended to measure the speed of the Earth relative to the ether, a priori fundamentally immobile, within which all objects in the cosmos are supposed to move. But the sensitivity remained insufficient to draw a conclusion, and it was by repeating the experiment in 1887 with the American Edward Morley that the result was obtained [12]. And then, a dramatic turn of events: this speed maintained a constant value regardless of the direction considered.

In 1894, the Dutchman Hendrik Antoon Lorentz considered a solution to save both the ether and Maxwell's equations, which contain two parameters, ε_0 and μ_0, from which the speed of light, identified as an electromagnetic wave, is deduced at the speed $c = 1/\sqrt{\varepsilon_0\mu_0}$. He considered that motion alters the functioning of the measuring instrument. He began by considering a contraction of lengths [14]. Then, in 1895, an alteration of time [15]. These two elements were combined in 1904 in an article where Lorentz presented his famous transformations [16]. Poincaré immediately identified a group structure in these transformations, an idea he presented in 1905 in a lecture given in Palermo [17]. From the outset he associated spatio-temporal translations with this Lorentz group, which gave birth to the group that would bear his name.

In 1905, Einstein's interpretation was conceptually different [18]. He showed that recourse to the ether was no longer necessary. Instead of proposing transformations to acrobatically salvage both Maxwell's equations and the concept of the ether, which disappeared, he manipulated "inertial frames of reference" and presented a kinematic vision based on two principles: relativity and the invariance of the speed of light. But while Einstein's vision accounted for the measurements, he did not, at this stage, introduce a clear geometric vision.

At that time, mathematics and geometry were also present, but the link with physics was not yet established. With Bernhard Riemann [19], the concept of the manifold emerged. Only then did people begin to consider the existence of spaces that escape our mental imagery, spaces that deviate significantly from the Euclidean. These objects exist, for mathematicians, who know how to equip them with metrics, associate them with isometry groups, and link them to invariants. The vertiginous leap that Minkowski dared to make was to understand, initially somewhat confusedly, that this approach was not simply a game for mathematicians, but that it touched upon the geometry of the real world.

The Norwegian mathematician Sophus Lie gradually developed the group theory tools between 1870 and 1874. A summary of his monumental works was published with Friedrich Engel between 1888 and 1893 [20]. The pieces are there, scattered. All that remains is to assemble them, to understand that they constitute the building blocks of a new physics and represent the conceptual leap of refounding the geometric structure in which we operate. But the mental attachment to the Euclidean model remains very strong. In 1907, Hermann Minkowski published a first article [21] in which the idea of integrating the time dimension into a four-dimensional space (x_1 , x_2 , x_3 , x_4) is outlined. But the reference to the ether remains, and the Lorentzian character of space is managed by taking up the idea suggested by Poincaré in 1905

by introducing a purely imaginary time dimension: $x_4 = i\,t$. The following year [22] he declared in a conference:

> *Die Vorstellung, dass Raum und Zeit unabhängig voneinander sein können, ist nichts als eine eitle Illusion. Wir werden eine Vereinigung der beiden in Betracht ziehen müssen.*
>
> *The idea that space and time can be independent is nothing but a vain illusion. We will have to consider a union of the two.*

This is the birth of the entity that is space-time. Further on:

> *Das Weit wird die Menge aller denkbaren Werte von x, y, z, t sein.*
>
> *The universe will be the set of all imaginable values of x, y, z, t.*

And :

> *Das Zeitelement dt entspricht den Variationen dx, dy, dz dt der Koordinaten des Punktes im Raum.*
>
> *The time element dt will correspond to the variations dx, dy, dz, dt of the coordinates of the point in space.*

Further on, Minkowski explicitly gives the metric of his spacetime:

> *Wir betrachten eine Progression dx, dy, dz, dt entlang einer Weltgeraden und die Größe:*
>
> *We will consider a progression dx, dy, dz, dt along a world line and the quantity:*

$$d\tau = \frac{1}{c}\sqrt{c^2dt^2 - dx^2 - dy^2 - dz^2}$$

We couldn't put it more clearly, and this introduces a purely geometric definition.

> *Das Integral* $\tau = \int d\tau$ *dieser Größe entlang dieser Weltlinie, von einem beliebigen Ursprung Po bis zu einem variablen Endpunkt P, wird als Eigenzeit* τ *(Eingeizeit) des substanziellen Punktes bei P bezeichnet.*
>
> *The integral* $\tau = \int d\tau$ *of this quantity taken along this world line, from any origin* P_o*, to a variable extremity P, will be called the proper time* τ *(Eingeizeit) of the substantial point at P.*

This leads to a purely geometric definition of the photon. The fact that particles cannot travel along these world lines at speeds greater than c follows from the fact that s and τ are real.

The photon is therefore the particle that, in spacetime, follows a world line of zero length ($ds = d\tau = 0$). Incidentally:

$$(1)\quad c^2dt^2 - dx^2 - dy^2 - dz^2 \geq 0 \qquad or\ c \geq \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

In other words, dit $v \leq c$ and that says it all. Minkowski's universe is a space where coordinates are real, where time is measured in meters, where length, assimilated to proper time, is real.

If $ds^2 < 0$ *we are simply stepping outside the world of physics.*

.

Until now, physicists had never considered any space other than Euclidean space, with its metric:

$$(2)\qquad ds^2 = dx^2 + dy^2 + dz^2$$

It is understandable that this new geometry had difficulty taking root in the minds of men in general, and scientists in particular.

It was the Englishman Sylvester who introduced the concept of signature in 1852 [23], but by linking it to quadratic forms, and not systematically to a metric. In 1883, the Dane Jørge Gram [24] introduced a matrix presentation of bilinear forms. With X being a column vector (a column matrix) and ${}^tX$ a row vector (the transpose of this row matrix), a bilinear form can be written as:

$$(3)\qquad {}^tX\,G\,X$$

G being the Gram matrix:

$$(4)\qquad G = \begin{pmatrix} \pm 1 & 0 & \ldots & 0 \\ 0 & \pm 1 & \ldots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \ldots & \pm 1 \end{pmatrix}$$

In this context, the Lorentz metric is written:

$$(5)\qquad ds^2 = {}^tdX\,G\,dX$$

And the corresponding Gram matrix is then:

$$(6)\qquad G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

This matrix notation then allows us, in just a few lines of calculation, to obtain the elegant axiomatic definition of the element *L* of the Lorentz group:

$$(7)\qquad {}^tL\,G\,L = G$$

If we add the space-time translation column vector

$$(8)\qquad C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

We immediately obtain the representation, in the form of (5,5) matrix structures, of the Poincaré group:

(9) $$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}$$

If we switch to a Euclidean space, its metric is of the form (5) where its Gramma matrix becomes the identity matrix *I*. Its isometry subgroup is the group of matrices defined by:

(10) $$^{t}a\, I\, a = \; ^{t}a\, a = I$$

That is, the orthogonal matrices and the isometry group become the Euclid group :

(10) $$\begin{pmatrix} a & C \\ 0 & 1 \end{pmatrix}$$

Noether's theorem tells us that every subgroup is associated with the conservation of a quantity. For the Euclid group, we have the rotation subgroup SO(3), which conserves the radius. For the Lorentz group, a subgroup of the Poincaré group, we have conservation of the quantity:

(11) $$c^2t^2 - x^2 - y^2 - z^2$$

2 – **Regarding the phenomenon of metric signature inversion.**

Today, in all treatises on theoretical physics and cosmology, one reads:

(12) $$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

This "modern signature" $(-+++)$ gradually became widespread after the Second World War. Since the coordinates are real, defining world lines equipped with real proper time then requires a purely imaginary length element *ds*:

(13) $$d\tau = \sqrt{-ds^2}$$

Given that the solutions to Einstein's field equation are metrics, it would be logical to interpret these geometries as hypersurfaces whose geodesics of non-zero length are the paths followed by masses, with this very simple guiding principle:

- Where the length s is real: we are in the hypersurface, therefore in physical reality
- Where it is purely imaginary: we are outside the hypersurface, therefore we leave the world of physics.

By modifying the signature, we abandon this criterion and give rise to two families of trajectories:

- "Time-like" trajectories, with $ds^2 > 0$

- "Space-like" trajectories, with $ds^2 < 0$, which are actually located outside the hypersurface, are real, and represent an extension of spacetime into complex numbers.

3 – **The birth of the field equation.**

This Lorentz metric describes geometry in a vacuum. Ten years later, Einstein set out to describe the geometry created by the presence of masses, both outside and inside them. Initially, the great mathematician David Hilbert did not believe that this world of high mathematics in which he moved, and where he produced groundbreaking work, had any connection whatsoever with that of physics. During numerous meetings in 1915, Einstein managed to change his mind. It is important to remember that at that time, the dynamics of the universe were understood to be limited to the action of the electromagnetic force and the force of gravity. Hilbert then envisioned the possibility of carrying out an immensely ambitious project, constituting "the first theory of everything." When we read his 1915 memoir, entitled "The Foundations of Physics," we discover a space whose points are located by four coordinates $\{ w_1, w_2, w_3, w_4, \}$, time being the fourth. Let us quote him:

(14)

Es seien $w_s\ (s = 1, 2, 3, 4)$ *irgendwelch die Weltpunkte wesentlich eindentig benennende Koordinaten, die sogenannten Weltparameter (allgemeiste Rau-Zeit-Koordinaten)*

Traduction :

Let $w_s\ (s = 1, 2, 3, 4)$ *be any coordinates labeling the world's points essentially uniquely—the so-called world parameters (most general spacetime coordinates).*

Below is the complete action integral proposed by Hilbert:

(15)

und zwar mub die Variation des Integrals

$$\int H \sqrt{g}\ d\omega \quad \left(g = |g_{\mu\nu}|,\ d\omega = dw_1 dw_2 dw_3 dw_4\right)$$

für jedes der 14 Potentiale $g_{\mu\nu}$, q_s *verschwinden.*

Traduction :

where the variation of the integral

$$\int H \sqrt{g}\ d\omega \quad \left(g = |g_{\mu\nu}|,\ d\omega = dw_1 dw_2 dw_3 dw_4\right)$$

must vanish for each of the fourteen potentials $g_{\mu\nu}$, q_s .

Here we recognize its Lagrange function *H* and the four-dimensional hypervolume:

(16) $$\sqrt{g}\, d\omega = \sqrt{g}\ dw_1 dw_2 dw_3 dw_4$$

Hilbert represents by $|g_{\mu\nu}|$, the absolute value of the determinant constructed from the metric potentials $g_{\mu\nu}$. He will explicitly give the form of his Lagrange function:

(17) $$H = K + L$$

Where K is the Ricci scalar and L the Lagrangian of matter. What is crucial, when reading his first article from 1915, is to understand that Hilbert performs all his calculations with the coordinates w_s ($s = 1, 2, 3, 4$) and that at no point does either the signature of the metric or the metric itself appear in the form ds^2. For the mathematician David Hilbert, the solution to his field equation is not a metric, defining an element of length ds, but a bilinear differential form $g_{\mu\nu}\, dw_\mu dw_\mu$, whose sign, a priori, is of little importance to him. Given the historical significance of this event, here is Hilbert's field equation, as it appears in his 1915 article:

Unter Verwendung der vorhin eingeführten Bezeichnungsweise für die Variationsableitungen bezüglich der $g^{\mu\nu}$ erhalten die Gravitationsgleichungen wegen (20) die Gestalt

(21) $$[\sqrt{g}\,K]_{\mu\nu} + \frac{\partial \sqrt{g}\,L}{\partial g^{\mu\nu}} = 0.$$

Das erste Glied linker Hand wird

$$[\sqrt{g}\,K]_{\mu\nu} = \sqrt{g}\,(K_{\mu\nu} - \tfrac{1}{2} K g_{\mu\nu}),$$

Fig.1 : Hilbert's field equation (November 20, 1915) [26])

Traduction :

Using the notation introduced earlier for the variational derivatives with respect to the $g^{\mu\nu}$, the gravitational equations, because of (20), take the form

$$[\sqrt{g}K]_{\mu\nu} + \frac{\partial \sqrt{g}L}{\partial g^{\mu\nu}} = 0. \qquad (21)$$

The first term on the left hand side becomes

$$[\sqrt{g}K]_{\mu\nu} = \sqrt{g}\left(K_{\mu\nu} - \frac{1}{2} K g_{\mu\nu}\right),$$

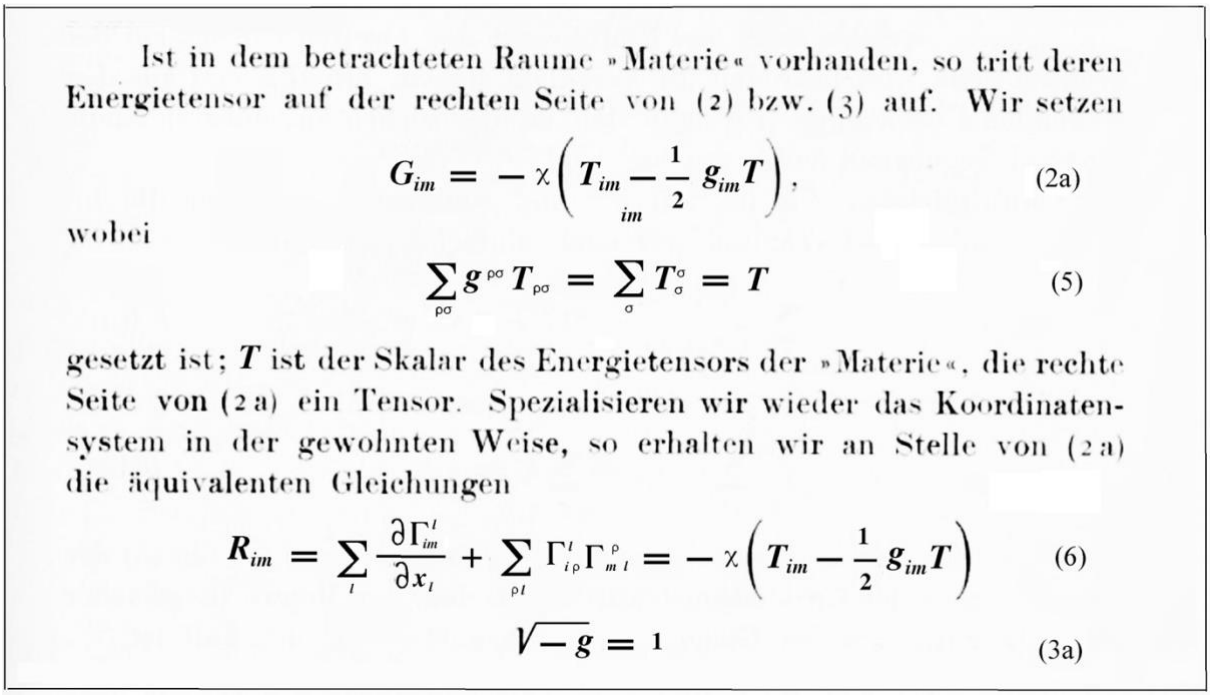
Fig.2 : English translation of the passage in Figure 1 [27].

What we can write:

(18) $$K_{\mu\nu} - \frac{1}{2} K\, g_{\mu\nu} = -\frac{1}{\sqrt{g}} \frac{\partial \sqrt{g}\, L}{\partial g_{\mu\nu}}$$

Simply replacing the letter K with the letter R (for Ricci) reveals that this equation is equivalent to the one published by Einstein five days later [28]! Hilbert's left-hand side results from his

choice to include the Ricci scalar, which he calls *K*, in the definition of its action. It's worth noting that Einstein published his equation in a form equivalent to the one that would later be considered standard, which is thus closer to the one published by Hilbert.



Ist in dem betrachteten Raume »Materie« vorhanden, so tritt deren Energietensor auf der rechten Seite von (2) bzw. (3) auf. Wir setzen

$$G_{im} = -\chi\left(T_{im} - \frac{1}{2} g_{im} T\right), \qquad (2a)$$

wobei

$$\sum_{\rho\sigma} g^{\rho\sigma} T_{\rho\sigma} = \sum_{\sigma} T^{\sigma}_{\sigma} = T \qquad (5)$$

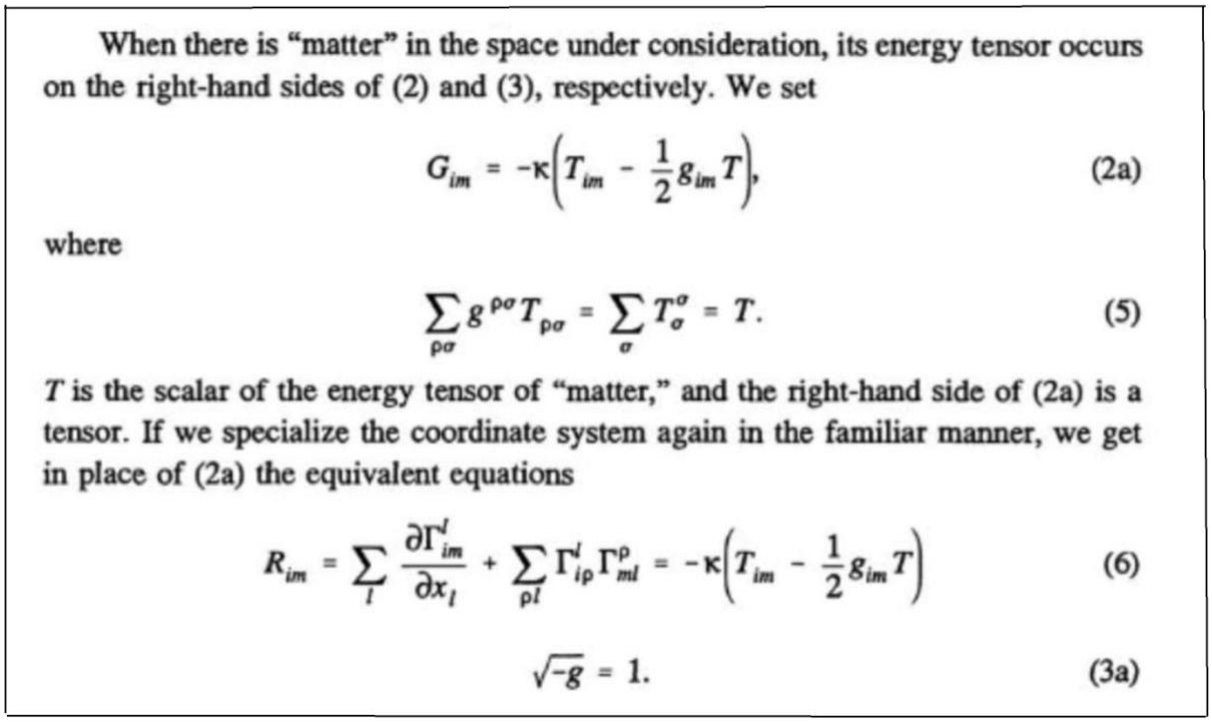
gesetzt ist; T ist der Skalar des Energietensors der »Materie«, die rechte Seite von (2a) ein Tensor. Spezialisieren wir wieder das Koordinatensystem in der gewohnten Weise, so erhalten wir an Stelle von (2a) die äquivalenten Gleichungen

$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^{l}_{i\rho} \Gamma^{\rho}_{ml} = -\chi\left(T_{im} - \frac{1}{2} g_{im} T\right) \qquad (6)$$

$$\sqrt{-g} = 1 \qquad (3a)$$

Fig.3 : Einstein's field equation [28]

Traduction :



When there is "matter" in the space under consideration, its energy tensor occurs on the right-hand sides of (2) and (3), respectively. We set

$$G_{im} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right), \qquad (2a)$$

where

$$\sum_{\rho\sigma} g^{\rho\sigma} T_{\rho\sigma} = \sum_{\sigma} T^{\sigma}_{\sigma} = T. \qquad (5)$$

T is the scalar of the energy tensor of "matter," and the right-hand side of (2a) is a tensor. If we specialize the coordinate system again in the familiar manner, we get in place of (2a) the equivalent equations

$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^{l}_{i\rho} \Gamma^{\rho}_{ml} = -\kappa\left(T_{im} - \frac{1}{2} g_{im} T\right) \qquad (6)$$

$$\sqrt{-g} = 1. \qquad (3a)$$

Fig4 : Translation of the previous figure.

In this construction of the field equation including gravitation, Hilbert preceded Einstein by five days. This fact placed Einstein in a delicate position. In June-July 1915, Einstein was invited by Hilbert, a request supported by Felix Klein, to give lectures at the University of Groningen where he endeavored to show that his physical ideas required profound mathematical tools, and that these were not arbitrary, but dictated by physics itself. During the summer and autumn of 1915, Einstein maintained a dense correspondence with Hilbert [29,30]. Einstein sensed that Hilbert was undertaking to tackle the problem from a variational and mathematical perspective. He insisted, sometimes nervously, that physics preceded form (what we would call modeling today), that equations were not an autonomous aesthetic construct. Their correspondence reached a peak in intensity in the last 6 to 8 weeks before their publication. Einstein sought not only to convince Hilbert that his mathematics was correct, but also that general covariance was not a formal luxury, that mathematical structure was imposed by the physical principle of equivalence. Hilbert, conversely, was fascinated by the possibility that the variational approach could encompass all of physics. Two converging approaches, two temperaments, and a few months during which theoretical physics was written at the pace of letters and lectures, with no finalized articles. Two approaches, pursued differently, at breakneck speed, but leading to the same result. On November 18, Einstein, without revealing his equation to Hilbert, informed him that his approach had succeeded and wrote:

> *"Ich bin sehr gespannt, ob Sie mit Ihrer Lösung ebenso zufrieden sein werden wie ich mit der meinen."*

Translation:

> *"I am very curious to see if you will be as satisfied with your solution as I am with mine."*

Their approaches were so similar, so parallel, that a joint publication could have been considered. But the stakes remained enormous, and they knew it. Especially since, for Hilbert, it was the equation of physics on which everything might depend: gravitation and electromagnetism. Nevertheless, Hilbert made the first move, sending his result for publication without informing Einstein, who immediately sent him a chilling letter on November 25, 2015 [29]. Einstein wrote, in particular:

> *« Die von Ihnen gegebene Lösung stimmt, soweit ich sehe, genau mit dem überein, was ich in meiner letzten Abhandlung gegeben habe."*

Translation:

> *"The solution you provided coincides, as far as I can tell, exactly with the one I presented in my last work."*

Their relationship nearly broke down. But the two managed to preserve their friendship and mutual respect. Hilbert made the first move on December 15, 1915, in a letter congratulating Einstein on what he considered his achievement. He noted the convergence of independent efforts but stated that he claimed no priority. It was a magnanimous gesture to which Einstein replied on December 21, 1915:

> *"Ich freue mich, dass zwischen uns keine Verstimmung entstanden ist, und hoffe, dass wir uns auch künftig als Freunde betrachten können."*

Translation:

> *“I am pleased that no awkwardness has arisen between us, and I hope that we can continue to consider each other as friends.”*

At the time, Einstein was already a member of the Royal Prussian Academy of Sciences, a professor at the University of Berlin (without a regular teaching obligation), and director of the Kaiser Wilhelm Institute for Physics (then being established). His academic position was therefore firmly established. He and Hilbert knew that this breakthrough was too important to be tainted by a question of paternity, and opted for an amicable solution.

4 – **La géométrisation de la physique.**

The French mathematician Jean-Marie Souriau, who is an exceptional example combining high mathematics [31] and a keen sense of physics, used to say:

> *"A little mathematics takes you away from reality, a lot brings you back to it."*

Cosmology is a perfect example, provided one knows how to use mathematical tools correctly. From what has been said above, we see that what guided Einstein was his extraordinary physical intuition. Didn't he say that in creating his special relativity, he strove to imagine how an observer "sitting on a photon" would perceive the universe? A few years later, Minkowski geometrized this vision, and everything became incredibly simple, expressed in just a few lines. Poincaré added to this the group tool and the group that bears his name (10), which became the isometry group of Minkowski space. The Lorentz metric, thanks to the use of the matrix tool, is reduced to a given Gramm matrix (6) and emerges from an extremely elegant axiomatic system (7), which evokes the extension of rotations to this 4D space. This is where we realize that this geometrization of reality, while impeccably accounting for the physical phenomena of the moment, provides a context that suggests looking beyond the world that gave birth to it, through the hidden symmetries it reveals.

Let us take Euclidean space as an example. Being in this space allows us to construct the operations of rotations and symmetries in three dimensions. By combining the two, we can then imagine the object that allows us to extract a cork from a bottle. By geometrizing these operations, we create the Euclid group, where rotations (subgroup SO(3)) are translated into the form of matrices a. These are then defined axiomatically (10) from a Gram matrix, which becomes the identity matrix. We then realize that the complete group possesses an operation that had not been considered: "mirror symmetry." The shaping of objects, using our machine tools, represents the implementation of group actions. This is how we can manufacture "straight" corkscrews. The group suggests an inversion, a P-symmetry. So our machine tools produce "face-and-grab corkscrews", "left-handed" corkscrews, which screw in the other direction.

Geometrizing special relativity, as Minkowski and Poincaré did, gives us a group structure that accounts for the two symmetries it comprises: 4D rotations, thanks to the Lorentz subgroup L, and spacetime translations. This subgroup contains the inversion of the orientation of space, the P-symmetry. So far, so good. This symmetry exists in our physics, even in the movements of the elementary objects that compose it. The phenomenon of polarization allows us to add helicity to the photon's motion. And so, there exist two photons of the same energy with

opposite helicities. But the complete Poincaré group possesses antichronous components, which reverse time. At this point, the geometrizing approach represented by taking motion into account in Minkowski space is no longer sufficient. An additional act of geometrization is required, represented by symplectic geometry [31]. And here our spatial intuition and our effort at analogical extrapolation completely fail us. While this geometric extension, of pure mathematics, was introduced by mathematicians such as the American Bertram Kostant [32, 33] and the Russian Alexander Kirilov [34, 35], it is Souriau [31], as the third member of the trio, who highlights the impact of the tool in the world of physics, which is expressed in the following sentence:

> *To consider that there is a movement of a particle which is inscribed along a geodesic which is deduced from the "normal movement" by inversion of the time coordinate, is equivalent to considering that this particle carries a negative energy and, when it has a mass, that this mass is negative.*

Assimilating special relativity meant admitting that particles—photons—could exist in spacetime, as presented by Minkowski, traveling along geodesics of zero length. This already required a considerable effort of mental representation. In a comic strip [36], I mention that the photon's death certificate is printed on the back of its birth certificate, a way of evoking the extreme brevity of its life, in its proper time. But imagining a spacetime resulting from the inversion of the temporal coordinate is beyond human imaginative possibilities—mine, at least. Yet it is this more advanced geometrization process, stemming from the implementation of all the symmetries of the Poincaré group [33], that gave rise to the Janus model [34]. And it is this new geometric conception of the universe that allows us to decode observations such as the acceleration of cosmic expansion, the large-scale, incomplete structure of the universe, the confinement of galaxies and the emergence of their spiral structure, the early birth of galaxies and stars, and the nature of the supermassive objects M87* and Sgr A* [39]. And, once again, it is up to Nature to decide. To better understand the topological aspects of the Janus model, refer to my comic strip Topo the World [40].

What the correspondence between Hilbert and Einstein reveals is the latter's skepticism towards this approach to pure mathematics, based on a variational model, and his fear of seeing this tool stray from reality. To Einstein's surprise, it enabled Hilbert to be the first to construct the tensorial field equation, geometrizing gravitation. Regarding the Janus Cosmological Model (JCM), while the topological aspects were described as early as 1994 [41], only the variational approach [38] allows this geometry to be equipped with its system of coupled field equations:

(19) $$R_{\mu\nu} - \frac{1}{2} \mathrm{R}\, g_{\mu\nu} = \chi \left(T_{\mu\nu} + \sqrt{\frac{\bar{g}}{g}}\ K_{\mu\nu} \right)$$

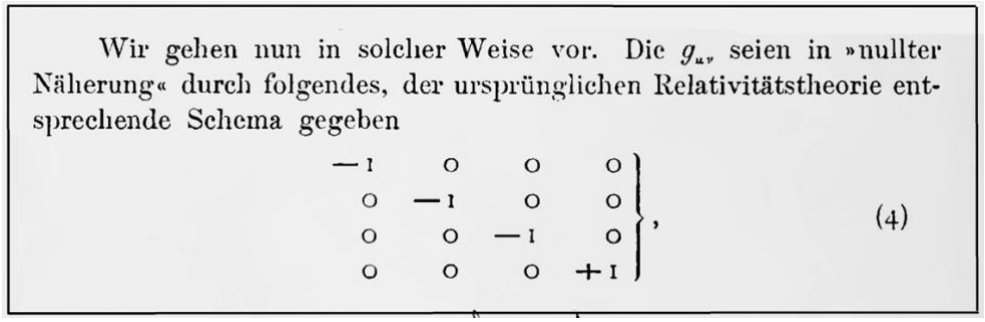
(20 $$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R}\, \bar{g}_{\mu\nu} = - \chi \left(\bar{T}_{\mu\nu} + \sqrt{\frac{g}{\bar{g}}}\ \bar{K}_{\mu\nu} \right)$$

If this resonates with the reader's intuition, we can attempt to summarize the approach taken in the Janus Cosmological Model as follows. In their embedding in $\mathbb{R}^3$, every surface, a two-dimensional object, locally possesses a front and a back. The solutions to Einstein's equation on four-dimensional hypersurfaces. The extension of general relativity to JCM amounts to considering that the 4D hypersurface "has a front and a back" (which follows from the fact that it is orientable), with positive masses and photons of positive energy traveling on the "front," and negative masses traveling on the "back." What is then difficult to imagine and marks the

limits of this creation of a mental image is the fact that at two points A and B of this hypersurface there exist two different geodesics, one inscribed on the front and the other on the back, which are by no means each the image of the other and that the distances traveled and the travel times are different depending on whether one travels on the front or the back of the hypersurface.

5 – **The Schwarzschild exterior metric solution**.

It is November 1915. Hilbert and Einstein have just developed the "field equation." At the same time, Einstein, a physicist first and foremost, has published his own version and produced an approximate, linearized solution [42] that accounts for the advance of the perihelion of Mercury's orbit. As early as January 1916, Karl Schwarzschild provided an exact, nonlinear solution to the field equation in a vacuum and with spherical symmetry [43]. This remarkable work answered the question raised as early as 1846 by Le Verrier [44, 45, 46]. Nowadays, faced with the impossibility of observing this new planet, which Le Verrier had named Vulcan, our modern theorists might have called it a dark planet. Nevertheless, a month later Schwarzschild completed his work by publishing a second paper describing the geometry inside the mass [47]. Before analyzing Hilbert's article, we will point out that at the time, all the scientists involved in these developments of relativity opted, without exception, for a signature (+ - - -). Below is an excerpt from Einstein's article on the advance of Mercury's perihelion [42]:

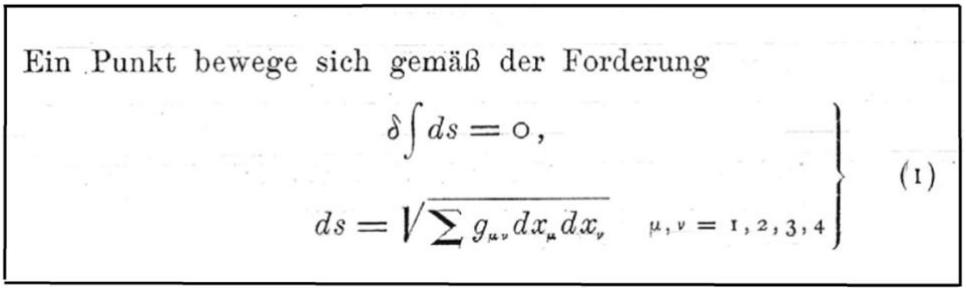


Wir gehen nun in solcher Weise vor. Die $g_{\mu\nu}$ seien in »nullter Näherung« durch folgendes, der ursprünglichen Relativitätstheorie entsprechende Schema gegeben

$$\left.\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +1 \end{matrix}\right\}, \qquad (4)$$

Fig.5 : The Gram matrix used by Einstein [42]

Unambiguously, if we decide to focus on the concept of signature, Einstein's choice is clearly $(-\,-\,-\,+)$ or $(+\,-\,-\,-)$, which amounts to exactly the same thing. The same options were used by contemporaries such as Karl Schwarzschild, Hermann Weyl [49], Ludwig Flamm [50], J. Droste, [51], etc. The only exception is Hilbert, in 1916 [48]. We need to understand why. As it appears explicitly in his first article of January 1916, the concept of geodesic is perfectly defined by Schwarzschild [43]:



Ein Punkt bewege sich gemäß der Forderung

$$\left.\begin{aligned} \delta \int ds &= 0, \\ ds &= \sqrt{\sum g_{\mu\nu} dx_\mu dx_\nu} \qquad \mu, \nu = 1, 2, 3, 4 \end{aligned}\right\} \qquad (1)$$

Fig.6: Geodesics, as defined by Schwarzschild [43].

In simpler terms, these are variationally defined curves, along which the length can only be real. The solution constructed by Schwarzschild is a metric, that is, a formula for calculating this real length element along a curve. He starts with a set of coordinates $\{ t , r , \theta , \varphi \}$ accounting for spherical symmetry, where $r = \sqrt{x^2 + y^2 + z^2}$ is automatically non-negative. Taking into account spherical symmetry and stationarity, this metric becomes the following bilinear form (its equation (6)):

(21) $$ds^2 = F\, dt^2 - (G + H)\, dr^2 - Gr^2(d\theta^2 + sin^2\theta\, d\varphi^2)$$

All that remains is for him to express, using this metric, the components of the Ricci tensor in order to inject them into Einstein's homogeneous equation:

(22) $$R_{\mu\nu} = 0$$

To obtain the functions $F(r)$, $G(r)$, and $H(r$, which, expressed using these coordinates and a positive integration constant α, gives

(23)
$$ds^2 = \frac{(r^3 + \alpha^3)^{1/3} - \alpha}{(r^3 + \alpha^3)^{1/3}} c^2 dt^2 - \frac{r^4\, dr^2}{(r^3 + \alpha^3)[(r^3 + \alpha^3)^{1/3} - \alpha]} - (r^3 + \alpha^3)^{2/3}(d\theta^2 + sin^2\theta d\varphi^2)$$

More precisely :

(24)
$$ds = \sqrt{\frac{(r^3 + \alpha^3)^{1/3} - \alpha}{(r^3 + \alpha^3)^{1/3}} c^2 dt^2 - \frac{r^4\, dr^2}{(r^3 + \alpha^3)[(r^3 + \alpha^3)^{1/3} - \alpha]} - (r^3 + \alpha^3)^{2/3}(d\theta^2 + sin^2\theta d\varphi^2)}$$

With this choice of coordinates, *ds* is always real. What cosmologists haven't yet understood is that a metric solution also contains "topology" information. The only intrinsic quantity is the length *s*. From this metric, we can calculate the surface area of a sphere corresponding to a given value of the variable r and with $dt = 0$. The metric then gives:

(25) $$d\sigma^2 = (r^3 + \alpha^3)^{2/3}(d\theta^2 + sin^2\theta d\varphi^2)$$

The determinant is then:

(26) $$\gamma = (r^3 + \alpha^3)^{4/3}\, sin^2\theta$$

The area of this sphere is:

(27) $$A(r) = \iint \sqrt{\gamma}\; d\theta\, d\varphi = 4\pi\, (r^3 + \alpha^3)^{2/3}$$

This coordinate *r* is not a "radius". This area has a minimum value for the value $r = 0$, which is:

(28) $$A_{min} = 4\pi\, \alpha^2$$

This means that the geometric object described by this metric solution is not contractible. We can, as Schwarzchild does, present this solution with a different set of coordinates by introducing the change of variable:

(27) $$r = (R^3 - \alpha^3)^{1/3} \quad \text{ou} \quad R = (r^3 + \alpha^3)^{1/3}$$

The metric then becomes:

(29) $$ds^2 = \left(1 - \frac{\alpha}{R}\right)c^2dt^2 - \frac{dR^2}{1-\frac{\alpha}{R}} - R^2(d\theta^2 + sin^2\theta\, d\varphi^2) \qquad R = (r^3 + \alpha^3)^{1/3}$$

And it was in this form that Schwarzschild presented his solution in January 1916. But it then becomes absurd to consider studying the hypersurface for values $R < \alpha$ since, under these conditions, we leave the domain of definition with imaginary values of r. We will not dwell on this, as this issue has already been raised in [52].

6 – **Calculation of geodesics using Lagrange's equations.**

As indicated by Schwarzschild, see figure 6, geodesics result from the variation:

(30) $$\delta \int \sqrt{\sum g_{\mu\nu}\, dx_\mu dx_\nu} = \delta \int \sqrt{\sum g_{\mu\nu} \frac{dx_\mu}{dp} \frac{dx_\nu}{dp}}\, dp$$

Posing : $$\frac{dx_\mu}{dp} = \dot{x}_\mu \qquad \frac{dx_\nu}{dp} = \dot{x}_\nu$$

(31) $$\delta \int \sqrt{\sum g_{\mu\nu}\, \dot{x}_\mu \dot{x}_\nu}\; dp = \delta \int F(x_i, \dot{x}_i)\, dp = 0$$

Which leads to Lagrange's equations:

(32) $$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{x}_i}\right) = \frac{\partial F}{\partial x_i}$$

However, in all articles, treatises, and books, the Lagrangian is presented in the following form (example: [52], page 98):

19. The geodesics in the Schwarzschild space-time: the time-like geodesics

We have shown in Chapter 1 (§6(*a*), equation (203)) that the equations governing the geodesics in a space-time with the line element,

$$ds^2 = g_{ij}\,dx^i\,dx^j, \tag{78}$$

can be derived from the Lagrangian

$$2\mathscr{L} = g_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}, \tag{79}$$

where τ is some affine parameter along the geodesic. For time-like geodesics, τ may be identified with the proper time, s, of the particle describing the geodesic.

For the Schwarzschild space-time, the Lagrangian is

$$\mathscr{L} = \frac{1}{2}\left[\left(1-\frac{2M}{r}\right)\dot{t}^2 - \frac{\dot{r}^2}{1-2M/r} - r^2\dot{\theta}^2 - (r^2\sin^2\theta)\dot{\phi}^2\right], \tag{80}$$

where the dot denotes differentiation with respect to τ.

Fig.7 : The Lagrangian, after Chandrasekhar [52].

And it is from this that we construct the Lagrange equations. This requires explanation. By writing:

(33)

$$\delta \int \left(g_{\mu\nu}\,\dot{x}_\mu \dot{x}_\nu\right) dp \;=\; 0$$

It seems that we are basing our calculations not on the Lagrange function derived from the expression for the length, but on its square, and that we are actually writing:

(34)

$$\delta \int [F(\,x_i\,,\dot{x}_i\,)]^2\, dp \;=\; 0$$

This then leads to the following equations:

(35)

$$\frac{1}{L}\left(\frac{dL}{dt}\right)\left(\frac{\partial L}{\partial \dot{x}_i}\right) + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_i}\right) = \frac{\partial L}{\partial x_i}$$

So we have an extra term. How do we get rid of it? By choosing that:

(36)

$$\frac{dL}{dt} \;=\; 0$$

That is, by making the function L equal to a constant of arbitrary value. If we give it the value $L = 1$, we obtain (we don't see where this coefficient $\frac{1}{2}$ that Chandrasekhar adds comes from, see figure 7), we get this:

(37) $$1 = \left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{1}{1-\frac{2M}{r}}\dot{r}^2 - r^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2\right)$$

Or, under these conditions:

(38) $$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{1}{1-\frac{2M}{r}}dr^2 - r^2\left(d\theta^2 + sin^2\theta\,d\varphi^2\right)$$

We can therefore see, by constructing a system of Lagrange equations, and linking it to a metric solution (38), that if we start from the actions:

(39) $$A = \int \sqrt{\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{1}{1-\frac{2M}{r}}\dot{r}^2 - r^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2\right)}\,ds$$

Or from :

(40) $$A' = \int \left(\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{1}{1-\frac{2M}{r}}\dot{r}^2 - r^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2\right)\right)ds$$

We arrive at the *same* system of Lagrange equations! And the same would be true, with any n, for the action:

(41) $$A' = \int \left(\left(1 - \frac{2M}{r}\right)\dot{t}^2 - \frac{1}{1-\frac{2M}{r}}\dot{r}^2 - r^2\left(\dot{\theta}^2 + sin^2\theta\,\dot{\varphi}^2\right)\right)^n ds$$

But (39) requires that the quantity under the radical remain positive, whereas (40) and (41) do not. By opting for (40), see Figure 7, we then construct real curves that "extend inside the sphere" of radius $R = 2M = \alpha$, the Schwarzchild sphere, but where the portions of the curves located "inside" are then equipped with purely imaginary lengths. Without realizing it, postwar theorists achieved an extension into the world of complex numbers.

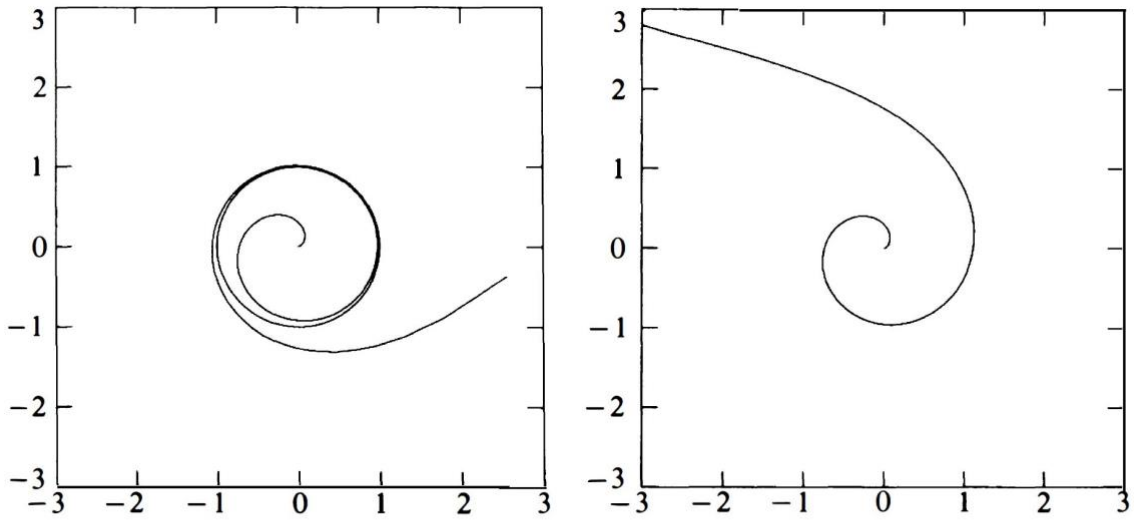


FIG. 7*b*. Various classes of time-like geodesics described by a test particle with $E^2 > 1$: (a), (b), (c): orbits of the first and the second kind with eccentricity $e = 3/2$ and latera recta, 4.5, 2.5, and 1.94 respectively ($M = 3/14$ in the scale along the coordinate axes); (d), (e): unbound orbits with $l = 1$ and with imaginary eccentricities $e = 0.001i$ and $0.1i$ ($M = 0.3$ in the scale along the coordinate axes).

121

Fig.8 : Chandrasekhar's "plunging geodesic" [52], page 121.

It should be noted that these curves are merely the projections of the geodesics onto the plane θ = constant where they lie. This distinction is important, as we will see later. Nevertheless, when we use the differential equation that gives these geodesics (equation (6.90) from reference [53]), derived from the system of Lagrange's equations, we do indeed find these projections, which evoke the crossing of the Schwarzschild sphere with superb continuity of the tangent to these curves (in projection!). Below is this equation with a Schwarzschild radius.

$$R_s = 2M = \alpha = \frac{2Gm}{c^2} = 1 \tag{42}$$

$$\varphi = \varphi_0 + \int \frac{dr}{r^2 \sqrt{\frac{l^2 c^2}{h^2} + \frac{1}{r\,h^2} - \frac{1}{r^2} + \frac{1}{r^3}}} \tag{43}$$

About thirty years ago, in an attempt to get to the bottom of this, we recalculated everything until we arrived at this equation. Plotting the curves immediately revealed these quasi-elliptical geodesics, with the phenomenon of perihelion precession. See below, in Chandrasekhar ([52] figure 7a):

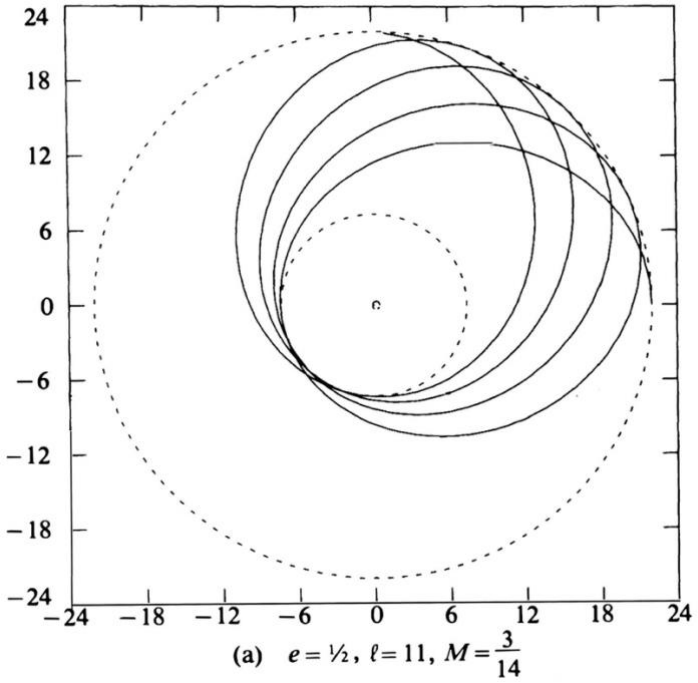


Fig.9 : Advance of perihelion ([52] figure 7a):

In the previous figure, the small circle with the dashed line is not the Schwarzschild circle but the circle along which the perihelion moves. Below is the curve I myself obtained, not without surprise:

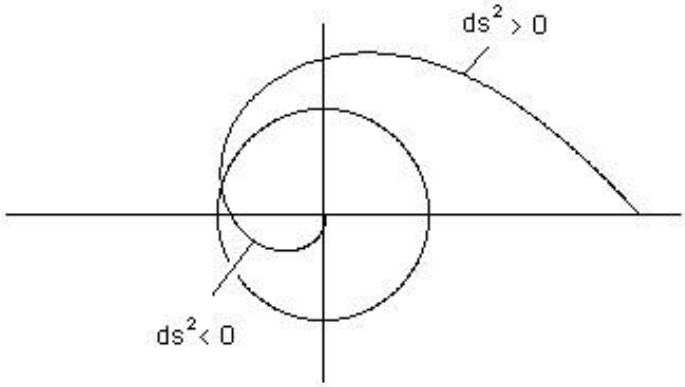


Fig.10: Real extension, in projection, of a curve equipped with an *imaginary length*.

The deceptive nature of this equation arises from the fact that, when one imagines extending the curve "inside the Schwarzschild sphere" using this equation, one obtains a perfect continuity of the tangent on the circle, which represents the projection of the sphere onto the plane $\theta = Cst$ In fact, as we will see later in a 3D representation (t, r, φ) and a perspective view, the geodesic never reaches the sphere! From this point, switching to the Kerr metric, which also has its own horizon, changes nothing. This has not prevented many authors from specifying "the structure of the “central singularity", *which exists only in their imagination* !

7- **The geodesics of the Flamm surface are not those of the Schwarzschild hypersurface!**

As early as 1916, the young Austrian mathematician Ludwig Flamm precisely analyzed [54] the structure of the solution hypersurface constructed by Schwarzschild. This constituted a brilliant demonstration of mastery of 4D geometry, a skill absent today among black hole specialists, who have not integrated the topological aspects of the solution. Since this solution does not depend on the time coordinate, Flamm deduced that the object is a 3D hypersurface undergoing a time translation. The metric of this 3D object is then simply, still with the indicated constraint:

$$(44) \qquad d\sigma^2 = \frac{dR^2}{1-\frac{\alpha}{R}} - R^2(\,d\theta^2 + sin^2\theta\, d\varphi^2\,) \qquad R = (\,r^3 + \alpha^3)^{1/3}$$

What is difficult to grasp is that our choice of coordinates is merely a *system of representation*. It so happens that this 3D hypersurface described by equation (44) and with $\alpha > 0$ is embeddable in $\mathbb{R}^3$ (which would not be the case for the hypersurface described by the same metric, equipped with $\alpha < 0$). Mentally, we therefore imagine this object embedded in the only representational space available to our minds: 3D Euclidean space. The Schwarzschild sphere then becomes a s throat sphere, the point of junction of this structure, which then appears, in the mental image we are trying so hard to construct, as a point connecting two 3D Euclidean spaces constituting a covering. The geodesics resulting from the metric (44) lie in planes. Below, on the left, is a planar geodesic that grazes the throat surface without touching it, and is therefore confined to the same 3D sheet. On the right, this geodesic "crosses the throat sphere" and thus extends into the other 3D sheet (shown as a dotted line):

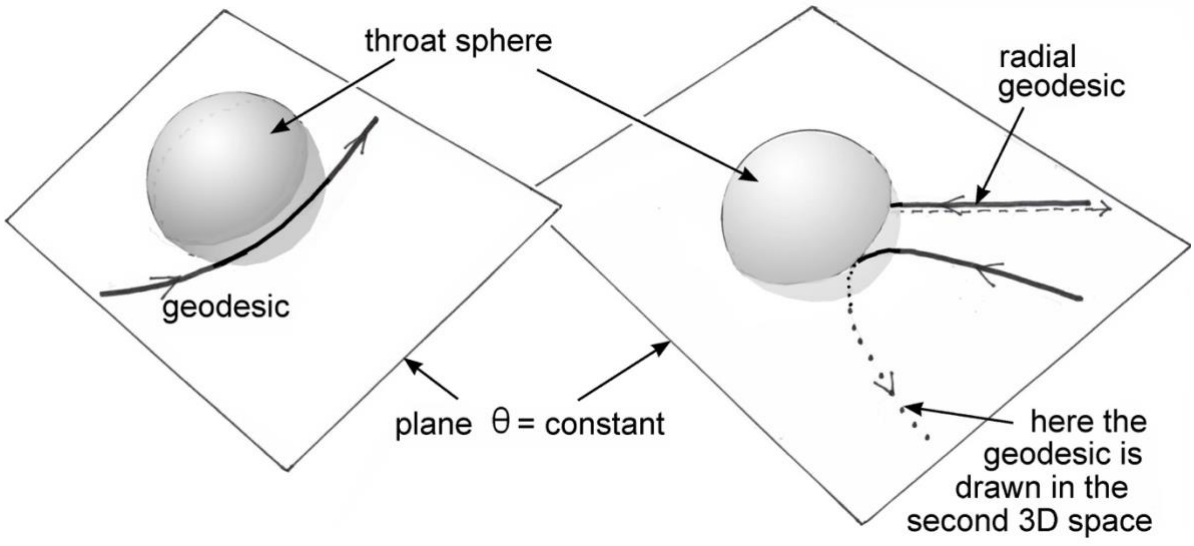


Fig.10: The hypersurface from equation (44) figured as a space bridge linking two 3D-spaces.

It is important to keep in mind that these are only mental images, derived from a representation in the specific coordinates $(R\,,\theta\,,\varphi)$. If we return to the initial coordinates chosen by Schwarschild, the 4D hypersurface, equation (23), is then the time translation of a 3D hypersurface defined by the metric:

(45) $$ds\sigma^2 = \frac{r^4\,dr^2}{(r^3+\alpha^3)\left[(r^3+\alpha^3)^{1/3}-\alpha\right]} + (r^3+\alpha^3)^{2/3}(d\theta^2+sin^2\theta d\varphi^2)$$

The throat sphere corresponds to the value $r = 0$. In other words, while its area is $4\pi\alpha^2$, in this representation it reduces to a point. This means that mental images have limits. But let's return to Flamm's analysis. If the hypersurface is three-dimensional, by setting, for example, $\theta = \pi/2$, we obtain a two-dimensional object, described by the metric:

(46) $$d\sigma^2 = \frac{dR^2}{1-\frac{\alpha}{R}} - R^2 d\varphi^2 \qquad R = (r^3 + \alpha^3)^{1/3}$$

Still with the same constraint $R \geq \alpha$. Flamm [54] observes that this object can be embedded in $\mathbb{R}^3$ and that it is then generated by the rotation of a horizontal parabola.

(47) $$z = \pm 2\sqrt{\alpha\,(r-\alpha)}$$

The length of a portion of a curve corresponds to the integral:

(48) $$\int \sqrt{\frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2}\quad ds$$

As we saw above, we obtain the Lagrange equations by basing the calculus of variations on:

(49) $$\delta \int \left(\frac{\dot{r}^2}{1-\frac{\alpha}{r}} + r^2\dot{\varphi}^2 \right) ds = 0$$

But this eliminates the constraint of non-negativity of the quantity under the radical. The equations giving the solution curves are the same:

(50) $$\frac{d\varphi}{dr} = \pm\frac{h}{r^2}\frac{1}{\sqrt{\left(1-\frac{\alpha}{r}\right)\left(1-\frac{h^2}{r^2}\right)}}$$

We see that for $r > \alpha$, we obtain real curves for $|h| < \alpha$ .. But for $r < \alpha$, we also obtain real curves with $|h| > \alpha$. However, along these curves, which are spirals that start tangent to the circle and wind in a spiral, the length is purely imaginary. When we embed the Flamm surface in $\mathbb{R}^3$, the absurdity becomes evident:

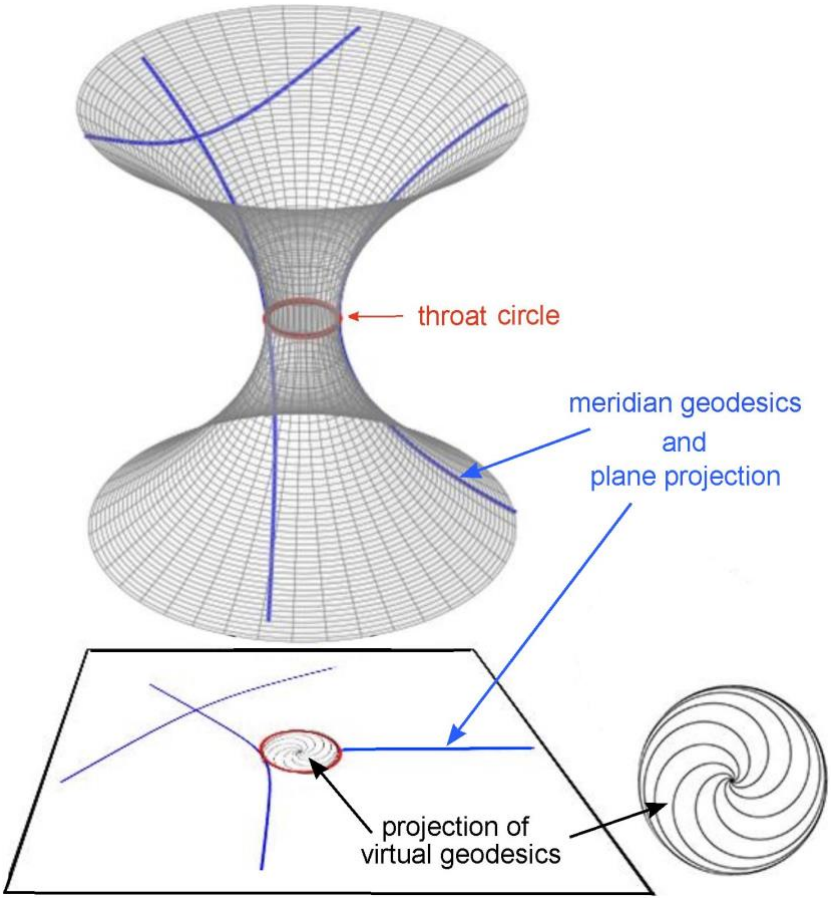


Fig. 11: Flamm surface, real and… virtual geodesics

But the geodesics of the Flamm surface are not those of a 4D hypersurface: the equations from which they are derived are different. Among other things, one cannot construct quasi-elliptic geodesics like those in Figure 9 on the Flamm surface. The virtual geodesics from (50) are tangent to the throat circle, while the curves from (43) intersect this circle at an angle. See Figures 8 and 10. But the idea is the same. Along these real curves, the length, and therefore proper time, is purely imaginary. It will probably be a long time before black hole specialists admit that these trajectories plunging into the horizon sphere and converging in a spiral toward the central singularity, like the singularity itself, have no physical reality. That being said, the Flamm surface remains a cross-section of Schwarzschild spacetime with constant θ. If we want to represent these geodesics in this embedding of the cross-section in $\mathbb{R}^3$, we must lift the curves from the plane towards the Flamm surface, where they belong. But then they are not geodesics of this surface.

8 – **The concept of "virtual geodesics"**.

It is easy to endow the torus and the sphere with these real curves, traversed by an imaginary length. What physicists have failed to grasp is that metrics like those derived from Einstein's equation possess not only the symmetries that preceded their construction, but also their own topology, revealed by an appropriate choice of coordinates. Let us illustrate this idea by taking the example of the torus, presented according to the metric:

(51) $$ds^2 = r_0^2\, d\theta^2 + (R + r_0 \cos\theta)^2\, d\varphi^2$$

The signature is (+ +). Considering the paths at $\theta = Cst$ and then at \ $\varphi = cst$, we immediately highlight the non-contractility of this surface. Let's now introduce the change of coordinate:

(52) $$\theta = arc\,cos\,\frac{r-R}{r_0}$$

The line element becomes :

(53) $$ds^2 = r^2 d\varphi^2 + \frac{dr^2}{-R^2 + {r_0}^2 - r^2 + 2\,r\,R}$$

It is immediately apparent that the signature (+ +) will only be maintained if:

(54) $$R - r \leq r \leq R + r_0$$

Otherwise, we are outside the torus. The length is expressed as follows:

(55) $$s = \int \sqrt{r^2\dot{\varphi}^2 + \frac{\dot{r^2}}{-R^2 + {r_0}^2 - r^2 + 2\,r\,R}}\; ds$$

But we know that we will obtain the same system of Lagrange equations by doing:

(56) $$\delta \int \left(r^2\dot{\varphi}^2 + \frac{\dot{r^2}}{-R^2 + {r_0}^2 - r^2 + 2\,r\,R} \right) ds = 0$$

Lagrange's equations give us the projection of the curves resulting from this variation calculation onto the plane (r, φ):

(57) $$d\varphi = \frac{\pm\, h\, dr}{r^2 \sqrt{(-R^2 + {r_0}^2 - r^2 + 2\,r\,R)\left(1 - \frac{h}{r^2}\right)}}$$

This allows us to plot the projection of a geodesic of the torus:

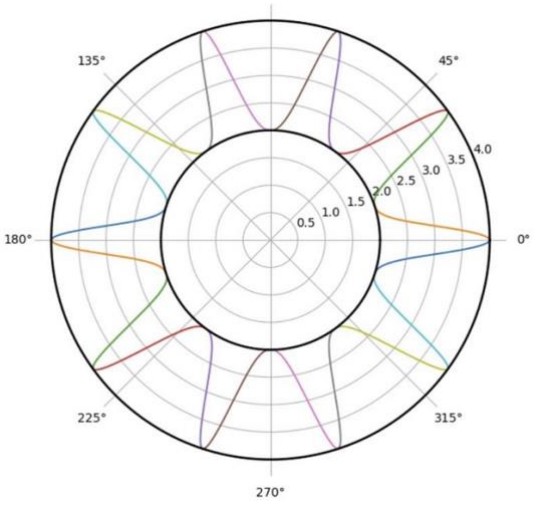


Fig.12: Projection of a geodesic of the torus.

Mais cette solution engendre également des courbes réelles qui sont la projection de courbes équipées d'une longueur imaginaire. Ci-après ces géodésiques virtuelles, à l'intérieur du tore:

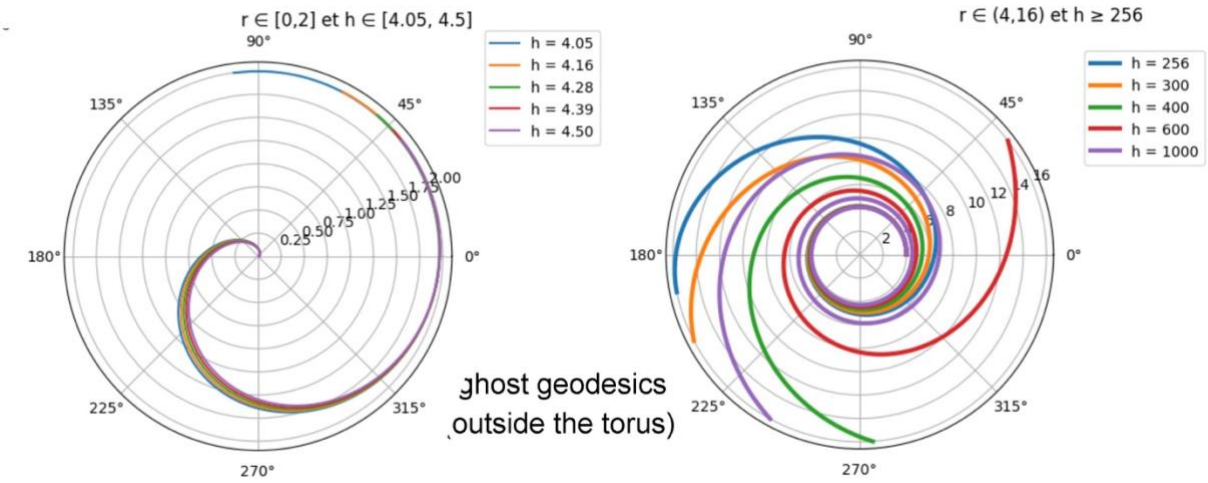


Fig. 13: Virtual geodesics of the torus "inside" its throat circle and "outside" its rim circle

A perspective view encompasses these results:

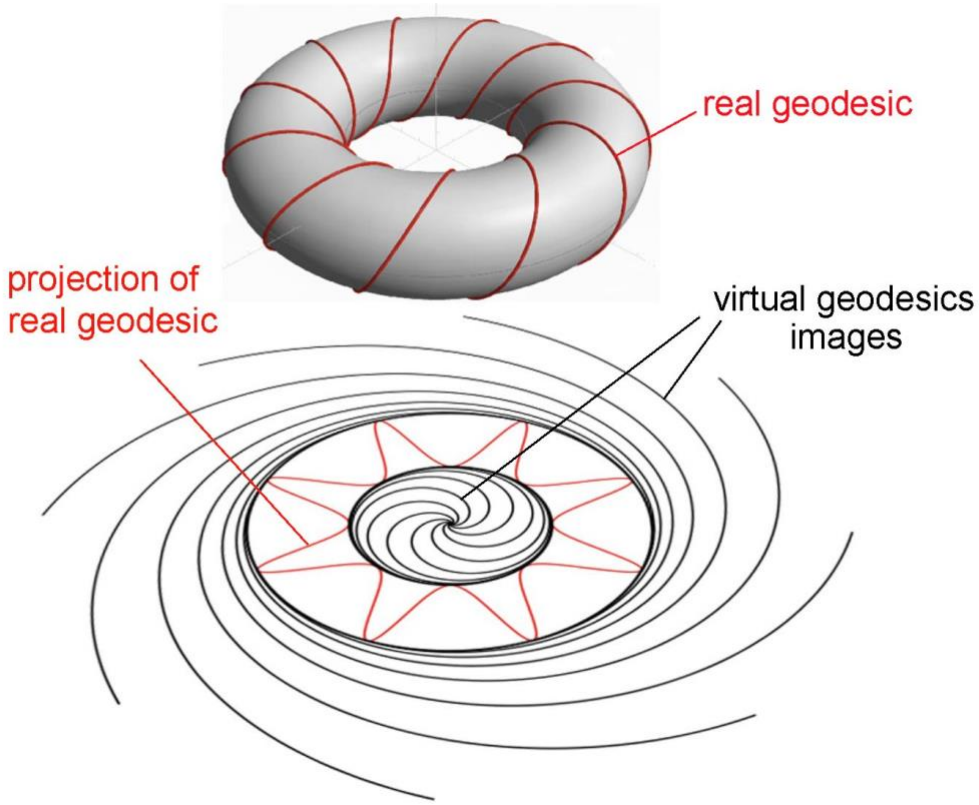


Fig.14: The torus, equipped with its (red) real and (black) virtual geodesics

Let's now turn to the sphere. The standard form of its metric is:

(58) $$ds^2 = R^2 (d\theta^2 + sin^2\theta \, d\varphi^2)$$

Let's make the following change of coordinate:

(59) $$\theta \; = \; arc\; sin\; \frac{r}{R}$$

The line element becomes :

(60) $$ds^2 \; = \; r^2 d\varphi^2 \; + \; \frac{R^2\, dr^2}{R^2 - r^2}$$

The variation calculation leads to the equation giving the projection of the geodesics:

(61) $$d\,\varphi \; = \; \frac{h\,R}{r} \frac{dr}{\sqrt{(R^2 - r^2)(\,r^2 - h^2)}})$$

We get that :

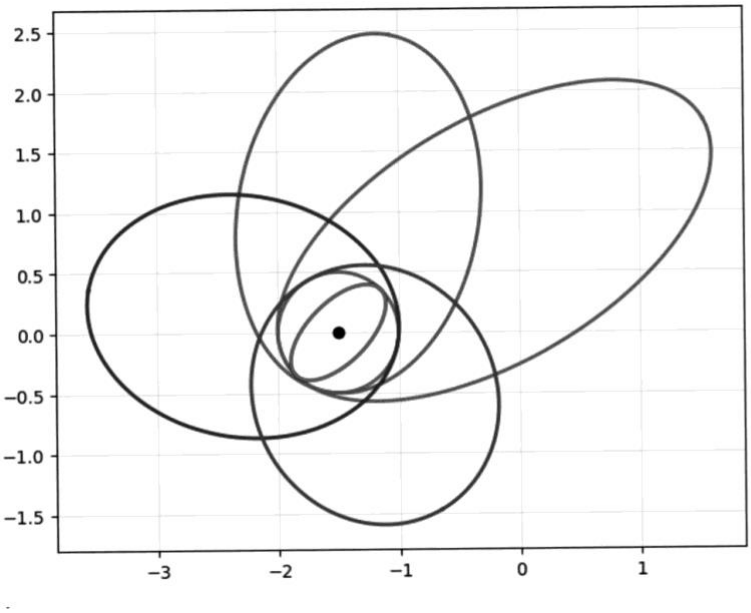


Fig.15 : Projections of geodesics and pseudo-geodesics of a sphere of radius 0.5

We have drawn, inside the circle, an ellipse, the projection of a geodesic of the sphere (these are "great circles"). But we have also drawn some of the projections of its "virtual geodesics". What are these curves, tangent to the sphere? Let's write:

(62) $$\frac{dr}{d\varphi} \; = \; \frac{r}{h\,R} \sqrt{(\,r^2 - \; h^2)\,(\,R^2 - \; r^2)}$$

Posing :

(63) $$r \; = \; \frac{1}{u}$$

(64) $$-\; \frac{d\varphi}{du} \; = \; \frac{1}{h\,R} \sqrt{(\;1 \; - \; h^2 u^2)\,(\,R^2 u^2 - \; \frac{11}{u^2}\,)}$$

(65) $$\frac{d^2 u}{d\varphi^2} \; + \; u \; = \; \frac{1}{h^2}$$

(66) $$u = \frac{1}{h^2} + A\cos(\varphi - \varphi_0)$$

The parameter R disappears.

(67) $$r = \frac{h^2}{1 + e\cos(\varphi - \varphi_0)}$$

We have just established the first theorem of *imaginary geometry*:

The projections of the virtual geodesics of the sphere are ellipses!

Below, a perspective view:

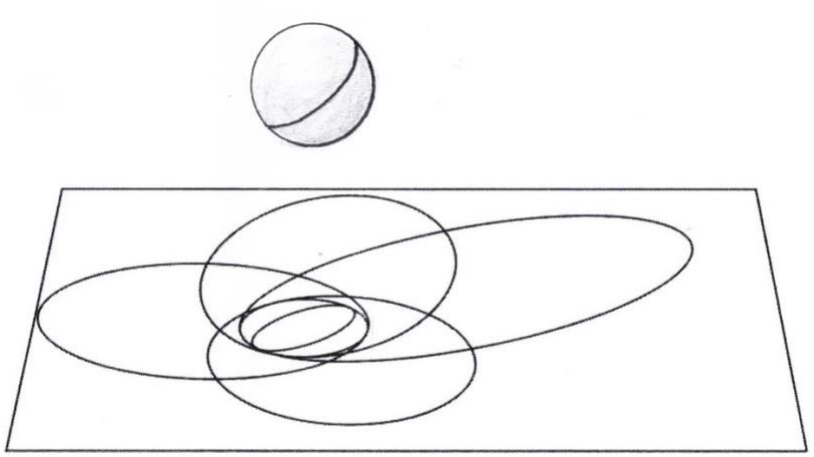


Fig.16 : Projection of geodesics and pseudo-geodesics of the sphere

Renowned scientists have produced some very elegant theorems about "the central singularity that black holes harbor at their center." Alas, these objects exist only in their imagination.

Using the Kerr metric would only complicate matters further. This object also possesses an event horizon, within which length, and therefore proper time, are purely imaginary. Thus, we would have the same phenomenon of extension through virtual geodesics, undoubtedly quite striking.
We can see, therefore, that the black hole model is a fantastic mathematical chimera, an artifact resulting from a flawed geometric interpretation. But one question remains: How is it possible to observe such continuity when, beyond the Schwarzschild circle, we construct the curve—real, but endowed with an imaginary length—that extends the geodesic?

9 – **The geodesics of the Schwarzschild solution in** 3D **space** (t, r, φ)

We will focus on the wormhole model, where two Schwarzschild solutions, in a form free of cross-terms in *drdt*, are associated to form a "space bridge". The curves then have the following schematic appearance:

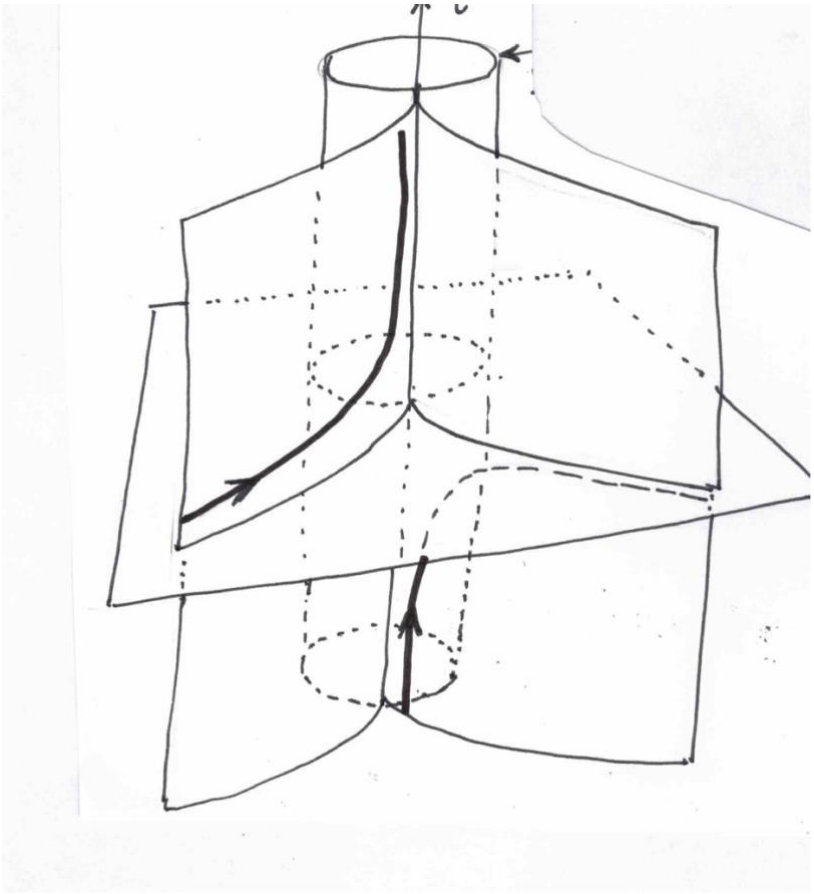


Fig.17 : Geodesics of the Schwarzschild wormhole in (t , r , φ)

We see that, evaluated using this coordinate t (called the "Schwarzschild time"), the free-fall and escape times are infinite. Crossing the throat sphere is then accompanied by a discontinuity in the geodesic. The figure shows the trace of these trajectories in their projection onto the plane. The break is therefore linked to the projection (r , φ). This explains why theorists were encouraged to extend this geodesic along a curve spiraling towards the center. The continuity was only apparent. We can now move on to a Koiran wormhole [55,51] where two Schwarzschild solutions are conjugate, with crossed terms of opposite signs;

In our own old (attractive field):

$$(68)\quad \boldsymbol{ds^2 = \left(1 - \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 + \frac{\alpha}{r}\right) dr^2 - \frac{2\alpha c}{r} dr dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}$$

In the adjacent fold (repulsive field):

$$(69)\quad \boldsymbol{ds^2 = \left(1 + \frac{\alpha}{r}\right) c^2 dt_E^2 - \left(1 - \frac{\alpha}{r}\right) dr^2 + \frac{2\alpha c}{r} dr dt_E - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)}$$

Since we are trying to model the embedding in the wormhole and its emergence in the adjacent sheet, we have represented the direction of time (this time coordinate t_E, of Eddington), downwards:

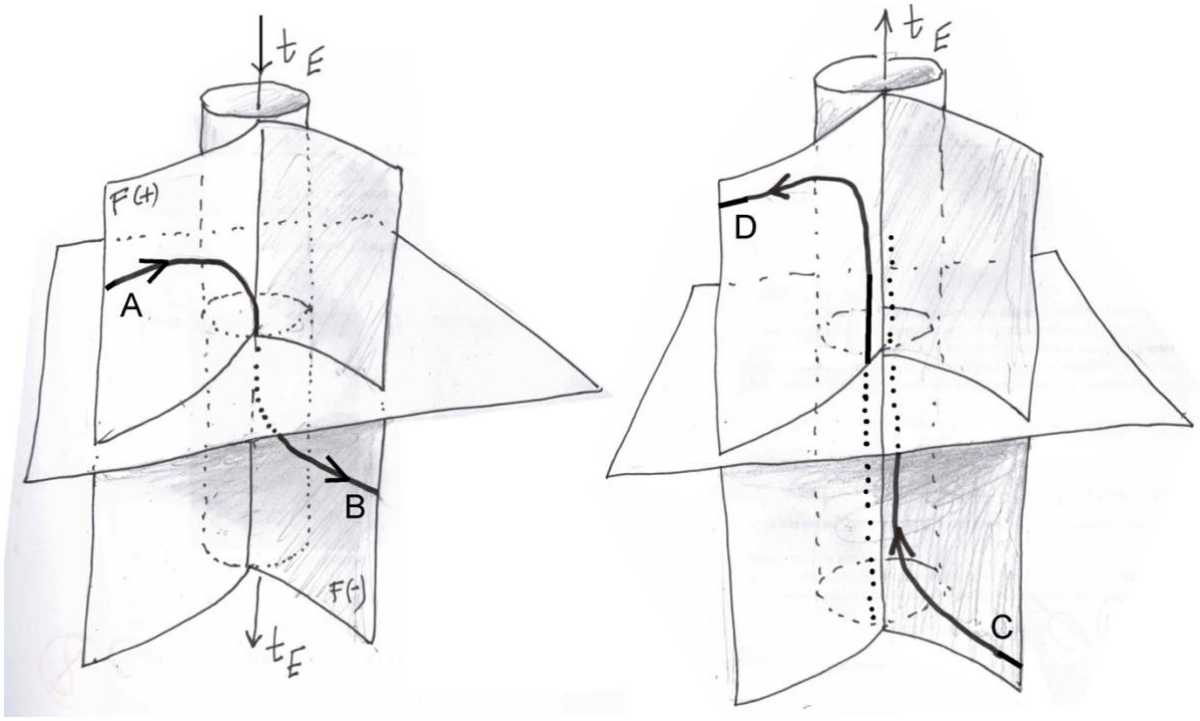


Fig.18 : Geodesics of the Koiran wormhole in (t_E , r , φ)

Wormholes do not belong to the realm of physics, as there, too, one would be considering describing a highly unsteady process using a steady-state solution. These images are simply intended to provide an interpretation of a geometric solution exhibiting continuity in geodesics. This interpretation of the solution illustrates the concept of a one-way membrane. Geodesics do not behave the same way depending on whether they are plunging or emerging.

On the left, at A, a control mass reaches the throat sphere in a finite time, and at B emerges from it into the second layer, also in a finite time. On the right, a control mass moving towards the throat sphere at C reaches it in an infinite time. At D, it emerges in an infinite time. Transit in the reverse direction is therefore impossible.

The figure on the left also shows the projection of the geodesics onto the plane (r , φ),, with their apparent break, which had led theorists to project these curves along a spiral converging towards the "center." The figure on the left, the only one depicting a physical phenomenon, shows a continuous geodesic, and it is its projection that produces this apparent break. To the cosmologists' credit, it must be admitted that this apparent continuity of the tangent in the plane (r , φ), was entirely convincing enough to encourage them to consider such an extension.

10 – **When the black hole model becomes, in the 21st century, the equivalent of the atom.**

In 1989, seventy-three years after Hilbert's publication, the Canadian mathematician L. Abrams was the first to publish an article entitled "The black holes: the legation of Hilbert's error" [56]. In 2004, A. Loinger published a new article entitled "Black holes do not exist. Thus spoke Karl

Schwarzschild" [57]. In 2011, C. Corda attempted to refute these arguments [58]. The fact that these exchanges are located exclusively in the arXiv database, moreover in its "physics" section and not in the section where cosmologists publish their preprints: "general relativity and quantum gravity, abbreviated gr-qc," shows that no peer-reviewed journal has seen fit to submit to its reviewers an article questioning the existence of black holes. Thus, this journal is the first to address this question in an article [25] where we have simply reiterated and expanded upon arguments raised in previous articles, to general indifference. Conversely, the article [59] published in 2019 by Robbert Dijkgraaf, current director of the prestigious Institute for Advanced Study in Princeton, a veritable Mecca of world science, founded in 1930 and to which Einstein and Oppenheimer belonged, reveals the general adherence of current theorists to this model. It is 2019. The EHT has just obtained the first image of the supermassive object located at the center of the galaxy M87 [60]. Note the title of the article: The Shadow of the Supermassive Black Hole. It is a black hole… "because we cannot see what else it could be." Dijkgraaf, who is very active in science education, writes, and we quote [59]:

> *The iconic black hole image released by the Event Horizon Telescope collaboration has gone viral these days.… It is difficult to overestimate the importance of this discovery. There is only one moment in human history when we will have seen a black hole for the first time … Black holes thus play a similar role as atoms did a hundred years ago … The black hole is just such a paradox, the atom of the twenty-first century.*

It is clear that it will prove very difficult to position this question within a scientific community that has made black holes the idols of a religion of dreary times, especially since an alternative interpretation of the available observational data : plugstars, has just emerged. [61]

11 – **David Hilbert's stange conception of space-time**.

We now return to 1916. Hilbert felt compelled to report, in an expanded version of his 1915 paper, on these solutions to the field equation, which had recently appeared in the literature. But physical applications remained quite new to him, as he had been navigating this world since July 1915 as if on another planet. When one reads his 1916 article [48], one realizes that he had not yet incorporated the concept of spacetime as defined by Minkowski in 1909, namely that the time coordinate *t*, modulo the multiplicative coefficient c, thus becoming the quantity *ct*, like *x, y*, and *z*, is expressed in meters. It's as if an ancient surveyor, after starting with land surveying and measuring length and width, suddenly realized that heights and altitudes can also be expressed in acres, miles, feet, and meters, and that by turning one's head 90 degrees, it's possible to consider things differently, from another angle. The same applies to relativity, except that it's the Lorentz group that allows us to perform these strange four-dimensional rotations.

In his first article [26], Hilbert constructed his field equation with his set of coordinates $\{ w_1, w_2, w_3, w_4\}$, without ever needing to specify the nature of these quantities, where the three coordinates $\{ w_1, w_2, w_3 \}$ refer to space and w_4 to time. Strangely, Hilbert's article makes no mention of the second solution published by Schwarzschild, describing the geometry inside the masses [47]. He never attempts to incorporate into his paper anything other than Schwarzschild's first solution, which refers to the vacuum [43].

In his article, attention should be focused on certain key passages that reveal Hilbert's conception of the universe. It is essential to keep in mind that in 1916 the universe was not evolving and relativistic physics did not yet exist. The curvatures affecting it were merely minute perturbations unfolding within a decidedly Euclidean framework, associated with metric potentials $\delta_{\mu\nu}$. We read:

Im Falle der pseudo-Euklidischen Geometrie haben wir

$$g_{\mu\nu} = \delta_{\mu\nu},$$

worin

$$\delta_{\mu\mu} = 1, \qquad \delta_{\mu\nu} = 0 \quad (\mu \neq \nu)$$

bedeutet. Für jede dieser pseudo-Euklidischen Geometrie benachbarte Maßbestimmung gilt der Ansatz

Fig. 19: Hilbert's "pseudo-Euclidean" view of spacetime [48].

Translation :

In the case of the pseudo-Euclidean geometry we have

$$g_{\mu\nu} = \delta_{\mu\nu},$$

where

$$\delta_{\mu\nu} = 1, \qquad \delta_{\mu\nu} = 0 \quad (\mu \neq \nu).$$

For every metric in the neighborhood of this pseudo-Euclidean geometry the ansatz

$$g_{\mu\nu} = \delta_{\mu\nu} + \varepsilon h_{\mu\nu} + \ldots \qquad (37)$$

is valid, where ε is a quantity converging to zero, and $h_{\mu\nu}$ are functions of the w_s.

Fig. 20: Spacetime, according to Hilbert, is pseudo-Euclidean.

The matrix $\delta_{\mu\nu}$ is the identity matrix. The spacetime metric, according to Hilbert, is therefore:

(70) $$ds^2 = g_{11}dw_1^{\,2} + g_{22}dw_2^{\,2} + g_{33}dw_3^{\,2} + g_{44}dw_4^{\,2}$$

Then, the « signature » of his bilinear for mis (+ + + +)

In Hilbert's theory, spacetime is not defined by a metric. *It is determined by a bilinear form*. The difference is crucial. If its points are located along a trajectory using a parameter *p*, he writes:

$$(71)\quad G\left(\frac{dw_1}{dp},\frac{dw_2}{dp},\frac{dw_3}{dp},\frac{dw_4}{dp}\right) = g_{11}\left(\frac{dw_1}{dp}\right)^2 + g_{22}\left(\frac{dw_2}{dp}\right)^2 + g_{33}\left(\frac{dw_3}{dp}\right)^2 + g_{44}\left(\frac{dw_4}{dp}\right)^2$$

In the following figures, Hilbert, in his own way, attempts to integrate into the new version of his memoir "Foundation of Physics" [48] the exact solution that Karl Schwarzschild had just published in January 1916 [43], the one describing the geometry of the void surrounding a mass. In the plate below, you will find gathered all the errors made by Hilbert that would become the foundation of "modern cosmology." Let us begin with the German text:

Nach Schwarzschild ist die allgemeinste diesen Annahmen entsprechende Maßbestimmung in räumlichen Polarkoordinaten, wenn

$$w_1 = r\cos\vartheta$$
$$w_2 = r\sin\vartheta\cos\varphi$$
$$w_3 = r\sin\vartheta\sin\varphi$$
a → $$w_4 = l$$

gesetzt wird, durch den Ausdruck

$$(42)\qquad F(r)\,dr^2 + G(r)\,(d\vartheta^2 + \sin^2\vartheta\,d\varphi^2) + H(r)\,dl^2$$

dargestellt, wo $F(r)$, $G(r)$, $H(r)$ noch willkürliche Funktionen von r sind. Setzen wir

b → $$r^* = \sqrt{G(r)},$$

so sind wir in gleicher Weise berechtigt r^*, ϑ, φ als räumliche Polarkoordinaten zu deuten. Führen wir in (42) r^* anstatt r ein und lassen dann wieder das Zeichen $*$ weg, so entsteht der Ausdruck

$$(43)\qquad M(r)\,dr^2 + r^2\,d\vartheta^2 + r^2\sin^2\vartheta\,d\varphi^2 + W(r)\,dl^2,$$

wo $M(r)$, $W(r)$ die zwei wesentlichen willkürlichen Funktionen von r bedeuten. Die Frage ist, ob und wie diese auf die allgemeinste Weise zu bestimmen sind, damit den Differentialgleichungen (36) Genüge geschieht.

Zu dem Zwecke müssen die bekannten in meiner ersten Mitteilung angegebenen Ausdrücke $K_{\mu\nu}$, K berechnet werden. Der erste Schritt hierzu ist die Aufstellung der Differentialgleichungen der geodätischen Linien durch Variation des Integrals

c → $$\int\left(M\left(\frac{dr}{dp}\right)^2 + r^2\left(\frac{d\vartheta}{dp}\right)^2 + r^2\sin^2\vartheta\left(\frac{d\varphi}{dp}\right)^2 + W\left(\frac{dl}{dp}\right)^2\right)dp.$$

Fig.21: The errors made by Hilbert, in their original version [48].

Translation :

The assumptions about the $g_{\mu\nu}$ are the following:

1. The metric is represented in a Gaussian coordinate system, except that g_{44} is left arbitrary, i.e. we have

$$g_{14} = 0, \qquad g_{24} = 0, \qquad g_{34} = 0.$$

2. The $g_{\mu\nu}$ are independent of the time coordinate x_4.
3. The gravitation $g_{\mu\nu}$ is centrally symmetric with respect to the origin of coordinates.

According to Schwarzschild the most general metric conforming to these assumptions is represented in polar coordinates, where

$$w_1 = r\cos\vartheta$$
$$w_2 = r\sin\vartheta\cos\varphi$$
$$w_3 = r\sin\vartheta\sin\varphi$$
$$w_4 = l,$$

a

by the expression

$$F(r)dr^2 + G(r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + H(r)dl^2 \qquad (42)$$

where $F(r)$, $G(r)$, $H(r)$ are still arbitrary functions of r. If we put

$$r^* = \sqrt{G(r)},$$

b

then we are equally justified in interpreting r^*, ϑ, φ as spatial polar coordinates. If we introduce r^* in (42) instead of r and then eliminate the sign *, the result is the expression

$$M(r)dr^2 + r^2d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 + W(r)dl^2, \qquad (43)$$

where $M(r)$, $W(r)$ mean the two essential, arbitrary functions of r. The question is whether and how these can be determined in the most general way so that the differential equations (36) enjoy satisfaction. |

To this end the well-known expressions $K_{\mu\nu}$, K given in my first communication must be calculated. The first step in this is the derivation of the differential equations for geodesic lines by variation of the integral

$$\int\left(M\left(\frac{dr}{dp}\right)^2 + r^2\left(\frac{d\vartheta}{dp}\right)^2 + r^2\sin^2\vartheta\left(\frac{d\varphi}{dp}\right)^2 + W\left(\frac{dl}{dp}\right)^2\right)dp.$$

c

Fig. 22: Hilbert's multiple errors, all on the same plate.

→ As you can see is **a** , the calculations will be undertaken without the specific nature of this fourth coordinate being defined. Hilbert feels no need for it at this stage. In his equation (42), we find its bilinear form, with three unknown functions $F(r)$, $G(r)$, $H(r)$, to be determined. He knows that in polar coordinates, the bilinear form that gives substance to Euclidean space is:

(73) $$dr^2 + r^2d\theta^2 + sin^2\theta\, d\varphi^2$$

→ In **b**, replacing the function G(r) with it therefore seems to be a way to simplify the calculation by converging more quickly towards the expected result. Especially since this form must be identified with (73) at infinity. This is the error that L. Abrams pointed out in 1989, *seventy-three years* later [56].

→ In **c**, Hilbert defines its action. We disregard the length. We saw the consequences above. It remains for him to calculate the coefficients of his Ricci tensor, which he denotes by $K_{\mu\nu}$, and to calculate the Christoffels coefficients. And the field equation is then, in a vacuum, simply:

(74) $$K_{\mu\nu} = 0$$

Having completed all calculations, here is its result, that is to say its bilinear form, still expressed using this fourth time coordinate *l*.

(75) $$G(dr, d\theta, d\varphi, dl) = \frac{r}{r-\alpha} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta \, d\varphi^2 + \frac{r-\alpha}{r} \, dl^2$$

with signature (+ + + +)

Since it was necessary to recover Schwarzschild's result, Hilbert posited $l = it$, which shows that he had not at all incorporated the vision of Einstein and Minkowski and remained attached to the hypothesis initially proposed by Poincaré in 1902 [16], then by Minkowski in 1907 [21] (which the latter would abandon the following year). Despite eight months of intense dialogue with Einstein:

→ For Hilbert, Special Relativity boils down to the fact that time is purely imaginary

This leads him to reformulate his result according to:

(76) $$G(dr, d\theta, d\varphi, dt) = \frac{r}{r-\alpha} dr^2 + r^2 d\theta^2 + r^2 sin^2\theta \, d\varphi^2 - \frac{r-\alpha}{r} \, dt^2$$

→ Thus, in December 1916, we have the birth certificate of the "modern" signature:

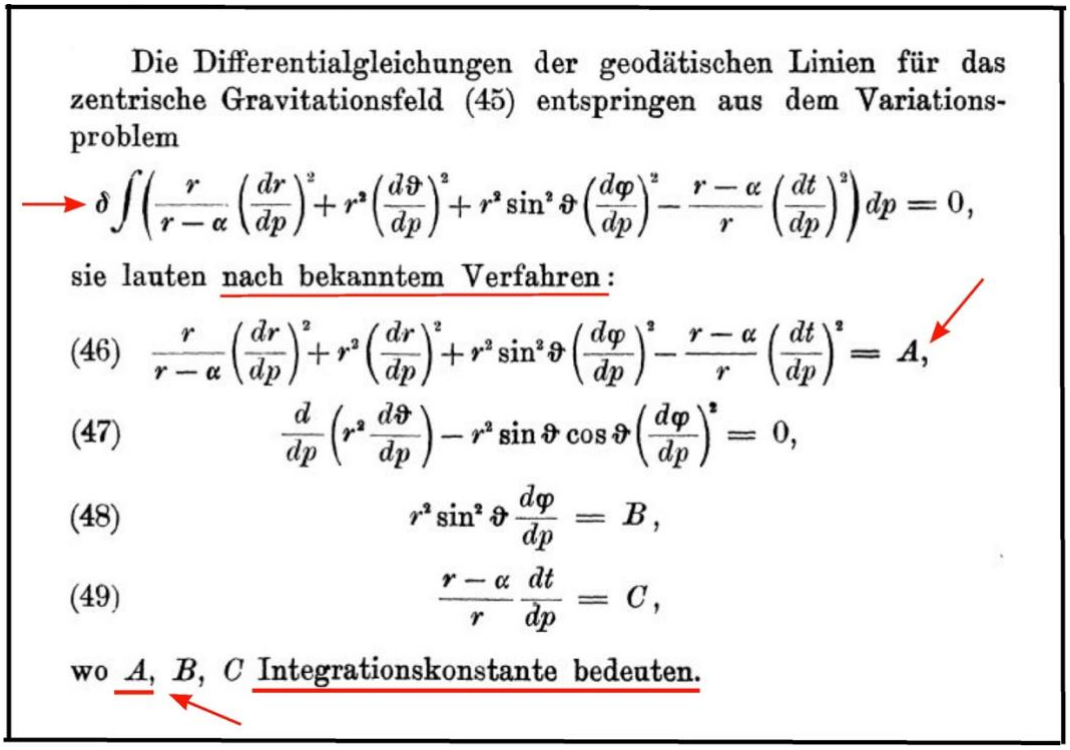
(+ ++ -)

Neglecting differences in sign, Hilbert concludes that this expression is, in his opinion, identical to the result published by Schwarzschild. He then mentions, in a footnote, this change of variable used by Schwarzschild:

(77) $$R = (r^3 + \alpha^3)^{1/3}$$

He considers this a way to bring the singularity (at $r = \alpha$) back to the origin (at $r = 0$), specifying that "there are simpler ways to do this." This is without realizing that his simplification **b** transforms his initial variable *r*, which he considers "radial," into this "intermediate quantity" *R* of Schwarzschild.

He still needs to construct the geodetic trajectories. He then performs the variation according to:



Die Differentialgleichungen der geodätischen Linien für das zentrische Gravitationsfeld (45) entspringen aus dem Variationsproblem

$$\delta \int \left(\frac{r}{r-\alpha} \left(\frac{dr}{dp}\right)^2 + r^2 \left(\frac{d\vartheta}{dp}\right)^2 + r^2 \sin^2\vartheta \left(\frac{d\varphi}{dp}\right)^2 - \frac{r-\alpha}{r} \left(\frac{dt}{dp}\right)^2 \right) dp = 0,$$

sie lauten nach bekanntem Verfahren:

$$\frac{r}{r-\alpha} \left(\frac{dr}{dp}\right)^2 + r^2 \left(\frac{dr}{dp}\right)^2 + r^2 \sin^2\vartheta \left(\frac{d\varphi}{dp}\right)^2 - \frac{r-\alpha}{r} \left(\frac{dt}{dp}\right)^2 = A, \tag{46}$$

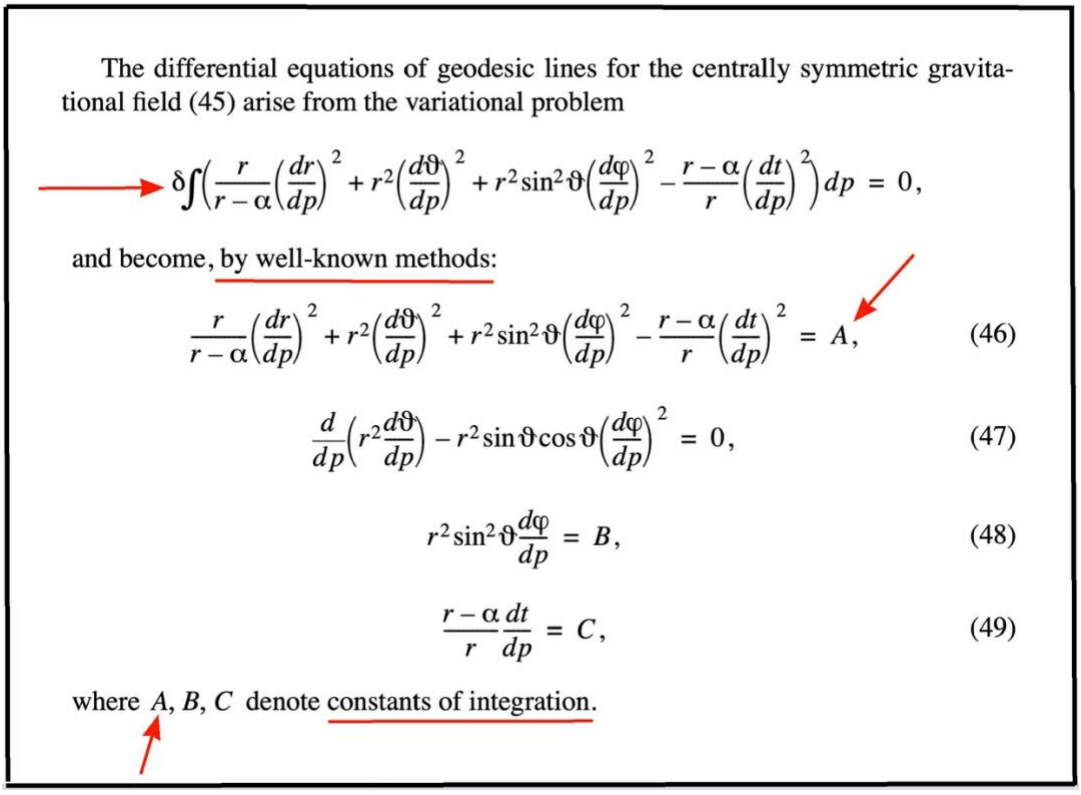
$$\frac{d}{dp}\left(r^2 \frac{d\vartheta}{dp}\right) - r^2 \sin\vartheta \cos\vartheta \left(\frac{d\varphi}{dp}\right)^2 = 0, \tag{47}$$

$$r^2 \sin^2\vartheta \frac{d\varphi}{dp} = B, \tag{48}$$

$$\frac{r-\alpha}{r} \frac{dt}{dp} = C, \tag{49}$$

wo A, B, C Integrationskonstante bedeuten.

Fig.23: Hilbert's variational approach [48]



The differential equations of geodesic lines for the centrally symmetric gravitational field (45) arise from the variational problem

$$\delta \int \left(\frac{r}{r-\alpha} \left(\frac{dr}{dp}\right)^2 + r^2 \left(\frac{d\vartheta}{dp}\right)^2 + r^2 \sin^2\vartheta \left(\frac{d\varphi}{dp}\right)^2 - \frac{r-\alpha}{r} \left(\frac{dt}{dp}\right)^2 \right) dp = 0,$$

and become, by well-known methods:

$$\frac{r}{r-\alpha} \left(\frac{dr}{dp}\right)^2 + r^2 \left(\frac{d\vartheta}{dp}\right)^2 + r^2 \sin^2\vartheta \left(\frac{d\varphi}{dp}\right)^2 - \frac{r-\alpha}{r} \left(\frac{dt}{dp}\right)^2 = A, \tag{46}$$

$$\frac{d}{dp}\left(r^2 \frac{d\vartheta}{dp}\right) - r^2 \sin\vartheta \cos\vartheta \left(\frac{d\varphi}{dp}\right)^2 = 0, \tag{47}$$

$$r^2 \sin^2\vartheta \frac{d\varphi}{dp} = B, \tag{48}$$

$$\frac{r-\alpha}{r} \frac{dt}{dp} = C, \tag{49}$$

where A, B, C denote constants of integration.

Fig. 24: Hilbert's variational approach (translation)

This part of his article [48] is perhaps the one that most deserves our attention. In 1915, Hilbert introduced [27] the variational technique into cosmology for the first time, which allowed him

to use it to construct the field equation. His genius lay in constructing the action based on the Ricci scalar, which he called *K*, and on the four-dimensional hypervolume $\sqrt{[g]}\, dx_1 dx_2 dx_3 dx_4$. The scientific community would later designate this approach as based on the "Hilbert-Einstein action." In these diagrams, the first red arrow shows that the action is constructed not on the elementary length, but using the bilinear form. Of the four equations (46), (47), (48), and (49) in Figures 23 and 24, the last three are the Lagrange equations, which are derived from the indicated variation. However, equation (46), which assumes the constancy of the bilinear form, is an additional assumption of Hilbert's, the very one that explains why his chosen action will indeed yield the geodesics, even though the action is not designed to minimize the length, but its square. See what was explained above. In the subsequent calculation, Hilbert concludes that this constant A must be negative.

Nach dem obigen Axiom soll die Weltlinie für die Bewegung eines Massenpunktes Zeitlinie sein; aus der Definition der Zeitlinie folgt mithin stets $A < 0$.

Translation :

According to the axiom above the world line for the motion of a point mass shall be a time line; from the definition of the time line it thus follows that always $A < 0$.

Fig.25: New implicit inversion of the signature of the bilinear form.

The solutions to these equations must be timelines. Now, in this same article, Hilbert defines the proper time associated with a timeline, adding in passing another "length λ" this time associated with trajectories that will later be described as "space-like":

Ist in dieser Geometrie eine Kurve

$$x_s = x_s(p) \qquad (s = 1, 2, 3, 4)$$

gegeben, wo $x_s(p)$ irgend welche reelle Funktionen des Parameters p bedeuten, so kann diese in Teilstücke zerlegt werden, auf denen einzeln der Ausdruck

a → $$G\left(\frac{dx_1}{dp}, \frac{dx_2}{dp}, \frac{dx_3}{dp}, \frac{dx_4}{dp}\right)$$

nicht sein Vorzeichen ändert: ein Kurvenstück, für welches

$$G\left(\frac{dx_s}{dp}\right) > 0$$

ausfällt, heiße eine *Strecke* und das längs dieses Kurvenstücks genommene Integral

$$\lambda = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\, dp$$

heiße die *Länge der Strecke*; ein Kurvenstück, für welches

b → $$G\left(\frac{dx_s}{dp}\right) < 0$$

ausfällt, heiße eine *Zeitlinie* und das längs dieses Kurvenstückes genommene Integral

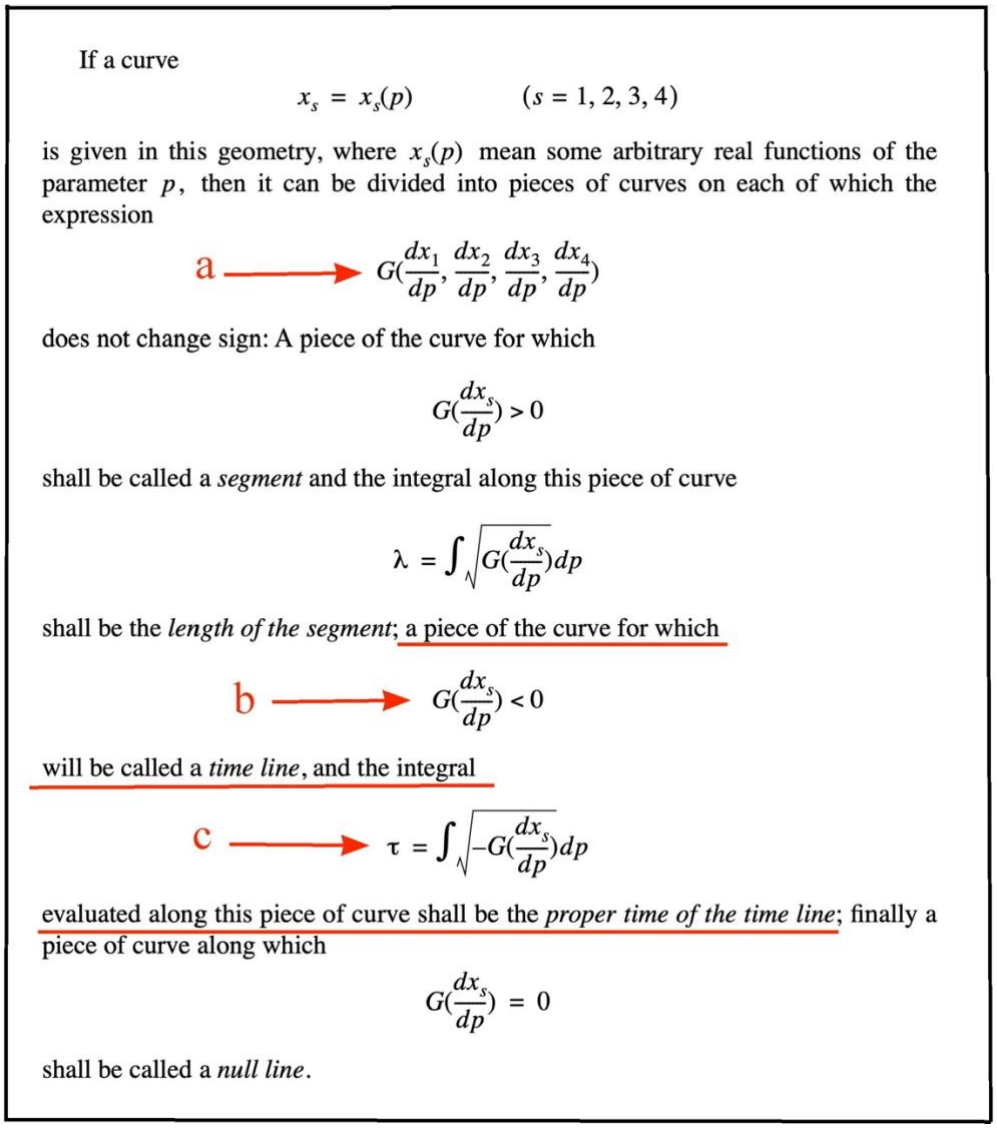
c → $$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}\, dp$$

heiße die *Eigenzeit der Zeitlinie*; endlich heiße ein Kurvenstück, längs dessen

$$G\left(\frac{dx_s}{dp}\right) = 0$$

wird, eine *Nullinie*.

Fig.26: The two lengths introduced by Hilbert [48]



If a curve

$$x_s = x_s(p) \qquad (s = 1, 2, 3, 4)$$

is given in this geometry, where $x_s(p)$ mean some arbitrary real functions of the parameter p, then it can be divided into pieces of curves on each of which the expression

a ⟶ $G(\frac{dx_1}{dp}, \frac{dx_2}{dp}, \frac{dx_3}{dp}, \frac{dx_4}{dp})$

does not change sign: A piece of the curve for which

$$G(\frac{dx_s}{dp}) > 0$$

shall be called a *segment* and the integral along this piece of curve

$$\lambda = \int \sqrt{G(\frac{dx_s}{dp})}dp$$

shall be the *length of the segment*; a piece of the curve for which

b ⟶ $G(\frac{dx_s}{dp}) < 0$

will be called a *time line*, and the integral

c ⟶ $\tau = \int \sqrt{-G(\frac{dx_s}{dp})}dp$

evaluated along this piece of curve shall be the *proper time of the time line*; finally a piece of curve along which

$$G(\frac{dx_s}{dp}) = 0$$

shall be called a *null line*.

Fig.27 : Translation.

In **a** its bilinear form **G**, which therefore corresponds to equation (76). Now, in Figure 25, he has just determined that, for these trajectories to have physical meaning, to be timelines, G is negative. We are therefore in case b. So, why introduce this catch-up c which allows us to obtain a real proper time? Wouldn't it be simpler and more logical, given this sign constraint, to return to the fundamentals introduced by Einstein, Schwarzschild, Wey, Droste, and all the others, and simply write:

(78) $$G(dr, d\theta, d\varphi, dt) = ds^2 = \frac{r-\alpha}{r}\,dt^2 - \frac{r}{r-\alpha}dr^2 - r^2 d\theta^2 - r^2 sin^2\theta\, d\varphi^2$$

with signature $(+ - - -)$

This would then align with Schwarzschild's result of January 1916 and, by taking a unit value for the speed of light as he did, would transform his expression **c** into:

(79) $$\tau = s = \int \sqrt{G\left(\frac{dx_s}{dp}\right)}\,dp$$

We see in this section that several things came into play, initially stemming from Hilbert's vision of spacetime as Euclidean geometry, suddenly accompanied by a time coordinate that reveals its relativistic nature, that of a purely imaginary entity. Then, while retaining his signature (+++), to recover a real proper time, he had to change the sign of his bilinear form to a radical sign. These are subtleties of which we find little trace in works devoted to cosmology. Adding this question of virtual geodesics, everything was then in place to give birth to a chimera: the model of the black hole. The transition from the original signature (+ + + -), where the guiding thread, allowing exploration of the solution hypersurface, defined by its metric being to stick to a real length s, to a metric, called modern (+ - - -) has caused this thread to be lost, leading to expressions like $d\tau = \sqrt{-ds^2}$ et and, on the level of a physical interpretation to this surrealistic conclusion that "inside the black hole the coordinates of time and space exchange their respective roles".

11- **Conclusion**.

This second article complements what was presented in the first [25], where the mathematical and geometric inconsistency of the black hole model was outlined. It discusses the approach that gradually brought to light the true geometric context of special relativity, Minkowski space, with its signature (+ - - -). It recalls that the requirement for subluminal speeds boils down to the imperative of considering only trajectories of real length s, that is, with $ds^2 \geq 0$. It is shown that this principle guided the approaches of all cosmologists for half a century, until the appearance of the signature (- + + +) which has now become widespread, even though, at least in cosmology, this change is not based on any ontological argument, and no article advocates such an option.

We return to the creation of the field equation by Hilbert in 1915, using the variational method. We recall that this was done within a four-dimensional framework without mentioning the particular role played by the time coordinate w_4.

Returning to the solution presented by Schwarzschild in January 1916, we recall that the spiral trajectories towards the geometric center are in fact merely an artifact resulting from an erroneous application of the solutions to Lagrange's equations, used to construct geodesics, and that these real portions of curves are endowed with an imaginary length (and therefore a proper time). We propose to designate them as virtual geodesics. To illustrate the concept, we construct the virtual geodesics of the torus and the sphere.

It is noted that while Schwarzschild spacetime geodesics are inscribed, in the planes $\theta = Cst$, on the Flamm surface, they differ from the geodesics of this surface. 3D images of spacetime geodesics are produced in a coordinate system (t, r, φ).

Finally, we examine carefully how, in his December 1916 article [48], David Hilbert attempted to integrate Schwarzschild's result, focusing exclusively on the stationary solution of the homogeneous field equation [43] and completely neglecting its logical complement, the interior metric [47]. He then treats this solution of the homogeneous field equation for a bilinear form, not as a metric; at no point in either of his articles does an equation begin with $s^2 = etc$.

This amounts to him granting himself the freedom to choose the elements of the signature, in a stationary solution with spherical symmetry, expressed using coordinates (x, y, z, l), and, considering that physical phenomena, whether special relativity or curvature effects, are merely small perturbations of a fundamentally Euclidean geometry, he opts for the signature (+ + + +). It is only after all calculations have been done that he translates his conception of relativity by giving his time coordinate l a purely imaginary nature it , thus adhering to an idea initially introduced in 1902 by Poincaré [62], taken up by Lorentz in 1904 [16] and then by Minkowski in 1907 [21]. The signature then becomes (+ + + -). Without attaching importance to the inversion of the signs, he concludes that he has thus rediscovered Schwarzschild's metric

To obtain the geodesics, he constructs the Lagrange equations using the bilinear form $G\left(\frac{dx_s}{dp}\right)$ as the Lagrange function. Since this form is constant (so that the geodesics can be derived from it and are time lines), and also negative, this observation should have led him to reverse the signs and opt for (- - - +). However, to obtain a real proper time, he chooses to add a minus sign under the radical:

$$\tau = \int \sqrt{-G\left(\frac{dx_s}{dp}\right)}\, dp$$

When, in 1916, Hilbert discovered that Einstein and Schwarzschild had produced linearized and non-linearized solutions to the field equation, he hastened to incorporate them by publishing, with the same title, a new version of his 1915 article, "Foundations of Physics." His choice to completely ignore the solution with a non-linear term suggests a certain haste on his part, inconsistent with his legendary rigor. It is worth noting that he sent this new article on December 23, 1916, Christmas Eve. The general impression is that, in this rather rough second article, two approaches clash. Clearly, Hilbert did not subscribe to this new view of the universe, as Minkowski had just specified it in 1909, which stemmed from the definition of length and was accompanied by a signature (+ - - -). It then becomes clear that for him, it is not this definition of length that structures space, but a bilinear form. What also emerges is that Hilbert considers these new contributions, such as relativity and curvature, to be only very small perturbations to a space he conceives as fundamentally Euclidean. Indeed, a phenomenon like the advance of Mercury's perihelion represents only a tiny perturbation compared to Newtonian dynamics. At that time, no one could imagine that a relativistic physics would one day emerge, where objects could move at speeds so close to that of light, like "cosmic rays," to the point that relativistic corrections must be taken into account to interpret their apparent lifespan. In 1916, astrophysicists were well aware that the Schwarzschild radius was negligible compared to the radius of the Sun, and probably of stars in general. Who would have imagined at that time that hyperdense objects, such as neutron stars, or those located at the centers of galaxies, would one day take their place in the cosmic bestiary?

The general impression, after the interlude of the Second World War, is that something was lost at that time: a certain mastery of geometry (think of the analysis published by Flamm in 1916). As noted in our two articles, this chimera of the black hole stems from a complete disregard for topological aspects, the basis of the Janus Cosmological Model, which originates in the work of the mathematician Werner Boy [63], who, like Schwarzschild, enlisted at the outbreak of the First World War and died almost immediately in France on September 6, 1914. If these two had remained alive, the face of science would undoubtedly have been different. Everything suggests that after the Second World War, those who called themselves cosmologists simply reproduced

these different elements identically, including the inversion of the signature, as if they were simple words of the Gospel, completely missing "the train of topology".

References.

[1] Huygens C. Traité de la lumière, où sont expliquées les causes de ce qui lui arrive dans la réflexion & dans la réfraction. Edité par Pieter van der Aa, libraire-éditeur. 1690.
[2] Newton, I. (1704). *Opticks*. London: Samuel Smith & Benjamin Walford.
[3] Young, T. (1802). *On the theory of light and colours* Philosophical Transactions of the Royal Society of London, 92, p. 12–48.
[4] Fraunhofer, J. (1814) *Bestimmung des Brechungs- und Farbenzerstreuungs-Vermögens verschiedener Glasarten* Denkschriften der Königlichen Akademie der Wissenschaften zu München. 5, p. 193–226.
[5] Fresnel A.J. Mémoire sur la diffraction de la lumière *Annales de chimie et de physique*, 11 (1819), p. 246–296.
[6] Bessel, F. W. (1838) *Bestimmung der Entfernung des 61sten Sterns des Schwans* Astronomische Nachrichten, 16, p. 65–96.
[7] Christian Doppler C. (1842). *Über das farbige Licht der Doppelsterne und einiger anderer Gestirne des Himmels* Abhandlungen der Königlichen Böhmischen Gesellschaft der Wissenschaften, 2, p. 465–482.
[8] Buys Ballot, C. (1845) *Akustische Versuche auf der Niederländischen Eisenbahn* Poggendorffs Annalen der Physik, 66, p. 321–351.
[9] Maxwell J.C. (1865). A Dynamical Theory of the Electromagnetic Field *Philosophical Transactions of the Royal Society of London*, **155**, p. 459–512.
[10] Fizeau H. *Sur l'effet Doppler dans les phénomènes lumineux* Comptes rendus hebdomadaires des séances de l'Académie des sciences, 26, p. 342–345.
[11] Huggins, W. (1868) *On the motions of some of the stars towards or from the Earth* Proceedings of the Royal Society of London, 16, p. 318–331.
[12] Michelson, A. A., & Morley, E. W. (1887). *On the Relative Motion of the Earth and the Luminiferous Ether*. American Journal of Science, 34, p. 333–345.
[13] Michelson, A.A. & Morley, E.W. (1887) *On the relative motion of the Earth and the luminiferous ether* American Journal of Science, 3rd Series, volume 34 (203), pages 333–345 (Novembre 1887). DOI : 10.2475/ajs.s3-34.203.333
[14] Lorentz, H. A. (1892) *La théorie électromagnétique de Maxwell et son application aux corps en mouvement* Archives Néerlandaises des Sciences Exactes et Naturelles, 25, p. 363–552.
[15] Lorentz, H. A. (1895) *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern*
Leiden: Livre édité par E. J. Brill.
[16] Lorentz, H. A. (1904) *Electromagnetic phenomena in a system moving with any velocity smaller than that of light* Proc. Royal Netherlands Academy, 6, 809–831.
[17] Einstein A. Zur Elektrodynamik bewegter Körpner.Annalen der Physik zei4 Vol 17 (1905) pp/ 89-92
[18] Poincaré H. *Sur la dynamique de l'électron. Rendiconti del Circolo Matematico di Palermo.* 23 juillet 1905 publicated : Dec 1905 / jan 1906 DOI: https://doi.org/10.1007/BF03013466
[19] Riemann, B. *Über die Hypothesen, welche der Geometrie zu Grunde liegen* (« Sur les hypothèses qui servent de fondement à la géométrie »). Publié de manière posthume

en 1867 dans la revue Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen, vol.13 pp 133-150
[20] Lie, S. & Engel, F. (1888–1893). *Theorie der Transformationsgruppen*.Leipzig: B. G. Teubner.
[21] Minkowski H. (1907) Die Grundgleichungen für die elektromagnetischen Vorgänge in bewegten Körpern (*Les équations fondamentales des phénomènes électromagnétiques dans les corps en mouvement*) Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse,
[22] Minkowski, H. (1909). *Raum und Zeit (space and time)*. *Physikalische Zeitschrift*, 10, 75–88. (conference in 1908, publication in 1909)
[23] Sylvester, J. J. (1852) *On the Law of Inertia of Quadratic Forms* Philosophical Magazine**,** 4e série**,** 4, p. 138–142.
[24] Gram, J. P. (1883) *Über die Entwicklung reeller Funktionen in Reihen mittelst der Methode der kleinsten Quadrate* Journal für die reine und angewandte Mathematik, 94, p. 41–73.
[25] J-P Petit & G. D'Agostini. Mathematical and geometrical inconsistency of the black hole model. Journal of Modern Physics, &&& 2026.
[26] D.Hilbert. Die Grundlagen der Physik (Esrte Mitteilung). Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse. (20 novembre 1915) 395-407.
[27] D.Hilbert. The foundations of Physics (first communication). English translation. Boston Series in Philosophy and Science. Copyright Springer, Vol.250 2007 https://link.springer.com/chapter/10.1007/978-1-4020-4000-9_44
[28] A.Einstein : Die feldgleichungen der gravitation Sitzungsberichte der Königlich Preubishen 25 November 1915
[29] Albert Einstein – David Hilbert: Briefwechsel (correspondance). Renn Jürgen and Sauer Tilman. *Wissenschaftliche Korrespondenz.* Springer-Verlag (Vieweg) Braunschweig / Wiesbaden. 1997.
[30] Renn Jürgen **:** The Genesis of General Relativity. Springer, 2007
[31] J.M.Souriau : Structure des systèmes dynamiques. Dunod Ed. France, 1970 and Structure of Dynamical Systems. Boston, Birkhaüser Ed. 1997
[32] Kostant B, "Quantization and Unitary Representations" *Lectures in Modern Analysis and Applications III*, Lecture Notes in Mathematics 170, Springer (1970).
[33] Kostant B. On the Definition of Quantization. *Géométrie symplectique et physique mathématique*, Coll. CNRS n°237, Paris (1975), p. 187–210.
[34] Kirillov A. Unitary representations of nilpotent Lie groups. *Russian Mathematical Surveys* **17**(4) (1962), 53–110
[35] Kirillov A. Merits and Demerits of the Orbit Method. *Bulletin of the AMS* (N.S.) **36** (1999), 433–488
[36] J-P Petit (1981). http://www.savoir-sans-frontieres.com/JPP/telechargeables/English/EVERYTHING_IS_RELATIVE.pdf
[37] J-P Petit, H.Zejli : Study of symmetries through the action on torsors of the Janus symplectic group. Reviews in Mathematical Physics. Vol. 37, n001, 2024.
[38] J-P Petit, F.Margnat, H.Zejli : A bimetric cosmological model on Andreï's twin universe approach. Th European Physical Journal. Vol. 84 :N°1126 (2024)
[39] J-P Petit , G.d'Agostini. Alternative to black holes, Gravastars and Plugstars. Journal of Modern Physics. Vol. 16, n°10. October 2025.
[40] J-P Petit, Topo the World, 1985. http://www.savoir-sans-frontieres.com/JPP/telechargeables/English/Topo_the_world_eng.pdf

[41] J-P Petit : The missing mass problem. Il Nuovo Cimento B Vol. 109 July 1994, pp. 697-710
[42] A.Einstein : Erlklärung der Perihelbewegung des Merkur aus allgemeinen Relarivitätstheorie. Königlish Preubische Akademie der Wissenschaften (Berlin) Sitzungberichet (25 nov. 1915) p. 831-838
[43] K. Schwarzschild : Über das Gravitationsfeld Messenpunktes nach der Einsteinschen Theorie. Sit. Deut. Akad. Wiss. 1916English translation by S.Antoci and A.Loinger. On the gravitational field of a mass point according to Einstein theory. arXiv :physics/9912033v1 [physics.hist-ph] 16 dec 1999.
[43] Le Verrier U. (1846) *Détermination de la position de la planète qui produit les perturbations d'Uranus* Comptes rendus**,** 23, p. 914–918. (1846)
[44] Le Verrier, U. *Théorie du mouvement de Mercure*. Comptes rendus hebdomadaires des séances de l'Académie des sciences**,** 49, p. 379–383. (1859)
[45] Le Verrier U. *Sur une planète qui pourrait exister en deçà de l'orbite de Mercure* Comptes rendus hebdomadaires des séances de l'Académie des sciences**,** 50, p. 7–13. (1860)
[47] K. Schwarzschild **:** Über das Gravitionalsfeld einer Kugel Aus incompressibler Flüssigkeit nach der Einsteinschen Theorie. Sitzung der phys. Math. Klasse v.23 märz 1916. English translation, by S.Antoci. On the gravitational field of a sphere of incompressible fluide according to Einstein theory. arXiv :physics/9905030v1 [physics.hist-ph] 12 may 1999.
[48] D.Hilbert. Die Grundlagen der Physik (Esrte Mitteilung). Nachrochten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-physikalische Klasse. (1916) . The foundations of Physics (second communication). English translation. Boston Series in Philosophy and Science. Copyright Springer, Vol.250 2007 https://link.springer.com/chapter/10.1007%2F978-1-4020-4000-9_45
[49] H.Weyl : Zu Gravitationstherie. Annalen der physik, Vol.35 n° 18 (1917) Reprinted: On the Theory of Gravitation. Reprinted in Gen. Relat. And Gravit. Vol.44, pp.779-810
[50] Ludwig Flamm. Beiträge zur Einsteinschen Gravitationstheorie, Physikalische Zestschrift XVII (1916) pp. 448-454.
[51] J.Droste – The field of a single centre in Einstein's theory of gravitation and the motion of a particle in that field. Ned. Acad. Wer. SA 1917 lutecium.fr. KNAW, Proceedings 19 I , 1917, Amsterdam, pp. 197-215
[51] J-P Petit, G.D'Agostini. Mathematical and geometrical inconsistency of the black hole model. Journal of Modern Physics &&&
[52] S.Chandrasekhar :The Mathematical Theory of Black Holes. Oxford University Press 1992
[53] R.Adler, M.Bazin & M.Schiffer : Introduction to General Relativity Mc Graw Hill Book 1965-1975. Equation (6.90).
[54] Ludwig Flamm. Beiträge zur Einsteinschen Gravitationstheorie, Physikalische Zestschrift XVII (1916) pp. 448-454.
[55] P.Koiran, H.Zejli, J.P.Levy, F.Margnat, MF. Duval : PT-symmetry in one-way wormholes. Annals of Physics, Vol. 470, November 2024, 169765
[56] L.S.Abrams : Black holes: the legacy of Hilbert's error. Phys.Rev.D20, 2474 (1989) arXiv :gr-qc/0201044
[57] A.Loinger : The black holes do not exist, "also sprach Karl Scwarzschild" (Thus spoke Karl Schwarzschild). arXiv:physics/04020088v1/(physics.nen.ph/ 18 Feb 2004

[58] C.Corda : A clarification on the debate on « the original Schwarzschild solution » (2011) 62-82. arxiv :1010 :6031v5 (gr-qc) 25 mar 2011.
[59] R.Dijgraaf. The black hole is the atom of the 21 st century. Publication of the Institute for Advanced Studies of Princeton. https://www.ias..edu/ideas/dijkgraaf-EHT-black-hole.
[60] First M87 Event Horizon Telescope Results. I.. The Shadow of the Supermassive Black Hole. K. Akiyama, A.Alberdi,W.Alef,K.Asada, R.Azulay, A.K.Bcako, D.Ball,M. Balakovic, J. Barrett. Astrophysical Journal, vol. 875, n°1,2019
[61] J.P.Petit , G.d'Agostini/ Alternative to black holes, Gravastars and Plugstars. Journal of Modern Physics. Vol. 16, n°10. Octobre 2025.
[62] Henri Poincaré **:** La mesure du temps"**,** *Revue de métaphysique et de morale, vol. 10, 1902, pp. 1–13*
[63] W.Boy :Über die curvatura integra und die Topologie geschlossener Flächen. Dissertation, Georg-August-Universitât, Gottingen, 1901

Remerciement.

Je remercie Jean-Marc Pechinot pour la réalisation d'images illustrant cet article.

As we have seen, I first introduced this idea of "variable-constant" evolution in an article published in 1988 in Modern Physics Letters A,

... 38 years ago.

A second, more comprehensive paper appeared in *Astrophysics and Space Science* in 1995,

27 years ago.

Why has this theme of "variable constants" failed to resonate within the scientific community?

In 2003, the Portuguese scientist João Magueijo published *Faster than the Speed of Light: The Story of a Scientific Speculation* in the U.S. with Basic Books, followed by *Plus rapide que la Lumière* in England with H. Henemann Ed—also in 2003—published by Dunod.

In 2006, he became a full professor at Imperial College in England.

He, like all those who have attempted to vary the speed of light (the other being the American Moffat), encountered two problems.

The Lorentz metric, which structures the space of special relativity—Minkowski space—is:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

What we need to understand is that this fourth dimension—let's call it:

$$x^0 = ct$$

It is measured, like the others, in meters! That was Minkowski's brilliant idea in 1909 (four months before his death):

Time is measured in meters!

This space has an isometry group that preserves length *s*. Within this group, the Lorentz subgroup is the 4D equivalent of the 3D Euclidean rotation group.

In 3D, you have the length x, the width y, and the height z. Imagine that you were to change z, or any one of these three dimensions. Rotation would be impossible.

It's the same in Minkowski space. If we vary c, then the coordinate x^0 is altered. 4D rotations become impossible.

To preserve these rotations, we would have to modify x, y, and z—introduce a "spatial gauge." Let's call it a. But, why not go all out? Shall we also introduce a time gauge? Let's call it T.

From the outset, all of this had been considered in the 1988 model. Yet today, no one has understood it.

But there's something else: the fine-structure constant, which is supposed to be an absolute constant.

$$\alpha = \frac{e^2}{4\,\pi\,\varepsilon_o \hbar\, c} \approx \frac{1}{137}$$

If we tamper with this constant… atoms can no longer form!!!

What do we find there?

e: elementary electric charge.

ε_o : dielectric constant of a vacuum

$\hbar$: Planck's constant

c : speed of light

Solution: Let's vary everything (JP Petit 1988, 38 years ago). Let's introduce a "generalized gauge theory" in which all constants vary, along with the space and time gauges.

To remain within the realm of physics: ensure that all equations of physics remain invariant.

We can then express, within these N quantities, the variations of N-1 of them in terms of the Nth.

Since we want to account for variations as a function of spatial expansion, we'll take *a* as *the guiding quantity*. We then obtain (since 1988–1995):

$$c = \hat{c}\, c_0$$

$$h = \hat{h}\, h_0$$

$$G = \hat{G}\, G_0$$

$$m = \hat{m}\, m_0$$

$$e = \hat{e}\, e_0$$

$$\varepsilon_0 = \hat{\varepsilon}_0\, \varepsilon_{0-act}$$

$$\mu_0 = \hat{\mu}_0\, \mu_{0-act}$$

$$a = \hat{a}\, a_0$$

$$t = \hat{t}\, t_0$$

$$v = \hat{v}\, v_0$$

We have before us all the constants of physics. But there is one too many, due to the relationship:

$$c = \frac{1}{\sqrt{\varepsilon_0\, \mu_0}}$$

Well… let's choose to eliminate ε_0

The speeds also vary$v$ like the speed of light *c,* to ensure the constancy of the geometric factor:

$$\sqrt{1 - \frac{v^2}{c^2}}$$

We are left with:

$$c = \hat{c}\, c_0$$

$$h = \hat{h}\, h_0$$

$$G = \hat{G}\, G_0$$

$$m = \hat{m}\, m_0$$

$$e = \hat{e}\, e_0$$

$$\mu_0 = \hat{\mu}_0\, \mu_{0-act}$$

$$a = \hat{a}\, a_0$$

$$t = \hat{t}\, t_0$$

The values with index zero are the "present-day" values. For this quantity a, "traditionally," this index is zero. Therefore, it will μ_{0-act} mean "the current value of μ_0»

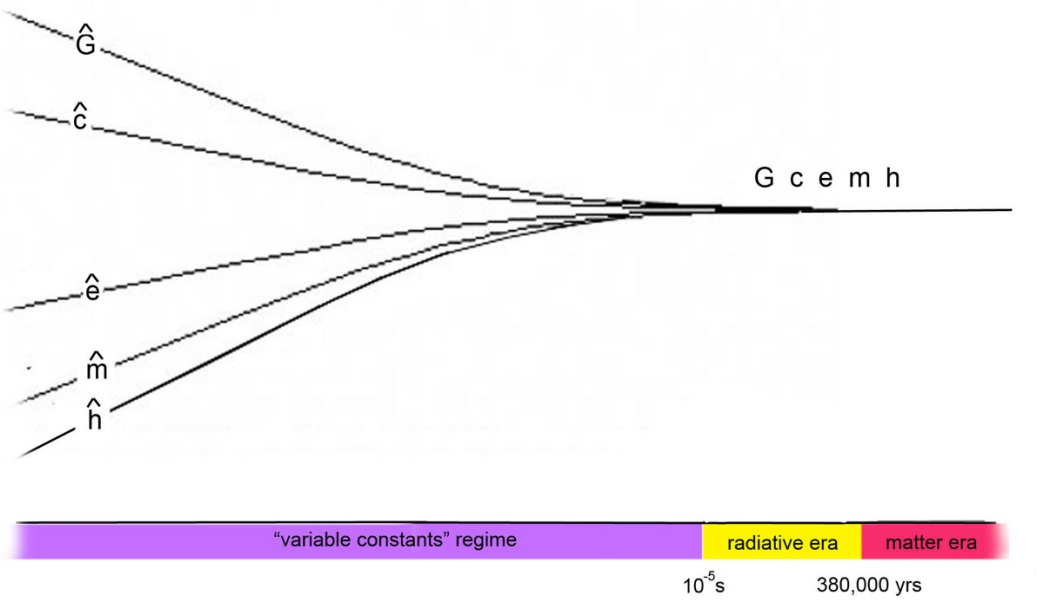
Constructing this generalized gauge theory will consist of linking these quantities together:

$$\left(\hat{c}\, , \hat{h}\, , \hat{G}\, , \hat{m}\, , \hat{e}, \hat{\mu}_o\, , \hat{a}\, , \hat{t}\, , \hat{v}\ \right)$$

by making them vary "jointly." The result is (1988–1995):

$$\hat{c} \sim \frac{1}{\sqrt{a}} \; ; \; \hat{h} \sim a^{3/2} \; ; \; \hat{G} \sim \frac{1}{a} \; ; \; \hat{m} \sim a \; ; \hat{e} \sim \sqrt{a} \; ; \; \hat{\mu}_0 \sim a \; ; \; \hat{t} \sim a^{3/2}$$

Here is how these "constants" evolve:



Schematic evolution of the "constants"

This evolution occurs before the era dominated by matter and before the era dominated by radiation. Afterward, the values stabilize at the values we know today.

Well, you might ask, if these variations go hand in hand with the invariance of the equations of physics, what exactly are we measuring?

As the article points out, measuring instruments are built based on the equations. And the equations reflect invariances. We find ourselves in a situation akin to physicists trying to measure the expansion of an iron table by taking a measurement with a ruler made of the same metal!

So we can no longer measure anything. Except this: from this world woven from intertwined invariants, to bring to light one that had eluded us.

The cosmological horizon:

$$H = \int^t c \, dt$$

But in this different evolution, dominated by this gauge process:

$$\hat{c} \sim \frac{1}{\sqrt{a}} \sim \hat{t}^{-1/3}$$

So, without this variable speed-of-light regime:

$$H = \int^{\hat{t}} \hat{c}\, d\hat{t} \sim \int^{\hat{t}} \hat{t}^{-1/3}\, d\hat{t} \sim \hat{t}^{2/3} \sim a$$

$$\boxed{Cosmological\ horizon\ H \sim a}$$

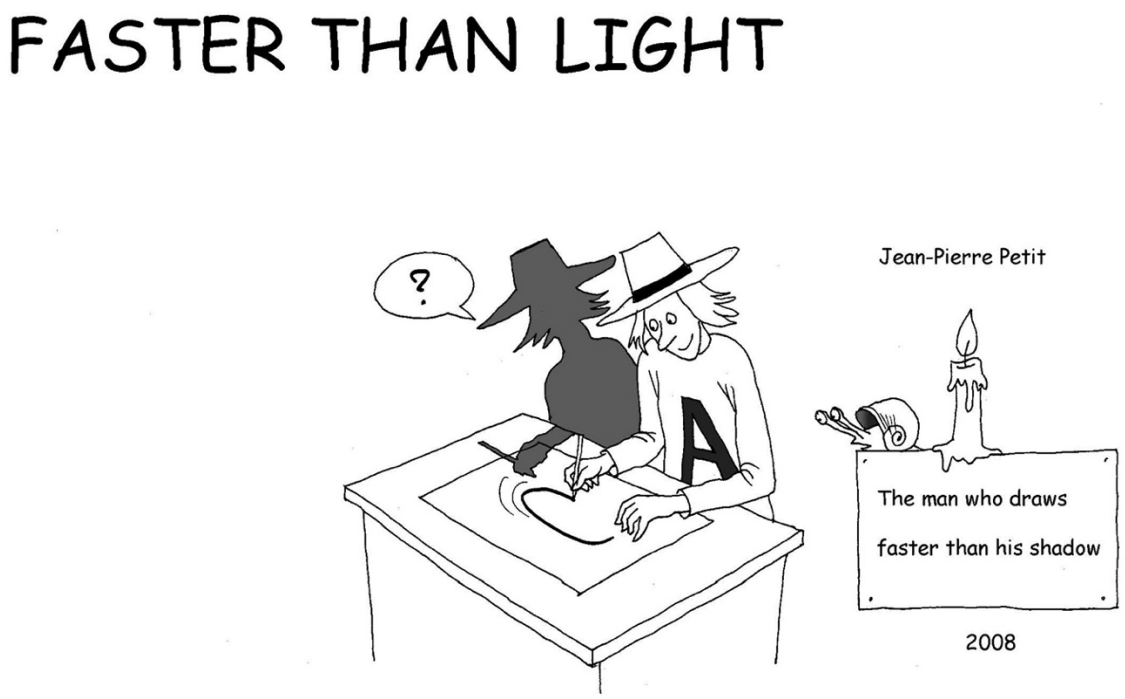
Observable: the extreme homogeneity of the early universe.

This is therefore an alternative to the inflationary model.

When will this work be "recognized" by the scientific community?

I created a comic book centered on this theme[98]. Its title:

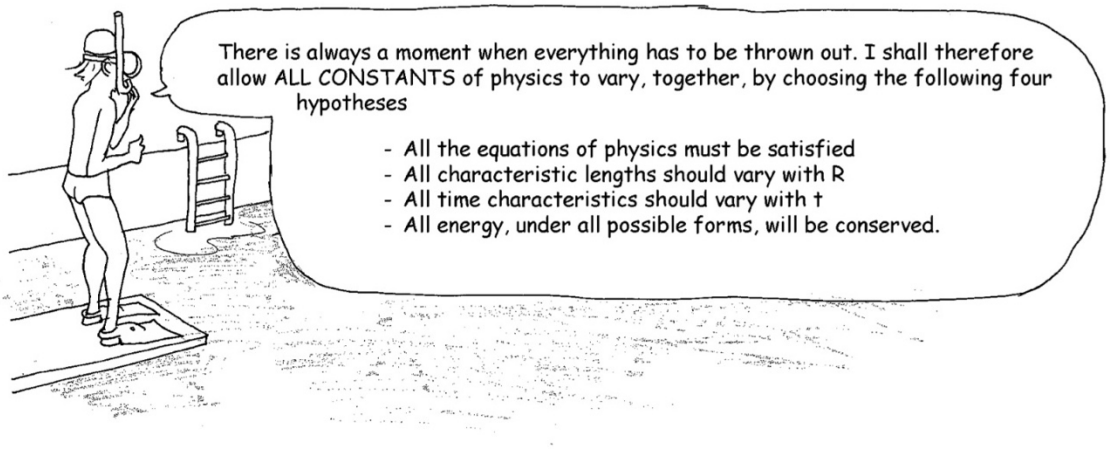
http://www.savoir-sans-frontieres.com

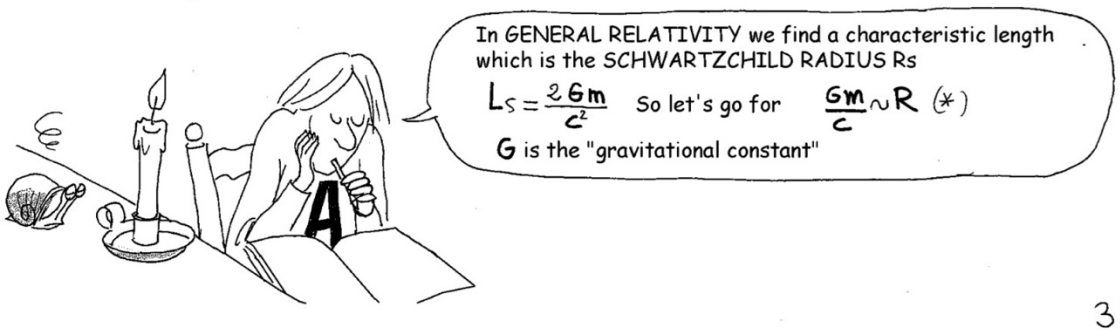


Comic Book "Faster Than Light"

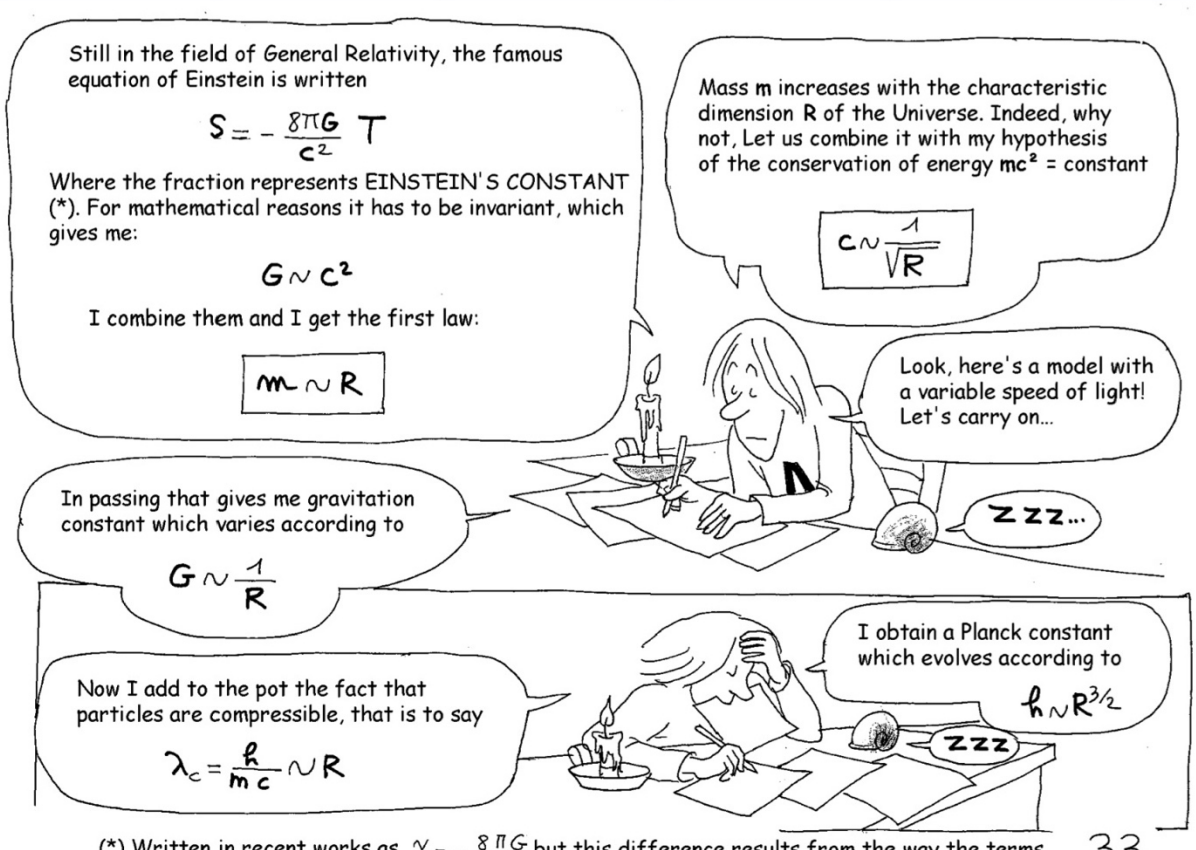
It has been translated into Portuguese. I asked Joao (who has become a national hero in Portugal) to write the foreword. He hasn't responded. A page from this comic:

[98] http://www.savoir-sans-frontieres.com/



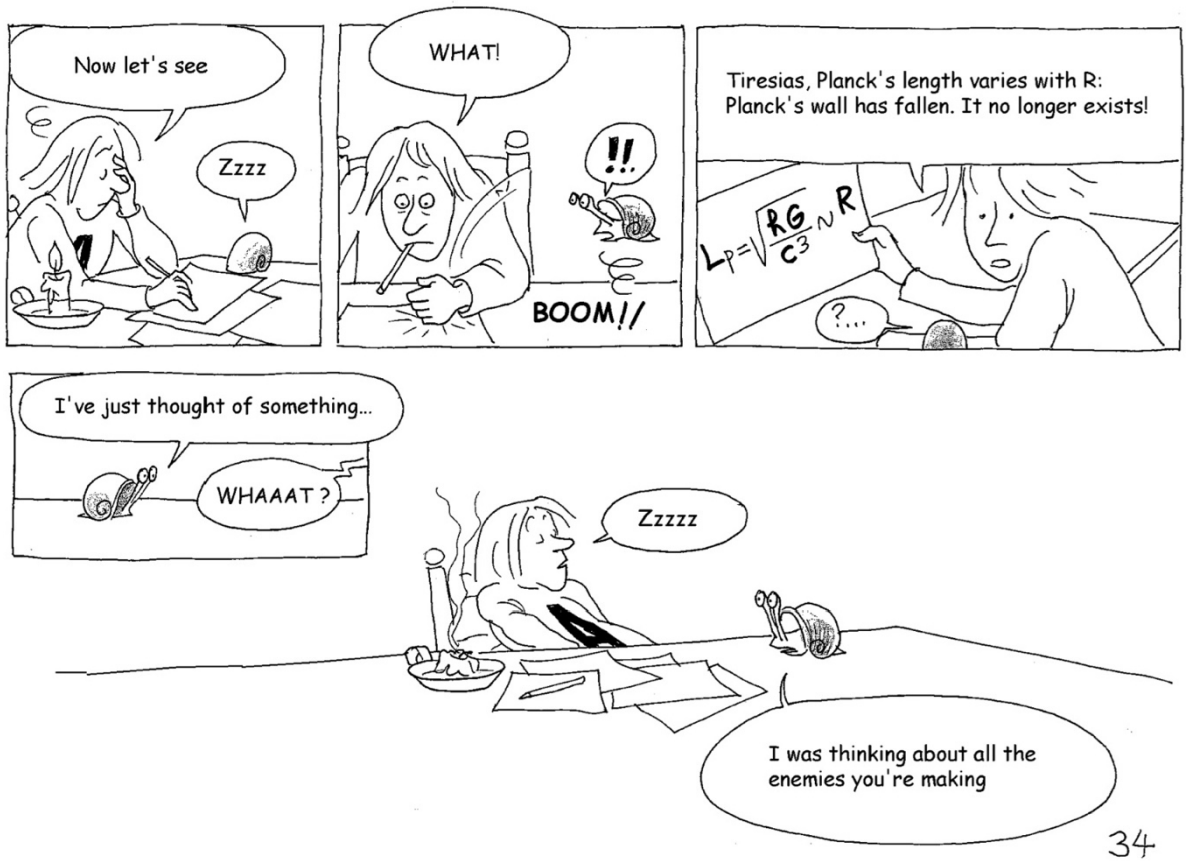


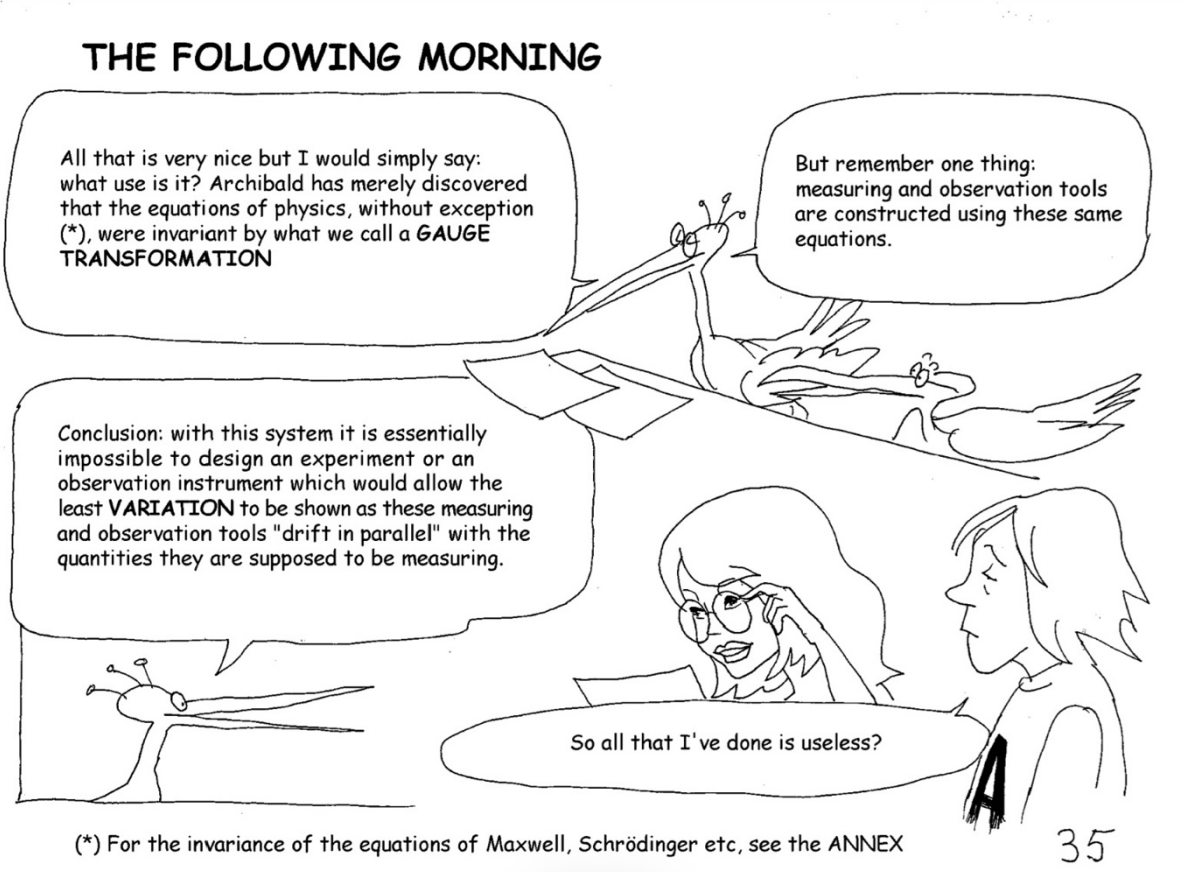
32



(*) Written in recent works as $\chi = -\frac{8\pi G}{c^4}$ but this difference results from the way the terms of the tensor T are written.

33


Now let's see
Zzzz
WHAT!
!!
BOOM!!
Tiresias, Planck's length varies with R:
Planck's wall has fallen. It no longer exists!
$L_P=\sqrt{\frac{RG}{c^3}}\sim R$
?...
I've just thought of something...
WHAAAT ?
Zzzzz
I was thinking about all the enemies you're making
34


THE FOLLOWING MORNING
All that is very nice but I would simply say: what use is it? Archibald has merely discovered that the equations of physics, without exception (*), were invariant by what we call a GAUGE TRANSFORMATION
But remember one thing: measuring and observation tools are constructed using these same equations.
Conclusion: with this system it is essentially impossible to design an experiment or an observation instrument which would allow the least VARIATION to be shown as these measuring and observation tools "drift in parallel" with the quantities they are supposed to be measuring.
So all that I've done is useless?
(*) For the invariance of the equations of Maxwell, Schrödinger etc, see the ANNEX
35

http://www.savoir-sans-frontieres.com

(knowledge without borders)

It made sense to extend this model to a bimetric universe. I tried to do so, on several occasions, in various journals. Generally, the editors refused to submit it to their reviewers.

- Variable constants?… a Frenchman… unknown… no affiliated lab (no lab email address)… 1937: so he's approaching 90… Oh my!

Straight into the trash!

A single mention in a "prestigious journal" in 2025.

First reviewer: rejection

Second reviewer: rejection

I request a third reviewer, arguing that the first two did not read the article.

The journal finds a third reviewer: rejection… the journal closes the review process. Months wasted.

I notify *The Journal of Modern Physics*. As with my other articles, this one falls "outside the mainstream." But the editor manages to find two reviewers who examine the article carefully, ask for explanations—which I provide. Most importantly: they grasp the significance of the work.

Everything went… as expected, and the paper has just been published.

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(10), 1461-1487
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Alternative to Inflation. Variable Speed of Light Cosmological Model with Lorentz and Fine Structure Constant Invariance Maintained

Jean-Pierre Petit, Gilles D'Agostini

Manaty Research Group, Glanon, France
Email: jppetit1937@yahoo.fr

How to cite this paper: Petit, J.-P. and D'Agostini, G. (2026) Alternative to Inflation. Variable Speed of Light Cosmological Model with Lorentz and Fine Structure Constant Invariance Maintained. *Journal of Modern Physics*, **17**, 1461-1487.
https://doi.org/10.4236/jmp.2026.1710061

Received: June 7, 2026
Accepted: October 25, 2026
Published: October 28, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

Within the framework of the Janus cosmological model, we investigate an alternative to the inflationary scenario based on the existence of an early phase preceding decoupling and the radiation-dominated era. During this phase, the six fundamental physical constants—along with the space and time gauge factors associated with the model's two sheets—evolve in a correlated manner through a generalized gauge process. This evolution is constrained by the invariance of the fundamental equations of physics, the Einstein constant appearing in the coupled field equations, and the fine-structure constant, while Lorentz invariance is preserved in both sheets. We show that this generalized gauge process is consistent with the conservation of various forms of energy and implies that characteristic physical lengths evolve in proportion to the corresponding spatial scale factors—notably the cosmological horizons. Since radiation itself acts as a source of the gravitational field, Jeans instability can occur during the radiation-dominated era. Due to the intrinsic asymmetry between the two sheets of the Janus model, their cosmological horizons evolve on vastly different scales. Consequently, fluctuations developing in the negative-mass sector can affect the positive-mass sector during its subsequent matter-dominated era. This mechanism provides a possible alternative origin for the primordial fluctuations observed in the cosmic microwave background (CMB), without invoking an inflationary phase. A quantitative test of this scenario requires determining the spectrum of non-linear fluctuations generated in the negative-mass sector, establishing their transfer to the positive-mass sector, and then comparing the resulting spectrum with CMB observations. This asymmetry also leads to a higher speed of light in the negative-mass sector. Should a material object transition to this sector by inverting the sign of its mass, this property would open up the possibility of significantly reducing interstellar travel times as measured in the positive sector. In this context, we outline a geometric mechanism for transition between the two sheets, based on a localized concentration of energy around the object in question.

Keywords

Lorentz Invariance, Variable Speed of Light, Variable Constants Regime, Radiative Era, Compton Length, Inflation, Generalized Gauge Phenomenon

1. Introduction

It is no exaggeration to say today that cosmology and astrophysics are facing an unprecedented crisis. For decades, efforts were made to salvage the Standard Model by augmenting it with two new components: dark matter and dark energy. By reintroducing the cosmological constant Λ into the general relativity equation, equating it to a constant negative energy density, the model was able to account for a constant acceleration of expansion, implying exponential growth over time. However, in 2025, initial results reported by the DESI collaboration [1] suggest that this acceleration was greater in the past—a topic that remains a subject of debate within the scientific community. This represents a phenomenon that the Standard Model cannot explain. In contrast, the Janus Cosmological Model (JCM), based on a system of coupled field equations, has predicted this cosmic dynamic since 2014 [2], relying on a twin-universe model characterized by a strong asymmetry between its two components.

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\chi\left(T_{\mu\nu}+\sqrt{\frac{\overline{g}}{g}}K_{\mu\nu}\right) \tag{1}$$

$$\overline{R}_{\mu\nu}-\frac{1}{2}\overline{R}\overline{g}_{\mu\nu}=-\chi\left(\overline{T}_{\mu\nu}+\sqrt{\frac{g}{\overline{g}}}\overline{K}_{\mu\nu}\right) \tag{2}$$

Presented below is the expansion law, the exact solution to the system of JCM field equations [2].

$$a(u)=\alpha^2\cosh^2 u \tag{3}$$

$$t(u)=\alpha^2\left(1+\frac{\mathrm{sh}2u}{2}+u\right) \tag{4}$$

On **Figure 1** the corresponding curve:

It should be noted that this solution, in which the negative energy density decreases over time, yields an expansion that gradually approaches a linear time dependence, whereas in the ΛCDM model—where dark energy density is invariant—the expansion is exponential. **Figure 2** shows the evolution of the acceleration:

It is well known that, in the late 1980s, the discovery of the extreme homogeneity of the early universe—specifically the CMB—immediately posed a major problem. **Figure 3** illustrates the situation in which the distance traveled by the electromagnetic wave—propagating at a constant speed creaches neighboring particles only after a certain time t_h.

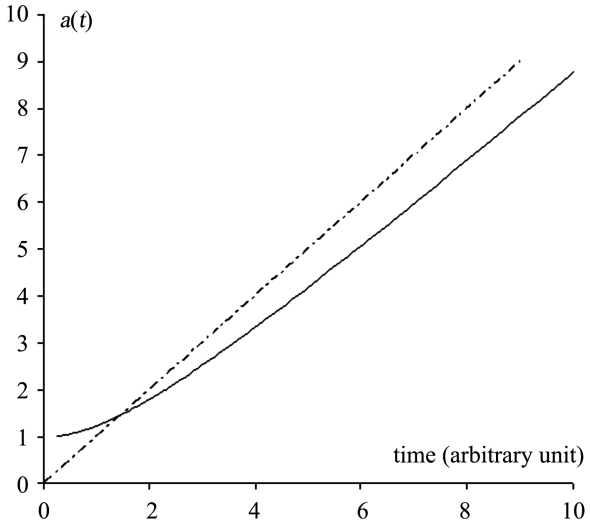


Figure 1. Janus expansion law, in the form of an exact solution [2].

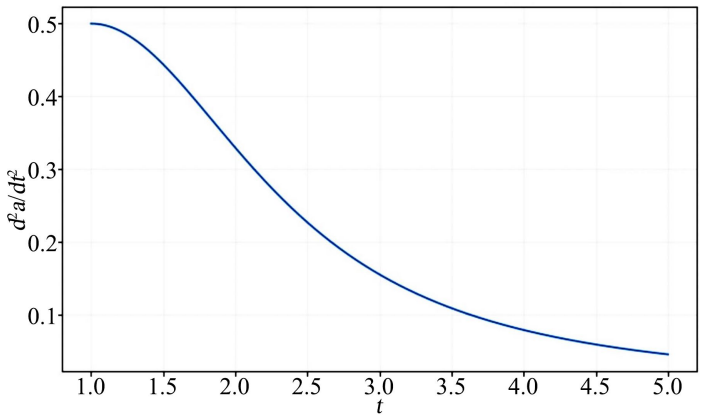


Figure 2. Evolution of acceleration [2].

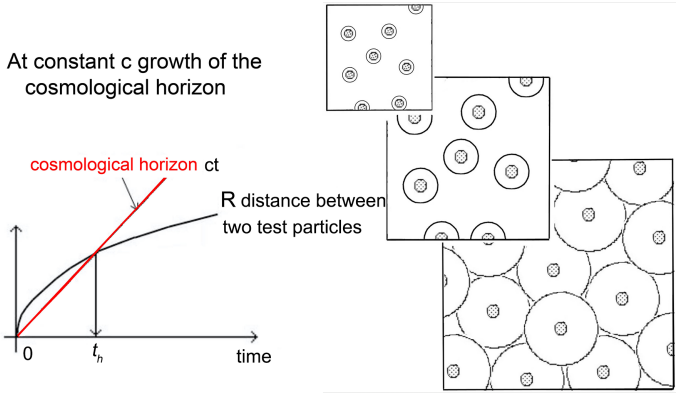


Figure 3. The problem of the homogeneity of the early universe.

For more than three decades, the inflation model has served as the scientific community's answer to this problem, forming part of the standard cosmological scenario. Between 10^{-36} and 10^{-32} seconds, the universe, thought to be populated by inflatons, is believed to have undergone an expansion by a factor of 10^{26}. At the

end of this phase, these inflatons would have given rise to the menagerie of particles we know today. Unfortunately, there is no complete description of this entire scenario.

An alternative solution would be to consider that the speed of light had a higher value in the past, in order to account for such homogeneity. However, such variability in this fundamental physical parameter immediately breaks the invariance of the fine-structure constant, which is then no longer guaranteed.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{5}$$

The entire field of atomic physics then immediately suffers an impact that is difficult to manage.

There is another objection, of a purely geometric nature. Special relativity imposes local invariance under the Lorentz group. This is a subgroup of the Poincaré group, the isometry group of Minkowski space. Relativistic length is given by the Lorentz metric:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) \tag{6}$$

Its invariance is thus immediately broken if one allows c to vary. To draw a comparison: imagine that one of the dimensions of 3D Euclidean space were affected by a gauge variation. It would then no longer be possible to perform a rotation within it. The rotation group would "cease to function". A variation in c has a similar impact in 4D, with the Lorentz group "ceasing to function". In such an affected spacetime, it becomes impossible to perform those 4D rotations that constitute the very source of special relativity.

As early as 1988 [3] [4], the idea was proposed of varying a certain set of constants, but all of them simultaneously. There are six physical constants:

- Speed of light: c
- Planck's constant: h
- The constant of Gravitation: G
- Unit electrical charge: e
- Magnetic permeability of free space: μ_0
- The unit masses m

The dielectric constant of the vacuum ε_0 can be derived from the relation:

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \tag{7}$$

Consequently, if simultaneous variations of μ_0 are introduced, the quantity ε_0 must then be treated as an absolute constant (an arbitrary choice, given that one could just as well take ε_0 as variable and μ_0 as constant).

But that is not enough. Lorentz invariance must still be maintained. This can only be achieved by introducing a gauge phenomenon that affects space and time variables alike. Let us now look for guiding ideas that would allow us to envisage the early universe undergoing a phase of evolution characterized by "variable constants and space-time gauge factors".

2. The Principles behind the Construction of Such a Model

A first idea, in the search for a generalized gauge phenomenon, is to construct it in such a way that all the equations of physics remain invariant—namely, the field equation and the equations of quantum mechanics, electromagnetism, and fluid mechanics.

This framework leads us to consider the state of the universe in the distant past. Directly usable information, carried by light, only becomes available after decoupling, at t = 380,000 years. For the period prior to this, we can only make inferences based on what we observe today. In 1920, the discovery of cosmic expansion by E. Hubble and the solutions provided by A. Friedmann compelled scientists to consider—by tracing time backward—that the universe had once been much hotter and denser. The model showed that photon wavelengths evolved in step with the cosmic scale factor, *a*. Observations had demonstrated that high-energy photons could give rise to matter-antimatter pairs. This scenario was vividly described by S. Weinberg in his famous book "The First Three Minutes" [5]; he depicts the universe as filled with "all sorts of radiation"—using the plural to indicate that massive components (both matter and antimatter) were moving with thermal agitation speeds approaching the speed of light.

It was here that the standard cosmological model found an observational anchor, aligning with the discovery of cosmic expansion. The universe—teeming with high-energy photons—was constantly giving rise to matter-antimatter pairs; these particles, bearing opposite electric charges and thus attracting one another, would quickly annihilate each other a short distance away, transforming back into radiation. Consequently, as the photons lost energy simply due to the expansion, they were no longer able to compensate for the losses caused by annihilation. This explains the existence of the 2.7 K background radiation, serendipitously detected by Penzias and Wilson in 1967. Detecting this "ash" from cosmic expansion reinforced the hypothesis of a primordial explosion, while also making it possible to trace events back to $t = 10^{-2}$ second.

It is worth noting in passing that this advance in knowledge—this ability to look further back into the past—brought with it a new and significant problem: why did one particle in a billion remain? Why was the annihilation of this primordial matter and antimatter not total? It is a question that, to this day, still awaits a satisfactory answer. Setting this new problem aside, a question then arises:

- *And before* $t = 10^{-2}$ *second*?

On what factual evidence can a description of cosmic history in an earlier era be based? Scientists envisioned that experiments conducted in their particle accelerators would enable them to propose answers to this question. These experimental devices were intended to recreate the state of the universe as it existed in earlier epochs. The first success was, of course, the ability to create antimatter in the laboratory. However, the progressive increase in energy levels now faces a new problem: the particles that this escalation was expected to produce, superparticles, categorically refuse to appear. The impasse is so glaring that those proposing to

invest vast sums in expanding the LHC find themselves unable to answer the question:

- *And what do you hope to find with this device*?

In any case, what is missing from these experiments is the density parameter, which these setups cannot reproduce. After initially inducing proton collisions, the approach is now being extended to collisions involving heavy ions, specifically lead or gold. However, we are still far from the mark. A rarefied or moderately dense medium does not, a priori, behave like a hyperdense one. We have no reliable model to offer for this state, found at the core of hyperdense neutron stars and described as a "soup of quarks and gluons".

The calculation then shows that the average distance separating the hadrons approaches their Compton wavelength:

$$\lambda_C = \frac{\hbar}{mc} \tag{8}$$

The standard explanation for how density can increase is that hadrons can fragment into smaller components—specifically, the appearance of free quarks. Indeed, this general concept applies as one delves deeper into a neutron star. After the constituent atoms have disintegrated into a plasma of protons and electrons, and the distance between these components falls below the electron Compton wavelength, the electrons vanish by combining with protons to form neutrons. At even greater depths, it is the mesons that, in turn, disappear.

Is there, then, a mechanism that could serve as an alternative to this hierarchical fragmentation? To illustrate this idea, we employ a 2D toy model. The masses represent local concentrations of curvature. Let us imagine a closed universe containing eight masses. The resulting shape of such a universe is a truncated cube. The total curvature of this cube is 4π. The eight truncated vertices of the cube are spherical octants, each concentrating a curvature of π/2. These elements are joined by curvature-free surfaces consisting of quarter-cylinders and flat sections; the latter represent the "void" separating the eight masses. The expansion of this universe—which possesses the topology of an S^2 sphere—results from the stretching of the flat sections. This reflects the standard model, in which photons bear the cost of expansion, with their wavelengths tracking the pace of cosmic stretching. Figure 4 shows the toy model used to illustrate the idea.

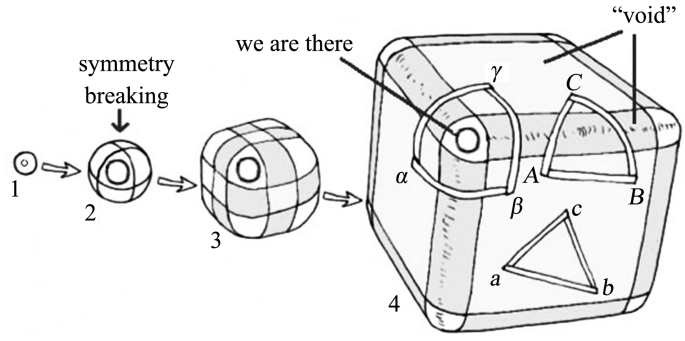


Figure 4. A toy model to schematize evolution.

From this perspective, masses can be likened to "frozen space". Figure 5 shows the didactic image illustrating the idea that masses are equivalent to regions of space that are unaffected by the phenomenon of expansion.

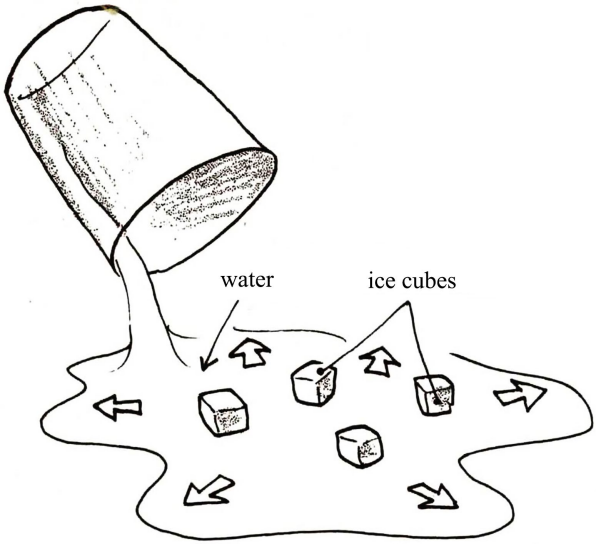


Figure 5. Didactic model in which the masses are represented by ice cubes.

Tracing the history of this universe back, we observe in Figure 4 that the eight sphere-octants come into contact with one another. This corresponds to the moment when the masses make contact, when the distance separating them becomes comparable to their Compton wavelength. Instead of seeing these "masses" fragment into smaller components, we can envision a transition to a regime with variable constants, where the Compton wavelength varies in proportion to the spatial scale factor a:

$$\lambda_C = \frac{\hbar}{mc} \sim a \tag{9}$$

We thus have here an initial idea suggesting a possible link between various constants, specifically Planck's constant, mass, and the speed of light, through what we will term a "gauge link". Let us now see what can be derived from this invariance of the equations of physics.

3. Invariance of the Field Equation

Let us begin by considering Einstein's field equation in the absence of a cosmological constant:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \tag{10}$$

The Einstein constant will be given the form:

$$\chi = -\frac{8\pi G}{c^2} \tag{11}$$

This search for a generalized gauge relationship is analogous to the dimensional

analysis used by physicists, in which elements such as mass, electric charge, and energy are replaced by dimensionless quantities. Here, we apply the same approach to the variables of time and space.

For space, we set: $r = \hat{a}a_0\zeta$, a_0 is the standard scale factor. Let t_{cr}. be the moment when evolution driven by gauge fluctuations takes over. At this moment, all quantities have the following values:

$$\left(c_0, h_0, G_0, m_0, e_0, \mu_{0-act}, r_0, t_0, v_0\right) \tag{12}$$

We thus have a snapshot of the state of the universe, with the values of the various constants.

We can extend this concept back in time by envisioning a different evolutionary mechanism—a generalized gauge process in which all these quantities vary in tandem. While this is a rather unusual scenario to picture, we must first bear in mind that when this mechanism takes over, we are in the era prior to decoupling. Consequently, the universe has effectively returned to a state of homogeneity and isotropy. Let us illustrate this scenario with a simple analogy. Imagine a viewer watching a film—played in reverse—that depicts the history of a universe bounded by a circular edge. On the screen, they first see galaxies disintegrating and stars vanishing. At the moment of decoupling, the interior of the disk appears homogeneous. However, the story then unfolds differently before their eyes as the bounding circle begins to shrink. Inside the circle, one can discern a regular distribution of points representing particles; these appear and disappear at an accelerating rate, acquiring increasingly high random agitation velocities. Simultaneously, the hue within the circle shifts in color and grows brighter.

In short, we are witnessing a scene where "something is happening", yet where the universe nonetheless retains a high degree of homogeneity.

All these quantities will then be modulated and weighted by functions:

$$\left(\hat{c}, \hat{h}, \hat{G}, \hat{m}, \hat{e}, \hat{\mu}_0, \hat{a}, \hat{t}, \hat{v}\right) \tag{13}$$

These functions all equal 1 at the time t_{cr} of the evolutionary transition. Looking further back into the past, some will be increasing while others are decreasing. What we seek is for their variations to leave all the equations of physics invariant. We can express this by writing that, in this "variable-constant" phase:

$$c = \hat{c}c_0 \tag{14}$$

$$h = \hat{h}h_0 \tag{15}$$

$$G = \hat{G}G_0 \tag{16}$$

$$m = \hat{m}m_0 \tag{17}$$

$$e = \hat{e}e_0 \tag{18}$$

$$\mu_0 = \hat{\mu}_0\mu_{0-act} \tag{19}$$

$$a = \hat{a}a_0 \tag{20}$$

$$t = \hat{t}t_0 \tag{21}$$

$$v = \hat{v}v_0 \tag{22}$$

Let us apply this to the field equation (without a cosmological constant):

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi T_{\mu\nu} \tag{23}$$

The Ricci tensor $R_{\mu\nu}$ will evolve according to:

$$R_{(0)\mu\nu} \to \hat{R}_{\mu\nu}R_{(0)\mu\nu} \tag{24}$$

However, we know that the components of the Ricci tensor have the dimension of the inverse of a length. Its gauge variation will therefore result from the variation of the spatial scale factor *a*, and we can write:

$$\hat{R}_{\mu\nu} = \frac{1}{\hat{a}^2} \tag{25}$$

A physicist will have their own way of expressing this in writing, more directly:

$$R_{\mu\nu} \sim \frac{1}{a^2} \tag{26}$$

The line element is:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} \tag{27}$$

In both members, we find the square of a length. This means that the quantities, which are simple numbers, are not altered by the gauge variation, a fact we can write as follows:

$$\hat{g}_{\mu\nu} = 1 \tag{28}$$

The Ricci scalar *R* also has the dimension of the inverse of the square of a length.

We shall assume that Einstein's constant χ, taken not in its "modern" form but in its "historical" form, behaves as an absolute constant.

$$\chi = -\frac{8\pi G}{c^2} = Cst \tag{29}$$

This corresponds, within the gauge variations under consideration, to:

$$\hat{G} = \hat{c}^2 \tag{30}$$

Or:

$$\boxed{G \sim c^2} \tag{31}$$

We will refer to this type of relationship as a "gauge link". In these joint gauge variations:

The gravitational constant varies as the square of the speed of light.

With this choice of formulation for the Einstein constant χ, the terms of the source tensor have the dimension of a mass density ρ, which undergoes gauge variation through the gauge variations of the mass $\hat{m}$ and the spatial scale factor $\hat{a}$. This results in:

$$\rho = \hat{\rho}\rho_0 = \frac{\hat{m}}{\hat{a}^3}\rho_0 \tag{32}$$

When we incorporate all this into the field equation, we obtain:

$$\frac{1}{\hat{a}^2} R_{(0)-\mu\nu} = \chi \frac{\hat{m}}{\hat{a}^3} T_{(0)-\mu\nu} \tag{33}$$

This immediately gives us two new gauge links, $\hat{m} = \hat{a}$; however:

$$\boxed{m \sim a} \tag{34}$$

With its corollary: Mass varies as the space gauge factor *a*

$$\hat{\rho} = \frac{1}{\hat{a}^2} \tag{35}$$

The mass density varies inversely with the square of the space factor.

By introducing our hypothesis that treats Einstein's constant as an absolute constant, we are led to:

$$\frac{1}{\hat{a}^2} = \frac{\hat{G}}{\hat{c}^2} \frac{\hat{m}}{\hat{a}^3} \tag{36}$$

The various gauge variations lead to a variation in the characteristic length associated with gravitation, the Schwarzschild length R_s:

$$R_s = \frac{2Gm}{c^2} = \frac{\hat{G}\hat{m}}{\hat{c}^2} \frac{2G_0 m_0}{c_0^2} = \hat{a} R_{s0} \tag{37}$$

$$R_s \sim a \tag{38}$$

In other words, in this evolution via joint gauge variations:

The Schwarzschild radius R_s varies as the space gauge factor *a.*

To proceed, we will add the two operators ∇ and Δ by setting:

$$\nabla = \frac{1}{\hat{a}} \delta \tag{39}$$

$$\Delta = \frac{1}{\hat{a}^2} \delta^2 \tag{40}$$

4. Invariance of Maxwell's Equations

They are:

$$\nabla \times B = \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \tag{41}$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{42}$$

$$\nabla \cdot B = 0 \tag{43}$$

Let us set:

$$B = \hat{B} B_0 \tag{44}$$

$$E = \hat{E} E_0 \tag{45}$$

The functions $\hat{B}$ and E represent the impact of the generalized gauge process on the values of the magnetic field B and the electric field E (which arise from variations in more fundamental quantities, such as electric charge *e*, charge density, and velocity, all of which are affected by this gauge process).

Maxwell's first equation becomes:

$$\frac{\hat{B}}{\hat{a}}\left(\delta\times E_0\right)=\frac{\hat{E}}{\hat{c}^2\hat{t}}\left(\frac{\partial E_0}{\partial t_0}\right) \tag{46}$$

Invariant, if:

$$\frac{\hat{B}}{\hat{E}}\sim\frac{a}{\hat{c}^2\hat{t}} \tag{47}$$

The second equation is written as:

$$\frac{\hat{E}}{a}\left(\delta\times E_0\right)=-\frac{\hat{B}}{\hat{t}}\left(\frac{\partial B_0}{\partial \tau}\right) \tag{48}$$

Invariant if:

$$\frac{\hat{B}}{\hat{E}}\sim\frac{\hat{t}}{a} \tag{49}$$

From this, we deduce the gauge relationship between $\hat{a}$, $\hat{c}$, and t:

$$\boxed{\hat{a}=\hat{c}\hat{t}} \tag{50}$$

5. Lorentz Invariance

The line element is:

$$\mathrm{d}s^2=c^2\mathrm{d}t^2-\mathrm{d}x^2-\mathrm{d}y^2-\mathrm{d}z^2 \tag{51}$$

The five terms of this expression undergo the same gauge variation of the quantities $\left(s,c,t,x,y,z\right)$, resulting in variations $\left(\hat{a},\hat{c},\hat{t}\right)$ of the gauge coefficients affecting each of them. These five terms have the dimension of the square of a length. It follows that:

We get:

$$\hat{a}^2\left(\mathrm{d}s_0^2\right)=\hat{c}^2\hat{t}^2\left(\mathrm{d}t_0^2\right)-\hat{a}^2\left(\mathrm{d}x_0^2+\mathrm{d}y_0^2+\mathrm{d}z_0^2\right) \tag{52}$$

Invariance is ensured if $\hat{a}=\hat{c}\hat{t}$. Yet this is precisely what we have just obtained.

Satisfying Maxwell's equations through the joint gauge fluctuations $\left(\hat{a},\hat{c},\hat{t}\right)$ of the quantities $\left(a,c,t\right)$ has geometric implications. The metric is preserved (Lorentz invariance). This means that the resulting isometry group—the Lorentz group—can still act: it remains possible to perform four-dimensional rotations, which is not the case if, as in other proposed approaches [6]-[9], the variation is restricted to *c*.

This action on the metric is equivalent to considering a "conformal" 4D evolution, as suggested by the 2D image in the diagram of **Figure 4**.

6. Invariance of the Schrödinger Equation

This equation is written as:

$$\frac{h^2}{2m}\Delta\psi+U\psi=-\frac{h}{2\pi}\frac{\partial\psi}{\partial t} \tag{53}$$

The wave function (everything in this set of physics equations is subject to the effect of this process of generalized gauge variation). We can express this by writ-

ing, a priori:

$$\psi = \hat{\psi}\psi_0 \tag{54}$$

On pose:

$$U = \frac{h^2}{2ma^2}u \tag{55}$$

Since the equation is linear in ψ, the gauge condition on the wave function drops out, and we obtain:

$$\frac{\hat{h}}{2\hat{m}}\left(\frac{1}{\hat{a}^2}\right)\left(\delta^2\psi_0 + u\psi_0\right) = -\frac{\hat{h}}{2\pi}\left(\frac{1}{\hat{t}}\right)\frac{\partial\psi_0}{\partial t_0} \tag{56}$$

But: $a = \hat{c}\hat{t}$. We get:

$$\left(\frac{\hat{h}}{\hat{t}}\right)\left(\frac{1}{\hat{m}\hat{c}^2}\right)\left[\left(\frac{\theta}{2\eta}\right)\left(\delta^2\mathcal{H} + u\mathcal{H}\right)\right] = -\left(\frac{1}{2\pi}\frac{\partial\mathcal{H}}{\partial\tau}\right) \tag{57}$$

But $\frac{\hat{h}}{\hat{t}} = \hat{h}\hat{\nu}$ is a form of energy, just as $\hat{m}\hat{c}^2$.

Thus, the conclusion of this passage, reflecting the invariance of the Schrödinger equation under our generalized gauge phenomenon, is that two forms of energy vary conjointly.

$$\hat{h}\hat{\nu} \sim \hat{m}\hat{c}^2 \tag{58}$$

7. Invariance of the Boltzmann Equation

This equation is:

$$\frac{\partial f}{\partial t} + v\cdot\frac{\partial f}{\partial r} + F\cdot\frac{\partial f}{\partial v} = \frac{\partial_e f}{\partial t} \tag{59}$$

By examining the first two terms and knowing that the velocities undergo the same gauge variation as the speed of light, so as to preserve the quantity $\sqrt{1-v^2/c^2}$, we have:

$$\frac{\partial}{\partial t} \sim \frac{1}{\hat{t}} \tag{60}$$

$$v\cdot\frac{\partial}{\partial r} \sim \frac{\hat{c}}{\hat{a}} \tag{61}$$

For the first two terms to vary jointly, we recover the relationship $\hat{a} = \hat{c}\hat{t}$.

F is a force acting on a unit mass, which can represent the gravitational force or the electromagnetic force. Let us choose the gravitational force. Gauge invariance is expressed as:

$$\frac{1}{\hat{t}} \sim \frac{\hat{c}}{\hat{a}} \sim \frac{\hat{G}\hat{m}}{\hat{c}} \tag{62}$$

We find that the characteristic gravitational length, the Schwarzschild radius \$ R_s, varies in the same way as the spatial scale factor *a.*

8. The Hypothesis of the Invariance of Einstein's Constant (in Its Traditional Form) and Its Consequences

We hypothesize that this constant χ, in its traditional form, is a gauge invariant:

$$\chi = -\frac{8\pi G}{c^2} \quad \text{absolute invariant} \tag{63}$$

$$\hat{G} \sim \hat{c}^2 \tag{64}$$

By combining this with what was obtained earlier, we derive the gauge transformation law for the elementary mass (and thus for "masses" in general).

$$\boxed{\hat{m} \sim a} \tag{65}$$

And the gauge variation of the gravitational constant:

$$\boxed{\hat{G} \sim \frac{1}{a}} \tag{66}$$

Assuming the number of species remains constant, the evolution of the density n corresponds to:

$$\hat{n} \sim \frac{1}{a^3} \tag{67}$$

This immediately gives us the variation of the mass density $\hat{\rho} = \hat{n}\hat{m}$ in this gauge process:

$$\hat{\rho} \sim \frac{1}{a^2} \tag{68}$$

The speed of light, higher in the past, then follows the gauge law:

$$\boxed{\hat{c} \sim \frac{1}{\sqrt{a}}} \tag{69}$$

This leads to the constancy of energies:

$$\hat{h}\hat{\nu} \sim \hat{m}\hat{c}^2 \sim Cst \tag{70}$$

For the reason cited above—Lorentz contraction—velocities follow the same variation as c:

$$\hat{v} \sim \hat{c} \tag{71}$$

We therefore have conservation of thermal agitation energy:

$$\frac{1}{2}\hat{m}\left\langle \hat{v}^2 \right\rangle = Cst \tag{72}$$

The same applies to all forms of energy: magnetic, gravitational, etc. All characteristic lengths, such as mean free paths, scale with the gauge factor a, and all characteristic times, such as mean free-path times, scale with $\hat{t}$. This then yields—and this will be of major importance later on—the scaling of the Jeans length:

$$L_J \sim \frac{\langle \hat{v} \rangle}{\sqrt{\hat{G}\hat{\rho}}} \sim a \tag{73}$$

The Jeans time t_J varies as $\hat{t}$:

$$t_J \sim \frac{1}{\sqrt{\hat{G}\hat{\rho}}} \sim \hat{t} \tag{74}$$

The invariance of the energy $\hat{m}\hat{c}^2$ immediately gives us the gauge link between $\hat{a}$ and $\hat{t}$.

$$\hat{m}\hat{c}^2 = \hat{m}\frac{\hat{a}^2}{\hat{t}^2} = \frac{a\hat{a}^3}{\hat{t}^2} = Cst \tag{75}$$

We recover Kepler's law, and we will have parabolic evolution:

$$\boxed{a \sim t^{2/3}} \tag{76}$$

Conservation of energy $\hat{h}\hat{\nu}$ immediately yields the gauge-variation law for Planck's "constant":

$$\boxed{h \sim t} \tag{77}$$

Adding $\hat{t} = \hat{a}/\hat{c}$, we get:

$$\boxed{h \sim a^{3/2}} \tag{78}$$

La constante de Planck varie comme la puissance 3/2 du space gauge a

This gives us the gauge law for the Compton wavelength:

$$\hat{\lambda}_C = \frac{\hat{h}}{\hat{m}\hat{c}} \sim a \tag{79}$$

This aligns with the 2D schematic diagram presented at the beginning of the article. It also follows immediately that the Planck length varies as a:

$$L_P = \sqrt{\frac{\hat{h}\hat{G}}{\hat{c}^3}} \sim a \tag{80}$$

And that the Planck time varies as:

$$t_P = \sqrt{\frac{\hat{h}\hat{G}}{\hat{c}^5}} \sim \hat{t} \tag{81}$$

Similarly, the Jeans time varies as $\hat{t}$, and this phenomenon holds generally for all characteristic lengths and timescales. Furthermore, since we have:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{82}$$

And since we have chosen, a priori, to treat ε_0 as an absolute constant (unaffected by the generalized gauge process), it follows that:

$$\hat{\mu}_0 \sim \frac{1}{\hat{c}^2} \tag{83}$$

This gives us the law governing the variation of the magnetic permeability of vacuum:

$$\boxed{\hat{\mu}_0 \sim a} \tag{84}$$

All that remains to complete the picture is the law governing the variation of the electric charge.

9. Gauge Invariance of the Fine-Structure Constant

The inability to ensure its constancy has been the weak link in all theories positing variations in the constants. Its invariance provides the missing element: the impact on electric charge.

$$\alpha = \frac{e^2}{hc} \tag{85}$$

Which gives:

$$\boxed{e \sim \sqrt{a}} \tag{86}$$

And this will give us a Bohr radius varying as a:

$$R_{\text{Bohr}} = \frac{4\pi\varepsilon_0 h^2}{m_e e^2} \sim a \tag{87}$$

10. Generalized Conservation of All Forms of Energy

At one time, it was proposed to attribute the redshift to a secular variation of Planck's constant [10]. Based on what was established earlier, the energy $h\nu$ of photons propagating through a sufficiently rarefied space is conserved.

$$\boxed{h\nu = Cst} \tag{88}$$

Now, a redshift measurement is simply a measurement of the energy loss of photons. Note that we place the phenomenon termed a "generalized gauge process" prior to decoupling.

Matter-energy is also conserved.

$$\boxed{mc^2 = Cst} \tag{89}$$

The electric field created by an electric charge e is:

$$E = \frac{e}{4\pi\varepsilon_0 r^2} \sim a^{-3/2} \tag{90}$$

The corresponding volumetric energy density is:

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \tag{91}$$

Therefore, electrostatic energy is conserved:

$$\boxed{u_E a^3 = Cst} \tag{92}$$

The magnetic field created by a current I at a distance r is:

$$B = \frac{\mu_{0I}}{r} \tag{93}$$

We have:

$$I \sim \frac{e}{t} \sim a^{-1} \tag{94}$$

Whence:

$$B \sim a^{-1} \tag{95}$$

The magnetic energy density is:

$$u_B = \frac{B^2}{2\mu_0} \sim a^{-3} \tag{96}$$

Thus, energy in magnetic form is conserved in the generalized gauge process:

$$\boxed{u_B a^3 = Cst} \tag{97}$$

The gravitational energy density is:

$$u_G = \frac{1}{a^3}\frac{Gm^2}{a} \tag{98}$$

Here again:

$$\boxed{u_G a^3 = Cst} \tag{99}$$

11. What Would Drive the Evolution in a Universe in Such a Situation?

We essentially have eight quantities involved in a gauge process:

$$(c, h, G, m, e, \mu_o, a, t) \tag{100}$$

Any one of them can be chosen as the governing variable. The one whose physical significance appears most relevant to us is the space scale factor, *a*. This yields the following evolution scheme:

$$\hat{c} \sim \frac{1}{\sqrt{a}}; \hat{h} \sim a^{3/2}; \hat{G} \sim \frac{1}{a}; \hat{m} \sim a; \hat{e} \sim \sqrt{a}; \hat{\mu}_0 \sim a; \hat{t} \sim a^{3/2} \tag{101}$$

Since this evolution occurs at constant energy and the measurement of redshift relates essentially to measuring the energy loss of photons, this process necessarily takes place at an epoch prior to decoupling, and even prior to a radiation-dominated phase, when physical constants behave as invariant quantities. The schematic evolution of the so-called "constants of physics" is therefore be presented as shown on Figure 6.

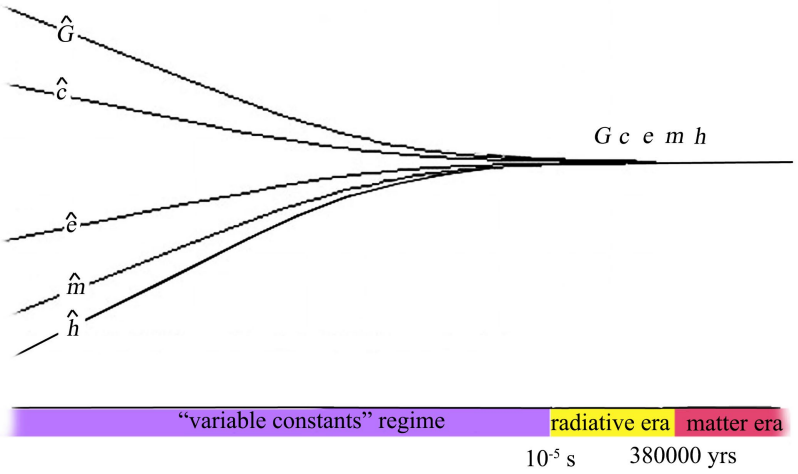


Figure 6. Schematic evolution of the "constants".

Looking back in time—after the transition from the matter-dominated era to the radiation-dominated era—we hypothesize that this "variable-constant" era emerges when the distance between baryons (and antibaryons) becomes comparable to their Compton wavelength, a point in time we place at around 10^{-5} second.

In the context of the Janus cosmological model [11], the evolutionary scheme during the matter-dominated era has already been described in reference [2]. This will be complemented by a description of the evolution during the radiation-dominated era in a subsequent paper. This presents no inherent difficulty; regarding positive mass, the evolution remains one of positive acceleration, dominated by the negative contribution of the negative mass.

As a first step toward integrating this "variable-constant" evolutionary scheme into the Janus cosmological model, and assuming that the negative world, composed of negative-mass antimatter, is subject to the same physical laws, we consider the following quantities:

$$\left(\overline{c},\overline{h},\overline{G},\overline{m},\overline{e},\overline{\mu}_o,\overline{a},\overline{t}\right) \tag{102}$$

Assuming that it obeys the same physical laws, we will have similar laws for the second species:

$$\hat{\overline{c}}\sim\frac{1}{\sqrt{a}};\hat{\overline{h}}\sim a^{3/2};\hat{\overline{G}}\sim\frac{1}{a};\hat{\overline{m}}\sim a;\hat{\overline{e}}\sim\sqrt{a};\hat{\overline{\mu}}_0\sim a;\hat{\overline{t}}\sim a^{3/2} \tag{103}$$

12. An Alternative to Inflation

How, then, does the cosmological horizon evolve? This is where the benefit of this approach lies—the link to observation. The horizon is the distance traveled by light:

$$H=\int^t c\mathrm{d}t \tag{104}$$

But in this different evolution, dominated by this gauge process:

$$\hat{c}\sim\frac{1}{\sqrt{a}}\sim\hat{t}^{-1/3} \tag{105}$$

So, without this variable speed-of-light regime:

$$H=\int^{\hat{t}}\hat{c}\mathrm{d}\hat{t}\sim\int^{\hat{t}}\hat{t}^{-1/3}\mathrm{d}\hat{t}\sim\hat{t}^{2/3}\sim a \tag{106}$$

$$\boxed{\text{Cosmological horizon } H\sim a} \tag{107}$$

We are presented here with an evolutionary scheme in which the homogeneity of the universe is maintained throughout this evolutionary phase characterized by variable constants; this constitutes an alternative to the inflation model, based on the single hypothesis of a generalized gauge process centered on the requirement of invariance for all equations of physics. On **Figure 7** we refer to this expansion, where the horizon grows like the space scale factor.

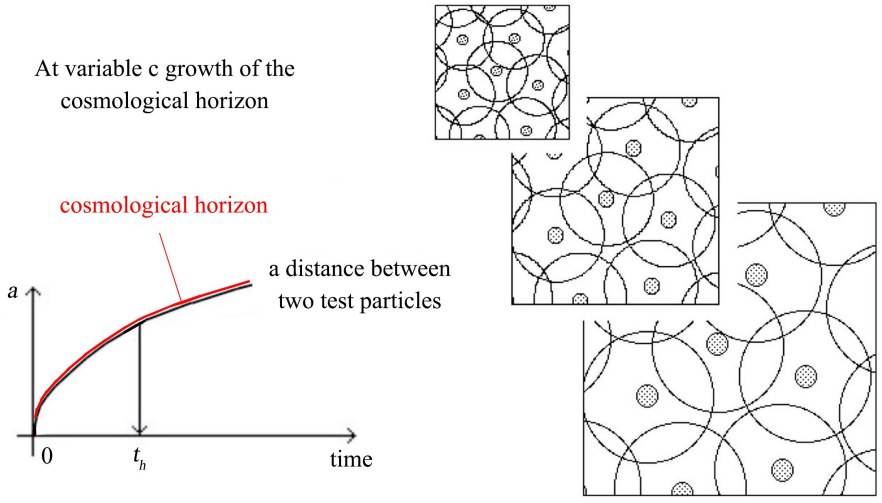


Figure 7. Homogeneity ensured by a variation in the speed of light.

13. The Question of Measuring Time

Cosmological models describe the evolution of the universe in terms of a quantity corresponding to the letter *t*, intended to represent time. However, this relies on a specific choice of coordinate system. Only a quantity that can be measured, even if only conceptually, using a clock belonging to the realm of physics qualifies as time. In the matter-dominated era, one can envision a conceptual clock consisting of two masses moving along the same circular orbit, centered on their common center of gravity. Measuring time then reduces to measuring the number of revolutions or an angle. Since the geodesics lie within planes in this scenario, this concept appears to be coordinate-independent. Similarly, the day corresponds to the Earth's rotation on its axis, the solar year to the Earth's revolution around the Sun, and what might be termed the galactic year to the solar system's revolution around its galaxy, the Milky Way.

If we follow this conceptual, non-relativistic clock back into the past—assuming it has avoided any collisions or exchanges with dangerous neighbors whose speeds (of thermal agitation) increase and approach the speed of light as we go further back—this clock corresponds to the classical conception of time.

But if the past encompasses an era of evolution with variable constants, then the components of our clock—its masses—are altered, as are the forces binding them. The geometric framework along which these masses trace their path varies as well. What, then, of the number of revolutions it completes? Figure 8 shows our elementary clock, consisting of masses orbiting their common center of gravity.

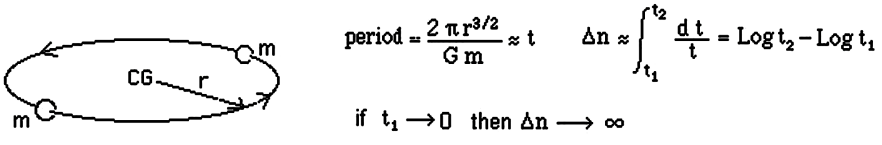


Figure 8. Our conceptual clock.

If we apply the effects of our generalized gauge phenomenon to the calculation of the period, that period varies as $a^{3/2}$. It therefore decreases, and as we look back into the past, our conceptual clock ticks faster and faster. As we approach a hypothetical time zero, the number of cycles completed tends toward infinity.

This yields the temporal equivalent of the myth of Achilles and the tortoise. Let us liken the history of the universe to a book. The character in my comic strips then leafs through the book backwards, attempting to discover, in the preface, the intentions its author might have recorded there. Figure 9 shows a man attempting in vain to reach the first page of a book whose pages—becoming increasingly thin—are infinite in number.



Figure 9. On the impossibility of accessing the "initial moment".

But he never reaches that preface, because the pages of the book tend to become increasingly thin and are therefore infinite in number.

This simply suggests that, when attempting to trace the universe's history back in time, quantum physics eventually comes into play and exerts its effects. Clarity will only be achieved when tools derived from general relativity and those from quantum mechanics can finally be combined through the quantization of the gravitational field—a task that, after decades, remains merely a project. This is an advance that, we believe, will entail constructing the geometric context of quantum mechanics—an extension that will, we believe, involve a systematic complexification of that geometric context.

14. Integration of This Variable-Constant Phase Model into the Janus Cosmological Model

In a future article, we will demonstrate how—starting from an initial state of perfect symmetry where the constants and the spatial and temporal scale factors of the universe's two components are identical—an instability inherent to this variable-constant regime causes the two systems to diverge violently (exponentially). Consequently, they come to possess distinct constants and space-time scales:

$(\hat{a},\hat{t})$ et $(\hat{\bar{a}},\hat{\bar{t}})$. This drift in constants ceases once the bayons and antibayons move apart by a distance exceeding their Compton wavelength (a point that would require further clarification, given that this Compton wavelength would itself increase in step with the scale factor). However, the tools currently employed—lacking a genuine geometric fusion that incorporates quantum mechanics—do not allow us to shed light on this specific issue ("For the possible, we act immediately; for the impossible, we ask for time").

Schematically, as shown in Figure 10, below:

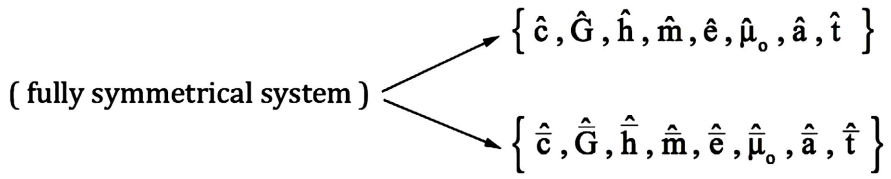


Figure 10. Diagram of the appearance of asymmetry in the phase with variable constants.

We might now have a way to specify the parameters linked to the second system. We already know one thing [2] regarding the scale factors (which, moreover, evolve differently, with acceleration in the positive sector and deceleration in the negative world):

$$a > \bar{a} \tag{108}$$

But in what proportion?

Gravitational instability can occur during the radiation-dominated era, as radiation acts as the source of the gravitational field. One can then adopt the Jeans approach and seek to determine the minimum wavelength for radiation density fluctuations such that the accretion time for additional radiation is shorter than the dissipation time for the radiation already accreted. This yields a value corresponding to the cosmological horizon.

Thus, these fluctuations would be present, yet we would have no way of detecting their existence.

And this brings us back to what emerges from this era of variable constants: the order of magnitude of the cosmological horizon is comparable to the scale factor in each of the two populations. However, we have $a > \bar{a}$, so:

$$\bar{H} < H \tag{109}$$

The two entities interact gravitationally—or, to be more precise, anti-gravitationally—at every stage of cosmic history. Consequently, fluctuations occurring in the negative realm must leave their imprint on the positive realm across all epochs: during the matter-dominated era, immediately following decoupling, the organization of negative mass into a regular distribution of vast, massive spheroidal conglomerates imparts a void-like structure to the positive mass, resembling "contiguous bubbles". Figure 11 illustrates how very rapid energy loss via radiative dissipation—facilitated by a large-scale structure of flat sheets—enables galaxies and stars to form very quickly.

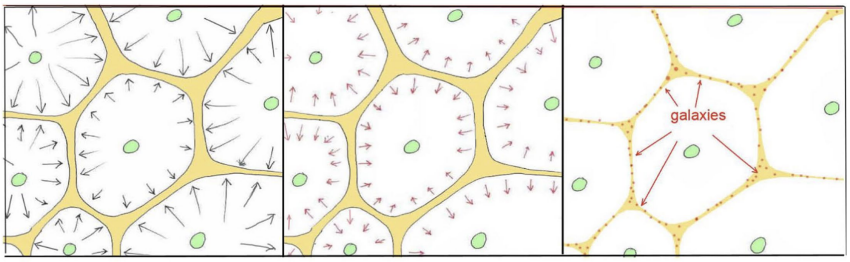


Figure 11. Formation of the large-scale structure. (a): The negative mass (green) forms a regular distribution of spheroidal clusters, imparting a void-like, contiguous-bubble structure to the positive mass (yellow). (b): The positive mass, having been strongly heated, undergoes rapid cooling via radiative losses. (c): Galaxies form [11].

However, this interaction persists in the epochs prior to decoupling. We therefore propose a different interpretation of the CMB fluctuations. **Figure 12** is an image intended to show how the slight fluctuations of the CMB represent the weak response of the positive world to fluctuations of the field arising in the negative world—fluctuations whose characteristic wavelength corresponds to the horizon within that universe sheet.

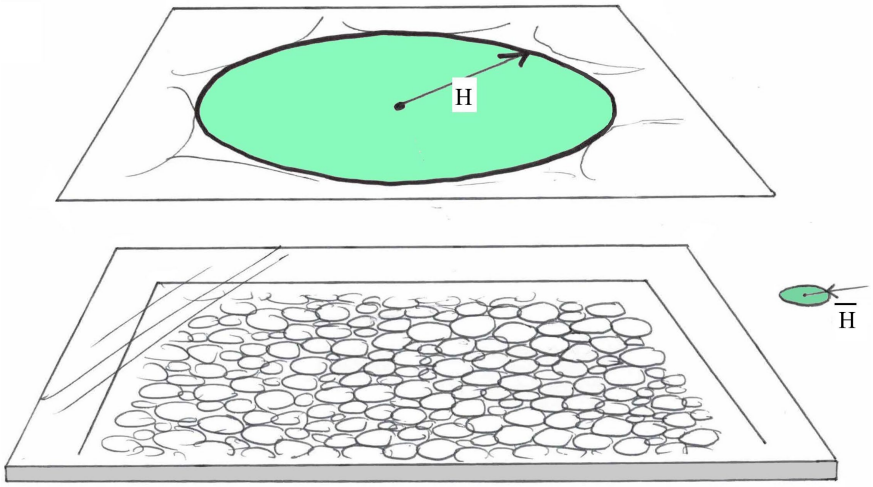


Figure 12. Our interpretation of the CMB inhomogeneity. H and $\bar{H}$ are the cosmological horizons of the two universes.

The non-linear radiation density fluctuations arising in the negative world leave a faint imprint on the positive world. In our view, this explains the CMB fluctuations. In very schematic terms, this yields the order of magnitude of the ratio between the two horizons (and thus the two scale factors), which differ by two orders of magnitude.

$$\frac{\bar{a}}{a} \sim \frac{1}{100} \tag{110}$$

Work is currently underway, focusing on the study of these fluctuations in the radiation density field within the "negative world" via gravitational instability

(specifically Jeans instability), that will enable a comparison of the two models.

That being said, in our model this discrepancy arose during the dual regime of variable constants, in which we have:

$$c \sim \frac{1}{\sqrt{a}} \quad \overline{c} \sim \frac{1}{\sqrt{\overline{a}}} \tag{111}$$

Whence:

$$\frac{\overline{c}}{c} \sim 10 \tag{112}$$

The scientific community's view that interstellar travel—on a human scale and at necessarily non-relativistic speeds—is forever impossible stems from the sheer duration involved. However, based on this new perspective, if a vehicle could invert its mass, it might undertake the journey via a sort of parallel universe, operating in a negative-mass regime, thereby reducing travel times by a factor of 1,000.

$$\text{travel time} \times \frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} \tag{113}$$

This would then place neighboring star systems—at a distance of around ten light-years, within reach, accessible via voyages undertaken at one-tenth the speed of light within this other sheet, with a duration on the order of an Earth month.

Regarding the phenomenon of mass inversion, we find a natural version of it in the phenomenon of plugstars [12].

For a long time, authors have envisioned the possibility of passing from one sheet to another via wormholes. However, for this to be feasible, such structures would need to exist naturally in the cosmos, involving sufficient mass to ensure that tidal forces do not tear apart the vessels venturing into them. Yet, a different scenario can be envisaged.

15. Mass Transfer via Surgery

If inverting a mass M required an energy expenditure on the order of Mc^2, the process would clearly have to be relegated to the realm of science fiction, given our current capabilities. However, let us examine the formula below.

In differential topology, surgery is an operation that consists of excising certain regions of a manifold and reattaching the resulting boundaries according to a specific identification. This operation alters the manifold's topology. In differential geometry, the resulting manifold must then be endowed with a metric compatible with the imposed physical conditions. When applied to two initially disjoint sheets, such an operation conceptually allows for the excision of a region from each sheet and the local reattachment of their boundaries.

Let us illustrate, as shown on **Figure 13**, the principle of such topological surgery using a series of drawings. Imagine two adjacent, flat sheets, F_1 and F_2. One is white, the other blue. A closed curve has been drawn on F_1, defining a region, a simple disk, that we wish to transfer to sheet F_2.

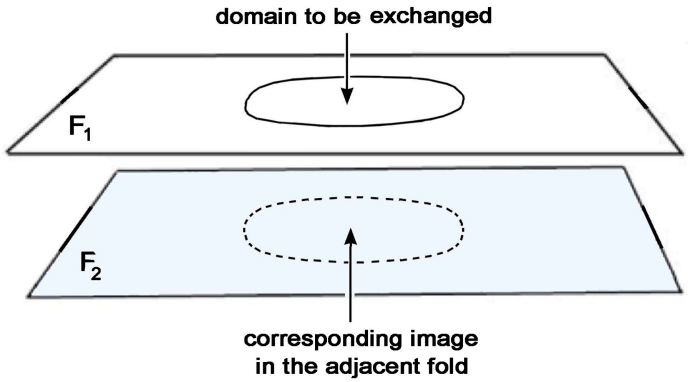


Figure 13. Two adjacent sheets of 2D Euclidean space.

To carry out this operation, we will create a curvature in the vicinity of this boundary. To visualize this, we will cut these two planes. See Figure 14. The curvature introduced into sheet F_1 produces an "induced curvature" in the adjacent sheet F_2, which would correspond to two geometries with coupled metrics, as in the Janus cosmological model [12].

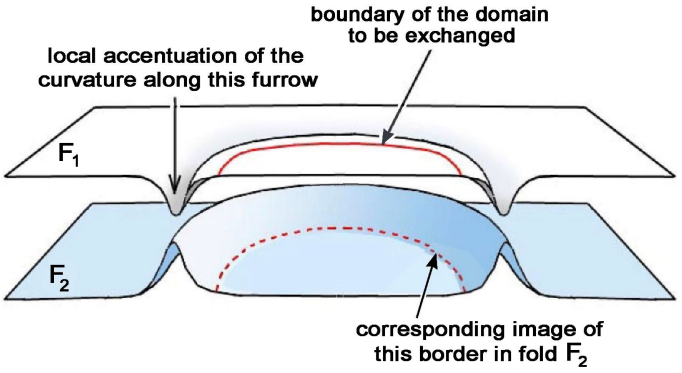


Figure 14. Creation of a curvature in the vicinity of a curve delimiting a domain and its image in the adjacent sheet.

The two sheets are depicted as immersed in $\mathbb{R}^3$. A critical state is reached when the curvature becomes infinite along the boundary of the domain where two tangent planes exist. One can visualize the two sheets coming into contact along the closed curve, with the induced curvature causing the tangent planes to align along that curve. This situation lends itself to what is known in topology as "surgery". The white portion of sheet F_1, lying outside the boundary curve, then connects to the interior of the blue disk located on sheet F_2. A symmetrical phenomenon occurs where the blue part of sheet F_2. (outside the image of the circular curve) suddenly extends into the white disk of sheet F_1. See Figure 15.

The curvatures disappear, resulting in a final configuration with two adjacent planar regions where the interiors of the boundary curve of domain F_1 and its image in the adjacent portion of F_2 have been swapped. In Figure 16, the exchange between the two adjacent regions is complete. A blue disk has been grafted into sheet H1, while a white disk has taken its place in sheet F2.

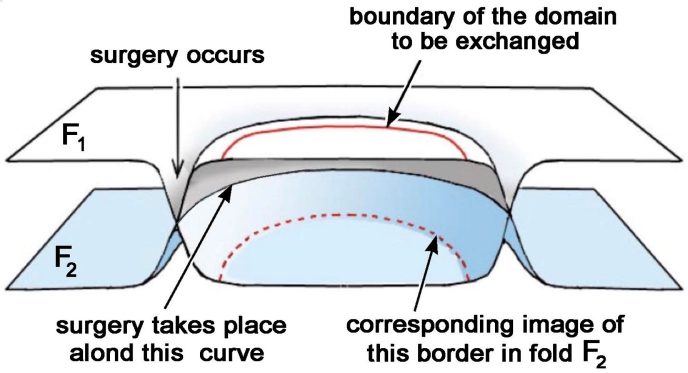


Figure 15. 2D surgery.

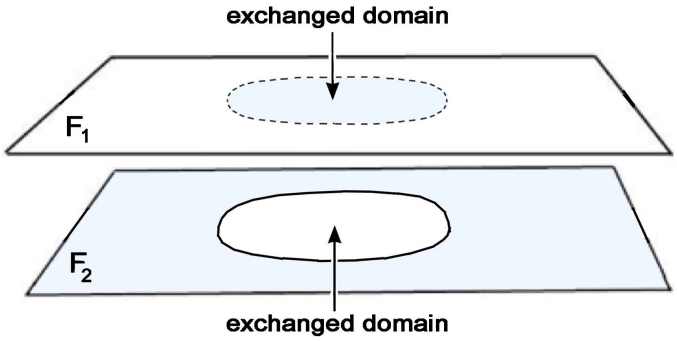


Figure 16. The two domains have been exchanged.

One can then envision such a situation in 3D, where balls B_3, located within two 3D sheets bounded by S2 spheres, are exchanged.

The surgical approach envisaged here therefore suggests a possibility of a different nature: the geometric transformation would not necessarily involve the entire volume occupied by the mass, but could be triggered by a very high concentration of energy within an extremely thin layer surrounding it.

One can envision, as a prospective example, a material whose nuclei are brought into metastable excited states. Certain nuclear isomers do indeed store excitation energies on the order of MeV per nucleus for exceptionally long periods. Their controlled, large-scale production could—in a still-speculative future—rely on coherent gamma-ray photon sources tuned to appropriate nuclear transitions. Such sources do not currently exist with the required characteristics. The fundamental question then becomes quantitative: what volumetric energy density—or, in the limit of a very thin layer, what surface energy density—would need to be achieved to trigger the envisioned topological change?

At this stage, general relativity and quantum mechanics, when used separately, likely cannot provide a meaningful answer. Classical general relativity describes the gravitational effect of an energy-momentum distribution but offers no microscopic mechanism for such controlled topological surgery. Conversely, quantum mechanics describes nuclear states and their transitions without providing a quantized dynamics of geometry itself. The answer to this question will only

emerge once general relativity, and more specifically the Janus Cosmological Model, has been merged with quantum mechanics within a single, complex geometric framework.

If such a transfer were to become physically feasible and controllable, the technological consequences would be immense. In particular, one could envisage transferring materials that currently pose major storage challenges—foremost among them, high-level radioactive waste. An unexpected consequence of such a process would immediately become apparent. Once the mass had been transferred to the conjugate sector, the container that previously held it would find itself in the physical state corresponding to the adjacent region of spacetime: a near-perfect vacuum. By then allowing the external atmosphere to rush into this chamber through a turbine, a portion of the energy associated with this expansion could be converted into electrical energy. This would not, of course, constitute the creation of energy. In the Janus model, transfers between sectors operate within a dynamic framework where the energy balance must be viewed globally. Consequently, conservation cannot be assessed based solely on the energy content of one of the two sectors. Initial applications of the Janus model equations [2] immediately reveal a law of energy conservation that extends across both sheets. This constraint would govern the mass transfer resulting from the exchange of contents between adjacent domains.

16. Conclusions

Within the framework of the Janus cosmological model, we propose, as an alternative to the inflationary scenario, the existence of a phase preceding decoupling and the radiation-dominated era, during which the six fundamental physical constants, as well as the space and time gauge factors of the two sheets, are involved in a generalized gauge process. This process arises simultaneously from the invariance of the fundamental equations of physics, the invariance of the Einstein constant appearing on the right-hand side of the model's coupled field equations, and the invariance of the fine-structure constant. Lorentz invariance is thus preserved within each of the two sheets.

This correlated evolution is accompanied by the conservation of all forms of energy. All characteristic lengths evolve in proportion to the space scale factors of the two sheets, notably their respective cosmological horizons.

Since radiation itself acts as a source of the gravitational field, Jeans instability can also occur during the radiation-dominated era. When the characteristic wavelength is on the order of the cosmological horizon, the corresponding fluctuations cannot be observed within a single causal domain. However, the fundamental asymmetry of the Janus model alters this situation: because the cosmological horizon of the negative-mass sector is much smaller than that of the positive-mass sector, fluctuations developing in the former can produce effects in the latter as it enters its matter-dominated era. This mechanism thus offers a possible origin, alternative to inflation, for the fluctuations recorded in the CMB.

Since radiation itself acts as a source of the gravitational field, Jeans instability can also occur during the radiation-dominated era. When the characteristic wavelength is on the order of the cosmological horizon, the corresponding fluctuations cannot be observed within a single causal domain. However, the fundamental asymmetry of the Janus model alters this situation: because the cosmological horizon of the negative-mass sector is much smaller than that of the positive-mass sector, fluctuations developing in the former can produce effects in the latter as it enters its matter-dominated era. This mechanism thus offers a possible origin, alternative to inflation, for the fluctuations recorded in the CMB.

Naturally, this hypothesis must be tested against observation. In particular, it requires constructing the spectrum of non-linear fluctuations developing in the negative sector during its radiation-dominated era, then calculating their transfer to the positive sector and comparing the resulting spectrum with CMB data. The difference of approximately two orders of magnitude found between the scale factors constitutes, in this regard, a first quantitative characteristic of the proposed mechanism.

Within the framework of the model, the asymmetry arising from phases with variable constants also leads to a speed of light that is an order of magnitude higher in the negative sector. If a material object could transition to this sheet by reversing the sign of its mass, the latter would then act as a "parallel universe", allowing for a considerable reduction in apparent interstellar travel times.

Finally, we have outlined a geometric transition mechanism between adjacent domains, akin to a form of "surgery", that relies on energy concentration within a layer surrounding the object in question. Mechanisms for the storage and release of nuclear energy, particularly involving long-lived metastable states excited by gamma radiation, have been proposed as conceptual avenues. Within this hypothetical framework, such a transition would also necessitate considering the transfer of unwanted material, specifically radioactive waste.

However, these latter considerations must be clearly distinguished from the cosmological results developed in this article. At this stage, it is not possible to calculate the value of the critical energy required for such a transition. Such an assessment would require a theory in which quantum mechanics and gravitation are integrated into a single geometric framework. Constructing such a framework, as well as quantitatively calculating the spectrum of cosmological fluctuations predicted by the mechanism presented here, therefore represents a natural avenue for future work.

Author Contributions

The theoretical section is by Jean-Pierre Petit. Gilles d'Agostini checked the calculations and produced some of the illustrations.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Lodha, K., et al. (2025) Extended Dark Energy Analysis Using DESI DR2 BAO Measurements. arXiv: 2503.14743v1. https://arxiv.org/html/2503.14743v1

[2] Petit, J.P. and d'Agostini, G. (2014) Negative Mass Hypothesis in Cosmology and the Nature of Dark Energy. *Astrophysics and Space Science*, **354**, 611-615.

[3] Petit, J. (1988) An Interpretation of Cosmological Model with Variable Light Velocity. *Modern Physics Letters A*, **3**, 1527-1532. https://doi.org/10.1142/s0217732388001823

[4] Petit, J. (1995) Twin Universes Cosmology. *Astrophysics and Space Science*, **226**, 273-307. https://doi.org/10.1007/bf00627375

[5] Weinberg, S. (1993) The First Three Minutes: A Modern View of the Origin of the Universe. Basic Books.

[6] Albrecht, A. and Magueijo, J. (1999) Time Varying Speed of Light as a Solution to Cosmological Puzzles. *Physical Review D*, **59**, Article ID: 043516. https://doi.org/10.1103/physrevd.59.043516

[7] Magueijo, J. (2003) New Varying Speed of Light Theories. *Reports on Progress in Physics*, **66**, 2025-2068. https://doi.org/10.1088/0034-4885/66/11/r04

[8] Moffat, J.W. (1993) Superluminary Universe: A Possible Solution to the Initial Value Problem in Cosmology. *International Journal of Modern Physics D*, **2**, 351-365. https://doi.org/10.1142/s0218271893000246

[9] Moffat, J.W. (2016) Variable Speed of Light Cosmology, Primordial Fluctuations and Gravitational Waves. *The European Physical Journal C*, **76**, Article No. 130. https://doi.org/10.1140/epjc/s10052-016-3971-6

[10] Milne, E.A. (1948) Kinematic Relativity: A Sequel to Relativity, Gravitation and World-Structure. Oxford University Press.

[11] Petit, J., Margnat, F. and Zejli, H. (2024) A Bimetric Cosmological Model Based on Andreï Sakharov's Twin Universe Approach. *The European Physical Journal C*, **84**, Article No. 1226. https://doi.org/10.1140/epjc/s10052-024-13569-w

[12] Petit, J. and D'Agostini, G. (2025) Alternatives to Black Holes: Gravastars and Plugstars. *Journal of Modern Physics*, **16**, 1479-1490. https://doi.org/10.4236/jmp.2025.1610072

We're still on the Ummo case.

During the night of October 18, 2018, during a phone call at 3 a.m., here is the content of that exchange.

> Me: *The situation on planet Earth is truly pitiful and worrisome*
>
> My interlocutor: *What you need is a theory of belief.*
>
> Me: *What do you mean by that?*
>
> My interlocutor: *For about twenty years, you've been following the teachings of the mathematician and geometer Jean-Marie Souriau. You yourself have expanded on his work by exploring its implications: the significance of T-symmetry as an inversion of mass and energy. More generally, Souriau's key idea boils down to saying, "Give me a space and its geometric properties, and I'll tell you what it contains." This is achieved through the study of the group's action on the dual of its Lie algebra. In 1970, by applying this to Minkowski space—by letting its restricted isometry group act on it—he brought forth the physical quantities as invariants." Repeat the same operation with a Hermite space.*

This dialogue gives a sense of the conciseness, precision, and density of the messages. There was no need to say more. All of this was within my grasp. I knew that Hermite space was a complexified Minkowski space. So I set out to extend what I had learned and what I had already extended (with extended groups, including T-symmetry) by calculating the action of the isometry group of Hermite space on the dual of its Lie algebra.

The results came very quickly.

Souriau's work can[99] be summarized succinctly

- Minkowski space (from special relativity) contains positive masses and photons with positive energy.
- Extending the model by introducing additional compact dimensions leads to the quantization of quantum[100] charges.

My own work extends this to the Janus symplectic group. The conclusion is:

- Both sides of the Janus universe contain populations of CPT-symmetric particles.
- To this, we add the hypothesis that their interaction is limited to antigravity.

The extension to Hermitian space requires us to consider its isometry group in its most general form, including T-symmetry.

---

[99] J.M. Souriau: Structure des systèmes dynamiques. Dunod, France, 1970, and Structure of Dynamical Systems. Boston: Birkhäuser, 1997

[100] Ibid., Chapter V, Geometric Quantification.

Space then becomes complex. All its dimensions—both time and space—become complex. The group becomes complex.

- Energy becomes complex
- Masses become complex.

If we proceed with dimensional extensions, charges become complex and quantized.

A comprehensive conception of the universe begins to take shape, one that blends physics with metaphysics. Its implications extend far beyond those arising from physics and cosmology.

All that remained was to transform these calculations into scientific publications. My friend Nathalie Debergh first published a paper in 2021[101]

In 2026, again thanks to the rigor and professionalism of the *Journal of Modern Physics*, I published another paper on this topic. A reproduction of this article will conclude this scientific appendix.

Over the past few days, I have attempted to upload this article to arXiv. It was immediately

- put on hold
- Rejected (deleted)
- Deleted (the file disappears from the account and no trace of the submission remains).

Same reason:

[101] N. Debergh, G. D'Agostini, J-P Petit: On the Poincaré Algebra in a Complex Space-Time Manifold. Journal of Modern Physics 2021, 12, pp. 218–228

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(7), 905-927
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Is the Real World, as a Part of Complex Reality?

Jean-Pierre Petit, Gilles d'Agostini

Manaty Research Group, Glanon, France
Email: Jean-Pierre.petit@manaty.net

How to cite this paper: Petit, J.-P. and d'Agostini, G. (2026) Is the Real World, as a Part of Complex Reality? *Journal of Modern Physics*, **17**, 905-927.
https://doi.org/10.4236/jmp.2026.177040

Received: March 6, 2026
Accepted: July 21, 2026
Published: July 24, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

By constructing the action of the Poincaré group on the dual of his Lie algebra, Souriau revealed, in 1970, the quantities of physics as objects of pure geometry. A first draft of a complex extension is presented.

Keywords

Hermite Space, Minkowski Space, Adjoint Action, Coadjoint Action, Lie Algebra, Dynamic Group, Complex Lorentz Group, Complex Poincaré Group

1. Introduction

The approach of mathematician Jean-Marie Souriau [1] provided his own path through the difficult process of systematically geometrizing physics, by applying the tools of symplectic geometry. At the end of his work, he established a link with quantum mechanics through its geometric quantization. We propose to attempt to follow this path through a complex extension. Before that, we present a description of his construction of the classical elements of physics in a form more accessible to the widest possible audience. At the end of this article, the reader will find a first draft of the complex extension, whose sole merit is to show that, as soon as one undertakes this type of approach, new elements naturally appear in the formalism. These should not be interpreted as established results, but rather as indications, signals suggesting that this path deserves to be explored further, which will be the subject of the following article.

This article is aimed at engineers, students, and teachers in scientific disciplines. Its ambition is simple and explicit: to show that by using familiar tools such as matrices and differential calculus, it is possible to reach very high levels of understanding of modern physics.

The reader does not need prior mastery of differential geometry or notoriously difficult abstract formalisms. The starting point is deliberately elementary: matri-

ces, linear transformations, invariants. Step by step, these tools lead to a deep geometric structure where the fundamental quantities of physics—energy, momentum, angular momentum, spin—naturally appear; that is to say, precisely the objects that experiment allows us to measure.

Some technical steps will require the reader to provisionally accept results without immediately having all the justifications. But the essential point lies elsewhere: at the end of the process, we discover that these physical quantities are not added from outside the model. They are already present, inscribed in the initial, very simple geometric structure.

In 1970, Jean-Marie Souriau built what can be considered a veritable cathedral of geometric physics. The aim of this work is not to present its complete architecture, but to offer a staircase of access. By following it, the reader can gradually ascend to a viewpoint where the overall structure becomes visible, and where one can perceive how geometry organizes and unifies seemingly disparate physical concepts.

This text is therefore intended as an invitation: to observe, through the experience of calculation, that simple tools, used rigorously, allow access to the upper platform, at the top of the cathedral, and to enjoy the view. The reader will be tempted to say to themselves, "*I didn't think it was possible to climb so high*, *to understand so much*, *with such simple tools*." The reward for this effort is a renewed vision of physics, where geometry reveals the hidden framework within which the laws of motion operate.

2. The Construction of Euclid's Group

3D Euclidean space is defined by its metric:

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{1}$$

We choose to introduce the column vector:

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2}$$

And its transpose, the line vector:

$$X^t = (x, y, z) \tag{3}$$

This allows us to translate the expression of the metric (1) into the matrix form of a scalar product, using row-column multiplication:

$$\langle X, X \rangle = X^t X \tag{4}$$

We will now search for a group of matrices a that preserve this length. We have:

$$X' = aX \tag{5}$$

Which means:

$$\langle X', X' \rangle = \langle X, X \rangle \tag{6}$$

First, consider the translations:

$$X' = X + \Delta X = X + C \quad \text{with} \quad \Delta X = C = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \tag{7}$$

Consider two points A and B in space. Translations preserve the length of the segment $X = X_B - X_A$ simply because:

$$X' = X'_B - X'_A = X_B + \Delta X - X_A - \Delta X = X_B - X_A = X \tag{8}$$

Therefore, (6) is automatically satisfied. We will represent this "translation" action in the form of this matrix with format (4, 4):

$$X' = TX = \begin{pmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x + \Delta x \\ y + \Delta x \\ z + \Delta x \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & C \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} X \\ 1 \end{pmatrix} = \begin{pmatrix} X + C \\ 1 \end{pmatrix} \tag{9}$$

We have just constructed, in matrix form, the group of 3D translations. Let us take another matrix a, and express that it preserves the length:

$$X^{t\prime} X' = X^t X \tag{10}$$

$$\left(aX\right)^t \left(aX\right) = X^t X \tag{11}$$

But:

$$\left(AB\right)^t = B^t A^t \tag{12}$$

Therefore, relation (11) is satisfied if the matrices a obey the relation:

$$a^t a = I \quad \text{(identity matrix)} \tag{13}$$

This implies that the inverses a^{-1} of matrices a have their transposes ${}^t a$. The determinant of these matrices is ±1. When it is +1, they are the matrices of 3D rotations. Since this set contains the identity matrix, it is a "special orthogonal" subgroup denoted by SO(3). When the set consists of elements whose matrices are −1, its elements associate a rotation with a P-symmetry, a "right-left" inversion. The union of the two forms the "three-dimensional orthogonal" group O(3). We will opt for another matrix representation of this group, using (4, 4) matrices:

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \tag{14}$$

By combining these two elements, we obtain the three-dimensional Euclidean group:

$$\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & C \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & C \\ 0 & 1 \end{pmatrix} \tag{15}$$

The Euclidean group is the isometry group of Euclidean space, as defined in (1).

3. Construction of the Poincaré Group

Let us consider the metric of space introduced in 1909 by Minkowski [2], whose points are located using the four coordinates $\left(x_0, x_1, x_2, x_3\right)$, where $\left(x_1, x_2, x_3\right)$

are the space coordinates and x_0 the time coordinate, the chronological coordinate.

$$ds^2 = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 \tag{16}$$

We will opt for the representation of quadratic forms introduced by the Dane Jørge Gram in 1883 [3]. We will use a Gram matrix for this:

$$\begin{pmatrix} \pm 1 & 0 & \cdots & 0 \\ 0 & \pm 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \pm 1 \end{pmatrix} \tag{17}$$

In this case, that of special relativity, presented by Einstein in 1915 [4], is:

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{18}$$

Which is equivalent to a signature:

$$(+---) \tag{19}$$

The metric then takes the form:

$$ds^2 = dX^t G dX \tag{20}$$

We can immediately see that in Euclidean space, the Gram matrix is simply the identity matrix:

$$I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{21}$$

We will similarly search for the matrices that will constitute the isometry group of the Minkowski space. We first find the subgroup of spacetime translations, which we will represent in the form of (5, 5) matrices:

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \Delta x_0 \\ 0 & 1 & 0 & 0 & \Delta x_1 \\ 0 & 0 & 1 & 0 & \Delta x_2 \\ 0 & 0 & 0 & 1 & \Delta x_3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & C \\ 0 & 1 \end{pmatrix} \tag{22}$$

We will proceed in the same way by searching for matrices L which, acting on the points X of 4D space, preserve length, that is to say, such that:

$$X''^t G X' = (LX)^t GLX = X^t L^t GLX = X^t GX \tag{23}$$

This will give us the axiomatic definition of the Lorentz group:

$$L^t GL = G \tag{24}$$

It is thus understood that the complete axiomatic definition of the orthogonal

group O(3) is in fact:

$$a^t I a = I \tag{25}$$

In 1905, Henri Poincaré identified a group structure in the transformation introduced by Lorentz in 1904 [5]. He included spacetime translations in the process [6], which gave the isometry group of Minkowski space, which would henceforth be called the Poincaré group.

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \tag{26}$$

We see that the Lorentz group represents 4D translations in a Minkowski space, which some interpreted as four-dimensional "Euclidean" rotations, where one of the dimensions, that of time, is then purely imaginary.

4. Minkowski Space Geodesics

Geodesics are the shortest paths, therefore falling under the variational approach:

$$\begin{aligned} &\delta\int\sqrt{\mathrm{d}x_0^2-\mathrm{d}x_1^2-\mathrm{d}x_2^2-\mathrm{d}x_3^2} \\ &=\delta\int\sqrt{\left(\frac{\mathrm{d}x_0}{\mathrm{d}p}\right)^2-\left(\frac{\mathrm{d}x_1}{\mathrm{d}p}\right)^2-\left(\frac{\mathrm{d}x_2}{\mathrm{d}p}\right)^2-\left(\frac{\mathrm{d}x_3}{\mathrm{d}p}\right)^2}\,\mathrm{d}p=0 \end{aligned} \tag{27}$$

or:

$$\delta\int\sqrt{\dot{x}_0^2-\dot{x}_1^2-\dot{x}_2^2-\dot{x}_3^2}\,\mathrm{d}p=0 \tag{28}$$

We are therefore in a situation that leads to the Lagrange equations, with the Lagrangian:

$$L=\sqrt{\dot{x}_0^2-\dot{x}_1^2-\dot{x}_2^2-\dot{x}_3^2} \tag{29}$$

and Lagrange equations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{x}_i}\right)=\frac{\partial L}{\partial x_i}$$

But we saw [7] that if the derivatives were taken with respect to the length *s*, we also obtained the geodesics with the Lagrangian:

$$L'=L^2=\dot{x}_0^2-\dot{x}_1^2-\dot{x}_2^2-\dot{x}_3^2 \tag{30}$$

The Lagrange equations then give:

$$\ddot{x}_0=\ddot{x}_1=\ddot{x}_2=\ddot{x}_3=0 \tag{31}$$

Geodesics are lines in spacetime:

$$x_0=a_0s+b_0 \tag{32}$$

$$x_1=a_1s+b_1 \tag{33}$$

$$x_2=a_2s+b_{21} \tag{34}$$

$$x_3=a_3s+b_3 \tag{35}$$

However, in shifting from the L-Lagrangian function to the L^2-Lagrangian function, the Lagrange equations reveal real curves, see Equations (32) to (35), but no longer satisfying the condition of non-negativity of the quantity under the radical. These real lines are then equipped with an imaginary length. They can be called virtual geodesics. They do not belong to the solution hypersurface. Light follows geodesics of zero length. From (16), by setting the condition of non-negativity of the length s or the proper time, and setting $s = c\tau$ and $x_0 = ct$, we obtain:

$$c^2 \geq \left(\frac{\mathrm{d}x_1}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}x_2}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}x_3}{\mathrm{d}t}\right)^2 = v^2 \tag{36}$$

The speed cannot exceed the value c.

5. Connected Components of the Isometry Group

We have seen that the orthogonal group has two components, called connected components. These are components that preserve the orientation of objects and components that include a P-symmetry, which reverses this orientation. We can perform the same analysis with the Lorentz group. In Minkowski space, we will therefore seek to identify components involving time inversion (T-symmetry) and space inversion (P-symmetry). The Lorentz group contains the identity element, which reverses neither time nor space:

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{37}$$

Among all the elements of this group of matrices, we will group together all those which, like the identity element, do not invert time or space. They will form the identity subgroup $\{L_n\}$. Among these elements, we will find an infinite number of elements that invert space, such as:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{38}$$

We will group them into a set $\{L_s\}$. We will also have the elements that reverse time, but not space, such as:

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{39}$$

We will group them into a set $\{L_t\}$. Finally, there will be the elements that invert both space and time. Among these:

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{40}$$

Elements that will constitute the set $\{L_{st}\}$. Thus, the complete group will be the union of its four connected components:

$$\{L\} = \{L_n\} \cup \{L_s\} \cup \{L_t\} \cup \{L_{st}\}. \tag{41}$$

The set:

$$\{L_o\} = \{L_n\} \cup \{L_s\} \tag{42}$$

Forms the orthochronous subgroup, or restricted Lorentz group. The set of time-reversing elements will form the orthochronous subset:

$$\{L_a\} = \{L_t\} \cup \{L_{st}\}. \tag{43}$$

From the orthochronous components, we can form the restricted Poincaré group:

$$\begin{pmatrix} L_o & C \\ 0 & 1 \end{pmatrix} \tag{44}$$

Until 1970, until the mathematician Jean-Marie Souriau [1] explained the function of these antichronous components of the group, the use of the Poincaré group was exclusively limited to the restricted group, equipped with its orthochronous components. Before addressing the central theme of this article, we will use the axiomatic definition of the group to construct its inverse, which we will need later. To do this, we start from (23) and multiply by G^1 on both sides. We obtain.

$$G^{-1}L^tGL = I \tag{45}$$

Taking into account that $G^1 = G$, we get:

$$L^{-1} = GL^tG \tag{46}$$

We will now move on to the main body of the article:

6. The Two Ways a Group Acts: Adjoint Action

It is easier, mentally, to refer to the Euclid group, which encompasses the operations of translation, rotation, and symmetry. The initial idea is that a group "transports". We have a point X in space, and an element a of the group. In the case of the Euclid group, we consider a segment AB. By acting on this segment, we transport it to another point in space by performing a translation.

$$\Delta X = (\Delta x, \Delta y, \Delta z) \tag{47}$$

And a rotation:

$$\Omega = \{\omega_1, \omega_2, \omega_3\} \quad \text{(three angles)} \tag{48}$$

At a point X, let a be the element of the group. This depends on the point, that is, the coordinates used to locate X, and on the six parameters defining the action

of this group, the quantities ΔX and Ω. Let's summarize this action from the perspective of "transport":

The group, acting on the segment AB, transports it to $X' = X + \Delta X$ by rotating it by angles $\Omega = \{\omega_1, \omega_2, \omega_{21}\}$. This gives the parameters of the segment $A'B'$ (this operation preserving the length of this segment $\overline{AB}$, or "$\overrightarrow{AB}$").

But we can consider it differently and say that these components of $\overrightarrow{A'B'}$ reflect how this vector is perceived from this new point of view $X' = X + \Delta X$, and this is the very essence of special relativity. Let's limit the Poincaré group to the action of its subgroup, the Lorentz group L. By applying an element of this group to a "motion," represented by a segment connecting two points X and R in four-dimensional Minkowski spacetime, we "see it from another angle" (the action of the Lorentz group being equivalent to this 4D rotation). We "change our frame of reference," and we observe that something is conserved (the quantity $\Delta s = s_B - s_A$). In the particular case of photons, when they travel from a point A to a point B in Minkowski spacetime, the quantity Δs is zero. By observing this motion of the photon from another angle, the quantity is conserved; therefore, it remains zero. And, referring to (16), this means that the measurement of the speed of these photons will also remain at c, whatever the "point of view" (the action of the Lorentz group), meaning that at the same point we observe this motion using a measuring instrument moving at any speed (Michelson experiment).

Returning to Euclid's group, this means that if an observer observes a segment $\overrightarrow{AB}$ "by turning their head", or "by observing it from a different point", or "by observing it through its image in a mirror", or all three at once, the length of this segment will be the same.

We can consider this vector $\overrightarrow{AB}$ as an object in Euclidean space. Now imagine that our operator changes its point of view, its frame of reference, but now observes not a vector but a matrix A_{ij}. This matrix, by maintaining more than one object, is an operator acting on objects in Euclidean space, a linear operator.

The group is also a linear operator, limited to rotations and translations. A linear operator is much more general. Let's imagine we decide to observe this matrix A from another viewpoint $\{\Delta X, \Omega\}$. That is, "from afar," by "turning our heads," and then possibly "using a mirror". How will the components of this matrix appear to us if we preserve its functionality, its identity?

The "identity" of a linear (matrix) operator is its determinant, its trace, and its eigenvalues. If we can conceive of a group action that preserves these three elements, then we will have what we are looking for, in particular, how a linear operator presents itself, seen from another point of view. If we consider the action:

$$aAa^{-1} \tag{49}$$

The first requirement is satisfied, since we know that the determinant of a^{-1} is equal to the inverse of the determinant of a.

The second follows from a theorem on matrices, which tells us that the trace of a product of n matrices, $Tr(ABC)$, is equal to the trace of a product constructed using a circular permutation: $Tr(ABC) = Tr(CAB)$, therefore:

$$Tr\left(aAa^{-1}\right)=Tr\left(a^{-1}aA\right)=Tr\left(A\right) \tag{50}$$

We recall the definition of the concept "eigenvectors v-eigenvalues λ".

v is an eigenvector of the linear operator A if:

$$Av=\lambda v \tag{51}$$

or:

$$\left(\lambda I-A\right)=0 \tag{52}$$

This is a system of n linear equations. This system gives a non-trivial solution $v\neq 0$ if and only if:

$$\det\left(\lambda I-A\right)=0 \tag{53}$$

This gives us the polynomial equation that yields the eigenvalues. These will be preserved since the polynomial equations giving the eigenvalues are identical.

$$\det\left(\lambda I-A\right)=\det\left(\lambda I-A'\right)=\det\left(\lambda I-aAa^{-1}\right) \tag{54}$$

But we can write that:

$$\lambda I=\lambda aa^{-1}=a\lambda Ia^{-1} \tag{55}$$

Thus:

$$\det\left(\lambda I-aAa^{-1}\right)=\det\left[a\left(\lambda I-A\right)a^{-1}\right]=\det\left(\lambda I-A\right) \tag{56}$$

The action in question does indeed retain its inherent values. A group, therefore, gives us two possible actions, which we must interpret as changes in perspective:

- With respect to vectors A, that is to say, the action aA called "regular left" (we call it "Right Regular Action")
- With respect to linear operators, the adjoint action (also called conjugation) is written aAa^{-1}

Tha action:

$$a^{-1}Aa \tag{57}$$

It is then only the adjoint action constructed with the element of the group a^{-1}.

7. Action of the Group on Its Lie Algebra

We will continue to clarify elements of calculation using only matrix calculus, without delving into the details of symplectic geometry—that is, without explaining why the coadjoint action of the group on the dual of its Lie algebra reveals this second space, which Souriau calls the momentum space and which complements the Minkowski space. We have just constructed a tool: the adjoint action, acting on matrices. We will now apply this tool to a set of matrices constituting what is called the Lie algebra of the group.

The Poincaré group is a Lie group, a group of matrices. The differential δa of the group is a set of matrices (which do not form a group since they do not contain

the identity element) called the Lie algebra of the group:

$$\delta a = \delta \begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} = Z = \begin{pmatrix} \delta L & \delta C \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} \Lambda & \gamma \\ 0 & 0 \end{pmatrix} \tag{58}$$

Let G be the Gram matrix of the Minkowski space, also called the "metric matrix". We will differentiate in the neighborhood of an element $L_{(0)}$, according to:

$$L = L_{(0)} + \varepsilon \delta L = L_{(0)} + \Lambda \tag{59}$$

And we will pose:

$$\Lambda = G\omega \tag{60}$$

By differentiating (19) and doing:

$$L_{(0)} = I \tag{61}$$

That is, by differentiating in the neighborhood of the group's identity element, we show that the matrix Λ is antisymmetric:

$$\Lambda^t = -\Lambda \tag{62}$$

We will construct the adjoint action of a^{-1} on this Lie algebra δa of the group *a*. That is to say, the expression:

$$Z' = \begin{pmatrix} G\omega' & \gamma' \\ 0 & 0 \end{pmatrix} = a^{-1} Z a \tag{63}$$

We need the inverse matrix a^{-1} of the one representing the Poincaré group. We find easily:

$$a^{-1} = \begin{pmatrix} L^{-1} & -L^{-1}C \\ 0 & 1 \end{pmatrix} \tag{64}$$

By forming (63), we easily find, after having made explicit and taken into account that $GG = I$

$$\omega' = L^t \omega L \tag{65}$$

$$\gamma' = GL^t \omega C + GL^t G\gamma \tag{66}$$

This result represents the result of the adjoint action of the element a^{-1} of the group on the element of its Lie algebra Z' (58).

8. Action of the Group on the Dual of Its Lie Algebra

The aim of this article is not to explain the path (symplectic geometry) by which J-M Souriau establishes this link between geometry and physics, but rather to demonstrate to students, engineers, and teachers who have access to matrix calculus, the simplicity, elegance, and clarity of the calculations. At this point, we must introduce duality. The element of the Lie algebra consists of an antisymmetric matrix ω, of size (4, 4), therefore with six components. Let us write:

$$Z = \left\{ \omega_{sx}, \omega_{sy}, \omega_{sz}, \omega_{fx}, \omega_{fy}, \omega_{fz}, \gamma_t, \gamma_x, \gamma_y, \gamma_z \right\} \tag{67}$$

It is therefore logical to imagine that the moment represents this set of ten com-

ponents.:

$$\mu = \left\{ s_x, s_y, s_z, f_x, f_y, f_z, E, p_x, p_y, p_z \right\} \tag{68}$$

The duality corresponding to the relationship:

$$\left\langle Z, \mu \right\rangle = Cst \tag{69}$$

The six components of an antisymmetric matrix M can be arranged in the form:

$$M = \begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix} \tag{70}$$

Let's multiply this matrix by an antisymmetric matrix:

$$\begin{pmatrix} 0 & -s_z & s_y & f_x \\ s_z & 0 & -s_x & f_y \\ -s_y & s_x & 0 & f_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix} \times \begin{pmatrix} 0 & \omega_{12} & \omega_{13} & \omega_{14} \\ -\omega_{12} & 0 & \omega_{23} & \omega_{24} \\ -\omega_{13} & -\omega_{23} & 0 & \omega_{34} \\ -\omega_{14} & -\omega_{24} & -\omega_{34} & 0 \end{pmatrix} \tag{71}$$

I simply calculate the diagonal terms to get the trace, and I obtain:

$$\begin{aligned} & s_z \omega_{12} - s_y \omega_{13} - f_x \omega_{14} \\ & s_z \omega_{12} + s_x \omega_{23} - f_y \omega_{24} \\ & -s_y \omega_{13} + s_x \omega_{23} - f_z \omega_{34} \\ & -f_x \omega_{14} - f_y \omega_{24} - f_z \omega_{34} \end{aligned} \tag{72}$$

When I add them up, I see that I have twice what I'm looking for. So I just need to form:

$$\frac{1}{2} Tr\left(M \omega \right) \tag{73}$$

With:

$$\mu\left(Z \right) = \frac{1}{2} Tr\left(M \omega \right) + P^t G \gamma \tag{74}$$

The combination $\left\{ M, P \right\}$ of an antisymmetric matrix and a vector forms what is called a torsor. This term will take on its full meaning when we apply all of the above to the Euclid group. It is then appropriate to express the duality in terms of scalar conservation (a simple trick for handling matrix calculations):

$$\frac{1}{2} Tr\left(M \cdot \omega \right) + P^t \cdot G\gamma = \frac{1}{2} Tr\left(M' \cdot \omega' \right) + P'^t \cdot G\gamma' \tag{75}$$

It will then suffice to substitute the quantities ω' and γ' into this equation, from (65) and (66), to construct the desired dual transformation:

$$\left\{ M, P \right\} \to \left\{ M', P' \right\} \tag{76}$$

Let's perform this replacement:

$$\frac{1}{2} Tr\left(M \cdot \omega \right) + P^t \cdot G\gamma = \frac{1}{2} Tr\left(M' \cdot L^t \omega L \right) + P'^t \cdot G\left(GL^t \omega C + GL^t G\gamma \right) \tag{77}$$

This equation must be satisfied for all ω and γ. Identification on γ gives, taking into account that $GG = I$:

$$P^t = P'^t L^t \tag{78}$$

Or:

$$P = LP' \tag{79}$$

The calculation for the application $M \to M'$ has been returned in the appendix. The set represents the expression of the coadjoint action of the Poincaré group on its moment space, a result which was first given by the mathematician J-M Souriau in 1970 [1].

9. The Euclid Group, a Particular Dynamic Group

If the reader has followed all these calculations closely, constructing the action of the Euclid group on the dual of its Lie algebra—that is, on its moment—will be a useful exercise. The result is absolutely similar to what emerges from the Poincaré group:

$$P = aP' \tag{80}$$

$$M = aM'a^t + CP'^t a - aP'^t C \tag{81}$$

The formulas are the same, with the matrix of the orthogonal group O(3) replacing that of the Lorentz group. The objects P and M then acquire an elegant physical meaning. P is a force, and M is a torsional moment. The Euclid group is therefore a special dynamic group, that of statics. Any change of reference frame in this uniform medium preserves its "state of stress".

10. Significance for the Physicist, the Engineer, and the Student

We started with a four-dimensional space, Minkowski spacetime, the spacetime of special relativity, defined by its metric (16).

In Section 2, we constructed its isometry group, the complete Poincaré group (26), formed from the spacetime translation subgroup (22) and the Lorentz subgroup (24).

In Section 3, we showed that the Lorentz group decomposes into four connected components: two orthochronous components (not reversing time), forming the subgroup known as the restricted Lorentz group, and two antichronous components (reversing time).

In Section 4, we considered two types of action: one acting on vectors and the other, the adjoint action, acting on matrices. The fundamental idea of special relativity was to show that masses and photons correspond to different ways of moving through Minkowski space, the former along geodesics of zero length, the latter along geodesics of zero length. Here we find Souriau's favorite expression.

- *Tell me how you move, and I'll tell you what you are.*

We thus identify particles with specific motions. However, joining two distinct

points A and B in spacetime with a geodesic is not enough to define the object in question. It lacks its attributes: energy, momentum, and spin. The idea is to group all these objects according to a matrix, constituting the object moment μ. We then consider a change of reference frame, which translates into a group action. The action on the left of the group preserves the length s. This action preserves the nature of the motion, the nature of the particle. We then seek to construct an action that preserves the identity of an object, which takes the form of a matrix, and we then define this operator, which is the coadjoint action. We then examine what this change of reference frame preserves: these other attributes of the motion that the action on the left does not allow us to "see". We obtain a wrench, an antisymmetric matrix, plus a four-vector P. Let's show that the coadjoint action preserves its length $P^tGP = \left(LP'\right)^t GLP$ is satisfied due to the axiomatic definition of L. Therefore, the dot product $\langle P, P \rangle$ is conserved. This is a property of motion. Let's introduce the scalar E and the 3-vector p by decomposing P according to:

$$P = \begin{pmatrix} E \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} \tag{82}$$

With $c = 1$ and:

$$p^2 = p_1^2 + p_2^2 + p_3^2 \tag{83}$$

We get:

$$E^2 - p^2 = Cst \tag{84}$$

We can therefore change our frame of reference, "consider a motion from another angle," using an observational or measuring instrument determined by this change of frame in spacetime. This quantity will be conserved and will thus constitute a characteristic of this particular motion. Souriau thus reveals, starting solely from the space where these motions occur, defined by his metric alone, essential characteristics of the entire range of motions that can correspond to the geodesics of this spacetime. Quantities familiar to the relativistic physicist, such as energy E and momentum p, emerge from this purely geometric approach. But these two parameters are not sufficient to categorize a particular motion. Spin is missing.

The spin 3-vector is one of the two 3-vectors involved in the construction of the antisymmetric matrix M, the second being the "passage" 3-vector. Souriau shows that this third vector is merely an artifact that must be eliminated by opting for the specific coordinate system that accompanies the particle's motion. For the coadjoint action of the group on the spin, we refer the reader to the appendix; we wish to focus here on the action on the energy-momentum pair. That being said, it is quite remarkable that Souriau, in 1970, brought spin to the fore as an object of pure geometry, leading to the concept of a dynamical group. We can say that the entire approach illustrates Souriau's statement:

- *A little mathematics takes you away from reality, a lot brings you back to it.*

11. Physical Meaning of T-Symmetry

What does the existence of antichronous components in the complete Poincaré group mean?

Geometrically, this means that, in Minkowski space, if we start with a set of motions oriented in a past-future direction, by introducing antichronous elements, we create their T-symmetrics, the same motions, but with an inverse temporal orientation, oriented in a future-past direction. Can we then consider that they belong to the world of physics?

What impact will this have on the parameters of the movement?

Equation (79) tells us that time reversal, T-symmetry, reverses both energy and momentum (referring to the appendix, we will see that it conserves spin).

Using the relation $E = mc^2$, we see that this reversal of energy is synonymous with a reversal of mass. Thus, the subsidiary question that arises is: can negative masses coexist with our positive masses in our spacetime?

In 1970, the emergence of objects with negative energy and mass was viewed with perplexity, as physicists preferred to continue limiting themselves to the restricted Poincaré group, restricted to its orthochronous components. The Janus cosmological model, on the other hand, exploited the complete group [8], involving the use of the geometric concept of a covering [9].

12. Geometric Translation of C-Symmetry

In chapter 5 of a work published in 1964 [10], J-M Souriau adds a fifth dimension, x_5, of the spacelike type, to spacetime. He notes that the inversion of this additional dimension accounts for charge conjugation. In Chapter 5 of [1], entitled "A Method of Quantization," the link between quantization and the compactness of the additional dimension is illustrated. Article [9] is a simple outline, through the theory of dynamical groups, of the geometrization of quantum mechanics, by taking into account additional, closed dimensions. C-symmetry then follows from the inversion of the additional dimensions. By coupling this C-symmetry to P and T symmetries, through the Janus group, the Janus model then becomes the geometric translation of a CPT symmetry of the two sheets of the coating. But these additional dimensions remain real. It seems to us that a geometrization including gravitation and quantum mechanics could emerge from a fibration of spacetime using compact complex dimensions. Geometrically, if we add a single compact dimension (fibered in a circle), the symmetry can be illustrated in **Figure 1**.

13. When Quantum Field Theory Rejects a Priori States of Negative Energy

In quantum field theory [11], we find P and T operations of space and time inversion. Two options then present themselves, depending on whether these operators are:

- Linear and unitary
- Antilinear and antiunitary

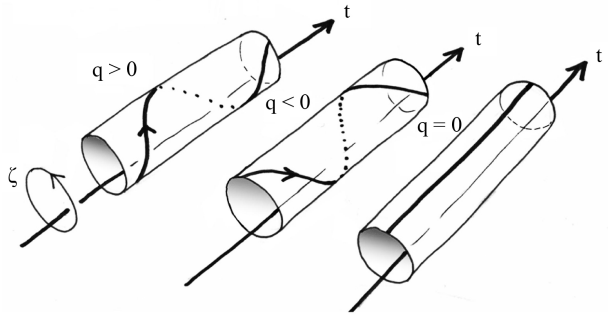


Figure 1. Educational image illustrating C-symmetry. *q* is the charge.

The imperative to avoid generating the existence of negative energy states, considered a priori impossible [12], leads this theory to the restrictive choice:

- Linear and unitary P
- Antilinear and antiunitary T

In 2014, in [12], it was suggested that the emergence of the Janus Cosmological Model calls into question this absolute principle of the a priori negation of negative energy states. The mathematician Nathalie Debergh takes up this idea, showing that with a linear and unitary operator T, such states naturally emerge from the Dirac and Schrödinger equations [13] [14]. In 2021 [15], the technique for constructing the elements of physics, based on calculating the action on the dual of its Lie algebra, was applied, extending it to the complex extension of the Poincaré group. This approach seems to us to constitute a tool for a geometrization of quantum mechanics, which remains to be done. However, this highly technical study has not garnered much attention from theoretical physicists. We will revisit this technique in the following section, attempting to make it more accessible.

14. Construction of the Coadjoint Action of the Dual of the Lie Algebra of the Complexified Poincaré Group

There are works in the literature that represent various attempts to translate quantum mechanics into a geometric approach. Our aim is not to be exhaustive in this regard, and interested readers can refer to the articles cited in reference [15]. What follows essentially reiterates part of that article, but with detailed calculations to make it more appealing to our target audience. It represents one way of presenting the extension of the group into complex numbers.

The real Poincaré group emerges as an isometry group of the Minkowski space, defined by its metric (16). Here we will focus on the complexified Minkowski space. In 1854, the French mathematician became interested in complex quadratic forms [16]:

$$\sum a_{ij} z^i \overline{z}^j \tag{85}$$

where z^i is a complex vector, defined in the set $\mathbb{C}^4$ and its conjugate (such that the imaginary part of each of these complex numbers is inverted):

$$z = a + ib \rightarrow \overline{z} = a - ib \tag{86}$$

The following metric corresponds to the reduced canonical form:

$$\mathrm{d}s^2 = z^i \overline{z}^i \tag{87}$$

In this case, the dot product is always real and equal to the square of the norm of the vector:

$$\langle z, z \rangle = \|z\|^2 = a^2 + b^2 \tag{88}$$

In the real case, considering matrices M, we introduced the transposition operation M^t, consisting of performing a symmetry with respect to the main diagonal:

$$\left(M_{ij}\right)^t = M_{ji} \tag{89}$$

The vector X could then be expressed in two forms. A "standard form": the column vector, and its transpose, the row vector. In reality, the Euclidean metric could then be written in the form:

$$\mathrm{d}s^2 = \mathrm{d}X^t I \mathrm{d}X \tag{90}$$

where I is the identity matrix (the Gram matrix of Euclidean space). We will use the adjoint of a matrix, where there is not only symmetry with respect to the main diagonal, but where these terms are also conjugate:

$$M = z_{ij} M_{ij} \rightarrow M^\dagger = \overline{z}_{ji} \tag{91}$$

This also applies to vectors. The adjoint of a column vector then becomes the row vector where the components are replaced by their conjugates. Thus, in a three-dimensional space:

$$\mathrm{d}X = \begin{pmatrix} \mathrm{d}x_1 \\ \mathrm{d}x_2 \\ \mathrm{d}x_3 \end{pmatrix} \rightarrow \mathrm{d}X^\dagger = \begin{pmatrix} \mathrm{d}\overline{x}_1 & \mathrm{d}\overline{x}_2 & \mathrm{d}\overline{x}_3 \end{pmatrix} \tag{92}$$

So, the complexified Euclidean space has the (real) metric:

$$\mathrm{d}s^2 = \mathrm{d}X^\dagger I \mathrm{d}X \tag{93}$$

Similarly, the complexified 4D Minkowski space $\mathbb{C}(1,3)$, will have the (real) metric:

$$\mathrm{d}s^2 = \mathrm{d}X^\dagger G \mathrm{d}X \tag{94}$$

where G is the classic Gram matrix (18). We will now search for the isometry group of this complexified Minkowski space. Let L be a complex matrix. Let's write that it preserves the length:

$$\mathrm{d}X'^\dagger G \mathrm{d}X' = \mathrm{d}\left(LX^\dagger\right) G \mathrm{d}\left(LX\right) = \mathrm{d}X^\dagger G \mathrm{d}X \tag{95}$$

Earlier, we saw how the transpose of a product is constructed. An analogy theorem works for the adjoint of a product:

$$\left(AB\right)^{\dagger} = B^{\dagger}A^{\dagger} \tag{96}$$

Thus, relation (95) becomes:

$$dX^{\dagger}L^{\dagger}GLdX = dX^{\dagger}GdX \tag{97}$$

This gives us the axiomatic expression:

$$L^{\dagger}GL = G \tag{98}$$

This is nothing other than the complexified Lorentz group. By adding the complexified 4D translations, we find that the isometry group of the complexified Minkowski space is simply the complexified Poincaré group, where L and C are complex. Formally, this is similar to (26).

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix} \tag{99}$$

We will then extend the calculations of the coadjoint action of the group on the dual of its Lie algebra. To perform this calculation, we will need to acquire new tools. In the real calculus, we saw the appearance of symmetric and antisymmetric matrices, defined by:

$$\text{Matrice symétrique:}\quad M + M^{t} = 0 \tag{100}$$

$$\text{Matrice antisymétrique:}\quad M - M^{t} = 0 \tag{101}$$

Here, we have:

$$\text{Matrice hermitienne:}\quad M + M^{\dagger} = 0 \tag{102}$$

$$\text{Matrice anti hermitienne:}\quad M - M^{\dagger} = 0 \tag{103}$$

In the real world, an antisymmetric matrix has its main diagonal composed of zeros. In the complex world, this diagonal will consist of purely imaginary numbers. Let's begin by constructing the inverse matrix of L:

$$L^{\dagger}GL = G \to \left(G^{-1}L^{\dagger}G\right)L = I \to L^{-1} = G^{-1}L^{\dagger}G \tag{104}$$

This is the same as (46), but with the transpose replaced by the adjoint. We will expand the matrix L around the identity element:

$$L = I + \varepsilon\delta L \tag{105}$$

By introducing into (104) and neglecting the second order:

$$\left(G\delta L\right)^{t} + G\delta L = 0 \tag{106}$$

This means that $G\delta L$ is an anti-Hermitian matrix which we will denote by ω.

$$G\delta L = \omega \text{ and } GG = I \to \delta L = G\omega \tag{107}$$

We will therefore write the element of the Lie algebra according to:

$$Z = \begin{pmatrix} \delta L & \delta C \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} G\omega & \gamma \\ 0 & 0 \end{pmatrix} \tag{108}$$

If we construct the group's action on its Lie algebra:

$$Z' = a^{-1} Z a \tag{109}$$

Using calculations entirely similar to those performed in the real domain, we obtain the same equations, where the adjoint replaces the transpose.

$$\omega' = L^{\dagger} \omega L \tag{110}$$

$$\gamma' = G L^{\dagger} \omega C + G L^{\dagger} G \gamma \tag{111}$$

This time, we will put the complex moment in the form of a pair consisting of an anti-Hermitian matrix M and a complex four-vector P.

$$\mu = \{M, P\} \tag{112}$$

An antisymmetric matrix of size (4, 4) depends, in complex numbers, on 12 scalars.

The element Z of the Lie algebra of the Poincaré group belongs to a vector space of dimension 20 (the dimension of the group). We can therefore write:

$$Z = \left\{\omega_{sx}, \omega_{sy}, \omega_{sz}, \omega_{fx}, \omega_{fy}, \omega_{fz}, \gamma_t, \gamma_x, \gamma_y, \gamma_z\right\} \tag{113}$$

It is therefore logical to imagine that the moment represents this set of ten complex components:

$$\mu = \left\{s_x, s_y, s_z, f_x, f_y, f_z, E, p_x, p_y, p_z\right\} \tag{114}$$

The duality corresponding to the relationship:

$$\left\langle Z, \mu \right\rangle = Cst \tag{115}$$

Let us form the anti-Hermitian matrix:

$$M - \begin{pmatrix} 0 & -\overline{s}_z & s_y & f_x \\ s_z & 0 & -\overline{s}_x & f_y \\ -\overline{s}_y & s_x & 0 & f_z \\ -\overline{f}_x & -\overline{f}_y & -\overline{f}_z & 0 \end{pmatrix} \tag{116}$$

Let's multiply this matrix by the anti-Hermitian matrix ω:

$$\begin{pmatrix} 0 & -\overline{s}_z & \overline{s}_y & \overline{f}_x \\ s_z & 0 & -\overline{s}_x & \overline{f}_y \\ -s_y & s_x & 0 & \overline{f}_z \\ -f_x & -f_y & -f_z & 0 \end{pmatrix} \times \begin{pmatrix} 0 & \overline{\omega}_{12} & \overline{\omega}_{13} & \overline{\omega}_{14} \\ -\omega_{12} & 0 & \overline{\omega}_{23} & \overline{\omega}_{24} \\ -\omega_{13} & -\omega_{23} & 0 & \overline{\omega}_{34} \\ -\omega_{14} & -\omega_{24} & -\omega_{34} & 0 \end{pmatrix} \tag{117}$$

Once again, keeping only the trace:

$$\begin{aligned} &\overline{s}_z \omega_{12} - \overline{s}_y \omega_{13} - \overline{f}_x \omega_{14} \\ &s_z \overline{\omega}_{12} + \overline{s}_x \omega_{23} - \overline{f}_y \omega_{24} \\ &-s_y \overline{\omega}_{13} + s_x \overline{\omega}_{23} - \overline{f}_z \omega_{34} \\ &-f_x \underline{\omega}_{14} - f_y \overline{\omega}_{24} - f_z \overline{\omega}_{34} \end{aligned} \tag{118}$$

These quantities are all of the form $a\overline{b} + \overline{a}b$. We obtain the desired scalar quantity by introducing: $\mu(Z) = \frac{1}{2} Tr(M\omega) + P^{\dagger} G \gamma$ où where M and γ are anti-

hermitian. We can therefore write:

$$\mu \equiv \{M, P\} \quad \text{with} \quad M^{\dagger} = -MP \in \mathbb{C}^4 \tag{119}$$

$$\frac{1}{2}Tr(M\omega) + P^{\dagger}G\gamma = \frac{1}{2}Tr(M'\omega') + P'^{\dagger}G\gamma' \tag{120}$$

We get:

$$\frac{1}{2}Tr(M\omega) + P^{\dagger}G\gamma = \frac{1}{2}Tr(M'L'^{\dagger}\omega L) + P'^{\dagger}G(GL^{\dagger}\omega C + GL'^{\dagger}G\gamma) \tag{121}$$

$$\frac{1}{2}Tr(M\omega) + P^{\dagger}G\gamma = \frac{1}{2}Tr(M'L'^{\dagger}\omega L) + P'^{\dagger}L^{\dagger}\omega C + P'^{\dagger}L'^{\dagger}G\gamma \tag{122}$$

Identifying the term γ gives:

$$P^{\dagger} = P'^{\dagger}L^{\dagger} \rightarrow P = LP' \tag{123}$$

In the trace, a circular permutation can be performed:

$$Tr(M'L'^{\dagger}\omega L) = Tr(LM'L'^{\dagger}\omega) \tag{124}$$

Separating the terms into ω:

$$\frac{1}{2}Tr(M\omega) = \frac{1}{2}Tr(LM'L'^{\dagger}\omega) + P'^{\dagger}L^{\dagger}\omega C \tag{125}$$

- The term $P'^{\dagger}L^{\dagger}\omega C$ is the dot product of two vectors
- There is a row vector $P'^{\dagger}$
- And the column vector $L^{\dagger}\omega C$

If I have a scalar that is the product of two vectors (the components are complex this time):

$$\begin{pmatrix} A & B \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = AC + BD \tag{126}$$

I can form the matrix:

$$\begin{pmatrix} C \\ D \end{pmatrix}\begin{pmatrix} A & B \end{pmatrix} = \begin{pmatrix} AC & BC \\ AD & BD \end{pmatrix} \tag{127}$$

We can see that:

$$\begin{pmatrix} B & B \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = Tr\begin{pmatrix} AC & BC \\ AD & BD \end{pmatrix} \tag{128}$$

The second term of the second member is equal to the product of a line-matrix by a column-matrix. This is equal to the reversed product. Hereafter, schematically, the product of a line-matrix by a column-matrix:

$$P'^{\dagger}L^{\dagger}\omega C = Tr(L^{\dagger}\omega CP'^{\dagger}) \tag{129}$$

In the trace, one can achieve a circular permutation:

$$P'^{\dagger}L^{\dagger}\omega C = Tr(CP'^{\dagger}L^{\dagger}\omega) \tag{130}$$

whence:

$$\frac{1}{2}Tr(M\omega) = \frac{1}{2}Tr(LM'L'^{\dagger}\omega) + Tr(CP'^{\dagger}L^{\dagger}) \tag{131}$$

$$\boxed{\begin{array}{c} M = LM'L^{\dagger} + 2CP'^{\dagger}L^{\dagger} \\ P = LP' \end{array}} \tag{132}$$

We then see that the group's action on the anti-Hermitian matrix is expressed in the form:

$$\boxed{\mu' = a\mu a^{\dagger}} \tag{133}$$

This is the action of the complex Poincaré group, the isometry group of the complexified Minkowski space, on the dual of its Lie algebra. The formula is derived from that of the action, in real numbers (see the appendix), by replacing transposed with adjoint.

15. Conclusions

In this work, we have shown that, starting from the sole geometric structure of Minkowski space and its isometry group, it is possible to derive, through essentially algebraic means, the fundamental quantities of relativistic physics. The approach introduced by Jean-Marie Souriau, based on the action of the Poincaré group on the dual of its Lie algebra, thus reveals energy, momentum, and spin as intrinsic geometric objects. The value of this presentation lies in the fact that it relies on accessible tools, while leading to a unified understanding of physical concepts generally introduced independently. It highlights the structuring role of symmetry groups and their actions in the organization of physical laws.

The extension of this construction to the complex framework, proposed here as an exercise, constitutes only a first sketch. It nevertheless shows that, as soon as one undertakes this type of approach, new elements naturally appear in the formalism. These should not be interpreted as established results, but rather as indications, signals suggesting that this path deserves to be explored further, which will be the subject of the following article.

We have deliberately kept the present work within a pedagogical framework intended to remain accessible to engineers and physics students, using only finite-dimensional matrix methods and avoiding the technical formalism of infinite-dimensional functional spaces.

In the following article, the situation will become substantially different. The transition toward quantum mechanics and quantum field theory naturally leads to new classes of groups acting on infinite-dimensional spaces. It is only within this broader framework that questions related to asymptotic symmetry groups, infinite-dimensional representations, spinorial structures, or formalisms such as Newman–Penrose may naturally arise and be meaningfully discussed.

Author Contributions

The body of the article is by J.-P. Petit. G. d'Agostini verified the calculations.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Souriau, J.M. (1977) Structure des systèmes dynamiques. Birkhaüser.

[2] Minkowski, H. (1909) Raum und Zeit. *Physikalische Zeitschrift*, **10**, 75-88.

[3] Gram, J.P. (1883) Über die Entwicklung reeller Funktionen in Reihen mittelst der Methode der kleinsten Quadrate. *Journal für die reine und angewandte Mathematik*, **94**, 41-73. https://doi.org/10.1515/crll.1883.94.41

[4] Einstein, A. (1915) Erlklärung der Perihelbewegung des Merkur aus allgemeinen Relarivitätstheorie. Königlish Preubische Akademie der Wissenschaften.

[5] Lorentz, H.A. (1904) Electromagnetic Phenomena in a System Moving with Any Velocity Smaller than That of Light. *The Proceedings of the Royal Netherlands Academy of Arts and Sciences*, **6**, 809-831.

[6] Poincaré, H. (1906) Sur la dynamique de l'électron. Rendiconti del Circolo Matematico di Palermo.

[7] Petit, J.P. and D'Agostini, G. (2026) Mathematical and Geometrical Inconsistency of the Black Hole Model. Part I. *Journal of Modern Physics*, **17**, 199-239. https://doi.org/10.4236/jmp.2026.172014

[8] Petit, J.P. and Zejli, H. (2024) Study of Symmetries through the Action on Torsors of the Janus Symplectic Group. *Reviews in Mathematical Physics*, **37**, Article 2450054. https://doi.org/10.1142/s0129055x24500545

[9] Petit, J.P., Margnat, F. and Zejli, H. (2024) A Bimetric Cosmological Model Based on Andreï Sakharov's Twin Universe Approach. *The European Physical Journal C*, **84**, Article No. 1226. https://doi.org/10.1140/epjc/s10052-024-13569-w

[10] Souriau, J.M. (1964) Géométrie et Relativité. Hermann (French Edition Only).

[11] Steven, W. (2005) The Quantum Theory of Fields. Cambridge University Press.

[12] Petit, J.P. and D'Agostini, G. (2014) Negative Mass Hypothesis in Cosmology and the Nature of Dark Energy. *Astrophysics and Space Science*, **29**, 145-182.

[13] Debergh, N., Petit, J.P. and D'Agostini, G. (2018) On Evidence for Negative Energies and Masses in the Dirac Equation through a Unitary Time-Reversal Operator. *Journal of Physics Communications*, **2**, Article 115012. https://doi.org/10.1088/2399-6528/aaedcc

[14] Debergh, N. and Petit, J.P. (2022) On Some Reasons Why the Time Reversal Operator Could Be Unitary. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3940205

[15] Debergh, N., D'Agostini, G. and Petit, J.P. (2021) On the Poincaré Algebra in a Complex Space-Time Manifold. *Journal of Modern Physics*, **12**, 218-228. https://doi.org/10.4236/jmp.2021.123017

[16] Hermite, C. (1854) Sur la théorie des formes quadratiques. *Journal de Mathématiques Pures et Appliquées*, **1**, 215-234.

Appendix

This appendix represents the completion of the calculation of the coadjoint action of the group on the dual of its Lie algebra, with respect to the matrix *M*, a result presented by Souriau in 1970 [1]. As will be seen, this formula shows that the antichronous components of the group do not modify the spin. This section makes use of theorems concerning matrices.

We therefore start from Equation (77) of the text, which gives us:

$$\frac{1}{2}Tr\left(M\cdot\omega\right)=\frac{1}{2}Tr\left(M'\cdot L^{t}\omega L\right)+P'^{t}L^{t}\omega C \tag{1}$$

We will use the fact that the trace of a matrix product does not change if we perform a circular permutation within that product.

$$\frac{1}{2}Tr\left(M\cdot\omega\right)=\frac{1}{2}Tr\left(LM'\cdot L^{t}\omega\right)+P'^{t}L^{t}\omega C \tag{2}$$

The second term of this equation is equal to the product of a row matrix P'^{t} by a column matrix $L^{t}\omega C$. Hereafter, schematically, this is equal to the trace of a row-matrix by a column-matrix:

$$P'^{t}L^{t}\omega C=Tr\left(L^{t}\omega CP'^{t}\right) \tag{3}$$

And, within this trace, a new circular permutation is performed.

$$Tr\left(L^{t}\omega CP'^{t}\right)=Tr\left(CP'^{t}L^{t}\omega\right) \tag{4}$$

Whence:

$$\frac{1}{2}Tr\left(M\cdot\omega\right)=\frac{1}{2}Tr\left(M'\cdot L^{t}\omega L\right)+Tr\left(CP'^{t}L^{t}\omega\right) \tag{5}$$

where ω is an antisymmetric matrix. We know that a matrix's trace is equal to the product of another matrix by a symmetrical matrix. Any matrix can be symmetrized or antisymmetrized. In addition, the trace of the product of a matrix by an antisymmetric matrix is zero. Whence:

$$Tr\left(A\omega\right)=Tr\left[antisym\left(A\right)\times\omega\right] \tag{6}$$

We can apply that to the matrix $CP'^{t}L^{t}$ where we take the trace of its product by an antisymmetric matrix ω.

$$CP'^{t}L^{t}=sym\left(CP'^{t}L^{t}\right)+antisym\left(CP'^{t}L^{t}\right) \tag{7}$$

But:

$$Tr\left[sym\left(CP'^{t}L^{t}\right)\times\omega\right]=0 \tag{8}$$

So that:

$$Tr\left(CP'^{t}L^{t}\omega\right)=Tr\left[antisym\left(CP'^{t}L^{t}\right)\times\omega\right] \tag{9}$$

$$antisym\left(CP'^{t}L^{t}\right)=\frac{1}{2}\left[CP'^{t}L^{t}-\left(CP'^{t}L^{t}\right)^{t}\right] \tag{10}$$

$$\left(CP'^{t}L^{t}\right)^{t}=LPC^{t} \tag{11}$$

$$Tr\left(CP'^{t}L^{t}\right)=\frac{1}{2}Tr\left(CP'^{t}L^{t}-LPC^{t}\right) \tag{12}$$

Finally:

$$M=LM'L^{t}+CP'^{t}L^{t}-LPC^{t} \tag{13}$$

The intrinsic invariance of the magnitude s of the spin vector then becomes apparent when the motion is inscribed along a world line (along a geodesic of Minkowski space where the length is real). Under these conditions, the vector C is collinear with P. In this case, the last two terms of the right-hand side are zero. But the transition vector of M is also zero, which gives the matrix M a row and a column composed of zeros. What remains is the omega submatrix, the spin submatrix. The action then represents an orthogonal rotation of the spin vector, which preserves its magnitude s. Incidentally, let us recall that the momentum matrix is

$$\mu=\begin{pmatrix} M & -P \\ P^{t} & 0 \end{pmatrix} \tag{14}$$

So that the action of the group on its momentum is:

$$\boxed{\mu=a\mu' a^{t}} \tag{15}$$

In 2019, Academician Thibaud Damour, known as “Mr. Cosmology” in France, posted an article on his IHES page criticizing my Janus cosmological model[102].



Academician Thibaud Damour

At the same time, he sent me a certified letter to my personal address:

[102] Damour’s personal website: https://www.ihes.fr/~damour/
http://www.jp-petit.org/papers/cosmo/2019-Damour-IHES.pdf

T. Damour, Letter to J.-P. Petit concerning the Janus model, January 7, 2019. English translation of the original signed letter, reproduced as supplementary material.

The original French document is reproduced together with the English translation.

Traduction letter Damour :

Thibault Damour
IHES, Bures-sur-Yvette

to Jean-Pierre Petit, BP 55, 84122 Pertuis

7 January 2019

Copies: I reserve the right to send copies of this letter to all the persons whom you cite on your website, in your videos, and in your letters, as well as to any person interested in the "Janus model".

Subject: "Janus model"

Sir,

I have duly received your letter of 7 December 2018. On the other hand, I have no recollection of having previously received from you a response to the objections and criticisms that I communicated to you in my letter of 12 March 2014, written after receiving the preprints of your recent papers, in particular: J. P. Petit and G. d'Agostini, "Negative mass hypothesis in cosmology and the nature of dark energy", *Astrophys. Space Sci.*, DOI 10.1007/s10509-014-2106-5; and J. P. Petit and G. d'Agostini, "Cosmological bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe", *Mod. Phys. Lett. A*, Vol. 29 (No. 34) (2014) 145082.

The receipt of your letter prompted me to take a closer look at the structure and consequences of the field equations of what you call the "Janus model". I have in particular pursued in greater detail the reasoning and arguments that I had indicated to you in my 2014 letter. I have thus demonstrated that the "Janus model" was **physically (and mathematically) inconsistent**. This inconsistency is particularly manifest in the quasi-Newtonian limit, where the "Janus model" implies that ordinary matter must, at the same time, gravitationally attract itself and repel itself. I have put my reasoning and conclusions on my web page (link "Sur le 'modèle Janus' de J. P. Petit" at http://www.ihes.fr/~damour).

You will understand, after reading it, that there can be no question of inviting you to give a seminar at the IHES, since you do not have a coherent theory, mathematically and physically well defined, to present. [Note, moreover, in passing, that I am not, as you write, "responsible for the cosmology seminar at the IHES". There is in fact no regular cosmology seminar at the IHES.]

Yours sincerely,

The registered letter with return receipt from Thibaud Damour

As explained in the article, he seized upon the Janus system of equations as presented in the 2014 article published in *Modern Physics Letters A*[103], in which—due to a typographical error—the "hats" were missing from the interaction tensors, which were then confused with the field tensors:

$$\hat{T}_{\mu\nu} \text{ and } \hat{\bar{T}}_{\mu\nu} \text{ were confused with } T_{\mu\nu} \text{ and } \bar{T}_{\mu\nu}$$

Under these conditions, the system degenerates, and for $T_{\mu\nu} \neq \bar{T}_{\mu\nu}$ the Bianchi conditions:

$$\nabla^{\nu}\bar{T}_{\mu\nu} = 0 \qquad \bar{\nabla}^{\nu}T_{\mu\nu} = 0$$

are no longer satisfied. The system is therefore mathematically inconsistent.

I immediately wrote that this was merely a typo and that, being fully aware of this condition of mathematical consistency, I had just published an article in the journal *Progress in Physics* at the same time (January 2019)[104].

But Damour had already sent a copy of his article to all French scientists, to the French popular science press, and to numerous foreign colleagues. It was therefore out of the question for him to retract what he had written.

Furthermore, he considers this journal to be minor and unreliable, and simply does not read the article.

Moreover—and this will be reported to us—he believes that a man who says and writes that his work is inspired by letters sent by extraterrestrials can only be mentally ill, a paraphrenic. See, on this subject, the telephone recording of a prominent figure in his circle.[105]

Since I received no response—including to my request for a meeting—I sent him the details of my calculations[106]. Still no response.

I then posted these calculations on my website so that my colleagues could review them. Some did so and found that the calculations were correct. In November 2022, they sent Damour a registered letter—bearing numerous signatures and with a return receipt requested—stating their objections and requesting an explanation.[107]

---

[103] J.-P. Petit, G. D'Agostini: Cosmological Bimetric Model with Interacting Positive and Negative Masses and Two Different Speeds of Light, in Agreement with the Observed Acceleration of the Universe. Modern Physics Letters A, Vol. 29, No. 34, 2014, Nov. 10 DOI: 10.1142/So21773231450182X http://www.jp-petit.org/papetrs/cosmo/2014-ModPhysLettA.pdf

[104] Jean-Pierre Petit, Gilles D'Agostini, and Nathalie Debergh. Physical and Mathematical Consistency of the Janus Cosmological Model (JCM). Progress in Physics, Vol. 15, Issue 1 (January 2019)

[105] http://www.jp-petit.org/AUDIOS-Staune-sur-Damour.mp3

[106] http://www.jp-petit.org/Damour/2019-Petit-to-Damour-en.pdf

[107] http://www.jp-petit.org/Damour/2022-Duval-to-Damour-nov-30-en-fr.pdf

In response, exasperated, Damour posted a new article[108] on his IHES page on December 12, 2022, this time titled "Physical and Mathematical Inconsistencies of the Janus Cosmological Model."

But the errors contained in this article show that Damour never made the effort to read the articles.

I made the mistake of pointing this out to him[109].

He deleted his file and posted a new one[110], in which he had removed those errors.

I eventually succeeded in getting a 36-page article published in the peer-reviewed journal:

J.P. Petit, F. Margnat, H. Zejli: A bimetric cosmological model on Andreï's twin universe approach. The European Physical Journal. Vol. 84: No. 1126 (2024**)** 18,000 views, cited 18 times[111]. The Newtonian satisfaction of the Bianchi conditions is presented there, as is the derivation of the equations from an action.

In January 2025, my friend, the retired astronomer Guy Monnet[112], wrote to Françoise Combes, an astrophysicist and President of the French Academy of Sciences, to bring these points to her attention. She replied to him via email with the following response:

[108] http://www.jp-petit.org/Damour/2022-Damour-IHES-dec-12-en-fr.pdf

[109] http://www.jp-petit.org/Damour/2022-Petit-to-Damour-dec-14-en-fr.pdf

[110] http://www.jp-petit.org/Damour/2022-Damour-IHES-dec-28-en-fr

[111] https://link.springer.com/content/pdf/10.1140/epjc/s10052-024-13569-w.pdf

[112] Former Head of Instrumentation at the **E**uropean **S**pace **O**bservatory

English translation

Questioned by Guy Monnet, former Head of ESO's Instrumentation Division, subsequently Head of Telescope Systems and Project Scientist for the Extremely Large Telescope (ELT), about the Janus model, Françoise Combes, President of the French Academy of Sciences, gave him the following reply on 24 January 2025, which he forwarded to me:

Dear Guy,

I understand your concern, but it must be acknowledged that the JANUS model has no physical consistency.

Thibault had the patience to write a letter in 2019 and to explain it through the equations, concerning their 2014 papers.

They immediately acknowledged their errors and wrote another version, which was even worse, and Thibault had the patience to explain why. The equations do not satisfy covariance. Their model includes negative masses, and masses of opposite signs repel each other.

As you can see, there is already a violation of the equivalence principle.

Although they acknowledge that it is not possible to have negative masses in GR, they claim that negative masses constitute all of dark matter and dark energy (of course, in the Solar System there are only positive masses).

Do you think such a model can be defended?

As you have seen, they tried to publish in various places, and ultimately this model is known and does not suffer from any impossibility of being presented publicly, since they are in Eur Phys C.

More than 99% of the theoretical astrophysicists I know share the same opinion about this model.

Warm regards,

Françoise

English translation of Françoise Combes's letter to Guy Monnet

All the academicians concerned have closed ranks in support of Damour, without even reading his posts on Janus, but "placing their complete trust in him."

A request to exercise the right of reply, addressed to the Academy's Presidency and co-signed by some fifty scientists—both French and foreign—remains unanswered[113].

[113] http://www.jp-petit.orgg/Damour/2025-Request-for-Exercise-of-Scientific-Right-of-Reply-before-the-French-Academy-of-Science.pdf

The years pass without the slightest response from the academician. Seven years later, his two articles are still prominently displayed on his page at the Institut des Hautes Études Scientifiques.

221 *Thibaud Damour's page on the IHES website*

In November 2026, I published a 56-page article in the journal *The Journal of Modern Physics*. See below. I plan to ask the Institut des Hautes Études to add a link on Damour's personal page pointing to this article, in the exercise of my right of scientific reply.

I do not know if my request will be granted.

November 13, 2026

Scientific Research Publishing

Journal of Modern Physics, 2026, 17(11), *-*
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Physical and Mathematical Inconsistency of T. Damour's Criticisms of the Janus Cosmological Model

Jean-Pierre Petit

Manaty Research Group, Glanon, France
Email: Jppetit1937@yahoo.fr

How to cite this paper: Petit, J.-P. (2026) Paper Title. *Journal of Modern Physics*, **17**, *-*.
https://doi.org/10.4236/***.2026.*****

Received: **** **, ***
Accepted: **** **, ***
Published: **** **, ***

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

Since 2019, two articles by T. Damour—published on his personal webpage hosted by the IHES site and addressing the alleged physical and mathematical inconsistency of the Janus cosmological model—have posed a major obstacle to the scientific community's consideration of this model, despite several points of qualitative agreement with astrophysical and cosmological observations. As these criticisms were not published in peer-reviewed journals, the controversy could not unfold within the standard editorial framework where critiques and responses are subject to review by referees. We first outline the independent origins of the Janus cosmological model, its theoretical foundations, its main results, and its qualitative observational consequences. We revisit the simultaneous satisfaction of the constraints arising from the two Bianchi identities—established in an explicit solution published as early as 2019 but overlooked in T. Damour's subsequent critiques. We then re-examine, point by point, the objections he raised. In particular, although a variational derivation of the coupled Janus field equations has already been published, we present a particularly direct illustration of their variational origin. In 2002, Damour and Kogan introduced an action describing two interacting dynamical metrics within the framework of non-linear massive bigravity. We show that by choosing gravitational coupling constants equal in magnitude but opposite in sign, the characteristic structure of the Janus model's coupled field equations emerges naturally from this construction. Regarding the contradiction concerning the nature of gravitational interactions raised by T. Damour in 2019—and maintained in his 2022 article—we demonstrate that his reasoning requires the same four-velocity field of a test particle to simultaneously satisfy the geodesic equations associated with both metrics. This prescription is not among the assumptions defining the Janus model: positive-mass test particles follow the geodesics of the first metric, whereas negative-mass test

particles follow those of the second. In the stationary regime, spherically symmetric exterior metrics thus allow for the direct recovery of mutual attraction between masses of the same sign and mutual repulsion between masses of opposite signs.

Keywords

Bimetric Gravity, Janus Cosmological Model, Negative Mass, Positive and Negative Masses, Gravitational Coupling Constants, Opposite Gravitational Couplings, Damour-Kogan Bigravity, Hossenfelder Bimetric Theory, Runaway Problem, Interacting Metrics, Bianchi Identities, FLRW Cosmology, Accelerated Expansion, Tolman-Oppenheimer-Volkoff Equations, Newtonian Limit.

1. Introduction

The standard cosmological model is based on the field equation introduced in 1915 by A. Einstein [1]. It assumes that the universe is a four-dimensional manifold associated with a Lorentzian metric $g_{\mu\nu}$, wherein masses follow non-null geodesics derived from that metric, while light follows null geodesics. In its most general form, this equation includes a constant Λ—known as the cosmological constant—whose role was long overlooked, as there was no clear observational data to which it could be linked.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \chi T_{\mu\nu} \tag{1}$$

By adding the assumption of homogeneity and isotropy—where k is the curvature index, (u,θ,φ) represents the dimensionless forms of the spatial coordinates under spherical symmetry, and $a(t)$ is a scale factor—the metric takes the FLRW form:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a\left(\frac{\mathrm{d}u^2}{1-ku^2} + u^2\mathrm{d}\theta^2 + u^2\sin^2\theta\mathrm{d}\varphi^2\right) \tag{2}$$

When introduced into the field equation, this metric leads to three possible solutions, depending on the value of the curvature index $k \in \{-1,0,+1\}$

$\rho > 10^{-29}\ \mathrm{gr/cm^3} \rightarrow k = +1$ Positive curvature, elliptic model

$\rho < 10^{-29}\ \mathrm{gr/cm^3} \rightarrow k = -1$ Negative curvature, hyperbolic model

$\rho = 10^{-29}\ \mathrm{gr/cm^3} \rightarrow k = 0$ Zero curvature, parabolic model

In all scenarios, the expansion proceeded with a deceleration:

$$\frac{\mathrm{d}^2a}{\mathrm{d}t^2} < 0 \tag{3}$$

One problem remained—and still remains unresolved: this solution confirmed the observational discovery of the universe's expansion. Consequently, tracing

back in time implied the existence of a hyper-dense, extremely hot state in which the universe's content consisted of a mixture of antimatter in equilibrium with radiation; high-energy photons were constantly creating pairs of matter and antimatter particles, which would almost immediately annihilate each other, regenerating those same high-energy photons. As the wavelength of these photons increased with the scale factor, the persistence of such primordial radiation would manifest as the observation of centimeter-wavelength photons forming an isotropic, homogeneous background—a phenomenon indeed observed as early as the 1960s. However, since these photons could no longer offset the annihilations, the result should have been the complete disappearance of those matter and antimatter particles. Yet:

- One particle of matter in a billion survived.
- No trace of its antimatter counterpart is found.

All attempts to detect this primordial antimatter have ended in failure. Thus, in this model, one begins by losing half the universe's content—no small matter. To date, no theoretical justification has been proposed to account for this state of affairs. The situation has reached the point where there is no debate comparing competing theories, nor any international conference where this issue is addressed.

In the 1970s, the shape of galactic rotation curves—exhibiting a plateau in the outer regions—could no longer be reconciled with the gravitational field generated solely by visible matter. Specialists were thus led to propose that this field might be attributed to a vast halo of a new, unknown component termed "dark matter." Despite decades of experimental research, no particle constituting dark matter has yet been conclusively identified.

Twenty years later, measurements taken from distant sources led specialists to acknowledge that the universe was undergoing accelerated expansion. The cosmological constant was simply reintroduced into the field equation, on the grounds that its mere presence accounted for this phenomenon—though the nature of this new component, termed "dark energy," remained unexplained. In any case, within this model, the expansion and the acceleration are exponential.

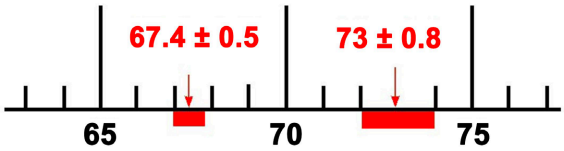
The DESI results from the first three years, announced in March 2025 [2], reinforce the indication that dark energy might evolve over time. DESI indicates that the preference for dynamic dark energy strengthens when BAO, CMB, and Type Ia supernova data are combined. Notably, extended analyses show that this indication is robust at low redshift ($z < 0.3$). This remains a statistical indication, not a definitively established discovery.

Once the map of the cosmic microwave background radiation was produced, its extreme uniformity became apparent. Consequently, the standard model was supplemented by a phase known as inflation, during which—in a timespan ranging from 10^{-36} to 10^{-30} seconds—the universe is thought to have expanded by a factor exceeding 10^{26} under the influence of a field generated by particles dubbed "inflatons." To date, there is no model for such particles, nor any mechanism ex-

plaining their conversion into the known and measured constituents of the universe.

Nevertheless, in-depth analysis of CMB fluctuations has given rise to a new discipline based on the use of a six-parameter model—namely, baryon density, cold dark matter density, the acoustic angular scale, the optical depth of reionization, and the amplitude and spectral index of primordial perturbations.

This involved adjusting the value of one of the parameters—the Hubble constant—to a very specific figure: $H_0 = 67\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}} \pm 0.5\ \mathrm{km\cdot s^{-1}}$. For a long time, this value could be considered to fall within the margin of uncertainty associated with the observational data. However, improved precision leading to a value of $H_0 = 73\ \mathrm{Mpc^{-1}} \pm 0.8\ \mathrm{km\cdot s^{-1}}$ has given rise to what has been termed the "Hubble tension". See **Figure 1**.



OK

Figure 1. The Hubble tension. The two ranges do not overlap [3].

All these factors justify considering a shift in the cosmological paradigm. This is what the Janus Cosmological Model proposes; it essentially involves introducing particles with negative mass and energy into the model.

2. Negative Masses? Impossible, Because of the Runaway Effect!

Such an introduction was proposed as early as 1975 by Hermann Bondi [4]. General relativity, through its Newtonian approximation, immediately reveals the nature of the forces acting on test particles, leading to an immediate observation: in a field created by a given source, there exists only a single family of geodesics, which are followed by any test particle—whether it possesses positive or negative mass. The simplest approach is to focus on non-null geodesics associated with a field created by a mass *M*. If this mass is positive, the resulting metric is:

$$\mathrm{d}s^2 = \left(1-\frac{2GM}{c^2 r}\right)c^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\dfrac{2GM}{c^2 r}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) \tag{4}$$

The geodesics corresponding to positive or negative test masses then have the form as shown on **Figure 2**.

Let us now imagine that this field is created by a negative mass. The external metric gives geodesics paths as shown on **Figure 3**.

This leads to the force configuration shown below, which immediately gives rise to a paradox known as "runaway": when two masses of equal absolute value but opposite signs are placed near each other, the negative mass flees, pursued by the positive mass. Both acquire kinetic energy (negative for the negative mass), and

their absolute values increase indefinitely. Furthermore, the action-reaction principle is violated. See the schema of **Figure 4**.

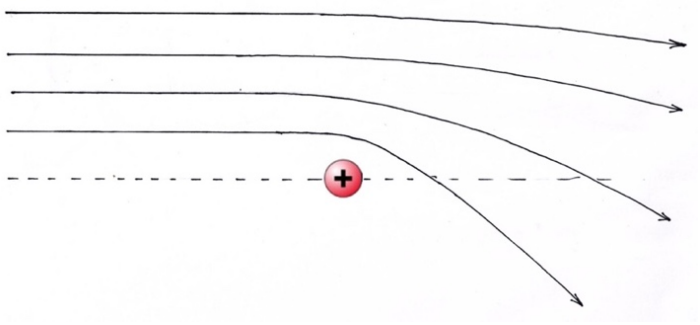
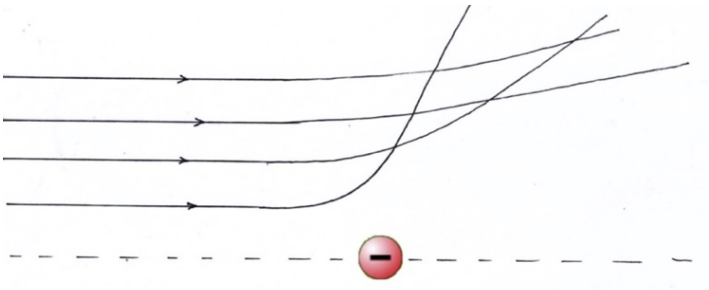
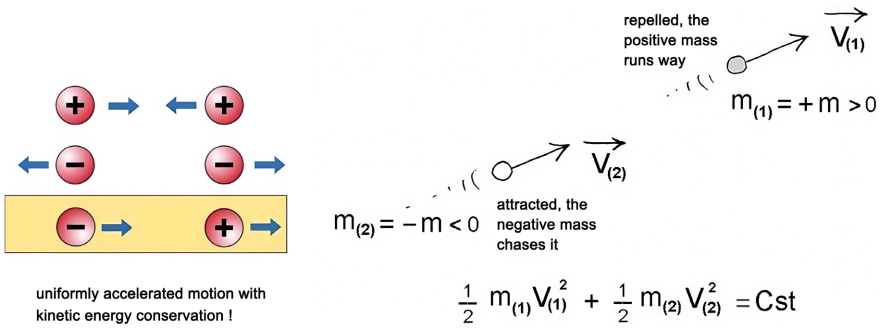


Figure 2. Geodesics in a field created by a positive mass in general relativity.



Geodesics in a field created by a negative mass, in general relativity

Figure 3. Géodésiques dans un champ créé par une masse négative en relativité générale.



OK **Figure 4.** Runaway phenomenon.

Hence the rejection by the entire scientific community of the idea of introducing negative masses into the cosmological model.

3. In Search of a Different Geometric Context

As early as 1974, in the now-defunct journal Il Nuovo Cimento, we published [5] what can be considered an initial sketch of the geometric framework of the Janus model.

The article proposed a rudimentary field equation:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \chi \left[T_{\mu\nu} - A\left(T_{\mu\nu}\right) \right] \tag{5}$$

A evoking antipodality.

The central geometric idea is to view a closed spacetime as the double covering of a projective space. The involution that swaps the two sheets of this covering is the antipodal involution. We will show how this can induce a PT symmetry.

To make this construction immediately visualizable, let us begin with its two-dimensional analogue.

Let us consider a compact two-dimensional spacetime, comprising one spatial dimension and one temporal dimension. We represent it as a sphere, S^2. The meridians represent the lines of time, while the parallels represent space. Its extent is zero at the first pole—representing the Big Bang—increases to a maximum value at the equator, and then decreases to the second pole, representing the Big Crunch. See **Figure 5**.

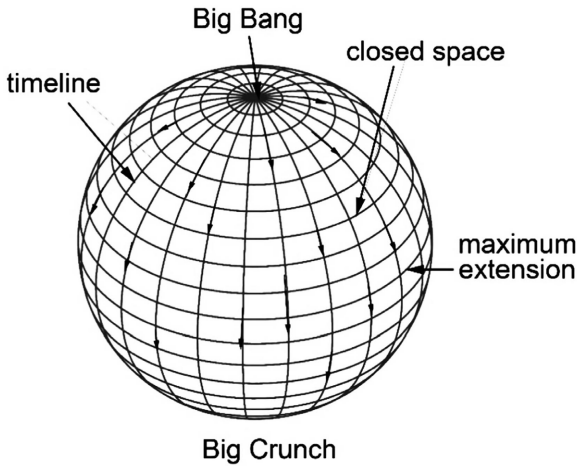


Figure 5. A two-dimensional spacetime.

Let us now consider the antipodal involution.

$$A: X \mapsto -X \tag{6}$$

It maps every point M on the sphere to its antipode M'. Identifying all pairs of antipodal points defines the real projective plane:

$$\mathbb{RP}^2 = S^2 / (X \sim -X) \tag{7}$$

Its canonical projection

$$\pi : S^2 \to \mathbb{RP}^2 \tag{8}$$

is a two-sheeted covering. For any point $q \in \mathbb{RP}^2$

$$\pi^{-1}(q) = \{X, -X\} \tag{9}$$

The text appears two times. The french text must be cancelled

Le plan projectif $\mathbb{RP}^2$ ne peut être plongé dans $\mathbb{R}^3$ sans auto-intersections. Il peut en revanche y être immergé, et la surface de Boy constitue l'une de ses représentations classiques.

The projective plane $\mathbb{RP}^2$ cannot be embedded in $\mathbb{R}^3$ without self-intersections. It can, however, be immersed, and Boy's surface is one of its classical repre-

sentations.

Figure 6 provides a visual representation of this construction. On the left are two antipodal points. In the center and on the right is an immersion of the projective quotient. In this particular representation, the neighborhood of the equator takes the form of a Möbius strip with three half-twists. See **Figure 6**.

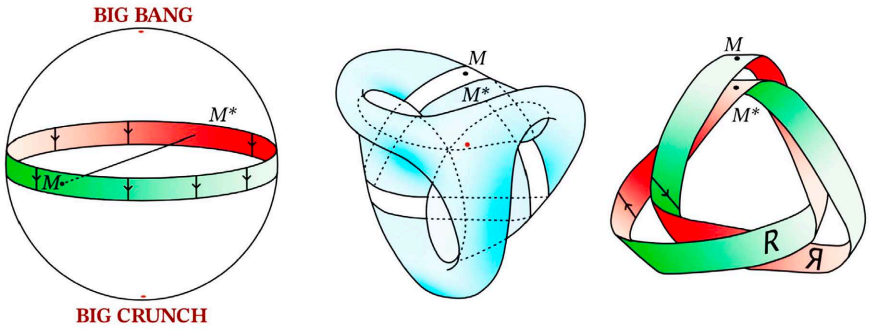
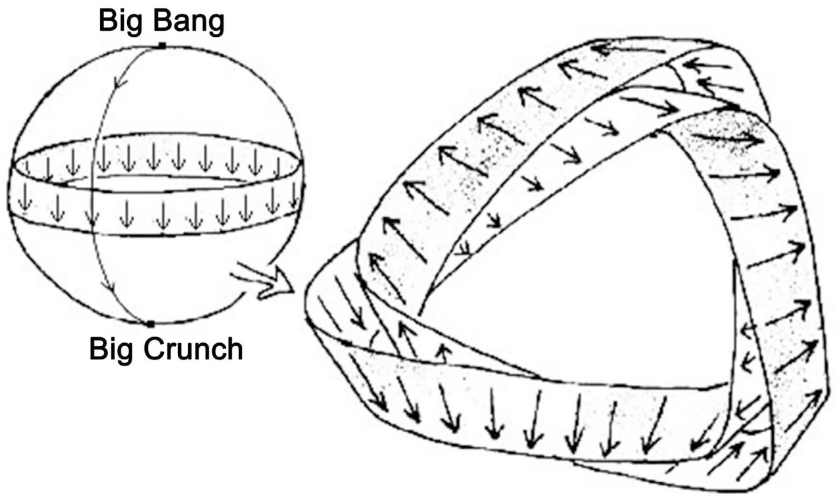


Figure 6. Identification of antipodal points on S^2 and representation of the projective quotient by a Boy immersion [6].

This construction has an interesting consequence when the meridians are interpreted as timelines. The two points X and $-X$, which correspond to the same point of the projective quotient, belong to regions of the covering in which the time orientations are opposite.

This is illustrated in **Figure 7**: antipodal identification brings into correspondence portions of spacetime endowed with antiparallel arrows of time. See **Figure 7**.



OK

Figure 7. Opposite time orientations induced by the two-sheeted covering of a spherical 2D spacetime [6].

Let us now examine this property in the neighborhood of the meridians.

On a sphere, all meridian curves issuing from one pole converge at the diametrically opposite pole. These two poles are antipodal and therefore correspond to

the same point of the quotient $\mathbb{RP}^2$. In the Boy immersion, they consequently appear joined at a single point.

Figure 8 shows the meridian lines and the neighborhood of each of them. Whereas the neighborhood of the equator is represented in this immersion by a Möbius strip with three half-twists, the neighborhood of a meridian curve has the structure of a Möbius strip with one half-twist. See **Figure 8**.

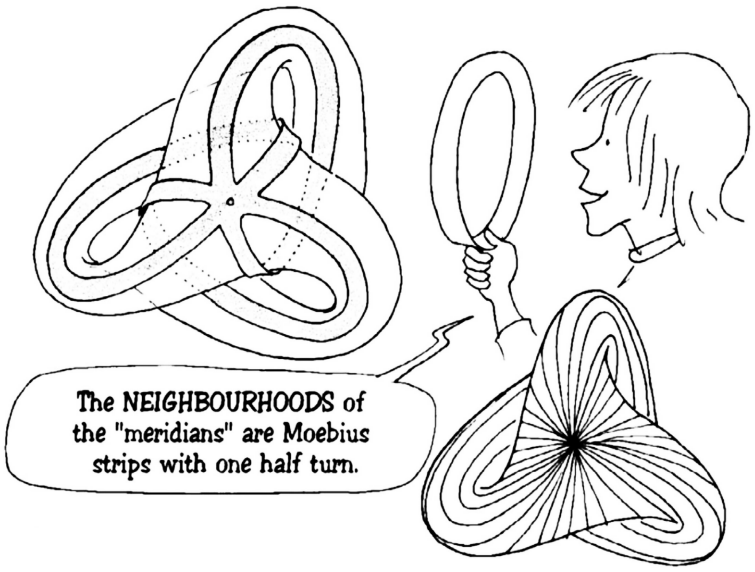


Figure 8. Meridian lines of Boy's surface and their neighborhoods [6].

This representation also makes it possible to visualize separately the two families of curves derived from the meridians and parallels of the initial sphere.

As shown in **Figure 9**. **Figure 9(a)** shows the meridians, **Figure 9(b)** the parallels, and **Figure 9(c)** their superposition.

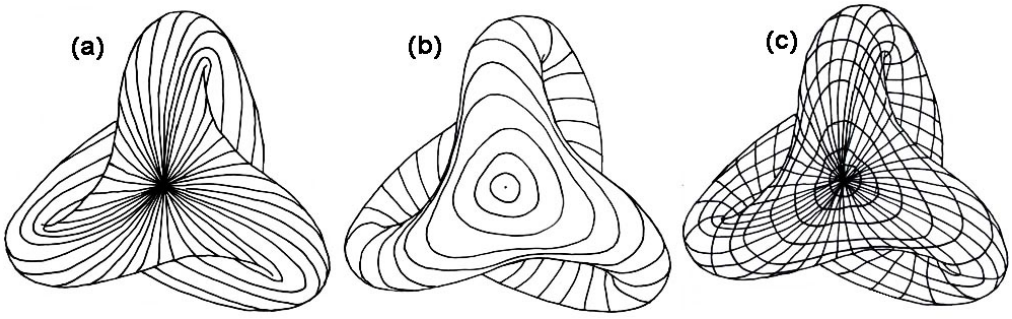


Figure 9. Meridian and parallel curves on Boy's surface. (a) Meridians; (b) parallels; (c) superposition of the two families [6].

We can now recover on the meridians the phenomenon already observed in **Figure 7** in the neighborhood of the equator.

Figure 10 shows how the two determinations of the covering related by the antipodal involution carry opposite time orientations. See **Figure 10**.

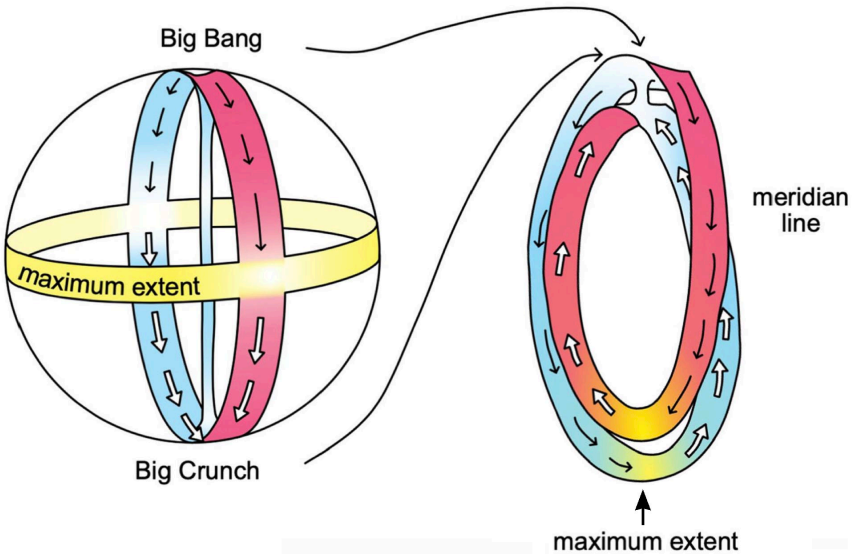


Figure 10. Opposite time orientations on the two determinations of the covering in the neighborhood of a meridian of Boy's surface.

This two-dimensional construction thus geometrically exhibits a reversal of time orientation, which we denote by T.

The non-orientability of the projective quotient likewise entails a reversal of spatial orientation. **Figure 11** gives an elementary representation of this effect: two objects carried by the two determinations of the covering appear as enantiomorphs. This spatial reversal corresponds to the parity operation P. See **Figure 11**.

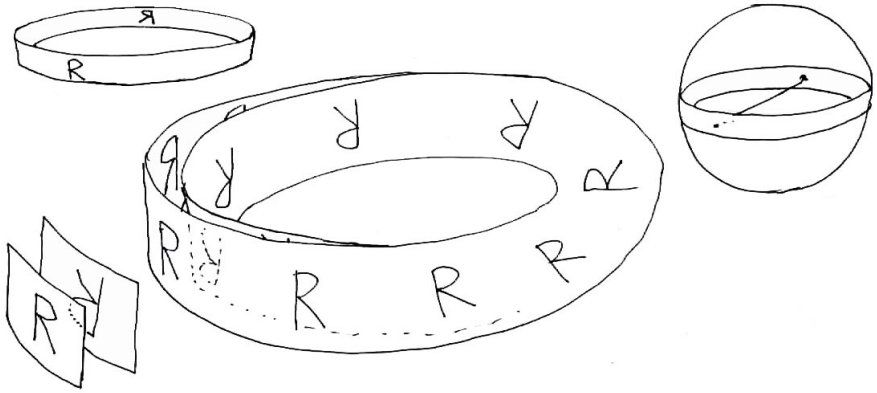


Figure 11. Enantiomorphism induced by the covering of a non-orientable structure [6].

We can now make the dimensional transition that constitutes the essential element of this construction.

Consider a four-dimensional spacetime, with three spatial dimensions and one time dimension. We represent it by a sphere immersed in a five-dimensional space.

We write

$$X = \left(x^0, x^1, x^2, x^3, x^4\right) \tag{10}$$

With

$$\left(x^0\right)^2 + \left(x^1\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2 + \left(x^4\right)^2 = R^2 \tag{11}$$

The four intrinsic coordinates are locally identified with

$$\left(x^0, x^1, x^2, x^3\right) = \left(t, x, y, z\right) \tag{12}$$

the fifth coordinate being required to represent the sphere S^4 in the ambient space.

We then consider the four-dimensional real projective space:

$$\mathbb{RP}^4 = S^4 / \left(X \sim -X\right) \tag{13}$$

whose S^4 canonical two-sheeted covering is given by:

$$S^4 \xrightarrow{2:1} \mathbb{RP}^4 \tag{14}$$

The antipodal involution is

$$A: X \mapsto -X \tag{15}$$

On the four intrinsic coordinates, it therefore acts as

$$\left(t, x, y, z\right) \mapsto \left(-t, -x, -y, -\right) \tag{16}$$

Let us define separately the time reversal

$$T: \left(t, x, y, z\right) \mapsto \left(-t, x, y, z\right) \tag{17}$$

and the spatial inversion

$$P: \left(t, x, y, z\right) \mapsto \left(t, -x, -y, -z\right) \tag{18}$$

Thus, on the intrinsic spacetime coordinates,

$$A = PT \tag{19}$$

The antipodal transformation that exchanges the two determinations of the covering $S^4 \to \mathbb{RP}^4$ therefore induces a PT transformation.

This direct passage from two to four dimensions is not arbitrary. One might be tempted to introduce an intermediate step involving a three-dimensional real projective space $\mathbb{RP}^3$. However, real projective spaces satisfy

$$\mathbb{RP}^4 \text{ orientable } \Leftrightarrow \text{ odd} \tag{20}$$

Thus, the even-dimensional cases $\mathbb{RP}^2$ and $\mathbb{RP}^4$ considered here are non-orientable, whereas the three-dimensional real projective space $\mathbb{RP}^3$ is orientable. The property sought here would therefore disappear at this intermediate step.

Let us now return to the two-dimensional model.

In **Figure 10**, the two polar singularities correspond to the Big Bang and the Big Crunch. One may consider removing these two singularities by connecting their neighborhoods. The preceding projective construction is then replaced by a topology whose classical two-dimensional model is the two-sheeted covering of the Klein bottle by the torus:

$$T^2 \xrightarrow{2:1} K^2 \tag{21}$$

Figure 12 illustrates this construction.

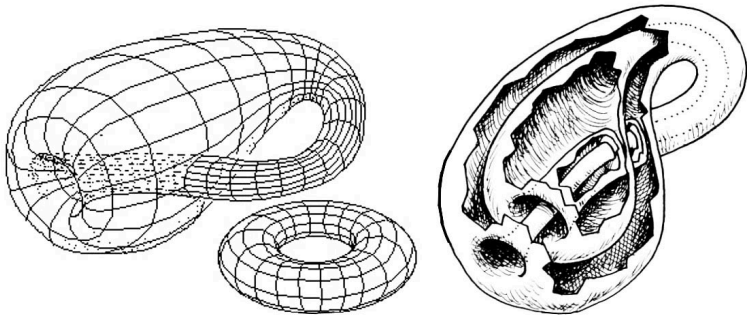


Figure 12. Two-sheeted covering of the Klein bottle by the torus [6].

An analogous generalization may be sought in four dimensions in order to replace the initial and final singularities by a regular structure. In four dimensions, however, the quotient manifold under consideration and the free involution generating its covering must be defined explicitly. We shall return to this point.

This construction is conceptually related to that envisaged by Andrei Sakharov, who considered two conjugate universes related by a CPT symmetry, without introducing between them the gravitational interaction considered here. CPT cancel

Figure 13 gives a didactic representation of this cosmology, first with a singularity and then in a form in which the singularity disappears.

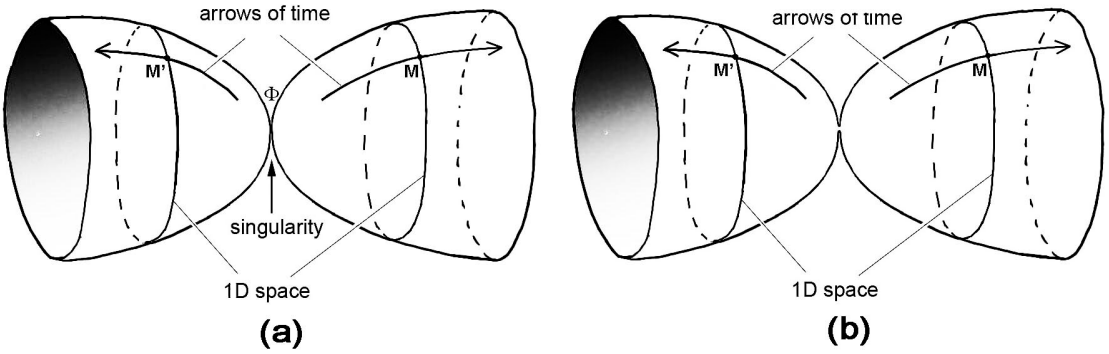


Figure 13. Didactic representation of Andrei Sakharov's cosmology: (a) with a singularity; (b) without a singularity.

It remains to examine how the C symmetry, *i.e.* charge conjugation, could be incorporated into this geometrical construction.

In [7], drawing on Souriau's theory of dynamical groups and on the Janus group, an extension of the full Poincaré group [8], we proposed introducing a fibration involving p compact dimensions.

To each spacetime event x is thus associated a compact fiber

$$F = T^p = \left(S^1\right)^p \tag{22}$$

described by p angular coordinates

$$\theta_1, \theta_2, \cdots, \theta_p \tag{23}$$

The motion of a material point then has two components: its motion in four-dimensional spacetime and its motion along the compact internal dimensions.

The continuous translations

$$\theta_i \mapsto \theta_i + \varepsilon_i \tag{24}$$

constitute symmetries $U(1)$. When the dynamics is invariant under these translations, Noether's theorem associates conserved quantities with them

$$Q_i = \frac{\partial L}{\partial \dot{\theta}_i} \tag{25}$$

which may play the role of charges or quantum numbers.

These continuous translations must then be distinguished from a discrete transformation of the fiber.

An involution

$$C_F : F \to F \qquad C_F^2 = id \tag{26}$$

may be chosen so as to induce

$$Q_i \mapsto -Q_i \tag{27}$$

Such an involution provides a geometrical candidate for representing charge conjugation C.

Figure 14 shows the didactic picture originally proposed to illustrate this idea: motion in a compact dimension is added to the trajectory in spacetime, with the two orientations associated with the two signs of the charge. See **Figure 14**.

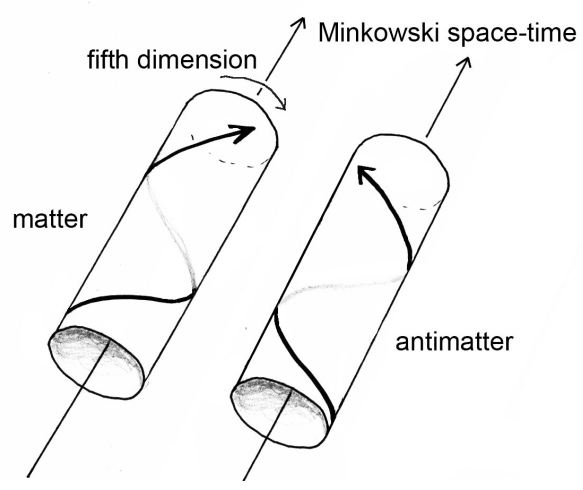


Figure 14. Outline of a geometrical interpretation of matter-antimatter symmetry based on a compact internal dimension [9].

The proposed construction thus has two distinct components

$$PT : \left(X \mapsto -X \right) \tag{28}$$

in spacetime geometry, and

$$C:(Q\mapsto -Q) \tag{29}$$

in the internal dimensions.

It therefore suggests the mapping

$$\boxed{(X,Q)\mapsto(-X,-Q)} \tag{30}$$

combining the transformations PT and C.

4. Result of a Heuristic Exploration

Pending the development of a more sophisticated system of field equations, a heuristic exploration is attempted using the following force scheme, which satisfies the action-reaction principle, as shown in **Figure 15**:

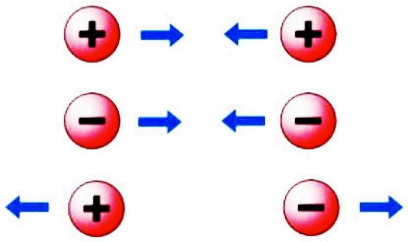


Figure 15. The heuristic hypothesis concerning the force field.

The initial trials in 1994 [5], involving the interaction of sets of positive and negative point masses, produced a percolation phenomenon that did not appear to be readily linkable to observations. See **Figure 16**.

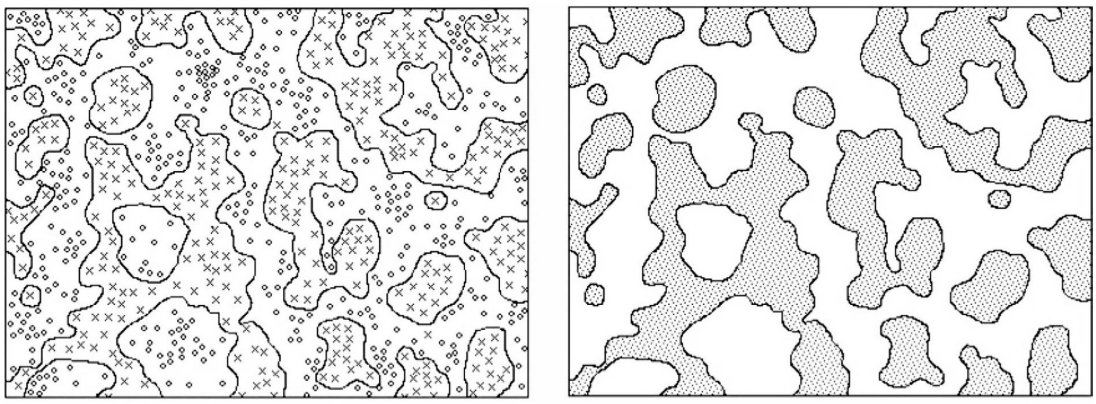


Figure 16. Full symmetry $(|\bar{\rho}|=\rho)$: percolation phenomenon [5].

However, by choosing to introduce significant asymmetry $(|\bar{\rho}|>\rho)$, this simulation immediately results in the formation of a very large-scale structure, in which the negative mass forms a regular network of spheroidal clusters, while the positive mass is confined to the remaining space, taking the form of contiguous bubbles. See **Figure 17**.

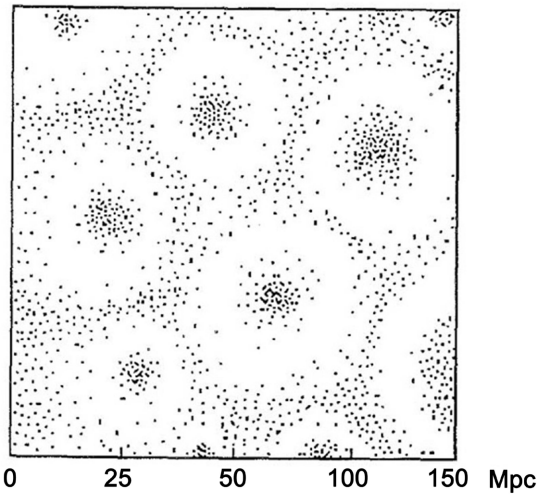


Figure 17. Formation of the very large structure [10].

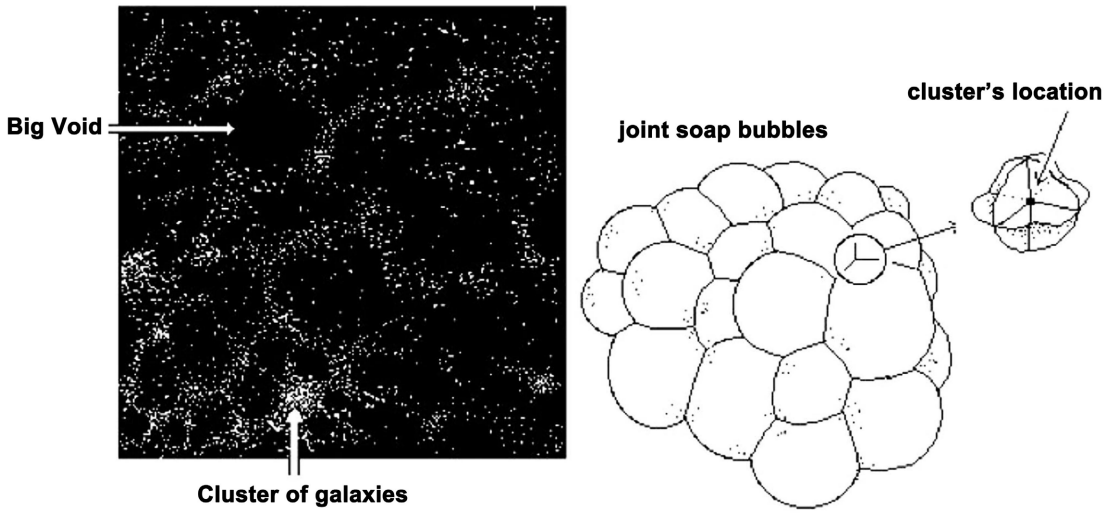
OK

The explanation is immediately apparent. Since the Jeans timescales of the two populations differ significantly, it is the one with the shorter timescale—the negative population—that gives rise to an organized structure first.

$$\bar{t}_J = \frac{1}{\sqrt{8\pi G\bar{\rho}}} \ll t_J = \frac{1}{\sqrt{8\pi G\rho}} \qquad (31)$$

(31)

Even before considering the geometric and mathematical context, the approach appears promising. The two subsystems mutually confer stability upon one another. The contiguous spheres of matter entrap the negative-mass clusters, preventing them from meeting and merging; these clusters, in turn, serve as anchor points for this porous structure. See **Figure 18**.



OK

Figure 18. Joint soap bubbles VLS structure.

This suggests a novel scenario for the formation of stars and galaxies. The scientific community is currently faced with the appearance of massive, fully formed

spiral galaxies at an extremely early stage—phenomena that standard models cannot explain. The primary obstacle lies in the time required to dissipate the heat generated as matter compresses into spheroidal structures. Here, however, the situation is quite different. The positive mass is not left to its own devices but undergoes intense compression between sheets; this geometry is optimal for maximizing energy dissipation through radiation. While 3D simulations could quantify this phenomenon, we lack the resources to carry out such a project. We believe that, in this context, massive spiral galaxies could form within the first hundred million years. See **Figure 19**.

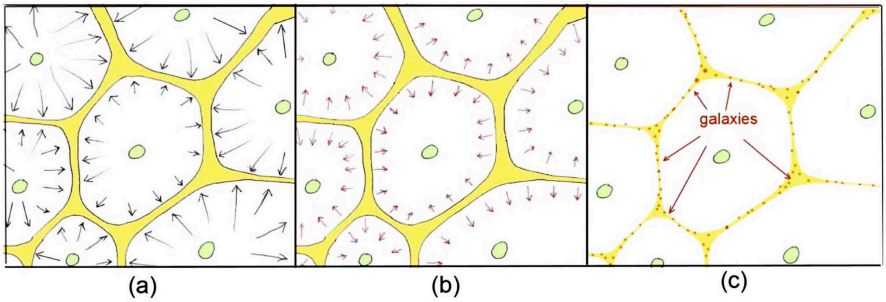


Figure 19. Rapid galaxy formation [9].

In **Figure 12(a)**, the matter (in yellow), subjected to the back-compression exerted by negative-mass clusters (in green), undergoes violent heating. In **Figure 12(b)**, this energy is rapidly dissipated through radiation. In **Figure 12(c)**, having become unstable, the positive mass gives rise to all the galaxies as we observe them today.

There is no subsequent galaxy formation; all galaxies are born within the first hundred million years. As early as 1992, 2D simulations—with initial conditions describing a rotating galaxy confined by its negative-mass environment—showed the very rapid emergence (within a few rotations) of a barred spiral structure. See **Figure 20**.

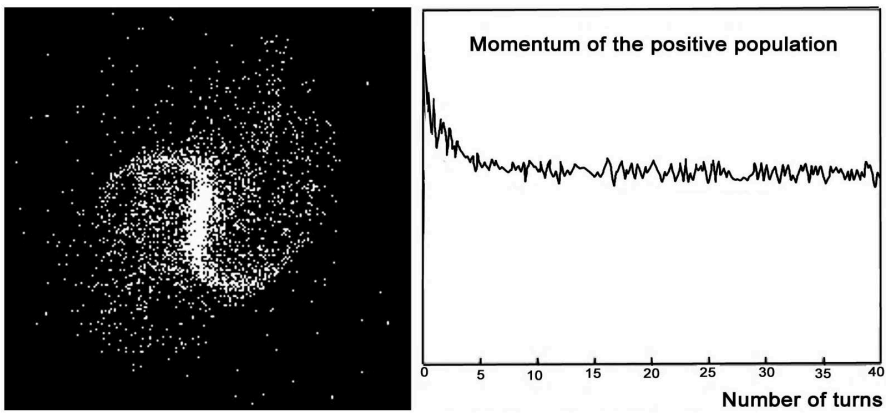


Figure 20. Barred spiral and the evolution of its angular momentum as a function of the number of rotations [9].

The initial conditions governing these simulations were based on solving a system of two Vlasov equations coupled by the gravitational field, yielding these density profiles [5]. See **Figure 21** and **Figure 22**.

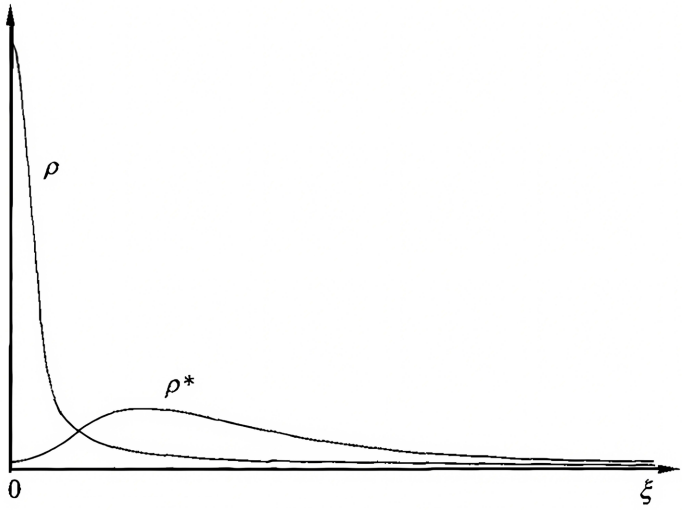


Figure 21. Elliptic solution of two coupled Vlasov equations. Densities of positive masses ρ and negative masses ρ^* (*i.e.*, of negative mass) [5].

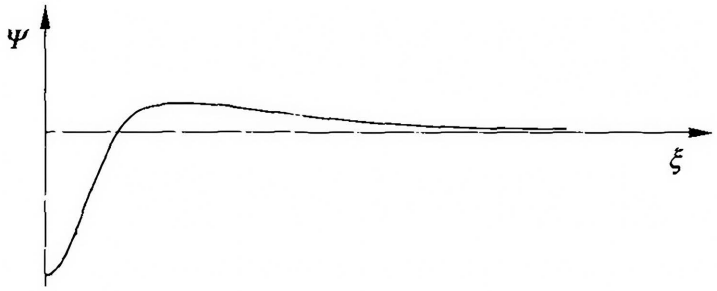


Figure 22. The corresponding gravitational potential [5].

Thirty-four years ago, these simulations thus provided the answer to the question that the entire community of specialists continues to ask: why does this spiral structure form, and why does it persist?

- The answer is that it is the way in which collisionless systems exchange energy and angular momentum with their environment, through the agency of density waves. Thus, this spiral structure is destined to persist as long as the galaxy contains gas—that is, virtually indefinitely.

Despite the very positive result obtained using this simple heuristic hypothesis regarding the direction of the forces, all specialized journals refused to review the submitted article, replying, "Sorry, we don't publish speculative works." We can only hope that today, thirty-four years later, a team with sufficient resources will take up this research again.

In this regard, one might wonder whether there is a fairly simple reason why other teams have not attempted—even on an exploratory basis—this type of com-

putational experiment. Simulation programs—evolving from 2D to 3D—have undergone considerable development over recent decades. Thanks to tree-coding, it has become possible to handle increasingly large numbers of point masses and to incorporate both multiple stellar populations and gas. Yet, is there any program that asks the user to "specify the sign of this component's mass"? This seems unlikely, given that the scientific community operates on the basis of an irrevocable decision to exclude negative masses from cosmology and astrophysics.

However, a cosmological model cannot be based on mere heuristic speculations, however fruitful and encouraging they may be. Since a dynamic based on a single field equation immediately ran up against the "runaway" phenomenon, a bimetric approach was considered.

5. First Bimetric Description by T. Damour and I. Kogan [11]

In 2002, a 40-page article presenting the first bimetric approach was published in Physical Review D, the highest-ranked journal in the field. The authors initially envisioned the interaction of *n* branes, each equipped with its own metric, before narrowing the scope of their project to two branes: "Left" and "Right." Their aim was not to introduce negative masses. From the outset, the authors referred to the concept known as "massive gravity." Back in 1939, Fierz and Pauli [12] had added a mass term to the tensor field, thereby assigning a non-zero mass to the graviton. This gives rise to a characteristic wavelength equivalent to the Compton wavelength.

$$\lambda_g = \frac{\hbar}{m_g} \tag{7}$$

(32)

By linking this to the Hubble constant and associating it with a modification of gravity at large distances, this yields an estimate of the order of magnitude of this hypothetical graviton mass:

$$m_g c^2 = \hbar H_0 \sim 10^{-33}\ \text{eV} \tag{8}$$

(33)

In 2011, C. de Rham, G. Gabadadze, *et al.* [13] linked this graviton mass to the recently discovered phenomenon of accelerated expansion, leading them to a graviton mass value of the same order of magnitude.

In 1972, Boulware and Deser [14] investigated the extension of Fierz-Pauli theory to the non-linear regime. They demonstrated that such an extension generically gives rise to a sixth degree of freedom—absent from the linear theory—associated with a "ghost" instability. In this context, any cosmological model based on a modification of gravitational dynamics must therefore satisfy a crucial requirement:

- *Is this model ghost-free*?

This question lies at the heart of the concerns of the authors, T. Damour and I. Kogan [11]. Not only do their gravitons possess mass—distributed according to a specific spectrum—but they also posit a hypothetical gap separating light gravitons from heavy ones. See **Figure 23**.

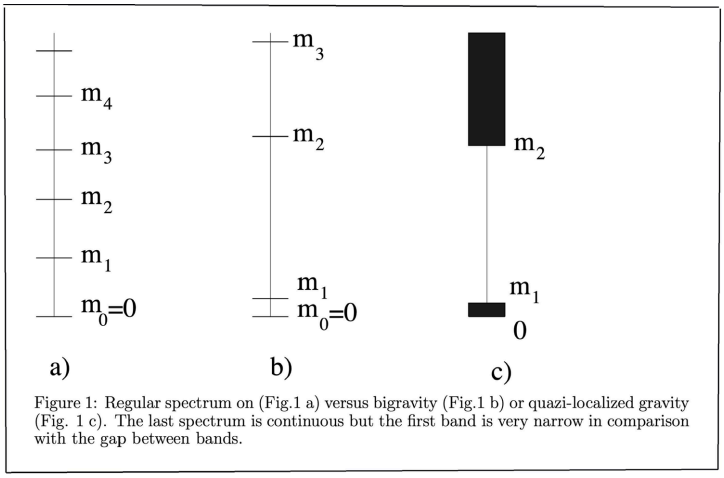


Figure 1: Regular spectrum on (Fig.1 a) versus bigravity (Fig.1 b) or quazi-localized gravity (Fig. 1 c). The last spectrum is continuous but the first band is very narrow in comparison with the gap between bands.

Figure 23. The hypothetical gap between light gravitons and heavy gravitons [11].

The fact that this article has become a major reference in the bimetric literature has contributed to the frequent association of bigravity with *massive gravity*—often referred to as "*massive bigravity*"—yet this association is not a mathematical necessity. The bimetric nature of a theory does not, in itself, imply the existence of a massive graviton. The Janus Cosmological Model is precisely not based on such an assumption. Consequently, the specific problem of the Boulware-Deser ghost, which arises in certain non-linear extensions of massive gravity, cannot be applied to it without first demonstrating that the mechanism generating this degree of freedom is indeed present in the Janus system.

Therefore, the question of whether the Janus cosmological model is free from the Boulware-Deser ghost is irrelevant in this context, since the Janus model is not constructed as a non-linear theory of massive gravitons. This specific instability, which is a central concern in massive gravity, is thus not the one governing its construction.

The same applies to a second bimetric model, introduced between 2006 and 2008 by S. Hossenfelder [15] [16]. Here, the focus is different, aiming to construct an antigravitational interaction capable of having cosmological consequences. Indeed, the 2006 paper explicitly frames the approach within the context of "antigravity." Hossenfelder demonstrates that the kinetic energy of the new fields remains positive, while the fields themselves can contribute negatively to the source term in Einstein's equations.

Damour-Kogan, Hossenfelder, and the Janus model thus propose constructions involving two gravitational sectors and coupled field equations, albeit with different physical motivations. The question of interest here is the construction of a Lagrangian that allows such a system to be obtained variationally. To this end, we will adopt the construction proposed by Damour and Kogan.

6. Construction of the System of Coupled Field Equations by T. Damour and I. Kogan [11]

The authors consider two sectors, "left: L" and "right: R". The proposed action is

then:

$$S=\int\left(M_R^2R^R-L^R-\Lambda^R\right)\sqrt{\left|g^R\right|}\mathrm{d}^4x+\int\left(M_L^2R^L-L^L-\Lambda^L\right)\sqrt{\left|g^L\right|}\mathrm{d}^4x -\mu^4\int\mathrm{d}^4x\left(g_Lg_R\right)^{\frac{1}{4}}V\left(g_L,g_R\right). \quad (9)$$ (34)

The constants M_L and M_R are "Planck masses." The inverses of their squares play the role of the Einstein-Hilbert gravitational coupling constants. What emerges from this formalism is the fact that the authors assign them the same sign.

In this action we find two elementary volumes … and …, and and elementary hypervolume … involved in the interaction between the tow populations

Dans ~~cette~~ action on trouve deux hypervolumes élémentaires $\sqrt{\left|g_R\right|}\mathrm{d}^4x$ an $\sqrt{\left|g_L\right|}\mathrm{d}^4x$ \ ainsi qu'un hypervolume élémentaire équivalent $\mathrm{d}^4x\left(g_Lg_R\right)^{\frac{1}{4}}$ lié au terme traduisant l'interaction entre les deux populations. μ is an interaction constant and $V\left(g_L,g_R\right)$ is an interaction potential.

Let us forget the two cosmological constants. Λ^R and Λ^L.

When performing the variational calculation, the terms $R^R\sqrt{\left|g_R\right|}$ and $R^L\sqrt{\left|g_L\right|}$ yield the "geometric tensors" $G_{\mu\nu}^R$ and $G_{\mu\nu}^L$ \$ that is, the left-hand sides of the two field equations.

The two Lagrangian densities of the two materials $L^R\sqrt{\left|g^R\right|}$ et $L^L\sqrt{\left|g_L\right|}$ give source tensors $T_{\mu\nu}^R$ and $T_{\mu\nu}^L$

The variations on the interaction term $-\mu^4\int\mathrm{d}^4x\left(g_Lg_R\right)^{\frac{1}{4}}V\left(g_L,g_R\right)$.

Gives interaction tensors $K_{\mu\nu}^{L/R}$ and $K_{\mu\nu}^{R/L}$

- The notation $K_{\mu\nu}^{L/R}$ represents the contribution of matter *L* to the field acting on matter *R*.
- The notation $K_{\mu\nu}^{R/L}$ represents the contribution of matter *R* to the field acting on matter *L*.

Hence the system of two field equations coupled by their interaction terms:

$$G_{\mu\nu}^R=R_{\mu\nu}^R-\frac{1}{2}R^Rg_{\mu\nu}^R=\frac{1}{M_R^2}\left[T_{\mu\nu}^R+t_{\mu\nu}^{L/R}\right] \quad (10a)$$ (35a)

$$G_{\mu\nu}^L=R_{\mu\nu}^L-\frac{1}{2}R^Lg_{\mu\nu}^L=\frac{1}{M_L^2}\left[T_{\mu\nu}^L+t_{\mu\nu}^{R/L}\right] \quad (10b)$$ (35b)

Which can be written by explicitly showing the gravitational coupling constants χ_R and χ_L:

$$G_{\mu\nu}^R=R_{\mu\nu}^R-\frac{1}{2}R^Rg_{\mu\nu}^R=\chi_R\left[T_{\mu\nu}^R+t_{\mu\nu}^{L/R}\right] \quad (11a)$$ (36a)

$$G_{\mu\nu}^L=R_{\mu\nu}^L-\frac{1}{2}R^Lg_{\mu\nu}^L=\chi_L\left[T_{\mu\nu}^L+t_{\mu\nu}^{R/L}\right] \quad (11b)$$ (36b)

Noting that the Damour-Kogan approach posits that:

$$\chi_R\chi_L>0 \quad (12)$$ (37)

Based on the two metrics, we have covariant derivative operators. ∇_R^ν and ∇_L^ν

As we have, by construction:

$$\nabla_R^\nu G_{\mu\nu}^R=\nabla_L^\nu G_{\mu\nu}^L=0 \quad (13)$$ (38)

The mathematical consistency of the system requires:

$$\nabla_R^{\nu}\left(T_{\mu\nu}^{R}+t_{\mu\nu}^{L/R}\right)=0 \quad (14a) \quad (39a)$$

$$\nabla_L^{\nu}\left(T_{\mu\nu}^{L}+t_{\mu\nu}^{R/L}\right)=0 \quad (14b) \quad (39b)$$

After reviewing various forms of interaction potentials, this 40-page article leads to no results that can be compared with observational data.

7. S. Hossenfelder's System of Coupled Field Equations [16]

S. Hossenfelder's approach also relies on an action.:

$$\begin{aligned} S=&\int \mathrm{d}^4x\left[\sqrt{-g}\left({}^{(g)}R/8\pi G+\mathcal{L}(\psi)\right)+\sqrt{-h}P_{\underline{h}}\left(\underline{\mathcal{L}}\left(\underline{\phi}\right)\right)\right]\\ &+\int \mathrm{d}^4x\left[\left({}^{(h)}R/8\pi G+\underline{\mathcal{L}}\left(\underline{\phi}\right)\right)+\sqrt{-\underline{g}}P_g\left(\mathcal{L}(\psi)\right)\right] \end{aligned} \quad (15) \quad (40)$$

We have added the brackets, which are missing from the article. Its way of managing the interaction between its two sectors g and h is different in that two terms appear representing:

- One, the action of the field ϕ, originating from the population h, on the elements of the sector g a term weighted by the elementary hypervolume $\sqrt{-h}\,\mathrm{d}^4x$
- The other, the action of the field ψ, originating from the population g, on the elements of the sector h a term weighted by the elementary hypervolume $\sqrt{-g}\,\mathrm{d}^4x$.

It is this approach that leads to the system of coupled field equations.:

$${}^{(g)}R_{\kappa\nu}-\frac{1}{2}g_{\kappa\nu}\,{}^{(g)}R=T_{\kappa\nu}-\underline{V}\sqrt{\frac{\underline{h}}{g}}a_{\nu}^{\underline{\nu}}a_{\kappa}^{\underline{\kappa}}\underline{T}_{\underline{\nu}\underline{\kappa}} \quad (16a) \quad (41a)$$

$${}^{(\underline{h})}R_{\underline{\nu}\underline{\kappa}}-\frac{1}{2}h_{\underline{\nu}\underline{\kappa}}\,{}^{(h)}R=\underline{T}_{\underline{\nu}\underline{\kappa}}-W\sqrt{\frac{\underline{g}}{\underline{h}}}a_{\underline{\kappa}}^{\kappa}a_{\underline{\nu}}^{\nu}T_{\kappa\nu} \quad (16b) \quad (41b)$$

It is the similarity in construction that also gives rise to the square root of the ratio of the determinants of the two metrics—and its inverse—in the Janus cosmological model, subject to a change of notation: the metrics are then $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$.

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\chi\left(T_{\mu\nu}+\sqrt{\frac{\overline{g}}{g}}\hat{\overline{T}}_{\mu\nu}\right) \quad (17a) \quad (42a)$$

$$\overline{R}_{\mu\nu}-\frac{1}{2}\overline{R}\overline{g}_{\mu\nu}=-\chi\left(\overline{T}_{\mu\nu}+\sqrt{\frac{g}{\overline{g}}}\hat{T}_{\mu\nu}\right) \quad (17b) \quad (42b)$$

A strong similarity is thus observed between the three systems of coupled field equations. But how do they differ? In the Janus model, the gravitational coupling constants are equal and opposite. In Hossenfelder's model, they are equal. In the Damour-Kogan model, they have the same sign.

8. Standardization of Notation

The shift to a bimetric system sheds new light on this issue. What about the ap-

proaches proposed by Damour and Hossenfelder? Since the components of their coupled field equation systems satisfy the mathematical constraints arising from the shared form of the Bianchi identities, the choice of free parameters allows for a wide range of possible models. Let us express both systems in a common form by changing the notation:

$$g^L_{\mu\nu} = g_{\mu\nu} \to g_{\mu\nu} \quad (18) \quad (43)$$

$$g^R_{\mu\nu} = h_{\mu\nu} \to \bar{g}_{\mu\nu} \quad (19) \quad (44)$$

$$R_{\mu\nu}\left(g^L\right) = {}^{(g)}R_{\mu\nu} \to R_{\mu\nu} \quad (20) \quad (45)$$

$$R_{\mu\nu}\left(g^R\right) = {}^{(h)}R_{\mu\nu} \to \bar{R}_{\mu\nu} \quad (21) \quad (46)$$

$$R\left(g^L\right) = {}^{(g)}R \to R \quad (22) \quad (47)$$

$$R\left(g^R\right) = {}^{(h)}R \to \bar{R} \quad (23) \quad (48)$$

$$T^L_{\mu\nu} = T_{\mu\nu} \to T_{\mu\nu} \quad (24) \quad (49)$$

$$T^R_{\mu\nu} = \underline{T}_{\mu\nu} \to \bar{T}_{\mu\nu} \quad (25) \quad (50)$$

$$t^L_{\mu\nu} = -\underline{V}\sqrt{\frac{\underline{h}}{g}}\, a^{\underline{\nu}}_{\nu} a^{\underline{\kappa}}_{\kappa} \underline{T}_{\underline{\nu}\underline{\kappa}} \to \bar{K}_{\mu\nu} \quad (26) \quad (51)$$

$$t^R_{\mu\nu} = -W\sqrt{\frac{g}{\underline{h}}}\, a^{\mu}_{\underline{\mu}} a^{\nu}_{\underline{\nu}} T_{\mu\nu} \to K_{\mu\nu} \quad (27) \quad (52)$$

In summary, in both models we have two types of matter associated with masses m and $\bar{m}$, which follow geodesics derived from metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. From these, Ricci tensors $R_{\mu\nu}$ and $\bar{R}_{\mu\nu}$ and corresponding Ricci scalars R and $\bar{R}$. are constructed. The source tensors associated with these two types of matter are $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$.

… is a tensor-referred as interaction tensor (denoted by … by Damour and Kogan) that accounts for the action of the matter field on the geodesics followed by the masses …

- $\bar{K}_{\mu\nu}$ est un tenseur que l'on qualifiera de tenseur d'interaction, (désigné par $t^L_{\mu\nu}$ chez Damour et Kogan), qui prend en charge l'action du champ de matière $\bar{m}$, sur les géodésiques empruntée par les masses m .

- $K_{\mu\nu}$ is a tensor—referred to as the interaction tensor (denoted by $t^R_{\mu\nu}$ by Damour and Kogan)—that accounts for the action of the matter field m on the geodesics followed by the masses $\bar{m}$.

Under these conditions, the two systems—those of Damour and Hossenfelder—can be written in the form:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu} + \bar{K}_{\mu\nu} \quad (28a) \quad (53a)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} = \bar{T}_{\mu\nu} + K_{\mu\nu} \quad (28b) \quad (53b)$$

In both approaches, the authors assume the conservation of both types of matter—that is to say:

$$\nabla^{\nu} T_{\mu\nu} = \bar{\nabla}^{\nu} \bar{T}_{\mu\nu} = 0 \quad (29) \quad (54)$$

∇^ν and $\bar{\nabla}^\nu$ being the two covariant derivative operators constructed from the metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ this immediately leads to the two constraints that determine the form of the interaction tensors:

$$\nabla^\nu \bar{K}_{\mu\nu} = \bar{\nabla}^\nu K_{\mu\nu} = 0 \quad (30) \quad (55)$$

Following the two cited articles, the discussions depend on the intended objectives. In [11], there is no mention of negative masses at any point. In that article, the observer is assumed to reside on one of the two branes, which therefore corresponds to matter—to standard mass. The authors note (in Section 4.3) that bigravity exactly satisfies the equivalence principle and that this model is compatible with local observational data. Regarding what is termed a "bi-cosmological configuration," they suggest that the action of this second type of matter could account for the recently discovered phenomenon of accelerating cosmic expansion. In Section VII of [11], which addresses potential observables, the author writes—we quote:

- *Since the matter contents of both types of fields is identical the source terms in the second members of the field equations cancel identically, no matter how large their values are.*

The equations of the Janus cosmological model also derive from an action (EPJC 2024 [9]). For reasons to be explained later, we will show that the Janus model equations can be derived from the Damour-Kogan action, subject to a slight modification.

9. Thanks to a Slight Modification, the Janus Model Becomes a Variant of [11]

The gravitational coupling constants χ_R and χ_L are already present in the action of [11].

$$S = \int \left(R^R - \chi_R L^R - \Lambda^R\right)\sqrt{|g^R|}\,\mathrm{d}^4x + \int \left(R^L - \chi_L L^L - \Lambda^L\right)\sqrt{|g^L|}\,\mathrm{d}^4x - \mu^4 \int \mathrm{d}^4x \left(g_L g_R\right)^{\frac{1}{4}} V\left(g_L, g_R\right). \quad (31) \quad (56)$$

The system of equations being:

$$R^R_{\mu\nu} - \frac{1}{2}R^R g^R_{\mu\nu} = \chi_R\left[T^R_{\mu\nu} + t^{L/R}_{\mu\nu}\right] \quad (32a) \quad (57a)$$

$$R^L_{\mu\nu} - \frac{1}{2}R^L g^L_{\mu\nu} = \chi_L\left[T^L_{\mu\nu} + t^{R/L}_{\mu\nu}\right] \quad (32b) \quad (57b)$$

Given that at Damour and Kogan $\chi_R\chi_L > 0$

Or, using our notation, by posing $\chi_R = \chi$ and $\chi_L = \bar{\chi}$:

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} = \bar{\chi}\left[\bar{T}_{\mu\nu} + K_{\mu\nu}\right] \quad (32c) \quad (57c)$$

As we are opting to preserve both types of materials:

$$\nabla^\nu T_{\mu\nu} = \bar{\nabla}^\nu \bar{T}_{\mu\nu} = 0 \quad (33) \quad (58)$$

Which will necessitate:

$$\nabla^{\nu}\overline{K}_{\mu\nu} = \overline{\nabla}^{\nu}K_{\mu\nu} = 0 \quad (34) \quad (59)$$

If this condition is met, one is then free to choose the signs of the source terms, except for the one that would be identified with standard, self-attractive matter.

This is where the benefit of the Janus model's heuristic approach lies; at the outset, this approach involved assigning a priori force directions corresponding to **Figure 9**. We therefore chose, heuristically, to adopt a system in which the two types of masses repel each other. We will adjust the signs of the terms in the various tensors and the Einstein constants to align with these force directions. Let us consider the Newtonian approximation regime. Under these conditions:

$$T_{00} = \rho > 0 \left(i \neq 0; j \neq 0\right) \to T_{ij} \cong 0 \quad (35) \quad (60)$$

$$\overline{T}_{00} = \overline{\rho} < 0 \left(i \neq 0; j \neq 0\right) \to \overline{T}_{ij} \cong 0 \quad (36) \quad (61)$$

For the second type of matter to be self-attractive as well, the coupling constants must have opposite signs. And we shall choose them to be equal and opposite:

$$\boxed{\overline{\chi} = -\chi} \quad (37) \quad (62)$$

We shall assume that the Newtonian approximation has the same effect on the interaction tensors and, that is to say:

$$K_{00} > 0 \left(i \neq 0; j \neq 0\right) \to K_{ij} \cong 0 \quad (38) \quad (63)$$

$$\overline{K}_{00} < 0 \left(i \neq 0; j \neq 0\right) \to \overline{K}_{ij} \cong 0 \quad (39) \quad (64)$$

Under these conditions, the heuristic laws hold true. The principles of equivalence and action-reaction are satisfied. We recall the benefits gained from the heuristic approach. A similar procedure could be carried out starting from Sabine Hossenfelder's model and also adding Einstein constants of opposite signs $\left(\overline{\chi} = -\chi\right)$; however, we shall limit ourselves to the derivation presented in [9].

It now remains for us to revisit the construction of solutions for the Janus cosmological model and the issue of their satisfying the Bianchi conditions.

10. Unsteady-State Solution for Isotropic and Homogeneous Media

By assigning the following forms to the interaction tensors:

$$K_{\mu\nu} = T_{\mu\nu};\ \ \overline{K}_{\mu\nu} = \overline{T}_{\mu\nu} \quad (40) \quad (65)$$

We obtain the trivial unsteady solution:

$$g_{\mu\nu} = \overline{g}_{\mu\nu} \quad (41) \quad (66)$$

With:

$$\left|\overline{\rho}\right| = \rho \quad (42) \quad (67)$$

In other words, zero density and a zero gravitational field. This recovers the Dirac-Milne linear expansion model [17].

Still seeking a time-dependent solution under conditions of isotropy and homogeneity, we looked for a solution [18] such that:

$$K_{\mu\nu} = \phi T_{\mu\nu};\ \ \overline{K}_{\mu\nu} = \varphi \overline{T}_{\mu\nu} \tag{43}$$ (68)

where φ and ϕ are functions of time. Symmetry gives the metrics the following FLRW forms:

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a^2\left(\frac{\mathrm{d}u^2}{1-ku^2} + u^2\mathrm{d}\Omega^2\right) \tag{44}$$ (69)

$$\mathrm{d}\overline{s}^2 = \overline{c}^2\mathrm{d}t^2 - \overline{a}^2\left(\frac{\mathrm{d}u^2}{1-\overline{k}u^2} + u^2\mathrm{d}\Omega^2\right) \tag{45}$$ (70)

By introducing these FLRW metrics into our bi-metric system, we obtain a system of four second-order differential equations yielding the scale factors a and $\overline{a}$. Requiring the system to be consistent will provide the equivalent of the Bianchi condition for this type of solution. The system of field equations becomes:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi\left[T_{\mu\nu} + \varphi\overline{T}_{\mu\nu}\right] \tag{46a}$$ (71a)

$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\overline{g}_{\mu\nu} = -\chi\left[\overline{T}_{\mu\nu} + \phi T_{\mu\nu}\right] \tag{47b}$$ (71b)

With:

$$\chi = -\frac{8\pi G}{c^2} \tag{48}$$ (72)

By considering the two scenarios using the equation of state:

$$p = \frac{\beta\rho c^2}{3} \qquad \overline{p} = \frac{\beta\overline{\rho}\overline{c}^2}{3} \tag{49}$$ (73)

With $\beta = 0$ for the matter-dominated era and $\beta = 1$ for the radiation-dominated era, compatibility is ensured (**Appendix 1**) by the equation:

$$\boxed{\rho c^2 a^{(3+\beta)} + \overline{\rho}\overline{c}^2\overline{a}^{(3+\beta)} = E = Cst}$$ (74)

Which translates to a generalized conservation of energy. The system then gives (**Appendix 1**), for the matter-dominated era:

$$\dot{a} = c\sqrt{-k - \frac{8\pi GE}{3a}} \tag{51a}$$ (75)

$$\ddot{a} = -\frac{4\pi GE}{3a^2} \tag{52a}$$ (76)

$$\dot{\overline{a}} = \overline{c}\sqrt{-\overline{k} + \frac{8\pi GE}{3\overline{a}}} \tag{51b}$$ (77)

$$\ddot{\overline{a}} = \frac{4\pi GE}{3\overline{a}^2} \tag{52}$$ (78)

Observation yields $\ddot{a} > 0$, hence $E < 0$. Negative mass dominates and drives galactic dynamics. This provides an alternative interpretation to that of dark energy. It also implies $k = -1$.

The evolution of the negative species occurs for $\ddot{\overline{a}} < 0$ and corresponds to the hyperbolic Friedmann solution. At infinity, it tends toward $\overline{c}t$. Similarly, at in-

finity, *a* tends toward *ct*. The equations also impose $\bar{k} = -1$.

The exact solution to Equation (52a) was constructed by William Bonnor [19] in parametric form. With $x^o = ct$:

$$a(\mu) = \alpha^2 c \,\mathrm{ch}^2 u \tag{53}$$ (79)

$$t(\mu) = 2\alpha^2 c\left(1 + \frac{\mathrm{sh}\,2u}{2} + u\right) \tag{54}$$ (80)

Which give us the curves of **Figure 24**.

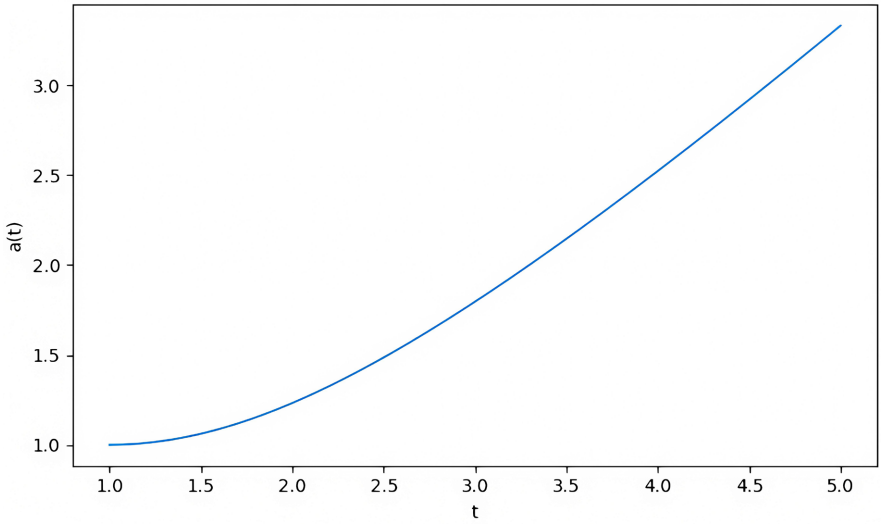


Figure 24. Expansion of the universe.

Profile corresponding to variable acceleration is shown on **Figure 25**.

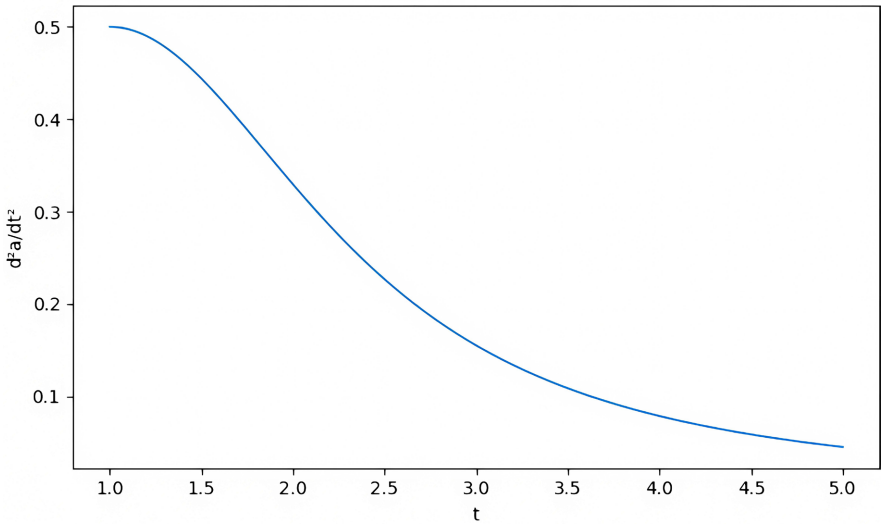


Figure 25. Evolution of acceleration.

This yields an interpretation of the origin of the accelerated expansion as an alternative to the dark energy hypothesis. The model predicts a significant accel-

eration value in the past—a finding seemingly supported by initial observational data from DESI [20], though confirmation from further observational data is still required.

Data regarding the dynamics of the negative species are not amenable to observation. For the radiation-dominated era and positive mass, the choice is made to extend the model with a curvature index of −1.

It is thus evident that the system of field equations is capable of yielding a mathematically consistent non-stationary solution describing isotropic and homogeneous media.

11. Spherically Symmetric Stationary Solution

In a vacuum, the satisfaction of the Bianchi conditions is automatic.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \qquad (55a) \quad (81a)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = 0 \qquad (55b) \quad (81b)$$

In a region of space with non-zero matter density, two cases must be considered.

First case: The field is created by a positive mass. The system becomes:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T_{\mu\nu} \qquad (56a) \quad (82a)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = -\chi K_{\mu\nu} \qquad (56b) \quad (82b)$$

The solution to the first equation is the classical solution—along with its TOV equation—which applies equally to the Newtonian approximation and to very dense objects such as neutron stars. The classical TOV equation is then:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left(\rho + p/c^2\right)\left(m + 4\pi G p r^3/c^4\right)c^2}{r\left(r - 2m\right)} \qquad (57) \quad (83)$$

And the interior metric is :

Et la métrique intérieure:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{1 - \frac{R_a^{\ 2}}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{R^2}{\hat{R}^2}}\right]^2 c^2 \dot{t}^2 - \frac{\dot{R}^2}{1 - \frac{R^2}{\hat{R}^2}} - R^2\left(\dot{\theta}^2 + \sin^2\theta \dot{\varphi}^2\right) \qquad (58) \quad (84)$$

If one were to consider constructing the interior metric $\bar{g}_{\mu\nu}^{int}$, it would then be necessary to define the form of the interaction tensor $K_{\mu\nu}$. However, this approach would not yield any elements capable of being compared with observational data, since our measuring instruments are unable to detect negative-energy photons. We are therefore not required to provide these elements.

Second case: the field is created by a negative mass. The system becomes:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi \bar{K}_{\mu\nu} \qquad (59a) \quad (85a)$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} = -\chi\bar{T}_{\mu\nu} \quad (59b)$$ (85b)

In this instance, we must define the interaction tensor $\bar{K}_{\mu\nu}$, as it is what yields the observables. The two equations will give rise to their own TOV equations—the equivalents of the Bianchi identities in this context. For the solution to be mathematically consistent, compatibility must be ensured.

As early as 2019 [21], it was shown that by opting for:

$$\bar{K}_{\mu}^{\nu} = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & \bar{q} & 0 & 0 \\ 0 & 0 & \bar{q} & 0 \\ 0 & 0 & 0 & \bar{q} \end{pmatrix} \quad (60)$$ (86)

Namely, the source tensor with the signs of the pressure terms inverted; this led to the TOV equation:

$$\bar{q}' = -\frac{\left(\bar{\rho}\bar{c}^2 + \bar{q}'\right)\left(-\bar{m} + 4\pi G\bar{q}' r^3/c^4\right)}{r\left(r + 2\bar{m}\right)} \quad (61)$$ (87)

Compatibility is then ensured—asymptotically, in the Newtonian approximation—with $\bar{q} = \bar{p}$. The two TOV equations then yield the same Euler equation, representing the equilibrium between pressure forces and the force of gravity:

$$\bar{p}' = \frac{\bar{\rho}\bar{c}^2\bar{m}}{r^2} = \frac{G\bar{\rho}\bar{M}}{r^2} \quad (62)$$ (88)

The Newtonian approximation means that this type of solution cannot handle cases where the negative-mass object—the source of the field—falls outside that framework. However, numerical simulations modeling the universe's large-scale structure produce vast, spherically symmetric negative-mass conglomerates made of negative-mass antihydrogen and antihelium. The cooling time for such objects exceeds the age of the universe; they do not evolve. Their temperature remains around 3000 K, and they simply radiate in the red and infrared spectra as negative-energy photons that escape detection. Yet, for travelers aboard a spacecraft capable of inverting its mass—thereby utilizing this negative realm as a sort of parallel universe—this is the spectacle that would meet their eyes. This world does not evolve and gives rise to neither stars nor galaxies; life is absent from it.

In conclusion, this Newtonian solution suffices to describe the full range of phenomena, as the negative-mass realm does not harbor superdense objects. From an observational standpoint, the interior metric $g_{\mu\nu}^{int}$ will be used to calculate the luminosity attenuation effect for objects located behind a negative-mass conglomerate—specifically, the one we believe lies at the center of the formation known as the "dipole repeller" [22]. Details regarding the construction of this metric are provided in **Appendix 2**.

In the vicinity of this negative-mass object, the exterior metric is:

$$ds^2 = \left(1 + \frac{8\pi G\rho r^2)}{3c^2}\right)c^2dt^2 - \left(1 - \frac{8\pi G\rho r^2}{3c^2}\right)dr^2 - r^2d\Omega^2 \quad (63)$$ (89)

Which matches the external metric, also linearized.

$$ds^2 = \left(1+\frac{2G|\bar{M}|}{c^2 r}\right)c^2 dt^2 - \left(1-\frac{2G|\bar{M}|}{c^2 r}\right)dr^2 - r^2 d\Omega^2 \quad (64)$$ (90-

This result should make it possible to assess the loss of brightness of background sources due to negative lensing, in order to compare them with observational data once the latter become available.

12. Re-Examination of Objections Raised against the Consistency of the Janus Model

12.1. Chronology of Objections and Responses

On January 4, 2019, based on a paper published in 2014 [23], T. Damour posted an initial highly critical titled "On the Janus cosmological model" [24] on his personal page on the IHES website. He informed J.-P. Petit of this via a registered letter with acknowledgment of receipt [25] sent to his home address.

A reply was sent to him, pointing out that some of his criticisms concerned equations affected by a typographical error in the article on which he based his analysis—specifically, the notation distinguishing interaction tensors from matter tensors had been omitted in the published version. This reply was accompanied by an article titled "Physical and Mathematical Consistency of the Janus Model," published in *Progress in Physics* that same month [21], as well as a proposal for a meeting to examine the calculations in question directly. This proposal received no response.

It should be noted, however, that although T. Damour specifies that his analysis is based on the two articles published in 2014, he does not discuss—nor does he subsequently do so—the non-stationary, homogeneous, and isotropic cosmological solution constructed in one of them [18]. Yet this solution constitutes an explicit case in which both systems of field equations are simultaneously satisfied, including the constraints resulting from the Bianchi identities.

In September 2019, a much more detailed document—explicitly setting out the calculations and, in particular, the analysis of the Tolman-Oppenheimer-Volkoff equations in the Newtonian limit—was sent to T. Damour via registered mail with acknowledgment of receipt [26]. This letter also went unanswered. The document was made public in 2020, in both French and English, as an open letter posted on Jean-Pierre Petit's website, so that the calculations could be independently examined.

In the years that followed, the published document prompted several French and foreign scientists to examine the calculations. In 2022, Marie-France Duval, a former astrophysics lecturer at the University of Marseille, initiated a new approach to T. Damour together with several colleagues. A joint letter, accompanied by an analysis of the calculations in question, was sent to him via registered mail with acknowledgment of receipt on November 31, 2022 [27].

On December 12, 2022, T. Damour informed J.-P. Petit by email that he had

posted a new article [28] on the IHES website—bearing the same title as before, "Physical and mathematical inconsistency of the Janus cosmological model"—which, in his view, contained "serious inconsistencies."

While in this article—as in the 2019 one—he reiterates that his analysis focuses on the two papers published in 2014 K [18] [23], he says nothing about the paper published in "Astrophysics and Space Science" [18], which presents a non-stationary solution—under conditions of homogeneity and isotropy—that satisfies the Bianchi conditions. Everything suggests, therefore, that he has still not consulted it.

However, with a three-year delay, he shows that he has examined the details of the calculations published in January 2019 [21]—the details of which had been sent to him in September 2019 [26] and which demonstrated the possibility of constructing a solution in the Newtonian approximation under steady-state conditions—and writes, see **Figure 26**.

Starting from the Janus-2019 field equations, the analogues of the Tolman-Oppenheimer-Volkoff equations are obtained from Eqs. (1) by setting $T^{-}_{\mu\nu}$ to zero and solving the equations for the two metrics in a stationary, spherically symmetric case. These solutions were written in Eqs. (45), (46) of PDD19, namely (with $' = d/dr$)

$$p'^{+} = -G(\rho^{+} + p^{+}/c^2)\,[M^{+}(r) + 4\pi p^{+} r^3/c^2] \,/\, [r(r - 2GM^{+}(r)/c^2)],$$

$$p'^{+} = -G(\rho^{+} - p^{+}/c^2)\,[M^{+}(r) - 4\pi p^{+} r^3/c^2] \,/\, [r(r + 2GM^{+}(r)/c^2)]. \quad (7)$$

Here $p^{+}(r)$ is the pressure (of ordinary matter), $\rho^{+}(r)$ its density, and $M^{+}(r) = 4\pi \int_0^r dr\, r^2 \rho^{+}(r)$ is the (positive) mass contained within radius r. Note that one passes from the first equation (7) to the second by the substitutions $p^{+} \to -p^{+}$ and $G \to -G$.

It is true that if one formally takes the Newtonian limit $1/c^2 \to 0$ in Eqs. (7), these two equations become compatible, because they both reduce to the single Newtonian structure equation (6).

But it is physically unacceptable to neglect the fact that the Janus-2019 model predicts that the radial pressure variation in an ordinary-matter star must satisfy two mutually incompatible equations. Indeed, if one considers, for example, a neutron star, the additional relativistic terms in (7), namely $\pm p^{+}/c^2$, $\pm 4\pi p^{+} r^3/c^2$, and $\pm 2GM^{+}(r)/c^2$, are numerically very significant (of order 10%) and conceptually very important because they greatly modify the value of the maximum mass of a neutron star. As recalled in Eq. (5), the analogues of Eqs. (7) in Janus-2014 had right-hand sides differing by about 200%. The modification introduced in Janus-2019 (geometrically undefined because it violates covariance) has merely formally reduced the difference between the right-hand sides from 200% to, say, about 20% (for a neutron star). This does not alter the fundamental mathematical inconsistency that the Janus-2019 model predicts that the radial pressure variation in an ordinary-matter star must satisfy two mathematically (and physically) incompatible equations.

CONCLUSION: The field equations of the 2019 version of the "Janus model", Eqs. (1), have no logical derivation from a covariant variational principle and are mathematically and physically contradictory. Moreover, they predict (contrary to the basic idea of the Newtonian Janus model) that a negative central mass must repel the negative masses around it.

Figure 26. Excerpt from the article posted on the IHES website by T. Damour on December 12, 2022 [28].

However, in the last line (underlined in red), he writes that the Janus equations predict that negative masses must repel each other. In a letter dated December 14, 2022 [29], we pointed out his error. T. Damour then withdrew the version dated December 12 and replaced it with a new document dated December 28, 2022 [30], bearing the same title. This second version, like the January 2019 document, remains accessible—four years after its initial posting—via T. Damour's personal

page on the IHES website. The differences between the versions dated December 12, 2022 [28] and December 28, 2022 [30], as well as their relationship to the objection raised in the letter dated December 14, 2022 [29], will be examined later.

A further step was taken in 2024 with the publication in European Physical Journal [9] of a derivation of the Janus system from an action. This formulation notably allows for the definition of interaction tensors via variation of the action and enables the issue of compatibility with Bianchi identities to be revisited within this framework. The article also reviews the analysis conducted as early as 2019 [21] regarding the spherically symmetric stationary case and its Newtonian limit.

In 2025, a draft letter calling for a scientific debate on the Janus model to be held at the Academy of Sciences was made public on J.-P. Petit's website to gather co-signatures. This letter, co-signed by 54 French and international scientists, was subsequently sent to Françoise Combes, President of the Academy of Sciences. As of the time of writing, this request has received no response [31].

This chronology is presented neither as an argument in favor of the Janus model nor as a response to the objections raised against it. Its purpose is to identify the various documents to which we will refer and to specify the sequence in which arguments were formulated, responses were communicated, and texts were modified. The question of the model's consistency must be resolved independently of this history, through an examination of the field equations, the Bianchi identities, and the solutions they admit.

The two articles by T. Damour addressing the alleged inconsistency of the Janus cosmological model—published in January 2019 [24] and December 2022 [30] respectively, and still available on his personal IHES webpage—were not published in a peer-reviewed scientific journal but were instead posted online on that personal institutional page. This circumstance obviously implies nothing regarding the validity or invalidity of the arguments contained therein. However, it has significant implications for the conditions under which scientific debate takes place. When a critique explicitly targeting an author's published work appears in a scientific journal, editorial procedures generally allow the author in question to submit a response—such as a "Letter," "Comment," or "Reply," depending on the journal's specific guidelines. The author can then address the objections raised in detail and refer back to published articles where the issues in question were already discussed. This response is, in turn, subject to the journal's editorial process and can thus become part of the scientific literature on the same footing as the critique to which it responds. Posting T. Damour's two critiques on his personal IHES webpage did not provide such a framework for a structured exchange of views. Therefore, we will now examine the objections raised in these two articles one by one and compare them against the equations, solutions, and scientific publications presented or cited in the preceding sections.

12.2. First Objection: Compatibility of the Two Bianchi Identities

In his document dated January 4, 2019 [24], he examines the system of equations

he then attributes to the Janus model and concludes that it is physically and mathematically inconsistent.

This question must be distinguished from the issue of whether arbitrarily chosen source tensors could simultaneously satisfy two independent sets of differential constraints. In the model considered here, the interaction tensors are precisely not arbitrary; they are subject to the conservation conditions resulting from the Bianchi identities.

Using the notation employed previously, the field equations are written in the form:

$$G_{\mu\nu} = \chi\left[T_{\mu\nu} + \bar{K}_{\mu\nu}\right] \quad (65a) \quad (91a)$$

$$\bar{G}_{\mu\nu} = -\chi\left[\bar{T}_{\mu\nu} + K_{\mu\nu}\right] \quad (65b) \quad (91b)$$

Geometric identities:

$$\nabla_\nu G^{\mu\nu} = \bar{\nabla}_\nu \bar{G}^{\mu\nu} = 0 \quad (66) \quad (92)$$

therefore require:

$$\nabla_\nu\left[T_{\mu\nu} + \bar{K}_{\mu\nu}\right] = \bar{\nabla}_\nu\left[\bar{T}_{\mu\nu} + K_{\mu\nu}\right] = 0 \quad (67) \quad (93)$$

The question of compatibility is therefore not an external condition added to the system; it constitutes a constraint on the form of the interaction tensors.

Let us first consider the non-stationary case describing homogeneous and isotropic media. The initial 2014 paper [16] provides precisely such a mathematically consistent solution. Although this paper is explicitly cited by T. Damour in his two documents [24] [30], the solution itself is not examined therein.

For two homogeneous and isotropic FLRW-type metrics and for the equation of state:

$$p = \beta\frac{\rho c^2}{3} \qquad \bar{p} = \beta\frac{\bar{\rho}\bar{c}^2}{3} \quad (68) \quad (94)$$

The two constraints derived from the Bianchi identities lead, as shown in **Appendix 1**, to:

$$\varphi = \left(\frac{\bar{a}}{a}\right)^{3+\beta} \qquad \phi = \left(\frac{a}{\bar{a}}\right)^{3+\beta} \quad (69) \quad (95)$$

and consequently to the generalized conservation law:

$$\rho c^2 a^{3+\beta} + \bar{\rho}\bar{c}^2\bar{a}^{3+\beta} = E = Cst \quad (70) \quad (96)$$

Thus, within this class of solutions, the two Bianchi constraints are satisfied simultaneously. They do not lead to a contradiction but rather determine the relations that the interaction terms must satisfy.

The existence of such a solution obviously does not constitute a proof of the existence of solutions for any arbitrary distribution of matter. It suffices, however, to show that a general and necessary incompatibility between the two Bianchi identities cannot be deduced solely from the bimetric nature of the system.

This point is all the more important given that the homogeneous and isotropic cosmological solution had already been constructed in the 2014 paper Negative mass hypothesis in cosmology and the nature of dark energy [18]—explicitly cited in the January 2019 document—without that solution being examined therein.

12.3. Second Objection: Incompatibility of the Structural Equations

In his January 2019 paper [24], T. Damour then considers the spherically symmetric, stationary case. For the equations he employs, he derives—in the Newtonian limit—two structure equations with opposite signs. This objection was mathematically valid regarding the system as it appeared in the second 2014 paper [23]. And this is where an unfortunate set of circumstances comes into play: when the manuscript for that paper was typeset by the publisher, the hats we had included disappeared. See **Figure 27**.

In our model, the Universe is an M_4 manifold associated not to one single metric, but to two: $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$, the former linked to species of positive mass and energy, the latter to species of negative mass and energy. From these metrics, one can build the associated Ricci tensors, $R_{\mu\nu}^{(+)}$ and $R_{\mu\nu}^{(-)}$. A system of two coupled field equations was then proposed:[20]

$$R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi\left(T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right), \tag{2a}$$

hat missing

$$R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(+)} = -\chi\left(T_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right). \tag{2b}$$

where the tensors $T_{\mu\nu}^{(+)}$ and $T_{\mu\nu}^{(+)}$ represent positive and negative energy contents (and positive and negative mass contents as well). Previously, in 1957, Bondi[23] did study the possibility of introducing negative masses into the Einsteinian model with a single metric. As a result, positive masses attracted everything and negative masses repelled everything. It then led to a phenomenon that was called "runaway": when a mass $+m$ met a mass $-m$, the positive mass fled, pursued by the negative mass, both undergoing a continuous acceleration, with conservation of energy, since one of the energies was positive and the other negative.

Figure 27. The field equations, as presented in the second 2014 paper [23].

In our model, the Universe is an M_4 manifold associated not to one single metric, but to two: $g_{\mu\nu}^{(+)}$ and $g_{\mu\nu}^{(-)}$, the former linked to species of positive mass and energy, the latter to species of negative mass and energy. From these metrics, one can build the associated Ricci tensors, $R_{\mu\nu}^{(+)}$ and $R_{\mu\nu}^{(-)}$. A system of two coupled field equations was then proposed:[20]

$$R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi\left(T_{\mu\nu}^{(+)} + \widehat{T}_{\mu\nu}^{(-)}\right) \tag{2a}$$

interaction tensors with «hat»

$$R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(+)} = -\chi\left(\widehat{T}_{\mu\nu}^{(+)} + T_{\mu\nu}^{(-)}\right) \tag{2b}$$

where the tensors $T_{\mu\nu}^{(+)}$ and $T_{\mu\nu}^{(+)}$ represent positive and negative energy contents (and positive and negative mass contents as well). Previously, in 1957, Bondi[23] did study the possibility of introducing negative masses into the Einsteinian model with a single metric. As a result, positive masses attracted everything and negative masses repelled everything. It then led to a phenomenon that was called "runaway": when a mass $+m$ met a mass $-m$, the positive mass fled, pursued by the negative mass, both undergoing a continuous acceleration, with conservation of energy, since one of the energies was positive and the other negative.

Figure 28. These equations, in the original manuscript.

While in the submitted version, hats marked the fundamental difference between these interaction tensors $\hat{T}^{(+)}_{\mu\nu}$ and $\hat{T}^{(-)}_{\mu\nu}$ and the field tensors $T^{(+)}_{\mu\nu}$ and $T^{(-)}_{\mu\nu}$. See **Figure 28**.

Thus, upon receiving his registered letter with acknowledgment of receipt at our home address—dated January 4, 2019 [25] and pointing to the article he had just posted on his IHES webpage [24]—the first thing we did was to state that we were fully aware of this requirement for compatibility with Bianchi conditions; we noted that simply identifying interaction tensors with field tensors imposed the degeneracy $g^{(+)}_{\mu\nu} = g^{(-)}_{\mu\nu}$, a condition that had already been satisfied in the non-stationary case (assuming homogeneity and isotropy) in the form of a generalized energy conservation law (see **Appendix 1**). Regarding the stationary solution, we explained that, following discussions with the referee for the journal Progress in Physics, we had secured acceptance for an article—just published in the January 2019 issue [21]—presenting a solution consistent with the Newtonian approximation; we forwarded a copy of this article to him. At the same time, we proposed a meeting to discuss the points raised in his article.

However, in his registered letter with acknowledgment of receipt [25], we find, see **Figure 29**.

> **Copies:** I reserve the right to send copies of this letter to all the persons whom you cite on your website, in your videos, and in your letters, as well as to any person interested in the "Janus model".

Figure 29. Damour's 2019 paper recipients [25].

It turned out, in fact, that a great many people within the French and international scientific communities, as well as in the press, had already received a copy of his article. We then proposed a meeting to T. Damour to examine these various points and shared with him the January 2019 article [21], which presented a construction of interaction tensors satisfying the constraints considered in the Newtonian limit. We received no response to this proposal or to the document over the following three years.

The details of the calculations regarding this construction of interaction tensors—which satisfy the Bianchi conditions in the Newtonian approximation—are given in **Appendix 2**. For an object composed of negative-mass matter, the matter tensor of the negative sector takes the form:

$$\bar{T}^{\nu}_{\mu} = \mathrm{diag}\left(\bar{\rho}, -\frac{\bar{p}}{\bar{c}^2}, -\frac{\bar{p}}{\bar{c}^2}, -\frac{\bar{p}}{\bar{c}^2}\right) \quad (71)$$ (97)

The compatibility condition in the Newtonian limit is then satisfied if the interaction tensor of the positive sector is given the form:

$$\bar{K}^{\nu}_{\mu} = \mathrm{diag}\left(\bar{\rho}, +\frac{\bar{q}}{\bar{c}^2}, +\frac{\bar{q}}{\bar{c}^2}, +\frac{\bar{q}}{\bar{c}^2}\right) \quad (72)$$ (98)

With $\bar{q} = -\bar{p}$. In other words, if the pressure terms of the interaction tensor are equal and opposite to those of the corresponding matter tensor.

The two TOV equations then lead, in the Newtonian limit, to the same Euler equation. With

$$M(r) = 4\pi \quad (73) \quad (99)$$

We get:

$$\frac{d\bar{p}}{dr} = -\frac{G\bar{M}(r)\bar{\rho}}{r^2} \quad (74) \quad (100)$$

and, for the interaction tensor:

$$\frac{d\bar{q}}{dr} = \frac{G\bar{M}(r)\bar{\rho}}{r^2} \quad (75) \quad (101)$$

Avec

$$\bar{q} = -\bar{p} \quad (76) \quad (102)$$

The two relations then become compatible.

It is particularly worth noting that T. Damour's document of December 28, 2022, itself acknowledges that the modification introduced in 2019 eliminates the contradiction between the two Newtonian equations and leads to a single Newtonian structure equation. See **Figure 30**.

> It is true that if one formally takes the Newtonian limit $1/c^2 \to 0$ in Eqs. (7), these two equations become compatible, because they both reduce to the single Newtonian structure equation (6).

Figure 30. Sentence from the article posted on the IHES website by T. Damour on December 12, 2022 [28].

However, the scope of the result must be clarified. We do not claim here to have demonstrated the identity of the two exact TOV equations to all relativistic orders. The result established is their compatibility in the Newtonian limit, which is precisely the regime used here to describe the diffuse negative-mass structures considered in the model.

12.4. On the Management of Superdense Objects

In his 2022 paper, T. Damour acknowledges—three years after receiving the 2019 article presenting the elements discussed in the previous section—that the two structure equations become compatible in the Newtonian limit. Indeed, he writes that as $\frac{1}{c^2} \to 0$, both equations become identical to the single Newtonian structure equation. However, he maintains his objection regarding the relativistic regime, citing the example of a neutron star, for which the $\frac{1}{c^2}$ correction terms can no longer be neglected.

He cites the model's inability to describe supermassive objects. This is incorrect for two reasons. In the case of neutron stars, the first field equation—through the

exterior and interior metrics—yields the required geometry. However, there is no obligation to formulate the interaction tensor appearing in the second equation, given that pursuing this theoretical work leads to no confrontation with observational data. Indeed, our instruments do not detect negative-energy photons. Finally, developments within the Janus model show that its negative sector is populated solely by vast, low-density, spheroidal conglomerates made of light elements—devoid of structures such as galaxies, stars, or planets. Consequently, they contain no high-density objects. Conversely, the construction of the positive sector's Newtonian metric—generated by the interaction term—predicts a dimming of distant background objects, thereby offering a potential observational confirmation.

12.5. Fourth Objection: Variational Derivation of the System

In his December 2022 document, T. Damour raises two objections to the derivation proposed in the 2019 article. The first concerns the introduction of the matrix

$$\varphi = \mathrm{diag}(1,-1,-1,-1) \tag{76}$$ (103)

which he considers incompatible with general covariance. The second concerns the relation used in this derivation,

$$\delta g^{-}_{\mu\nu} = -\delta g^{+}_{\mu\nu} \tag{77}$$ (104)

which, according to him, would not allow for obtaining two independent field equations via variation.

It is not necessary, in the present article, to defend this 2019 variational construction. Since then, the problem has been revisited independently. The derivation published in 2024 in the European Physical Journal C starts from a bimetric action and defines the interaction tensors by varying with respect to the two metrics. The present article goes a step further by demonstrating the close relationship between this construction and the class of bimetric actions introduced by Damour and Kogan. As shown in sections 6 through 9, the essential difference considered here lies in the relative sign of the two gravitational coupling constants. Damour and Kogan use constants of the same sign, whereas the Janus system considered here corresponds to the choice

$$\overline{\chi} = -\chi \tag{78}$$ (105)

The criticism regarding the constant matrix and the relationship between variations used in 2019 does not, therefore, constitute an objection to the variational derivation employed in the current formulation. It concerns an earlier derivation that is no longer necessary for the definition of the system studied here.

12.6. Objection Regarding the Direction of Gravitational Interactions

In his critiques of the Janus model, T. Damour identifies a contradiction regarding the nature of gravitational interactions between positive and negative masses. In

his article dated December 28, 2022, his reasoning involves requiring a single four-velocity field—associated with a test particle—to simultaneously satisfy two geodesic equations, constructed respectively from the connections associated with the metrics g^{+} and g^{-}. In the document dated December 28, 2022 [30], T. Damour explicitly states that "the same [...] four-velocity field" must satisfy, in turn, geodesic Equations (5) and (6), which are associated with the two metrics.

This prescription is inconsistent with the foundational assumptions of the Janus model. In that model, the two species of particles are associated with two different metrics. Positive-mass test particles follow the geodesics of the metric g^{+} the solution to the first field equation—whereas negative-mass test particles follow the geodesics of the metric g^{-} the solution to the second field equation. A single test particle is therefore not assumed to follow the geodesics of both metrics simultaneously.

The reasoning leading to the contradiction put forward by T. Damour becomes clear when he takes the Newtonian limit. By successively applying the connections associated with the two metrics to the same four-velocity field u_{μ}^{+}, he obtains two accelerations of opposite signs.

$$\frac{\mathrm{d}v_{+}^{i}}{\mathrm{d}t} = +\partial_{i}U \qquad \frac{\mathrm{d}v_{+}^{i}}{\mathrm{d}t} = -\partial_{i}U \tag{79}$$ (106)

...then interprets this opposition as a contradiction within the model. Yet this contradiction arises precisely from the simultaneous application of the two geodesic prescriptions to the same test particle. It does not occur when each type of particle is subjected to the metric associated with it in the definition of the model.

The most direct way to determine the nature of gravitational interactions is to refer to the exterior metric solutions obtained under spherical symmetry, as was done in the preceding sections. In the weak-field limit of the corresponding Schwarzschild-type metrics, the sign of the Newtonian term directly determines the direction of the acceleration of the test particle in the sector under consideration.

This yields the four situations:

(+ source, + test particule) ⟶ Attraction

(− source, − test particule) ⟶ Attraction

(+ source, − test particule) ⟶ Repulsion

(− source, + test particule) ⟶ Repulsion

These four results reproduce the interaction laws that constitute one of the founding hypotheses of the Janus model: masses of the same sign attract each other, and masses of opposite signs repel each other.

The origin of the contradiction raised in T. Damour's analysis can thus be unambiguously identified. It lies not in the sign of the forces derived from the system's metric solutions, but in the assumption that a single test-matter motion must simultaneously satisfy the geodesic equations associated with both metrics. This assumption is not part of the definition of the Janus model.

13. Conclusions

Since 2019, the two texts by T. Damour addressing the alleged inconsistency of the Janus model have constituted—to the best of our knowledge—the most frequently cited explicitly developed mathematical critiques challenging the model's consistency.

Among these criticisms is T. Damour's assertion that the system of two coupled field equations upon which this model rests cannot be derived from an action.

Such a derivation has since been published. However, the present work sheds additional, particularly simple light on this issue. In 2002, Damour and Kogan themselves proposed an action describing the coupled dynamics of two metrics. Their construction was framed within a different context—that of non-linear bigravity—and was in no way intended to construct the Janus cosmological model. We have shown here that by considering, within a construction of this type, two gravitational coupling constants that are equal in absolute value but opposite in sign, the structure of the system of two coupled field equations characteristic of the Janus model emerges naturally.

This result does not imply that the Janus model should be viewed merely as an application or a historically derived variant of the work of Damour and Kogan. Its genesis is independent, and it addresses a different set of physical issues. However, the comparison drawn in this article demonstrates that the objection—claiming the structure of its field equations is inherently incompatible with a Lagrangian formulation—cannot be sustained.

We review the origins of the Janus model, tracing the path from the introduction of negative masses and the "runaway" problem to the construction of a bimetric system in which positive- and negative-mass particles are associated with two distinct metrics. This structure yields the desired interaction laws: masses of the same sign attract one another, while masses of opposite signs repel. The runaway effect inherent in introducing negative masses within a monometric framework is eliminated.

We also recall the key cosmological and astrophysical properties that motivated the model's development, as well as the various qualitative agreements with observations achieved during its formulation. Taken in isolation, these elements do not constitute a validation of the model. However, they demonstrate that it yields consequences amenable to observational testing, thereby justifying an examination of its mathematical consistency and physical predictions based on its effective equations.

This consistency has been examined here in two situations possessing symmetries high enough to allow for an explicit analysis. In the non-stationary, homogeneous, and isotropic case, the constraints arising from the two Bianchi identities lead to a generalized conservation law and to coupled cosmological solutions; thus, they do not reveal the a priori incompatibility that had been cited as an argument against the model. In the stationary, spherically symmetric case, an anal-

ysis of the Tolman-Oppenheimer-Volkoff equations demonstrates how the structure of the interaction tensors ensures the hydrostatic compatibility of the two sectors within the Newtonian limit under consideration.

We then re-examine the following criticisms raised by T. Damour. The objection that the two Bianchi identities are necessarily incompatible is countered by the existence of an explicit cosmological solution that simultaneously satisfies the constraints they impose. Finally, the apparent contradiction regarding the direction of the forces stems from the requirement that the same four-velocity field of a test-matter particle simultaneously satisfy the geodesic equations associated with both metrics. This is precisely how the argument is formulated in the document dated December 28, 2022.

This latter prescription is not among the foundational hypotheses of the Janus model. A positive-mass test particle follows the geodesics of the metric $g_{\mu\nu}$, whereas a negative-mass test particle follows those of the metric $\overline{g}_{\mu\nu}$ $. Directly determining the nature of the interactions from spherically symmetric exterior metric solutions thus unambiguously yields attraction between masses of the same sign and repulsion between masses of opposite signs. In the December 2022 text, by contrast, the contradictory accelerations are obtained by successively applying the two connections to the same test-matter motion.

cancel « dollar »

With this clarification, we hope that the Janus Cosmological Model can now be examined by the specialist community based on its equations, its solutions, and how they compare with observations.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Einstein, A. (1915) Die Feldgleichungen des Gravitation [The Field Equation of Gravitation]. Sitzungsber. XLIV, 8, 778 and XLVI s. 799.

[2] Chaudhary, H., Capozziello, S., Sharma, V.K., Gómez-Vargas, I. and Mustafa, G. (2026) Evidence for Evolving Dark Energy from DESI DR2 BAO and Pantheon+, DesDovekie, and Union3. *The European Physical Journal C*, **86**, Article No. 564. https://doi.org/10.1140/epjc/s10052-026-15806-w

[3] Riess, A.G., Yuan, W., Macri, L.M., Scolnic, D., Brout, D., Casertano, S., *et al.* (2022) A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 Km S $^{-1}$ Mpc $^{-1}$ Uncertainty from the Hubble Space Telescope and the SH0ES Team. *The Astrophysical Journal Letters*, **934**, L7. https://doi.org/10.3847/2041-8213/ac5c5b

[4] Bondi, H. (1957) Negative Mass in General Relativity: Negative Mass in General Relativity. *Reviews of Modern Physics*, **29**, 423-428. https://doi.org/10.1103/revmodphys.29.423

[5] Petit, J.P. (1994) The Missing-Mass Problem. *Il Nuovo Cimento B*, **109**, 697-709. https://doi.org/10.1007/bf02722527

[6] Petit, J.P. (1986) Le Topologicon. Editions Belin. Topo The World. (In French) http://www.savoir-sans-frontieres.com/JPP/telechargeables/Eng-

see on the .docx

lish/Topo_the_world

[7] Souriau, J.M. (1970) Structure des systèmes dynamiques. Dunod.

[8] Petit, J. and Zejli, H. (2024) Study of Symmetries through the Action on Torsors of the Janus Symplectic Group. *Reviews in Mathematical Physics*, **37**, Article ID: 2450054. https://doi.org/10.1142/s0129055x24500545

[9] Petit, J., Margnat, F. and Zejli, H. (2024) A Bimetric Cosmological Model Based on Andreï Sakharov's Twin Universe Approach. *The European Physical Journal C*, **84**, Article No. 1226. https://doi.org/10.1140/epjc/s10052-024-13569-w

[10] Petit, J. (1995) Twin Universes Cosmology. *Astrophysics and Space Science*, **226**, 273-307. https://doi.org/10.1007/bf00627375

[11] Damour, T. and Kogan, I.I. (2002) Effective Lagrangians and Universality Classes of Nonlinear Bigravity. *Physical Review D*, **66**, Article ID: 104024. https://doi.org/10.1103/physrevd.66.104024

[12] Fierz, M. and Pauli, W.E. (1939) On Relativistic Wave Equations for Particles of Arbitrary Spin in an Electromagnetic Field. *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, **173**, 211-232. https://doi.org/10.1098/rspa.1939.0140

[13] de Rham, C., Gabadadze, G. and Tolley, A.J. (2011) Resummation of Massive Gravity. *Physical Review Letters*, **106**, Article ID: 231101. https://doi.org/10.1103/physrevlett.106.231101

[14] Boulware, D.G. and Deser, S. (1972) Can Gravitation Have a Finite Range? *Physical Review D*, **6**, 3368-3382. https://doi.org/10.1103/physrevd.6.3368

[15] Hossenfelder, S. (2006) Anti-Gravitation. *Physics Letters B*, **636**, 119-125. https://doi.org/10.1016/j.physletb.2006.03.038

[16] Hossenfelder, S. (2008) Bimetric Theory with Exchange Symmetry. *Physical Review D*, **78**, Article ID: 044015. https://doi.org/10.1103/physrevd.78.044015

[17] Benoit-Lévy, A. and Chardin, G. (2012) Introducing the Dirac-Milne Universe. *Astronomy & Astrophysics*, **537**, A78. https://doi.org/10.1051/0004-6361/201016103

[18] Petit, J.P. and D'Agostini, G. (2014) Negative Mass Hypothesis in Cosmology and the Nature of Dark Energy. *Astrophysics and Space Science*, **29**, 145-182.

[19] Bonnor, W.B. (1989) Negative Mass in General Relativity. *General Relativity and Gravitation*, **21**, 1143-1157. https://doi.org/10.1007/bf00763458

[20] Chaudhary, H., Capozziello, S., Sharma, V.K., Gómez-Vargas, I. and Mustafa, G. (2026) Evidence for Evolving Dark Energy from DESI DR2 BAO and Pantheon$^+$, Des-Dovekie, and Union3. *The European Physical Journal C*, **86**, Article No. 564. https://doi.org/10.1140/epjc/s10052-026-15806-w

[21] Petit, J.P., D'Agostini, G. and Debergh, N. (2019) Physical and Mathematical Consistency of the Janus Cosmological Model (JCM). *Progress in Physics*, **15**, PAGE.

see .docx document

[22] Hoffman, Y., Pomarède, D., Tully, R.B. and Courtois, H.M. (2017) The Dipole Repeller. *Nature Astronomy*, **1**, Article No. 36. https://doi.org/10.1038/s41550-016-0036

[23] Petit, J.P. and D'Agostini, G. (2014) Cosmological Bimetric Model with Interacting Positive and Negative Masses and Two Different Speeds of Light, in Agreement with the Observed Acceleration of the Universe. *Modern Physics Letters A*, **29**, Article ID: 1450182. https://doi.org/10.1142/s021773231450182x

[24] Damour, T. (2019) On the "Janus Cosmological Model Model". On His Personal Page on the IHES Website. (In French) https://www.ihes.fr/~damour/publications/JanusJanvier2019-1.pdf

[25] http://www.jp-petit.org/Damour/2019-Damour-letter-to-Petit.pdf

[26] http://www.jp-petit.org/Damour/2019-Petit-to-Damour-en.pdf

[27] http://www.jp-petit.org/Damour/2022-Duval-to-Damour-nov-30-en-fr.pdf

[28] http://www.jp-petit.org/Damour/2022-Damour-IHES-dec-12-en-fr.pdf

[29] http://www.jp-petit.org/Damour/2022-Petit-to-Damour-dec-14-en-fr.pdf

[30] Damour, T. (2022) Physical and Mathematical Inconsistency of the "Janus Cosmological Model". On Damour's Personal Page. (In French) https://www.ihes.fr/~damour/publications/JanusDecembre2022c.pdf

[31] http://www.jp-petit.org/Damour/2025-Request-for-the-Exercise-of-a-Right-of-Reply-before-the-French-Academy-of-Science.pdf

[32] Adler, R., Bazin, M., Schiffer, M. and Romain, J.E. (1965) Introduction to General Relativity. *Physics Today*, **18**, 68-68. https://doi.org/10.1063/1.3047725

Appendix 1

Generalized law of energy conservation for unsteady conditions and in homogeneous, isotropic media.

1 – **In classical cosmology**:

In general relativity, when the symmetry assumptions of homogeneity and isotropy are applied to a non-stationary solution, the metric takes the FLRW form, and the field equation yields a system of two second-order differential equations involving the scale factor and its first and second time derivatives.

$$G_0^0 - \left(\frac{3k}{a^2} + \frac{3\dot{a}^2}{c^2a^2}\right) = \chi\rho \tag{1}$$

$$G_i^i = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{c^2a^2} + \frac{2\ddot{a}}{c^2a}\right) = -\chi\frac{p}{c^2} \tag{2}$$

We must then derive a compatibility condition from this system, which will be the equivalent of the Bianchi condition relating to the symmetries of this solution. With:

$$\chi = -\frac{8\pi G}{c^2} \tag{3}$$

$$\frac{8\pi G}{c^2}\rho = \frac{3k}{a^2} + \frac{3\dot{a}^2}{c^2a^2} \tag{4}$$

$$\frac{8\pi G}{c^2}\left(\frac{p}{c^2}\right) = -\left(\frac{k}{a^2} + \frac{\dot{a}^2}{c^2a^2} + \frac{2\ddot{a}}{c^2a}\right) \tag{5}$$

By adding and then subtracting these two equations, we obtain:

$$\frac{4\pi G}{c^2}\left(\rho + \frac{3p}{c^2}\right) = -\frac{3\ddot{a}}{c^2a} \tag{6a}$$

$$\frac{4\pi G}{c^2}\left(\rho + \frac{p}{c^2}\right) = \frac{k}{a^2} + \frac{a'^2 - a\ddot{a}}{c^2a^2} \tag{6b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{c^2}\frac{\dot{a}}{a}\right) = \frac{k}{a^2} - \frac{4\pi G}{c^2}\left(\rho + \frac{p}{c^2}\right) \tag{7}$$

We now differentiate (4) with respect to time, and obtain:

$$\frac{8\pi G}{c^2}\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\frac{6k}{a^3}\dot{a} + \frac{6\dot{a}}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{c^2}\frac{\dot{a}}{a}\right) \tag{8}$$

Inserting (7) into (8) and rearranging we obtain the differential identity

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho a^3\right) + \frac{p}{c^2}\frac{\mathrm{d}a^3}{\mathrm{d}t} = 0 \tag{9}$$

Let us take an element of volume in the co-moving coordinates. Its geometry content will be proportional to a^3, from the Robertson-Walker metric. This, if $V(t)$ denotes the actual volume content of the element considered, $M = \rho V$ would measure the mass content and (9) could be written as:

$$\mathrm{d}M + \frac{1}{c^2} p\mathrm{d}V = 0 \tag{10}$$

Since $c^2 M$ measures the energy content of the element and $p\mathrm{d}V$ the work done against the pressure forces, we see that the energy balance under the cosmic evolution, is preserved. If we prescribe the equation of state of the fluid $p = p(\rho)$, we can obtain from (9) the dependance of a upon ρ. Indeed we may write Equation (9) in the alternative form:

$$\frac{\mathrm{d}a}{a} + \frac{1}{3}\left(\frac{\mathrm{d}\rho}{\rho + p(\rho)/c^2}\right) = 0 \tag{11}$$

It is this relationship that ensures the compatibility of the system of equations (1) + (2)—that is, the mathematical consistency of the solution, or the equivalent of the Bianchi condition for the field equation—within this particular symmetry.

In the denominator, we have a sum of two terms. Pressure is simply a volumetric energy density, just like the term ρc^2. It is the volumetric energy density that determines cosmic geometry and dynamics. The relative importance of the term $p compared to the term ρc^2 determines the type of dynamics under consideration. If we treat the matter fluid as a perfect fluid, and if $\langle v^2 \rangle$ is the mean square velocity of thermal motion within it, the pressure is given by:

p in italic

$$p_m = \frac{\rho_m \langle v^2 \rangle}{3} \tag{12}$$

If the medium is ionized, radiation pressure is added to this material pressure:

$$p_r = \frac{\rho_r c^2}{3} \tag{13}$$

In the general case, pressures are additive:

$$p = \frac{\rho_m \langle v^2 \rangle}{3} + \frac{\rho_r c^2}{3} \tag{14}$$

In the matter-dominated era, pressure terms are negligible compared to ρc^2. In the radiation-dominated era, radiation pressure is comparable to ρc^2. In the calculation, both simulations can be handled by expressing the pressure terms in the form:

$$p = \beta \frac{\rho c^2}{3}$$

$\beta = 0$ corresponds to the matter-dominated phase and $\beta = 1$ to the radiation-dominated phase. Equation (11) becomes:

$$\frac{\mathrm{d}a}{a} + \frac{1}{3}\left(\frac{\mathrm{d}\rho}{\rho(1+\beta/3)}\right) = \frac{\mathrm{d}a}{a} + \frac{\mathrm{d}\rho}{(3+\beta)\rho} = 0 \tag{15}$$

This results in two distinct dynamics.

Matter-dominated era:

$$E = \rho c^2 a^3 = Cst \tag{16}$$

Radiation-dominated era:

$$E = \rho c^2 a^4 = Cst \tag{17}$$

Or, overall

$$E = \rho c^2 a^{3+\beta} \tag{18}$$

We have added the factor c^2 because this condition represents the conservation of energy. For a dust-filled universe—modeled as masses m—the decrease in volumetric energy density is solely due to expansion. The individual energy mc^2 of the masses remains constant. During the radiation-dominated era, it is the "photon gas" that expands. However, since these photons constitute the very fabric of space, their wavelength λ stretches in tandem with the scale factor a; consequently, their individual energy hc/λ varies as $1/a$. This explains the exponent 4 in Equation (17).

With E representing energy, the system becomes:

$$-\left(\frac{k}{a^2}+\frac{\dot{a}^2}{c^2a^2}\right)=\chi\frac{E}{3c^2a^{3+\beta}} \tag{19}$$

$$-\left(\frac{k}{a^2}+\frac{\dot{a}^2}{c^2a^2}+\frac{2\ddot{a}}{c^2a}\right)=-\chi\frac{\beta E}{3c^2a^{3+\beta}} \tag{20}$$

Which leads to the equation:

$$a^{2+\beta}\ddot{a}=\frac{\chi(1+\beta)E}{6}=-\frac{4\pi G(1+\beta)E}{3c^2} \tag{21}$$

Consequently, in general relativity, if $E>0$, then $\ddot{a}<0$.

When $\beta=0$, we obtain the Friedmann equation.

When $\beta=1$, we have:

$$a^3\ddot{a}=-\frac{8\pi GE}{3c^2} \tag{22}$$

This equation does not prejudge, a priori, which value to choose for the curvature index:

$$k\in\{-1,0,+1\} \tag{23}$$

In classical cosmology, a zero value is generally chosen. In this case:

$$a(t)=\left(\frac{32\pi GE}{3c^2}\right)^{1/4}\sqrt{t} \tag{24}$$

Let us consider (which will be justified later):

$$k=-1 \tag{25}$$

As we have:

$$\dot{a}=c\sqrt{-\left(\frac{\chi E}{3c^2a^{1+\beta}}+k\right)} \tag{26}$$

He is coming with:

$$A = \frac{8\pi GE}{3c^2} \tag{27}$$

$$\dot{a} = \frac{\sqrt{c^2a^2 + A}}{a} \tag{28}$$

$$\mathrm{d}t = \frac{a\mathrm{d}a}{\sqrt{c^2a^2 + A}} \tag{29}$$

By taking:

$$(t \to 0) \Rightarrow (a \to 0) \tag{30}$$

$$t = \frac{1}{c^2}\left(\sqrt{c^2a^2 + A} - \sqrt{A}\right) \tag{31}$$

$$a(t) = \sqrt{c^2t^2 + 2\sqrt{\frac{8\pi GE}{3c^2}}t} \tag{32}$$

When t is very small:

$$t \ll \sqrt{\frac{32\pi GE}{3c^6}} \tag{33}$$

Then:

$$a(t) \cong \left(\frac{32\pi GE}{3c^2}\right)^{1/4}\sqrt{t} \tag{34}$$

Near the origin, the three solutions—for the three values of the curvature index k exhibit the same asymptotic behavior.

Noticed:

Let us now take relation (18)— $E = \rho c^2 a^{3+\beta}$ —as a hypothesis. That is to say:

$$\rho = \frac{E}{c^2a^{3+\beta}} \tag{35}$$

To which is added:

$$\frac{p}{c^2} = \frac{\beta E}{3c^2a^{3+\beta}} \tag{36}$$

From the system of Equations (1) + (2), we can move to (1) + (6a), which becomes:

$$\dot{a} = c\sqrt{-\left(\frac{\chi E}{3c^2a^{1+\beta}} + k\right)} \tag{37}$$

$$\frac{\chi E(1+\beta)}{6a^{2+\beta}} = \ddot{a} \tag{38}$$

Differentiating (37) yields:

$$\ddot{a} = \frac{c(1+\beta)\left(\frac{\chi E}{3c^2}\right)a^{-(2+\beta)}\dot{a}}{2\sqrt{-\left(\frac{\chi E}{3c^2a^{1+\beta}} + k\right)}} = \frac{(1+\beta)\chi Ea^{-(2+\beta)}}{6} \tag{39}$$

Identical to equation (38). The consistency of the system is ensured.

C'est-à-dire l'équation (38). La cohérence du système est assurée.

2 – **In the Janus Cosmological Model**:

In the first part of the article, we opted to construct the coupled system of field equations using a variant of the Damour-Kogan action [11]. In this case, the system is written as follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi\left(T_{\mu\nu} + \bar{K}_{\mu\nu}\right) \tag{40a}$$

$$\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} = -\chi\left(\bar{T}_{\mu\nu} + K_{\mu\nu}\right) \tag{40b}$$

where $K_{\mu\nu}$ and $\bar{K}_{\mu\nu}$ are the interaction tensors. In reference [18], a non-stationary solution exhibiting isotropy and homogeneity was sought, introducing specific forms for the interaction tensors in which φ and ϕ are functions of time:

$$\bar{K}_{\mu\nu} = \varphi \bar{T}_{\mu\nu} \tag{41a}$$

$$K_{\mu\nu} = \phi T_{\mu\nu} \tag{41b}$$

We will seek solutions derived from symmetry hypotheses—isotropy and homogeneity—that is, FLRW metric solutions, using the same time variable $x°$ for both:

$$\mathrm{d}s^2 = \mathrm{d}x^{\circ 2} - a^2\left(\frac{\mathrm{d}u^2}{1-ku^2} + u^2\mathrm{d}\theta^2 + u^2\sin^2\theta\mathrm{d}\varphi^2\right) \tag{42a}$$

$$\mathrm{d}\bar{s}^2 = \mathrm{d}x^{\circ 2} - \bar{a}^2\left(\frac{\mathrm{d}u^2}{1-\bar{k}u^2} + u^2\mathrm{d}\theta^2 + u^2\sin^2\theta\mathrm{d}\varphi^2\right) \tag{42b}$$

Posons:

$$H = \frac{\dot{a}}{a} \qquad \bar{H} = \frac{\dot{\bar{a}}}{\bar{a}} \tag{43}$$

We write the equations of state, with $\beta \in \{O,1\}$, in the form:

$$p = \frac{\beta}{3}\rho c^2 = \frac{\beta}{3}\varepsilon \qquad \bar{p} = \frac{\beta}{3}\bar{\rho}\bar{c}^2 = \frac{\beta}{3}\bar{\varepsilon} \tag{44}$$

To simplify the calculation, let us set:

$$T_{00} + \bar{K}_{00} = X = \varepsilon + \varphi\bar{\varepsilon} \tag{45}$$

Thus, our first field equation is written as:

$$G_0^0 = -\left(\frac{3k}{a^2} + \frac{3\dot{a}^2}{c^2a^2}\right) = \chi X \tag{46}$$

And the second:

$$-\left(\frac{k}{a^2} + \frac{\dot{a}^2}{c^2a^2} + \frac{2\ddot{a}}{c^2a}\right) = -\frac{\chi\beta}{3}X \tag{47}$$

Let's set

$$F = \frac{k}{a^2} + \frac{\dot{a}^2}{c^2a^2} \tag{48}$$

Equation (47) becomes:

$$F+\frac{2\ddot{a}}{c^2a}=\frac{\chi\beta}{3}X \tag{49}$$

$$-\frac{\chi X}{3}+\frac{2\ddot{a}}{c^2a}=\frac{\chi\beta}{3}X \tag{50}$$

$$\frac{\ddot{a}}{c^2a}=\frac{\chi}{6}(1+\beta)X \tag{51}$$

Let us now start with the first of the two equations of the system, Equation (43), which becomes:

$$-3F=\chi X \tag{52}$$

Let's derive it:

$$-3\dot{F}=\chi\dot{X} \tag{53}$$

$$\dot{F}=-\frac{2k\dot{a}}{a^3}+\frac{2\dot{a}\ddot{a}}{c^2a^2}-\frac{2\dot{a}^3}{c^2a^3} \tag{54}$$

With (43):

$$\dot{F}=2H\left[-\frac{k}{a^2}-\frac{\dot{a}^2}{c^2a^2}+\frac{\ddot{a}}{c^2a}\right] \tag{55}$$

But:

$$-\frac{k}{a^2}-\frac{\dot{a}^2}{c^2a^2}=-F \tag{56}$$

hence:

$$\dot{F}=2H\left(-F+\frac{\ddot{a}}{c^2a}\right) \tag{57}$$

Using (51) and (52) we get:

$$-F+\frac{\ddot{a}}{c^2a}=\frac{\chi}{3}X+\frac{\chi}{6}(1+\beta)X \tag{59}$$

$$-F+\frac{\ddot{a}}{c^2a}=\frac{\chi}{6}(3+\beta)X \tag{60}$$

Thus, Equation (57) becomes:

$$\dot{F}=\frac{\chi}{3}(3+\beta)HX \tag{61}$$

Let us substitute it into (53).

$$\dot{X}+(3+\beta)HX=0 \tag{62}$$

That's to say:

$$\frac{\mathrm{d}X}{X}+(3+\beta)\frac{\mathrm{d}a}{a}=0 \tag{63}$$

Or:

$$Xa^{3+\beta}=Cst \tag{64}$$

But $X = \varepsilon + \varphi\bar{\varepsilon}\cdot I$:

$$\dot{\varepsilon} + \varphi\dot{\bar{\varepsilon}} + \dot{\varphi}\bar{\varepsilon} + (3+\beta)H\varepsilon + (3+\beta)H\varphi\bar{\varepsilon} = 0 \tag{65}$$

As $\varepsilon = \rho c^2$; $\bar{\varepsilon} = \bar{\rho}\bar{c}^2$

$$C_+ = c^2\dot{\rho} + \varphi\bar{c}^2\dot{\bar{\rho}} + \dot{\varphi}\bar{c}^2\bar{\rho} + (3+\beta)H\left(\rho c^2 + \varphi\bar{\rho}\bar{c}^2\right) = 0 \tag{66}$$

We must now perform the same calculation in the negative sheet.

$$\bar{X} = \bar{\varepsilon} + \phi\varepsilon \tag{67}$$

$$-3\left(\frac{\bar{k}}{\bar{a}^2} + \frac{\dot{\bar{a}}^2}{\bar{c}^2\bar{a}^2}\right) = -\chi\bar{X} \tag{68}$$

$$-\left(\frac{\bar{k}}{\bar{a}^2} + \frac{\dot{\bar{a}}^2}{\bar{c}^2\bar{a}^2} + \frac{2\ddot{\bar{a}}}{\bar{c}^2\bar{a}}\right) = \frac{\chi\beta}{3}\bar{X} \tag{69}$$

$$\bar{F} = \frac{\chi}{3}\bar{X} \tag{70}$$

$$\frac{\chi\bar{X}}{3} + \frac{2\ddot{\bar{a}}}{\bar{c}^2\bar{a}} - - = \frac{\chi\beta}{3}\bar{X} \tag{71}$$

$$\frac{\ddot{\bar{a}}}{\bar{c}^2\bar{a}} = -\frac{\chi}{6}(1+\beta)\bar{X} \tag{72}$$

On dérive: Let's derive

$$\dot{\bar{F}} = 2\bar{H}\left(-\bar{F} + \frac{\ddot{\bar{a}}}{\bar{c}^2\bar{a}}\right) \tag{73}$$

$$-\bar{F} = -\frac{\chi}{3}\bar{X} \tag{74}$$

$$-\bar{F} + \frac{\ddot{\bar{a}}}{\bar{c}^2\bar{a}} = -\frac{\chi}{6}(3+\beta)\bar{X} \tag{75}$$

$$\dot{\bar{F}} = -\frac{\chi}{3}(3+\beta)\bar{H}\bar{X} \tag{76}$$

$$-3\dot{\bar{F}} = -\chi\dot{\bar{X}} \tag{77}$$

$$\chi(3+\beta)\bar{H}\bar{X} = -\chi\dot{\bar{X}} \tag{78}$$

$$\dot{\bar{X}} + (3+\beta)\bar{H}\bar{X} = 0 \tag{79}$$

$$\frac{\mathrm{d}\bar{X}}{\bar{X}} + (3+\beta)\frac{\mathrm{d}\bar{a}}{\bar{a}} = 0 \tag{80}$$

Ou:

$$\bar{X}\bar{a}^{3+\beta} = Cst \tag{81}$$

$$\dot{\bar{\varepsilon}} + \phi\dot{\varepsilon} + \dot{\phi}\varepsilon + (3+\beta)\bar{H}\bar{\varepsilon} + (3+\beta)\bar{H}\phi\varepsilon = 0 \tag{82}$$

As $\varepsilon = \rho c^2$; $\bar{\varepsilon} = \bar{\rho}\bar{c}^2$

$$C_- = \bar{c}^2\dot{\bar{\rho}} + \phi c^2\dot{\rho} + \dot{\phi}\rho c^2 + (3+\beta)\bar{H}\left(\bar{\rho}\bar{c}^2 + \phi\rho c^2\right) = 0 \tag{83}$$

We had a system of two pairs of second-order differential equations. The com-

patibility of the first two (derived from the first field equation) yielded Equation (66). Let us call this equation C_+. The compatibility of the next two (derived from the second field equation) has thus yielded Equation (83). Let us call this equation C_- To ensure the consistency of the four equations, we must therefore determine the time-dependent functions φ and ϕ. Let us identify:

$$C_+ = 0 \quad \text{et} \quad \lambda C_- = 0 \tag{84}$$

$$\dot{\varepsilon} + \varphi\dot{\bar{\varepsilon}} + \dot{\varphi}\bar{\varepsilon} + (3+\beta)H\varepsilon + (3+\beta)H\varphi\bar{\varepsilon} = 0 \tag{85}$$

$$\lambda\left[\dot{\bar{\varepsilon}} + \phi\dot{\varepsilon} + \dot{\phi}\varepsilon + (3+\beta)\bar{H}\bar{\varepsilon} + (3+\beta)\bar{H}\phi\varepsilon\right] = 0$$

We will therefore perform a term-by-term identification. The coefficient of $\dot{\varepsilon}$ yields:

$$1 = \lambda\phi \tag{85}$$

le coefficient de $\dot{\bar{\varepsilon}}$ donne the coefficient … gives

$$\varphi = \lambda \tag{86}$$

Thus:

$$\varphi\phi = 1 \tag{87}$$

The two time-dependent functions φ and ϕ are inverses of each other. The coefficient gives:

$$(3+\beta)H = \lambda\left[\dot{\phi} + (3+\beta)\bar{H}\phi\right] \tag{88}$$

But $\lambda = \varphi$. Hence:

$$(3+\beta)H = \varphi\dot{\phi} + (3+\beta)\bar{H}\varphi\phi \tag{89}$$

But $\varphi\phi = 1$ thus:

$$(3+\beta)H = \frac{\dot{\phi}}{\phi} + (3+\beta)\bar{H} \tag{90}$$

$$\frac{\dot{\phi}}{\phi} = (3+\beta)\left(H - \bar{H}\right) = (3+\beta)\left(\frac{\dot{a}}{a} - \frac{\dot{\bar{a}}}{\bar{a}}\right) \tag{91}$$

$$\phi = C\left(\frac{a}{\bar{a}}\right)^{3+\beta} \tag{92}$$

Hence:

$$\varphi = \frac{1}{C}\left(\frac{\bar{a}}{a}\right)^{3+\beta} \tag{93}$$

Since the integration constant is absorbed in the normalization of the interaction tensors, we will take $C = 1$.

Let's go back to our equation C_+.

$$\dot{\varepsilon} + \varphi\dot{\bar{\varepsilon}} + \dot{\varphi}\bar{\varepsilon} + (3+\beta)H\varepsilon + (3+\beta)H\varphi\bar{\varepsilon} = 0 \tag{94}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\varepsilon + \varphi\bar{\varepsilon}\right) + (3+\beta)H\left(\varepsilon + \varphi\bar{\varepsilon}\right) = 0 \tag{95}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\left(\varepsilon+\varphi\bar{\varepsilon}\right)+\left(3+\beta\right)\frac{\mathrm{d}}{\mathrm{d}t}\ln a=0 \tag{96}$$

But:

$$\varepsilon+\varphi\bar{\varepsilon}=\frac{\varepsilon a^{(3+\beta)}+\bar{\varepsilon}\bar{a}^{(3+\beta)}}{a^{(3+\beta)}} \tag{97}$$

Whence:

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\left(\varepsilon a^{(3+\beta)}+\bar{\varepsilon}\bar{a}^{(3+\beta)}\right)=0 \tag{98}$$

$$\boxed{\rho c^2 a^{(3+\beta)}+\bar{\rho}\bar{c}^2\bar{a}^{(3+\beta)}=E=Cst} \tag{99}$$

Let's go back to our equation.

In the matter dominated era:

$$\rho c^2 a^3+\bar{\rho}\bar{c}^2\bar{a}^3=E=Cst \tag{100}$$

The radiation dominated era:

$$\rho c^2 a^4+\bar{\rho}\bar{c}^2\bar{a}^4=E=Cst \tag{101}$$

Thus we get, for matter dominated era

On obtient alors, pour le matter dominated era:

$$\dot{a}=\sqrt{-k-\frac{8\pi GE}{3a}} \tag{102a}$$

$$\ddot{a}=-\frac{4\pi GE}{3a^2} \tag{103a}$$

$$\dot{\bar{a}}=\sqrt{-\bar{k}+\frac{8\pi GE}{3\bar{a}}} \tag{102b}$$

$$\ddot{\bar{a}}=\frac{4\pi GE}{3\bar{a}^2} \tag{103b}$$

Appendix 2

Dual numbering: This calculation is, in fact, an adaptation of the classic calculation presented in Chapter 14 of the book "Introduction to General Relativity" by Adler, Schiffer, and Bazin. [32]. In what follows, the equations from the article are numbered with digits enclosed in parentheses. These are followed by the corresponding equations from Chapter 14 of the book, enclosed in square brackets. For instance, Equation (15c) is followed by [14.12c].

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

Satisfaction of Bianchi conditions in the stationary case and in the Newtonian approximation.

In the following, tensors are always defined in their "old form":

$$T_{\mu}^{\nu}=\begin{pmatrix}\rho & 0 & 0 & 0\\ 0 & -p/c^2 & 0 & 0\\ 0 & 0 & -p/c^2 & 0\\ 0 & 0 & 0 & -p/c^2\end{pmatrix} \tag{1}$$

And not in their "modern form":

$$T_{\mu}^{\nu}=\begin{pmatrix}\rho c^2 & 0 & 0 & 0\\ 0 & -p & 0 & 0\\ 0 & 0 & -p & 0\\ 0 & 0 & 0 & -p\end{pmatrix} \tag{2}$$

In order to be able to make use of the symmetry

$$\chi=-\frac{8\pi G}{c^2}=-\bar{\chi}=\frac{8\pi\bar{G}}{c^2} \tag{3}$$

This means that G and $\bar{G}$ have opposite signs but are not equal in absolute value. Assuming a stationary state and spherical symmetry, we can consider the two cases where only one of the two types of mass—positive or negative—is present. When the mass is positive, the system is described as follows:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\chi T_{\mu\nu} \tag{4}$$

$$\bar{R}_{\mu\nu}-\frac{1}{2}\bar{R}\bar{g}_{\mu\nu}=-\chi K_{\mu\nu} \tag{5}$$

The solution to (1)—comprising the exterior and interior metrics—is the classical Schwarzschild solution. To construct the solution to Equation (2), we would need to define the form of the interaction tensor $K_{\mu\nu}$. This would allow us to construct the metrics $\bar{g}_{\mu\nu}^{ext}$ and $\bar{g}_{\mu\nu}^{int}$. However, this would yield geodesics that cannot be compared with observational data. Consequently, such work would be pointless.

The situation is different when considering the field created by a negative mass, which corresponds to the system:

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=\chi\bar{K}_{\mu\nu} \tag{6}$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\bar{g}_{\mu\nu} = -\chi\bar{T}_{\mu\nu} \tag{7}$$

Which will be written in the form:

$$R_{\mu\nu} = \chi\left(\bar{K}_{\mu\nu} - \frac{1}{2}\bar{K}g_{\mu\nu}\right) \tag{8}$$

$$\bar{R}_{\mu\nu} = -\chi\left(\bar{T}_{\mu\nu} - \frac{1}{2}\bar{T}\bar{g}_{\mu\nu}\right) \tag{9}$$

This situation corresponds to the "Great Repeller" object, located at the center of the 100-million-light-year-diameter void known as the Dipole Repeller [22]. We conjecture that a negative mass lies at its center, modeled as a sphere filled with negative mass of constant density $\bar{\rho}$. We will need to derive the form of the interaction tensor $\bar{K}_{\mu\nu}$, followed by the solutions $g_{\mu\nu}^{ext}$ and $g_{\mu\nu}^{int}$. Light rays originating from the background—from distant sources—will undergo a negative gravitational lensing effect in this region, which will serve to attenuate their luminosity. This effect can then be compared with observational data. Under spherical symmetry and stationary conditions, the metrics take the form:

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}x^{\circ 2} - \mathrm{e}^{\lambda}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) \tag{10}$$

$$\mathrm{d}\bar{s}^2 = \mathrm{e}^{\bar{\nu}}\mathrm{d}x^{\circ 2} - \mathrm{e}^{\bar{\lambda}}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2\right) \tag{11}$$

Attention will focus on the compatibility between the two metric solutions—between these two geometries—which entails the need to identify the two hydrodynamic equations, specifically the TOV-type (Tolman-Oppenheimer-Volkoff) equations derived from them. The solutions to the second field Equation (7), describing the geometry of the negative sector, are in every respect analogous to the pair of Schwarzschild exterior and interior metric solutions. Taking as the source tensor:

$$\bar{T}_{\mu}^{\nu} = \mathrm{diag}\left(\bar{\rho}\bar{c}^2, -\bar{p}, -\bar{p}, -\bar{p}\right) \tag{12}$$

That's to say:

$$\bar{T}_{\mu\mu} = \mathrm{diag}\left(\bar{\rho}\bar{c}^2\mathrm{e}^{\bar{\nu}}, \bar{p}\mathrm{e}^{\bar{\lambda}}, r^2\bar{p}, r^2\sin^2\theta\bar{p}\right) \quad [14.8] \tag{13}$$

Alongside our own equation numbering, we will indicate in square brackets the corresponding numbering from Chapter 14 of reference [32]. The covariant components of the Ricci tensor, derived from the form of the metric (11), are:

$$\bar{R}_{00} = \mathrm{e}^{\bar{\nu}-\bar{\lambda}}\left[-\frac{\bar{\nu}''}{2} + \frac{\bar{\nu}'\bar{\lambda}'}{4} - \frac{\bar{\nu}'^2}{4} - \frac{\bar{\nu}'}{r}\right] \quad [14.11a] \tag{14a}$$

$$\bar{R}_{11} = \frac{\bar{\nu}''}{2} - \frac{\bar{\nu}'\bar{\lambda}'}{4} + \frac{\bar{\nu}'^2}{4} - \frac{\bar{\lambda}'}{r} \quad [14.11b] \tag{14b}$$

$$\bar{R}_{22} = \mathrm{e}^{-\bar{\lambda}}\left[1 + \frac{\bar{\nu}'r}{2} - \frac{\bar{\lambda}'r}{2}\right] - 1 \quad [14.11c] \tag{14c}$$

By calculating the right-hand sides from the source tensor (13), one obtains the classical system of equations:

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

$$e^{-\bar{\lambda}}\left[-\frac{\bar{\nu}''}{2}+\frac{\bar{\nu}'\bar{\lambda}'}{4}-\frac{\bar{\nu}'^2}{4}-\frac{\bar{\nu}'}{r}\right]=-\frac{\chi}{2}e^{\bar{\nu}}\left(\bar{\rho}+\frac{3\bar{p}}{\bar{c}^2}\right) \quad [14.12a] \tag{15a}$$

$$e^{-\bar{\nu}}\left[\frac{\bar{\nu}''}{2}-\frac{\bar{\nu}'\bar{\lambda}'}{4}+\frac{\bar{\nu}'^2}{4}-\frac{\bar{\lambda}'}{r}\right]=-\frac{\chi}{2}\left(\bar{\rho}-\frac{\bar{p}}{\bar{c}^2}\right) \quad [14.12b] \tag{15b}$$

$$e^{-\bar{\lambda}}\left[1+\frac{\bar{\nu}'r}{2}-\frac{\bar{\lambda}'r}{2}\right]-1=-\frac{\chi}{2}r^2\left(\bar{\rho}-\frac{\bar{p}}{\bar{c}^2}\right) \quad [14.12c] \tag{15c}$$

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

The two unknowns in this system of differential equations are the functions $\bar{\lambda}(r)$ and $\bar{\nu}(r)$. The problem of finding a solution is then (standardly) reduced to finding the single function $\bar{\nu}(r)$ by setting:

$$-\bar{g}_{11}=e^{-\bar{\lambda}}=1-\frac{2\bar{m}(r)}{r} \quad [14.15] \tag{16}$$

This is to match the external metric $\bar{g}_{\mu\nu}^{ext}$, which is:

$$d\bar{s}^2=\left(1-\frac{2\bar{m}_0}{r}\right)dx^{\circ 2}-\frac{dr^2}{1-\frac{2\bar{m}_0}{r}}-r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right) \tag{17C}$$

By combining Equation (16) [14.15] with the system (15a) [14.12a], (15b) [14.12b], (15c) [14.12c], what is known as a "geometric mass" emerges:

$$\bar{m}=-\frac{4\pi r^3}{3c^2}G\bar{\rho}>0 \quad [14.18] \tag{18}$$

This reflects the fact that negative masses attract negative masses (compare with metric (11), where $\bar{m}_0>0$). From this system, using standard manipulations, one obtains a TOV-type equation:

$$\bar{p}'=-\frac{\left(\bar{\rho}\bar{c}^2+\bar{p}\right)\left(\bar{m}-4\pi G\bar{p}r^3/c^4\right)}{r\left(r-2\bar{m}\right)} \quad [14.22] \tag{19}$$

This one, without the Newtonian approximation:

$$|\bar{p}|\ll|\bar{\rho}|\bar{c}^2 \qquad r\gg 2\bar{m} \qquad 4\pi G\bar{p}r^3/c^2\ll\bar{m} \tag{20}$$

If $\bar{M}(r)$ is the total Newtonian mass contained within a sphere of radius r:

$$\bar{M}(r)=\frac{4}{3}\pi r^3\bar{\rho}<0 \tag{21}$$

The TOV equation then becomes the Euler equation for negative mass.

$$\bar{p}'=-\frac{G\bar{M}(r)\bar{\rho}}{r^2} \quad [14.24] \tag{22}$$

We shall not proceed any further with the solution to this second field equation. While we could indeed go so far as to construct the form of the interior metric, doing so would be of purely formal interest, since the geodesics derived from the solutions $\bar{g}_{\mu\nu}$ cannot be compared with observational data.

However, the same does not apply to the metrics $g_{\mu\nu}^{ext}$ and $g_{\mu\nu}^{int}$,; this compels us to specify a form for the interaction tensor $\bar{K}_{\mu\nu}$—which acts as the source of the field—in a manner consistent with the system's mathematical coherence, that

is, leading to a TOV-type equation that can be identified with the one previously derived from the second equation.

It has not been possible to construct such a tensor $\bar{K}_{\mu\nu}$ in the most general form—that is, for a non-linear solution capable of describing objects far removed from the Newtonian regime. This will therefore remain an open problem for the mathematical geometer:

- *How can the interaction tensors $\bar{K}_{\mu\nu}$ and $K_{\mu\nu}$ which appear in the Janus field equations be constructed in their most general, non-linearized form, given that the form of such tensors is constrained by the requirement to satisfy the mathematical consistency of the system?*

Someone may eventually discover this most general, non-linear form. In the meantime, we shall content ourselves with asymptotic satisfaction under Newtonian conditions, while recalling the arguments of a physical nature that justify this approach:

- *If the source of the field is a very dense object with highly concentrated positive mass—such as a neutron star or one of the supermassive objects located at the centers of galaxies—then the Newtonian approximation is no longer valid. In this case, however, the theorist is not required to construct the corresponding interaction tensor $K_{\mu\nu}$ or the solution $g_{\mu\nu}$ to the second equation, since, in essence, such constructs cannot be compared against observational data in any scenario. At most, a mathematician might formulate a theorem guaranteeing the existence of this solution.*

However, if the source of the field is a negative mass, then the Newtonian approximation and the asymptotic satisfaction of the consistency condition suffice, since simulations indicate that these negative masses remain loosely concentrated, taking the form of immense conglomerates located at the centers of large voids in the large-scale structure.

This will therefore be the guiding thread for what follows. We will express the interaction tensor in the following form:

$$K_{\mu}^{\nu} = \mathrm{diag}\left(\bar{\rho}, -\frac{\bar{q}}{\bar{c}^2}, -\frac{\bar{q}}{\bar{c}^2}, -\frac{\bar{q}}{\bar{c}^2}\right) \tag{23}$$

Here, $\bar{q}(r)$ will be a function to be determined. We will have (we shall indicate, in square brackets, the corresponding numbers from Chapter 14 of reference [21] to make it easier for the reader to follow, as this is one of the tricky points in this calculation **Appendix**):

$$K_{\mu\mu} = \mathrm{diag}\left(\bar{\rho}\mathrm{e}^{\nu}, \frac{\bar{q}}{\bar{c}^2}\mathrm{e}^{\lambda}, r^2\frac{\bar{q}}{\bar{c}^2}, r^2\sin^2\theta\frac{\bar{q}}{\bar{c}^2}\right) \quad [14.8] \tag{A24}$$

From this tensor, the field equation, and the coefficients of the Ricci tensor constructed from the metric, we obtain the system of three equations:

$$\mathrm{e}^{-\lambda}\left[-\frac{\nu''}{2}+\frac{\nu'\lambda'}{4}-\frac{\nu'^2}{4}-\frac{\nu'}{r}\right] = \frac{\chi}{2}\mathrm{e}^{\nu}\left(\bar{\rho}+3\frac{\bar{q}}{\bar{c}^2}\right) \quad [14.12a] \tag{25a}$$

$$e^{-\nu}\left[\frac{\nu''}{2}-\frac{\nu'\lambda'}{4}+\frac{\nu'^2}{4}-\frac{\lambda'}{r}\right]=\frac{\chi}{2}\left(\bar{\rho}-\frac{\bar{q}}{\bar{c}^2}\right) \quad [14.12b] \tag{25b}$$

$$e^{-\lambda}\left[1+\frac{\nu' r}{2}-\frac{\lambda' r}{2}\right]-1=\frac{\chi}{2}r^2\left(\bar{\rho}-\frac{\bar{q}}{\bar{c}^2}\right) \quad [14.12c] \tag{25c}$$

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

For the student unfamiliar with this calculation, we will detail the steps. By adding (15a) [14.12a] and (15b) [14.12b], we obtain:

$$-\chi\left(\bar{\rho}+\frac{\bar{q}}{\bar{c}^2}\right)=e^{-\lambda}\left(\frac{\lambda'+\nu'}{r}\right) \quad [14.13] \tag{26}$$

Now, by combining (15c) [14.12c] and (26) [14.13] to extract $\bar{\rho}\bar{c}^2$ and $\bar{q}$, and by eliminating ν'' from (15b) [14.12b] and (15c) [14.12c], we obtain the system of three equations:

$$\chi\bar{\rho}=e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)-\frac{1}{r^2} \quad [14.14a] \tag{27a}$$

$$\chi\frac{\bar{q}}{\bar{c}^2}=\frac{1}{r^2}-e^{-\lambda}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right) \quad [14.14b] \tag{27b}$$

$$\frac{e^{\lambda}}{r^2}=\frac{1}{r^2}-\frac{\nu'^2}{4}+\frac{\nu'\lambda'}{4}+\frac{\lambda'+\nu'}{r}-\frac{\nu''}{2} \quad [14.14c] \tag{27c}$$

We will now reintroduce a "geometric mass" by ensuring a match with the external metric $g_{\mu\nu}^{ext}$, which reflects a *repulsion* (a negative mass repels positive masses) and thus takes the form:

$$ds^2=\left(1+\frac{2m_0}{r}\right)dx^{\circ 2}-\frac{dr^2}{1+\frac{2m_0}{r}}-r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right) \text{ with } m_0>0 \tag{28}$$

Earlier, $\bar{m}(r)$ was the "geometric mass" acting on negative masses. Here, we introduce the geometric mass $m(r)$ acting on positive masses. We must therefore have:

$$e^{-\lambda}=1-\frac{2m(r)}{r} \text{ with } m(r)<0 \quad [14.15] \tag{29}$$

If we calculate:

$$-\frac{1}{r^2}2m'=-\frac{1}{r^2}\left[r\left(1-e^{-\lambda}\right)\right]'=-\frac{1}{r^2}+e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right) \quad [14.16] \tag{30}$$

We see that the right-hand side of this equation is identical to the right-hand side of (27a) [14.14a]. But we have:

But:

$$\chi=-\frac{8\pi G}{c^2} \tag{31}$$

Thus:

$$m'=\frac{G\bar{\rho}}{c^2}4\pi r^2<0 \quad [14.17] \tag{32}$$

Then:

$$m = \frac{4\pi r^3}{3c^2} G\bar{\rho} < 0 \quad [14.18] \tag{33}$$

Let us return to (27b) [14.14b]. Applying (29) [14.15], we obtain:

$$\chi\bar{q} = \frac{2m}{r^3} - \mathrm{e}^{-\lambda}\frac{\nu'}{r} = \frac{2m}{r^3} - \left(1 - \frac{2m}{r}\right)\frac{\nu'}{r} \tag{34}$$

So:

$$\nu' = 2\frac{m + 4\pi G\bar{q}r^3/c^4}{r(r-2m)} \quad [14.19] \tag{35}$$

Comparing with (26) [14.13] we obtain a simple relationship between ν' and p'

$$\frac{\bar{p}'}{\bar{c}^2} = -\frac{\nu'}{2}\left(\bar{\rho} + \frac{\bar{p}}{\bar{c}^2}\right) \quad [14.21] \tag{36}$$

By differentiating (27b) [14.14b] and combining it with (27c) [14.14c], we obtain

$$\bar{q}' = -\frac{\left(\bar{\rho}\bar{c}^2 + \bar{q}\right)\left(m + 4\pi G\bar{q}r^3/c^4\right)}{r(r-2m)} \quad [14.21] \tag{37}$$

That is to say, the equivalent of the TOV equation for the first field equation. We have:

$$m(r) = -\bar{m}(r) \tag{38}$$

Il vient:

$$\bar{q}' = -\frac{\left(\bar{\rho}\bar{c}^2 + \bar{q}\right)\left(-\bar{m} + 4\pi G\bar{q}r^3/c^4\right)}{r(r+2\bar{m})} \tag{39}$$

Which cannot be identified with the TOV equation of the second equation, unless the Newtonian approximation is applied, yielding:

$$\bar{q}' = \frac{\bar{\rho}\bar{c}^2\bar{m}}{r^2} = \frac{G\bar{\rho}\bar{M}}{r^2} \tag{40}$$

To recover the equation:

$$\bar{p}' = -\frac{G\bar{M}(r)\bar{\rho}}{r^3} \quad [14.24] \tag{41}$$

That is to say, Euler's hydrostatic equation obtained using the second field equation—representing an asymptotic compatibility of the system; restricted to the Newtonian approximation, the following is required:

$$\bar{q} = -\bar{p} \tag{42}$$

Which gives the interaction tensor the form:

$$\bar{K}_\mu^\nu = \begin{pmatrix} \bar{\rho} & 0 & 0 & 0 \\ 0 & \frac{\bar{p}}{\bar{c}^2} & 0 & 0 \\ 0 & 0 & \frac{\bar{p}}{\bar{c}^2} & 0 \\ 0 & 0 & 0 & \frac{\bar{p}}{\bar{c}^2} \end{pmatrix} \tag{43}$$

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

In the Newtonian approximation, one then recovers (41) [14.24]. This is what was demonstrated in references [8] [21].

However, since this will lend itself to comparison with observations, we shall have to construct the linearized form of the interior metric.

As early as 1916, K. Schwarzschild had provided the expression for the interior metric. While the behavior of this metric has been extensively studied—particularly to reveal its geometric or physical criticality conditions—its application to the study of photon propagation through a distribution of transparent matter is not typically the aspect highlighted. In the Newtonian approximation, the interpretation becomes particularly straightforward: the trajectories of photons passing through such a transparent mass would undergo a deflection caused, within that mass, by the mass $M(r)$ contained inside a sphere of radius *r*.

$$M(r)=\frac{4\pi r^3\rho}{3} \tag{44}$$

Taking into account that this deflection would then be negative, we obtain:

$$ds^2=\left(1+\frac{2GM(r)}{c^2r}\right)c^2dt^2-\frac{dr^2}{1+\frac{2GM(r)}{c^2r}}-r^2d\Omega^2 \tag{45}$$

Taking linearization into account, the interior metric becomes:

$$ds^2=\left(1+\frac{8\pi G\rho r^2}{3c^2}\right)c^2dt^2-\left(1-\frac{8\pi G\rho r^2}{3c^2}\right)dr^2-r^2d\Omega^2 \tag{46}$$

Which matches the external metric, also linearized.

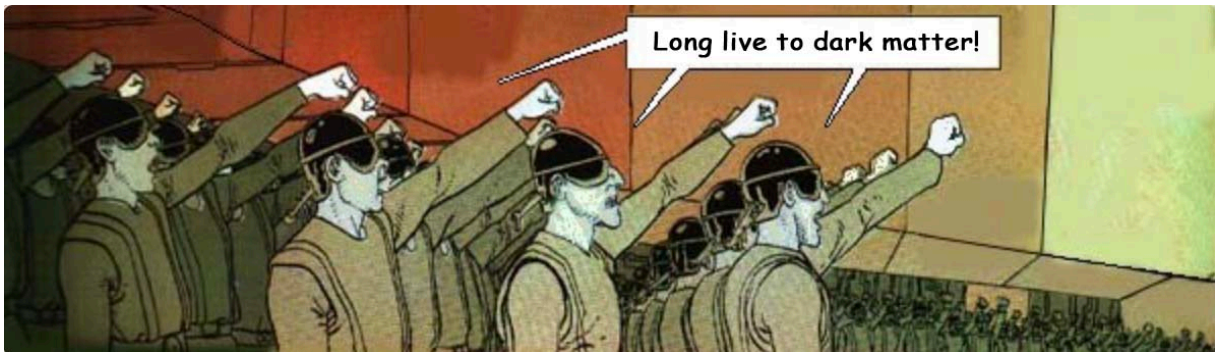
$$ds^2=\left(1+\frac{2G|\bar{M}|}{c^2r}\right)c^2dt^2-\left(1-\frac{2G|\bar{M}|}{c^2r}\right)dr^2-r^2d\Omega^2 \tag{47}$$

The equation numbers, enclosed in square brackets, are the correct ones. These numbers are "exotic" in the sense that they correspond to those found in the cited reference. This is provided so that the reader may see that the calculation presented in our paper is inspired by the classical calculation given in that reference.

2026-11-16 :

It is not known how long it will take before we see a reaction within the international scientific community.





In every country, the black hole has been a very lucrative business for over half a century. Our work represents a real danger to this community. For a long time, an effective strategy has been to feign complete ignorance of our work. With this book, this will become increasingly difficult. In France, for example, when Jean-Pierre Luminet continues to give lectures, people in his audience ask him questions:

- *What do you think of Jean-Pierre Petit's Janus Cosmological Model?*

So, at the end of 2026, he published an article in an obscure journal, Inference[114]. But since it was posted on arXiv, in the protected (gr-qc) section[115], it gets some attention, especially since, being blacklisted there, I can't respond to his allegations.

---

[114] JP Luminet. Twin Univers and Bimetric Gravity. Inference. International Review of Science. Vol.8 n°4. Sept 2026 https://inference-review.com/article/twin-universes-and-bimetric-gravity

[115] JP Luminet : Twin Universe and Bimetric Gravity : From Sakharov's Cosmological Symmetry to Modern Speculations on Negative Mass. https://arxiv.org/abs/2609.26834v1

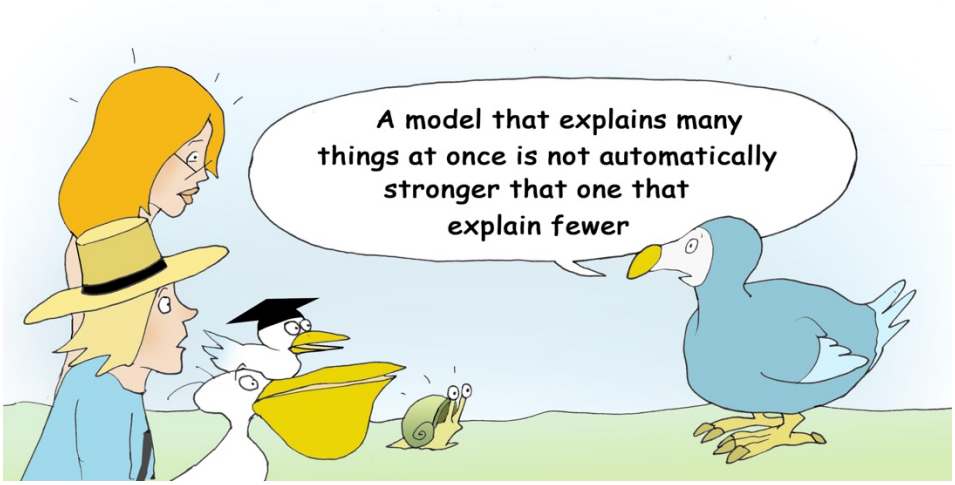
In my comics, all the characters have well-defined roles[116].



In the most recent one (The Metaphysicon), I tried to introduce a new character, using the Dodo bird as an image. But it didn't "work."

The content of Luminet's article changes everything and allows for the introduction of a character more in line with the Promises: Luminey.

No, you're not dreaming. This sentence is indeed present in the cited article:



[116] http://www.savoir-sans-frontieres.com