Experimental Data and Regimes at Stagnation of a Dense Z-Pinch

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Abstract. A wide range of experimental data from Z-pinches at stagnation are examined and categorised in terms of Reynolds' number, R, magnetic Reynolds' number, S, and the ratio of the electron-ion equilibriation time to Alfvén transit time. Both wire-array and gas-puff experiments are considered. Theory suggests that the experiments divide into two broad classes of pinches depending on the dominant damping mechanism for fast-growing, short wavelength m = 0 MHD instabilities. The first class is resistive stagnation, and the second is viscous. The latter is further divided for high Z into experiments with dominant electron viscosity, and those with dominant ion viscosity. This last category requires the equipartition time to be comparable or longer than the Alfvén transit time, and can lead to very high ion temperatures through ion viscous heating..

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INTRODUCTION

The phenomenon of viscous heating is a strong possibility to explain those experiments for which radiated X-ray energy is 3 or 4 times the kinetic energy of the implosion [1]. An elevated axial electric field consistent for this case results also in enhanced E/B drift of the more energetic electrons to the axis and localised harder X-ray emission [2]. The ratio S/R is the magnetic Prandtl number Pr_m and at stagnation depends on $I^8 a^2 / N^5$, and shows how strongly sensitive to current and line density is this division into the categories of resistive or viscous pinch. The significance of numerical simulations which only have an artificial Neumann viscosity will be discussed.

VISCOUS HEATING ASSOCIATED WITH SHORT WAVELENGTH m=0 MHD INSTABILITIES

The theory of a stagnated Z-inch requires that there is approximately a pressure balance, i.e. the Bennett relation

$$8\pi N_i e(T_i + ZT_e) = \mu_o I^2 \qquad (1)$$

holds, where N_i is the ion line density, and I is the current.

Provided the current density distribution in the stagnated Z-pinch is far from the Kadomtsev [3] profile for marginal stability, the pinched column is subject to m=0 MHD instabilities throughout the volume. The growth-rate γ is approximately $[kc_A^{\ 2}/a]^{1/2}$ for ideal MHD, i.e. proportional to $k^{1/2}$ where k is the axial wave number [4]. At large k there is damping either by viscosity or resistivity; or, indeed if these collisional effects are small the ion Larmor radius a_i will cause a cut-off at $ka_i \simeq 1$. It is well known that the fast magnetosonic Alfvén wave is critically damped by viscosity when the viscous Lunqvist number $L_\mu \equiv 2\rho(c_A^2 + c_S^{\ 2})^{1/2}/[(1/3\,\mu_{//} + \nu_1)k]$ is equal to unity. It is thus likely that the fastest growing mode has a value of L_μ close to 2, i.e. at double the wavelength at which the growth-rate is zero. In this formula $\mu_{//}$ is the isotropic viscosity $p_i\tau_{ii}$, while ν_1 is $\mu_{//}/(1+\Omega_i^2\tau_{ii}^2)$. Ω_i is the ion cyclotron frequency. It should be pointed out that dense Z-pinches are often only weakly magnetised, if at all.

The nonlinear amplitude of the perturbed velocity $\tilde{\underline{v}}$ is reached when the $\rho(\underline{v}.\nabla)\underline{v}$ term in the equation of motion is comparable with $\rho\gamma\tilde{\underline{v}}$. Thus $|\tilde{\underline{v}}| = \gamma/k$ leads to viscous heating per unit volume given in terms of the traceless stress tensor $\underline{\underline{\tau}}$ by

$$\underline{\underline{\tau}} : \nabla\underline{v} \approx \mu_{//}k^2\tilde{v}^2 = \mu_{//}kc_A^2/a \qquad (2)$$

It should be recalled that the m=0 mode uniquely is compressible, its growth rate depends on the ratio of specific heats, and viscous heating remains even for $\Omega_i\tau_{ii} \to \infty$. Neglecting ν_1 for the moment we find that fixing L_μ at 2 leads to a value of $\mu_{//}$k so that

$$\underline{\underline{\tau}} : \nabla\underline{v} \approx 3\rho c_A^2(c_A^2(c_A^2 + c_s^{\ 2})^{1/2}/a \qquad (3)$$

Thus every Alfvén transit time, $\tau_A \equiv a/c_A$ the internal energy will approximately double. The typical soft X-ray pulse is typically 1 or 2 τ_A, and it can be through viscous heating associated with short wavelength $(ka \sim 10^2)$ instabilities that much magnetic energy is converted to internal energy on this time-scale.

This fast conversion of magnetic energy into thermal energy can be represented as an effective resistance $R_{effective}$ given by

$$R_{effective} = \frac{\ell}{4(N_i m_i)^{1/2}}\left(\frac{\mu_0}{\pi}\right)^{3/2}\frac{I}{a} \qquad (4)$$

which is similar to that found in phenomenological models,[5-7] and arises from the $\underline{v}.\ \underline{J} \times \underline{B}$ energy conversion rate per unit volume.

ELECTRON VISCOSITY AND EQUIPARTITION

Very few text books in plasma physics mention viscosity; even fewer mention electron viscosity. The ratio of parallel electron viscosity $\mu_{e\,//}$ to ion viscosity $\mu_{i\,//}$ is given by

$$\frac{\mu_{e\,//}}{\mu_{i\,//}} = 0.02328\left(\frac{T_e}{T_i}\right)^{5/2} \frac{Z^3 \ell n\Lambda_{ii}}{A^{1/2}\left[\ell n\Lambda_{ei} + Z^{-1}\ell n\Lambda_{ee}\right]} \tag{5}$$

which for T_e=T_i and A = 2Z >>1 becomes 0.01646Z$^{5/2}$ or 291 for Z = 50.

For high mass Z-pinches the equipartition time τ_{eq} is somewhat less than the Alfvén transit time τ_A, i.e.

$$\frac{\tau_{eq}}{\tau_A} = \frac{3\pi^{1/2}}{64 e^4 c^4}\left(\frac{m_i}{m_e}\right)^{1/2} \frac{1}{Z^{5/2}(Z+T_i/T_e)^{3/2}} \frac{I^4 a}{N_i^{\,3}} \tag{6}$$

is less than one. Note the $I^4 a/N_i^{\,3}$ dependence [8].

METHOD TO INTERPRET DATA AT STAGNATION

We can obtain I, N_i and a and possibly T_e or T_i from experiment, and from eq.(1) identify $(T_i + ZT_e)$. The dimensionless parameters $R,S,$ T_{eq}/T_A, all depending[8] on $I^4 a/N^{2to3}$, can then be found, together with $\mu_{e\,//}/\mu_{i\,//}$ and $\mathrm{Pr}_m \equiv S/R$. Table 1 illustrates the results calculated for various large diameter stainless-steel wire arrays at 19MA from Coverdale et al [9]. Here the given experimental parameters are T_e, N_i, n_i *and* τ_x where τ_x is the width of the X-ray pulse. For all of the data it was found that $\mu_{e\,//} < \cdot 015\mu_{i\,//}$. Most notable is that τ_{eq}/τ_A is close to 1 and the calculated ion temperatures are 45 to 141 keV (omitting the 80mm diameter results) compared to T_e of 1.7 to 5.0 keV.

In such an experiment $R_{effective}$ *is* $\sim 10^4 \times$ Spitzer resistance. E_z and the loss rate of e.m. energy density E_zJ_z will be increased by the same factor leading to an enhanced E_z/B_θ drift of hot (collisionless) electrons to the axis in a time $\tau_A/4$, and hence harder X-ray emission there. This enhanced Ettingshausen effect will by this heat flow lead to a Nernst convection of magnetic field [10] (and current density) to the axis, which could explain the bright hot spots on axis. In ref.[1] the measured T_i increased from 240 to 320 keV in an Alfvén transit time. While the magnetic Prandtl number varied from 6 to 550 indicating a viscous stagnation.

In contrast, high mass, small radius (19mm) tungsten arrays by Sinars et al [11] with varying N_i and I show in Table 2 that τ_{eq}/τ_A varied from 0.3 to .03; $\mu_{e\,//}/\mu_{i\,//}$~200 and $\mathrm{Pr_m}$ varied from 2 x 10^{-3} to 6 x 10^{-5}. Thus these experiments are resistive at

TABLE 1. Large diameter stainless steel wire array experiments [9]

Diameter(mm)		45	55	60	65	70	75	80
T_e (keV)		1.7	1.8	1.6	2.0	3.5	5.0	1.1
N_i (10^{20}m^{-1})		12.7	7.98	6.76	5.86	4.88	4.36	4.04
n_i (10^{25}m^{-3})		9	14	20	11	4	4	2
$\tau_{\text{x-rays}}$ (ns)		4.4	3.9	4.4	4.2	5.6	8.0	14.2
τ_A (ns)		7.7	3.8	2.7	3.2	4.4	4.0	3.3
τ_{ii} (ps)		4.4	7.6	7.8	16	41	36	168
$\mu_{e//}$ / $\mu_{i//}$		.015	.0031	.0013	.0016	.0056	.015	.0001
T_i (keV)		45	94	125	141	140	129	250
R (10^2)		17	3.8	2.3	1.4	0.9	1.1	0.17
S (10^3)	10.9	9.8	7.8	12.2	38.3	60.6	11.9	
Pr_m		6.3	26	31	88	424	552	689
τ_{eq} / τ_A		0.47	0.67	0.59	1.13	4.26	7.54	1.59

TABLE 2 Small radius tungsten arrays (10mm) with pinch radius a =0.7mm[11]

Mass of 10cm long arrays (mg)	1.15	2.5	6.0
Current at stagnation, I (MA)	12.7	16.5	18
Line density N_i ($10^{20}m^{-1}$)	3.58	8.14	19.5
X-ray pulse FWHM (ns)	4.5	4.6	6.2
Alfvén transit time τ_A (ns)	3.7	4.3	6.0
$T_i = T_e$ (keV) assume Bennett relation	2.14	1.14	0.98
Z (calculated from Saha equilibrium at T)	65	61	52
τ_{ii} (fs)	2.38	0.95	0.33
$\mu_{e//}/\mu_{i//}$ (>>1)	192	186	150
Reynolds' number, $R_{e//}$ (10^5)	2.66	7.51	31.7
Magnetic Reynolds' number, S ($<<R_{e//}$)	516	391	188
τ_{eq}/τ_A	0.332	0.118	0.028
$\Omega_i\tau_{ii}$ (10^{-5})	14.5	7.06	2.30
$\Omega_e\tau_{ei}$	2.30	1.23	0.45
Magnetic Prandtl number, Pr_m (10^{-4})	19.4	5.21	0.59
Thermal energy at stagnation (kJ)	121	204	243
Thermal + magnetic energy at stagnation (kJ)	938	1584	1885
Total radiated energy measured (kJ)	832	1106	1278
Energy radiated in main X-ray pulse (kJ)	440	532	692

stagnation, i.e. the unstable Alfvén modes are limited at short wavelength by resistive damping, and any extra heating is by electron viscosity. Indeed it would appear that even here a significant amount of magnetic energy is converted into X-radiation. For all cases the ions are strongly coupled and unmagnetised.

TABLE 3. Low and high current gas-puffs and wire arrays

Experimental reference	[12]	[13]	[14]	[1]
Measured current at stagnation I (MA)	0.361	1.0	8.0	18
Line density N_i (m^{-1})	0.148	2.39	4.06	3.4
Measured T_e (keV)	0.25	0.109	1.25	3
Measured ion temperature (keV)	?	?	36	219
Measured pinch radius (mm)	0.45	0.75	1.31	0.75
τ_{eq}/τ_A	0.39	0.015	1.17	2.43
$\mu_{e//}/\mu_{i//}$	1.8	3.9	.0013	.0014
Reynolds' number R (10^2)	190	7900	0.45	0.606
Magnetic Reynolds' number S (10^2)	0.35	0.175	27.8	47.4
$\Omega_i\tau_{ii}$	.0021	.0001	1.6	1.65
$\Omega_e\tau_{ei}$	1.6	0.22	80.6	29.8
Magnetic Prandtl number Pr_m	.0018	2×10^{-5}	61.9	78.2
Alfvén transit time τ_A (ns)	5.58	15.5	4.0	1.49

In the lower current experiments of Kroupp et al [12] (neon gas-puff) and Lebedev et al [13] (Al wire arrays), $\tau_{eq} < \tau_A$ and $\mathrm{Pr_m}$ is 2 x 10^{-3} and 2 x10^{-5} respectively. These are resistive at stagnation. But Wong et al [14]($Ne + Ar$ gas puff at 8MA) has $\mathrm{Pr_m}$ = 62 and T_i = 36 keV with T_e = 1.25 keV leading to ion viscous heating as in [1] where $\mathrm{Pr_m}$ = 78 and T_i = 219 keV with T_e = 3keV.

The deuterium experiment of Coverdale et al [15] has $\tau_{eq}/\tau_A = 2.5$ leading to ion viscous heating, and enhanced neutron yield as calculated in a recent review of Z-pinches [16].

NUMERICAL SIMULATIONS

Real ion viscosity is $P_i\tau_{ii}$ and shock widths are a few mean-free-paths. Viscosity also has to provide thermalisation and the heating associated with short wavelength modes.

Numerical simulations cannot follow the short time step of τ_{ii} or the grid size of an ion mean-free-path, and instead they are essentially collisionless (apart from resistivity which is small at stagnation). Artificial viscosity is introduced only to prevent a large velocity jump across a grid cell. In this way shock waves can be modelled, as the resulting Rankine-Hugoniot conservation relations do not depend on the value of the viscosity (or thermal conductivity). But generally an artificial increase in τ_{ii} would lead to wilder behaviour as thermalisation is ~10^{-3} slower. There is also some doubtful physics introduced, e.g. no entropy generation for uniform compression or for any rarefaction [17]. At this conference Niasse et al [18] have found that the structure of the trailing mass varies with the grid cell size employed. At least two of the experimental presentations [19,20] required thermalisation into three dimensions to get agreement with data, while Jennings et al [21] would predict no ion Doppler broadening end-on in their simulations. An experimental measurement end-on of Doppler broadening is urgently required.

CONCLUSIONS

High current Z-pinches can be viscously heated through short wavelength, fast growing , m=0 MHD instabilities, especially if the mass is low and the original radius is large thus giving a high velocity of implosion and a high ion viscosity at stagnation.

If $\tau_{eq} \geq \tau_A$ ion viscosity will dominate, - but for $\tau_{eq} << \tau_A$ we find $T_e = T_i$ and for $Z>5$, electron viscosity will dominate over ion viscosity as in [11].

Viscous heating can be represented as a large additional resistance which drives up the E_z electric field and the $J_z E_z$ dissipation of magnetic energy. It also causes hot electrons to drift preferentially to the axis where harder X-rays are emitted. The lower MHD activity close to the axis would also result in a large transfer of current to the axis as its local impedance is low.

High mass and low current Z-pinches have a small magnetic Prandtl number, and their Alfvén spectrum is resistively limited. Low mass, high current Z-pinches have $Pr_m >>1$ and the spectrum is limited by viscous damping.

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