NEGATIVE ENERGIES AND TIME REVERSAL
IN QUANTUM FIELD THEORY

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Abstract

The theoretical and phenomenological status of negative energies is reviewed in Quantum Field Theory leading to the conclusion that hopefully their rehabilitation might only be completed in a modified general relativistic model.

1 Introduction

With recent cosmological observations related to supernovae, CMB and galactic clustering the evidence is growing that our universe is undergoing an accelerated expansion at present. Though the most popular way to account for this unexpected result has been the reintroduction of a cosmological constant or a new kind of dark matter with negative pressure, scalar fields with negative kinetic energy, so-called phantom fields, have recently been proposed [1] [2] [3] as new sources leading to the not excluded possibility that the equation of state parameter be less than minus one. Because such models unavoidably lead to violation of positive energy conditions, catastrophic quantum instability of the vacuum is expected and one has to impose an ultraviolet cutoff to the low energy effective theory in order to keep the instability at unobservable rate. Stability is clearly the challenge for any model trying to incorporate negative energy fields interacting with positive energy fields. But before addressing this crucial issue, it is worth recalling and analyzing how and why Quantum Field Theory discarded negative energy states. We shall find that this was achieved through several not so obvious mathematical choices, often in close relation with the well known pathologies of the theory, vacuum and UV loop divergences. Following another approach starting from the orthogonal alternative mathematical choices, the crucial link between negative energies, time reversal and the existence of discrete symmetry conjugated worlds will appear.

2 Negative energy and classical fields

2.1 Extremum action principle

Let us first address the stability of paths issue. Consider the path r(t) of a material point of mass m with fixed endpoints at time t_1 and t_2 in the potential

U(r,t). The action S is:

$$S = \int_{t_1}^{t_2} (1/2\, mv^2 \; - U(r,t))dt$$

The extremum condition (δS=0) is all we need to establish the equation of motion:

$$m\dot{v} = -\frac{\partial U}{\partial r}$$

S has no maximum because of the kinetic term positive sign. The extremum we find is a minimum. Let us try now a negative kinetic term:

$$S = \int_{t_1}^{t_2} (-1/2\, m\, v^2 \; - U(r,t))dt$$

The extremum condition (δS=0) is all we need to establish the equation of motion:

$$\text{-}m\dot{v} = -\tfrac{\partial U}{\partial r}$$

S has no minimum because of the kinetic term negative sign. The extremum we find is a maximum. Eventually, it appears that the fundamental principle is that of stationary (δS=0) action, the extremum being a minimum or a maximum depending on the sign of the kinetic term. In all cases we find stable trajectories.

2.2 Classical relativistic fields

We can also check that negative kinetic energy terms (ghost terms) in a free field action are not problematic. When we impose the extremum action condition the negative energy field solutions simply maximize the action. Now, in special relativity for a massive or mass-less particle, two energy solutions are always possible:

$$E = \pm\sqrt{p^2 + m^2},\; E = \pm\, |p|$$

In other words, the Lorentz group admits, among others, negative energy representations $E^2 - p^2 = m^2 > 0,\; E < 0$, $E^2 - p^2 = 0,\; E < 0$. Thus, not only can we state that negative energy free field terms are not problematic but also that negative energy field solutions are expected in any relativistic field theory. For instance the Klein-Gordon equation:

$$\left(\partial^\mu \partial_\mu + m^2\right) \overset{(\sim)}{\phi}(x) = 0$$

admits when $m^2 > 0$ (we shall not try to understand here the physical meaning of tachyonic ($m^2 < 0$) and vacuum ($E = p = m = 0$) representations) positive $\phi(x)$ and negative $\tilde{\phi}(x)$ energy free field solutions. Indeed, the same Klein-Gordon equation results from applying the extreme action principle to either the 'positive' scalar action:

$$\int d^4x\, \phi(x) \left(\partial^\mu \partial_\mu + m^2\right) \phi(x)$$

or the 'negative' scalar action:

$$-\int d^4x\, \tilde{\phi}(x) \left(\partial^\mu\partial_\mu + m^2\right) \tilde{\phi}(x)$$

From the former a positive conserved Hamiltonian is derived through the Noether theorem:

$$\int d^3x\left(\frac{\partial\phi^\dagger(\mathbf{x},t)}{\partial t}\frac{\partial\phi(\mathbf{x},t)}{\partial t} + \sum_{i=1,3}\frac{\partial\phi^\dagger(\mathbf{x},t)}{\partial x_i}\frac{\partial\phi(\mathbf{x},t)}{\partial x_i} + m^2\phi^\dagger(\mathbf{x},t)\phi(\mathbf{x},t)\right)$$

while a negative one is derived from the latter:

$$-\int d^3x\left(\frac{\partial\tilde{\phi}^\dagger(\mathbf{x},t)}{\partial t}\frac{\partial\tilde{\phi}(\mathbf{x},t)}{\partial t} + \sum_{i=1,3}\frac{\partial\tilde{\phi}^\dagger(\mathbf{x},t)}{\partial x_i}\frac{\partial\tilde{\phi}(\mathbf{x},t)}{\partial x_i} + m^2\tilde{\phi}^\dagger(\mathbf{x},t)\tilde{\phi}(\mathbf{x},t)\right)$$

3 Negative energy in relativistic Quantum Field Theory (QFT)

3.1 Creating and annihilating negative energy quanta

At first sight it would seem that the negative frequency terms appearing in the plane wave Fourier decomposition of any field naturally stand for the negative energy solutions. But as soon as we decide to work in a self-consistent quantization theoretical framework, that is the second quantization one, the actual meaning of these negative frequency terms is clarified. Operator solutions of field equations in conventional QFT read:

$$\phi(x) = \phi_+(x) + \phi_-(x)$$

with $\phi_+(x)$ a positive frequency term creating **positive** energy quanta and $\phi_-(x)$ a negative frequency term annihilating **positive** energy quanta. So negative energy states are completely avoided thanks to the mathematical choice of creating and annihilating only positive energy quanta and $\phi(x)$ built in this way is just the positive energy solution. This choice would be mathematically justified if one could argue that there are strong reasons to discard the 'negative action' we introduced in the previous section. But there are none and as we already noticed the Klein-Gordon equation is also easily derived from such action and the negative energy field solution:

$$\tilde{\phi}(x) = \tilde{\phi}_+(x) + \tilde{\phi}_-(x)$$

(with $\tilde{\phi}_+(x)$ a positive frequency term annihilating **negative** energy quanta and $\tilde{\phi}_-(x)$ a negative frequency term creating **negative** energy quanta) is only coherent with the negative Hamiltonian derived from the negative action through the Noether theorem (in the same way it is a standard QFT result that the usual positive energy quantum field $\phi(x)$ is only coherent with the above positive Hamiltonian [6] [7]). Therefore, it is mathematically unjustified to discard the negative energy solutions. Neglecting them on the basis that negative energy states remain up to now undetected is also very dangerous if we recall that antiparticles predicted by the Dirac equation were considered unphysical before they were eventually observed. If negative (or tachyonic) energy states are given a profound role to play in physics, this must be fully understood otherwise we might be faced with insurmountable difficulties at some later stage.

There is a widespread belief that the negative energy issue were once and for all understood in terms of antiparticles. Indeed, because charged fields are required not to mix operators with different charges, the charge conjugated creation and annihilation operators (antiparticles) necessarily enter into the game. Following Feynman's picture, such antiparticles can as well be considered as negative energy particles propagating backward in time. According S.Weinberg [8], it is only in relativistic (Lorentz transformation do not leave invariant the order of events separated by space-like intervals) quantum mechanics (non negligible probability for a particle to get from x_1 to x_2 even if x_1 - x_2 is space-like) that antiparticles are a necessity to avoid the logical paradox of a particle being absorbed before it is emitted. However, these antiparticles have nothing to do with genuine negative energy states propagating forward in time, whose quanta are by construction of the conventional QFT fields never created nor annihilated. Therefore, our deep understanding of the actual meaning of field negative frequency terms in QFT does not "solve" the negative energy issue since the corresponding solutions were actually neglected from the beginning. As we shall see, there is a heavy price to pay for having neglected the negative energy solutions: all those field vacuum divergences that unavoidably arise after quantization and may be an even heavier price are the ideas developed to cancel such infinities without reintroducing negative energy states.

3.2 A unitary time reversal operator

In a classical relativistic framework, one could not avoid energy reversal under time reversal simply because energy is the time component of a four-vector. But, when one comes to establish in Quantum Field Theory the effect of time reversal on various fields, nobody wants to take this simple picture serious anymore mainly because of the unwanted negative energy spectrum it would unavoidably bring into the theory. It is argued that negative energy states remain undetected and that their existence would necessarily trigger catastrophic decays of particles and vacuum: matter could not be stable. To keep energies positive, the mathematical choice of an anti-unitary time reversal operator comes to the rescue leading to the idea that the time-mirrored system corresponds to 'running the movie backwards' interchanging the roles of initial and final configurations. We shall come back to the stability issue later. But for the time being, let us stress that the running backward movie picture is not self–evident. In particular, the interchange of initial and final state under time reversal is very questionable. To see this, let us first recall that there are two mathematical possibilities for a time reversal operator; either it must be unitary or anti-unitary. These lead to two quite different, both mathematically coherent time reversal conjugated scenarios:

The process $i \rightarrow f$ being schematized as:

$$|i\rangle = a^{+}(E_{i1})...a^{+}(E_{in})\,|0\rangle \quad \overset{TIMEARROW}{\Rightarrow} \times \quad \langle f| = \langle 0|\,a(E_{f1})...a(E_{fp})$$

$$-\infty \leftarrow t \qquad\qquad t \rightarrow +\infty$$

the time reversed coordinate is $t_{rev} = -t$ and:

The conventional QFT anti-unitary time reversal scenario interchanges initial and final states:

$$i \rightarrow f \overset{T}{\Rightarrow} T^{A}(f) \rightarrow T^{A}(i)$$

$$|f\rangle = a^{+}(E_{f1})...a^{+}(E_{fp})\,|0\rangle \quad \overset{TIMEARROW}{\Rightarrow} \quad \langle i| = \langle 0|\,a(E_{i1})...a(E_{in})$$

$$-\infty \leftarrow t_{rev} \qquad\qquad t_{rev} \rightarrow +\infty$$

The unitary one does not interchange initial and final state but reverses energies

$$i \rightarrow f \overset{T}{\Rightarrow} T^{U}(i) \rightarrow T^{U}(f)$$

$$\left\langle \tilde{f} \right| = \langle 0|\,a(-E_{f1})...a(-E_{fp}) \quad \overset{TIMEARROW}{\Leftarrow} \quad \left|\tilde{i}\right\rangle = a^{+}(-E_{i1})...a^{+}(-E_{in})\,|0\rangle$$

$$-\infty \leftarrow t_{rev} \qquad\qquad t_{rev} \rightarrow +\infty$$

Our common sense intuition then tells us that the interchange of initial and final state, hence the anti-unitary picture stands to reason. This is because we naively require that in the time reverted picture the initial state (the ket) must come 'before' the final state (the bra) i.e for a lower value of t_{rev}. However, paying careful attention to the issue we realize that the time arrow, an underlying concept of time flow which here influences our intuition is linked to a specific property of the time coordinate which is not relevant for a spatial coordinate, namely its irreversibility or causality. But as has been pointed out by many authors, there are many reasons to suspect that such irreversibility and time arrow may only be macroscopic scale (or statistical physics) valid concepts not making sense for a microscopic time, at least before any measurement takes place. We believe that our microscopic time coordinate, before measurement takes place, should be better considered as a spatial one, i.e possessing no property such as an arrow. Then, the unitary picture is the most natural one as a time reversal candidate process simply because it is the usual choice for all other discrete and continuous symmetries.

But if neither t nor t_{rev} actually stand for the genuine flowing time which we experiment and measure, the latter must arise at some stage and it is natural to postulate that its orientation corresponding to the experimented time arrow is simply defined in such a way that, as drawn in the previous pictures, the initial state (the creator) always comes before the final state (the annihilator) in this flowing time. This clearly points toward a theoretical framework where the time will be treated as a quantum object undergoing radical transformations from the microscopic to the macroscopic time we measure. Let us anticipate that the observable velocities will be better understood in term of this new macroscopic flowing time variable which arrow (orientation) keeps the same under reversal of the unflowing space–like t coordinate.

Therefore, the interchange of initial and final states is only justified under the assumption that time coordinate reversal implies time arrow reversal. But this is not at all obvious and thus there is no more strong reason to prefer and adopt the QFT anti-unitary choice. At the contrary, we can now list several strong arguments in favor of the unitary choice:

- The mathematical handling of an anti-unitary operator is less trivial and induces unusual complications when applied for instance to the Dirac field.
- The QFT choice leads to momentum reversal, a very surprising result for a mass-less particle, since in this case it amounts to a genuine wavelength reversal and not frequency reversal, as one would have expected.
- Its anti-unitarity makes T really exceptional in QFT. As a consequence, not all basic four-vectors transform the same way under such operator as the reference space-time four-vector. In our mind, a basic four-vector is an object involving the parameters of a one particle state such as for instance its energy and three momentum components. The one particle state energy is the time component of such an object but does not reverses as the time itself if T is taken anti-unitary. This pseudo-vector behavior under time reversal seems nonsense and leads us to prefer the unitary scenario. At the contrary, we can understand why (and accept that) the usual operator four-vectors, commonly built from the fields, behave under discrete transformations such as unitary parity differently than the reference space-time four-vector. This is simply because, as we shall see, they involve in a nontrivial way the parity-pseudo-scalar 3-volume.
- Time irreversibility at macroscopic scale allows us to define unambiguously our time arrow. But, as we already noticed, the arrow of time at the microscopic scale or before any measurement process takes place may be not so well defined. The statement that the time arrow is only a macroscopic scale (or may be statistical physics) valid concept is not so innovative. We know from Quantum Mechanics that all microscopic quantum observables acquire their macroscopic physical status through the still enigmatic measurement process. Guessing that the time arrow itself only becomes meaningful at macroscopic scale, we could reverse our microscopic time coordinate t as an arrowless spatial coordinate. Reverting the time arrow is more problematic since this certainly raises the well known time reversal and causality paradoxes. But the good new is that reversing the time coordinate does not necessarily imply reversing the arrow of time, i.e interchanging initial and final state. In the unitary picture, you do not actually go backward in time since you just see the same succession

(order) of events counting the t_{rev} time "à rebours", with only the signs of the involved energies being affected and you need not worry anymore about paradoxes. Therefore, in a certain sense, the running backward movie picture was may be just a kind of entropy reversal picture, a confusing and inappropriate macroscopic scale concept which obscured our understanding of the time coordinate reversal and led us to believe that the anti-unitary scenario was obviously the correct one.

- Charge and charge density are invariant while current densities get reversed under a unitary time reversal (see section VI).
- Negative energy fields are natural solutions of all relativistic equations.
- The instability issue might be solved in a modified general relativistic model as we shall show in [5].

4 Negative energy quantum fields, time reversal and vacuum energies

We shall now explicitly build the QFT neglected solutions, e.g. the usual bosonic and fermionic negative energy fields, show how these are linked to the positive ones through time reversal and how vacuum divergences cancel from the Hamiltonians.

4.1 The neutral scalar field

The positive energy scalar field solution of the Klein-Gordon equation is:

$$\phi(x,t) = \int \frac{d^3p}{(2\pi)^{3/2}(2E)^{1/2}} \left[a(p,E)e^{i(Et-px)} + a^\dagger(p,E)e^{-i(Et-px)}\right]$$

with $E = \sqrt{p^2+m^2}$. The negative energy scalar field solution of the same Klein-Gordon equation is:

$$\tilde{\phi}(x,t) = \int \frac{d^3p}{(2\pi)^{3/2}(2E)^{1/2}} \left[\tilde{a}^\dagger(-p,-E)e^{i(Et-px)} + \tilde{a}(-p,-E)e^{-i(Et-px)}\right]$$

We just required this field to create and annihilate negative energy quanta. Assuming T is anti-unitary, it is well known that a scalar field is transformed according

$$T\phi(x,t)T^{-1} = \phi(x,-t)$$

where, for simplicity, an arbitrary phase factor was chosen unity. Then it is straightforward to show that:

$$Ta^\dagger(p,E)T^{-1} = a^\dagger(-p,E)$$

We do not accept this result because we want time reversal to flip energy, not momentum. If instead, the T operator is chosen unitary like all other discrete transformation operators (P, C) in Quantum Field Theory we cannot require $T\phi(x,t)T^{-1} = \phi(x,-t)$, but rather:

$$T\phi(x,t)T^{-1} = \tilde{\phi}(x,-t)$$

The expected result is then obtained as usual through the change in the variable p→-p:

$$Ta^{\dagger}(p,E)T^{-1} = \tilde{a}^{\dagger}(p,-E)$$

This confirms that a unitary T leads to energy reversal of scalar field quanta. Momentum is invariant. For a massive particle this may be interpreted as mass reversal coming along with velocity reversal. But in the unitary time reversal scenario it is not at all obvious that the velocity is built out of the time coordinate which gets reversed. Instead, as soon as this velocity is measured it seems more natural to build it out of the (as well measured) flowing time which never gets reversed. In this case, neither velocity nor mass get reversed. The Hamiltonian for our free neutral scalar field reads:

$$H = +\frac{1}{2}\int d^3x\,[\,(\frac{\partial\phi(x,t)}{\partial t})^2 + (\frac{\partial\phi(x,t)}{\partial x})^2 + m^2\phi^2(x,t)]$$

The Hamiltonian for the corresponding negative energy field is:

$$\tilde{H} = \tilde{P}^0 = -\frac{1}{2}\int d^3x\,[\,(\frac{\partial\tilde{\phi}(x,t)}{\partial t})^2 + (\frac{\partial\tilde{\phi}(x,t)}{\partial x})^2 + m^2\tilde{\phi}^2(x,t)]$$

The origin of the minus sign under time reversal of H will be investigated in sections VI. After replacing the scalar fields by their expressions, the computation then follows the same line as in all QFT books, leading to:

$$H = \frac{1}{2}\int d^3p\,p^0(a^{\dagger}(p,E)a(p,E) + a(p,E)a^{\dagger}(p,E))$$

$$\tilde{H} = -\frac{1}{2}\int d^3p\,p^0(\tilde{a}^{\dagger}(-p,-E)\tilde{a}(-p,-E) + \tilde{a}(-p,-E)\tilde{a}^{\dagger}(-p,-E))$$

With $p^0 = \sqrt{p^2+m^2}$ and the usual commutation relations,

$$[a_p^{\dagger}, a_{p'}] = \delta^4(p-p'), [\tilde{a}_p^{\dagger}, \tilde{a}_{p'}] = \delta^4(p-p')$$

vacuum divergences cancel (as we shall see, in a general relativistic framework, these only cancel as gravitational sources), and for the total Hamiltonian we get:

$$H_{total} = \int d^3p\,p^0\left\{a^{\dagger}(p,E)a(p,E) - \tilde{a}^{\dagger}(-p,-E)\tilde{a}(-p,-E)\right\}$$

It is straightforward to check that the energy eigenvalue for a positive (resp negative) energy ket is positive (resp negative), as it should. For a vector field, the infinities would cancel in the same way assuming as well the usual commutation relations.

4.2 The Dirac field

Let us investigate the more involved case of the Dirac field. The Dirac field is solution of the free equation of motion:

$$(i\gamma^\mu \partial_\mu - m)\psi(x,t) = 0$$

When multiplying this Dirac equation by the unitary T operator from the left, we get:

$$(iT\gamma^\mu T^{-1}\partial_\mu - TmT^{-1})T\psi(x,t) = (iT\gamma^\mu T^{-1}\partial_\mu - TmT^{-1})\tilde{\psi}(x,-t) = 0$$

If the rest energy term m is related to the Higgs field value at its minimum (or another dynamical field) its transformation under time reversal is more involved than that of a pure number. Rather, we have:

$$m = g\phi_0(x,t) \to \tilde{m} = TmT^{-1} = g\tilde{\phi}_0(x,-t)$$

Making the replacement, $\partial_0 = -\partial^0, \partial_i = \partial^i$ and requiring that the T conjugated Dirac and scalar fields at its minimum $\tilde{\psi}(x,-t) = T\,\psi(x,t)T^{-1}$, $\tilde{\phi}_0(x,-t) = T\,\phi_0(x,t)T^{-1}$ together should obey the same equation, e.g.

$$(i\gamma^\mu \partial^\mu - g\tilde{\phi}_0(x,-t))\tilde{\psi}(x,-t) = 0$$

as $\psi(x,t)$ and $\phi_0(x,t)$, leads to:

$$T\gamma^i T^{-1} = \gamma^i,\; T\gamma^0 T^{-1} = -\gamma^0$$

The T operator is then determined to be $T = \gamma^1\gamma^2\gamma^3$. Now assuming also that $\tilde{\phi}_0(x,t) = -\phi_0(x,t)$, the Dirac equation satisfied by $\tilde{\psi}(x,t)$ reads:

$$(i\gamma^\mu \partial_\mu + m)\tilde{\psi}(x,t) = 0$$

γ^0, γ^i being a particular gamma matrices representation used in equation $(i\gamma^\mu\partial_\mu - m)\psi(x,t) = 0$, then $(i\gamma^\mu\partial_\mu + m)\tilde{\psi}(x,t) = 0$ can simply be obtained from the latter by switching to the new gamma matrices representation $-\gamma^0$, $-\gamma^i$ and the negative energy Dirac field $\tilde{\psi}(x,t)$. As is well known, all gamma matrices representations are unitary equivalent and here γ^5 is the unitary matrix transforming the set γ^0, γ^i into $-\gamma^0$, $-\gamma^i$ ($\gamma^5\gamma^\mu\left(\gamma^5\right)^{-1} = -\gamma^\mu$). Thus $\tilde{\psi}(x,t)$ satisfies the same Dirac equation as $\gamma^5\psi(x,t)$. The physical consequences will be now clarified. Let us write down the positive (resp negative) energy Dirac field solutions of their respective equations.

$$\psi(x,t) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=\pm 1/2} \int_p \frac{d^3p}{(2E)^{1/2}} \{u(-E,m,-p,-\sigma)a_c(E,m,p,\sigma)e^{i(Et-px)}$$

$$+u(E,m,p,\sigma)a^\dagger(E,m,p,\sigma)e^{-i(Et-px)}\}$$

$$\tilde{\psi}(x,t) = \frac{1}{(2\pi)^{3/2}} \sum_{\sigma=\pm 1/2} \int_p \frac{d^3p}{(2E)^{1/2}} \{u(-E,-m,-p,-\sigma)\tilde{a}^\dagger(-E,-m,-p,-\sigma)e^{i(Et-px)}$$

$$+u(E,-m,p,\sigma)\tilde{a}_c(-E,-m,-p,-\sigma)e^{-i(Et-px)}\}$$

with $E = \sqrt{p^2+m^2}$. Classifying the free Dirac waves propagating in the x direction, we have as usual for the positive energy field spinors:

$$u(E,m,p_x,+1/2) = \begin{bmatrix} 1 \\ 0 \\ \frac{\sigma_x p_x}{m+E} \\ 0 \end{bmatrix}, \; u(-E,m,-p_x,-1/2) = \begin{bmatrix} 1 \\ 0 \\ \frac{\sigma_x p_x}{m-E} \\ 0 \end{bmatrix}$$

$$u(E,m,p_x,-1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{\sigma_x p_x}{m+E} \end{bmatrix}, \; u(-E,m,-p_x,+1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{\sigma_x p_x}{m-E} \end{bmatrix}$$

The negative energy field spinors are also easily obtained through the replacement m $\rightarrow$ -m

$$u(-E,-m,-p_x,-1/2) = \begin{bmatrix} 1 \\ 0 \\ \frac{\sigma_x p_x}{-m-E} \\ 0 \end{bmatrix}, \; u(E,-m,p_x,1/2) = \begin{bmatrix} 1 \\ 0 \\ \frac{\sigma_x p_x}{-m+E} \\ 0 \end{bmatrix}$$

$$u(-E,-m,-p_x,+1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{\sigma_x p_x}{-m-E} \end{bmatrix}, \; u(E,-m,p_x,-1/2) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \frac{\sigma_x p_x}{-m+E} \end{bmatrix}$$

We demand that:

$$T\psi(x,t)T^{-1} = \tilde{\psi}(x,-t)$$

This implies:

$$Ta^\dagger(E,m,p,\sigma)T^{-1}u(E,m,p,\sigma) = u(-E,-m,-p \rightarrow p,-\sigma)\tilde{a}^\dagger(-E,-m,p,-\sigma)$$

Hence:

$$Ta^\dagger(E,m,p,\sigma)T^{-1} = \tilde{a}^\dagger(-E,-m,p,-\sigma)$$

Thus, upon time reversal, energy, rest energy and spin are reversed. Because momentum is invariant helicity also flips its sign. Without having reverted the rest energy term in the negative energy Dirac field equation we could not have obtained this simple link through time reversal between the positive and negative energy creation operators. The rest energy reversal in the spinor expressions also reveals the difference between a true negative energy spinor $u(-E,-m,.,.)$ and a negative frequency spinor $u(-E,m,.,.)$. The Hamiltonian for $\psi(x,t)$ is:

$$H = P^0 = \int d^3x[\bar{\psi}(x,t)(-i\gamma^i.\partial_i + m)\psi(x,t)] \; + \; h.c$$

The negative energy field Hamiltonian will be built out of negative energy fields explicitly different from those entering in H. Hence, it is hopeless trying to obtain such kind of simple transformation relations such as $P^0 \Rightarrow \pm P^0$. On the other hand we can build the negative energy Hamiltonian and check that it provides the correct answer when applied to a given negative energy ket. We know that $T\gamma^i T^{-1} = \gamma^i$, $T\gamma^0 T^{-1} = -\gamma^0$, so that:

$$T\bar{\psi}(x,t)T^{-1} = T\psi^\dagger(x,t)\gamma^0 T^{-1} = -T\psi^\dagger(x,t)T^{-1}\gamma^0$$

$$= -(T\psi(x,t)T^{-1})^\dagger\gamma^0 = -\bar{\tilde{\psi}}(x,-t)$$

This will produce an extra minus sign in the negative energy Dirac field Hamiltonian. The origin of the other minus sign is the same as for the scalar field Hamiltonian and will be clarified later. The Hamiltonian for $\tilde{\psi}(x,t)$ is then:

$$\tilde{H} = \tilde{P}^0 = - - \int d^3x[\bar{\tilde{\psi}}(x,t)(-i\gamma^i\partial_i - m)\tilde{\psi}(x,t)] \,+\, h.c$$

Because the positive (resp negative) energy spinor satisfies $(i\gamma^\mu\partial_\mu - m)\psi(x,t) = 0$, (resp $(i\gamma^\mu\partial_\mu + m)\tilde{\psi}(x,t) = 0$) we have $(-i\gamma^i\partial_i + m)\psi(x,t) = i\gamma^0\partial_0\psi(x,t)$, (resp $(-i\gamma^i\partial_i - m)\tilde{\psi}(x,t) = i\gamma^0\partial_0\tilde{\psi}(x,t)$). The Hamiltonians then read:

$$H = P^0 = i\int d^3x[\psi^\dagger(x,t)\partial_0\psi(x,t)] \,+\, h.c$$

$$\tilde{H} = \tilde{P}^0 = i\int d^3x[\tilde{\psi}^\dagger(x,t)\partial_0\tilde{\psi}(x,t)] \,+\, h.c$$

Assuming for simplicity that we are dealing with a neutral field, the computation proceeds as usual for the positive energy Hamiltonian. With $p^0 = \sqrt{p^2+m^2}$:

$$H = \frac{1}{2}\sum_{\sigma=\pm 1/2}\int d^3p\, p^0(a^\dagger(E,p,\sigma)a(E,p,\sigma) - a(E,p,\sigma)a^\dagger(E,p,\sigma))$$

Negative energy spinors possessing the same orthogonality properties as positive energy spinors, the negative energy Hamiltonian is then obtained by the simple replacements $a^\dagger(E,p,\sigma) \to \tilde{a}(-E,-p,-\sigma)$; $a(E,p,\sigma) \to \tilde{a}^\dagger(-E,-p,-\sigma)$:

$$\tilde{H} = \frac{1}{2}\sum_{\sigma=\pm 1/2} - \int d^3p\, p^0(\tilde{a}^\dagger(-E,-p,-\sigma)\tilde{a}(-E,-p,-\sigma)$$

$$-\tilde{a}(-E,-p,-\sigma)\tilde{a}^\dagger(-E,-p,-\sigma))$$

Infinities cancel as for the boson fields when we apply the fermionic anti-commutation relations $\left\{a^\dagger_{p,\sigma}, a_{p',\sigma'}\right\} = \delta^4(p-p')\,\delta_{\sigma,\sigma'}$, $\left\{\tilde{a}^\dagger_{p,\sigma}, \tilde{a}_{p',\sigma'}\right\} = \delta^4(p-p')\,\delta_{\sigma,\sigma'}$, leading to:

$$H_{total} = \sum_{\sigma=\pm 1/2}\int d^3p\, p^0\left\{a^\dagger(p,E,\sigma)a(p,E,\sigma) - \tilde{a}^\dagger(-p,-E,-\sigma)\tilde{a}(-p,-E,-\sigma)\right\}$$

It is also easily checked that the energy eigenvalue for a positive (resp negative) energy ket is positive (resp negative), as it should. When we realize how straightforward are the cancellation of vacuum divergences for all fields it is very tempting to state that such infinities appeared only because half of the field solutions were neglected! We shall show in [5] that actually, in a general relativity context, our vacuum divergences only vanish as a source for gravitation. But the Casimir effect should still survive.

5 Phenomenology of the uncoupled positive and negative energy worlds

We shall now show that the uncoupled positive and negative energy worlds are both perfectly viable: no stability issue arises and in both worlds the behavior of matter and radiation is completely similar so that the negative signs may just appear as a matter of convention [9] [10]. Consider a gas made with negative energy matter particles (fermions) and negative energy photons. The interaction between two negative energy fermions is going on through negative energy photons exchange. Because the main result will only depend on the bosonic nature of the considered interaction field, let us compute and compare the simpler propagator of the positive and negative energy scalar fields.

-For a positive energy scalar field:

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}(2p^0)^{1/2}} \left[a(p)e^{ipx} + a_c^\dagger(p)e^{-ipx}\right]$$

we get as usual:

$$\begin{aligned}
\langle 0| T(\phi(x)\phi^\dagger(y)) |0\rangle &= \langle 0| \phi(x)\phi^\dagger(y) |0\rangle \ \theta(x_0 - y_0) + \langle 0| \phi^\dagger(y)\phi(x) |0\rangle \ \theta(y_0 - x_0) \\
&= \langle 0| \int \frac{d^3p}{(2\pi)^3 2p^0} a(p)a^\dagger(p)e^{ip(x-y)} |0\rangle \, \theta(x_0 - y_0) \\
&+ \langle 0| \int \frac{d^3p}{(2\pi)^3 2p^0} a_c(p)a_c^\dagger(p)e^{-ip(x-y)} |0\rangle \, \theta(y_0 - x_0) \\
&= \int \frac{d^3p}{(2\pi)^3 2p^0} e^{ip(x-y)}\theta(x_0 - y_0) + \int \frac{d^3p}{(2\pi)^3 2p^0} e^{-ip(x-y)}\theta(y_0 - x_0) \\
&= \Delta(y - x)\theta(x_0 - y_0) + \Delta(x - y)\theta(y_0 - x_0)
\end{aligned}$$

-For a negative energy scalar field:

$$\tilde{\phi}(x) = \int \frac{d^3p}{(2\pi)^{3/2}(2p^0)^{1/2}} \left[\tilde{a}^\dagger(p)e^{ipx} + \tilde{a}_c(p)e^{-ipx}\right]$$

we obtain:

$$\begin{aligned}
\langle 0| T(\tilde{\phi}(x)\tilde{\phi}^\dagger(y)) |0\rangle &= \langle 0| \tilde{\phi}(x)\tilde{\phi}^\dagger(y) |0\rangle \ \theta(x_0 - y_0) + \langle 0| \tilde{\phi}^\dagger(y)\tilde{\phi}(x) |0\rangle \ \theta(y_0 - x_0) \\
&= \langle 0| \int \frac{d^3p}{(2\pi)^3 2p^0} \tilde{a}_c(p)\tilde{a}_c^\dagger(p)e^{-ip(x-y)} |0\rangle \, \theta(x_0 - y_0)
\end{aligned}$$

$$+\langle 0|\int\frac{d^3p}{(2\pi)^3 2p^0}\tilde{a}(p)\tilde{a}^\dagger(p)e^{ip(x-y)}\,|0\rangle\,\theta(y_0-x_0)$$

$$=\int\frac{d^3p}{(2\pi)^3 2p^0}e^{-ip(x-y)}\theta(x_0-y_0)+\int\frac{d^3p}{(2\pi)^3 2p^0}e^{ip(x-y)}\theta(y_0-x_0)$$

$$=\Delta(x-y)\theta(x_0-y_0)+\Delta(y-x)\,\theta(y_0-x_0)$$

Summing the two propagators, the theta functions cancel:

$$\langle 0|\,T(\tilde{\phi}(x)\tilde{\phi}^\dagger(y))\,|0\rangle+\langle 0|\,T(\phi(x)\phi^\dagger(y))\,|0\rangle$$

$$=(\Delta(x-y)+\Delta(y-x))(\theta(x_0-y_0)+\theta(y_0-x_0)\,)$$

$$=\Delta(x-y)+\Delta(y-x)\propto\int(\delta\left(E-p^0\right)+\delta\left(E+p^0\right))e^{-iE(x_0-y_0)}dE$$

Therefore, if the two propagators could contribute with the same coupling to the interaction between two currents, the virtual particle terms would cancel each other. Only on-shell particles could still be exchanged between the two currents provided energy momentum conservation does not forbid it. For a photon field as well the two off-shell parts of the propagators would be found opposite. Hence the coulomb potential derived from the negative energy photon field propagator would be exactly opposite to the coulomb potential derived from the positive energy photon field propagator: as a consequence, the 1/r Coulomb potential and electromagnetic interactions would simply disappear. The interesting point is that in our negative energy gas, where we assume that only the exchange of negative energy virtual photons takes place, the coulomb potential is reversed compared to the usual coulomb potential generated by positive energy virtual photons exchange. However in this repulsive potential between oppositely charged fermions, these still attract each other, as in the positive energy world, because of their negative inertial terms in the equation of motion (as deduced from their negative terms in the action). The equation of motion for a given negative energy matter particle in this Coulomb potential is:

$$-m\dot{v}=--\frac{\partial U_c}{\partial r}$$

or

$$m\dot{v}=-\frac{\partial U_c}{\partial r}$$

We find ourselves in the same situation as that of a positive energy particles gas interacting in the usual way e.g through positive energy photons exchange. Hence negative energy atoms will form and the main results of statistical physics apply: following Boltzman law, our particles will occupy with the greatest probabilities states with minimum $\frac{1}{2}m\dot{v}^2$, thus with maximum energy$-\frac{1}{2}m\dot{v}^2$. Temperatures are negative. This result can be extended to all interactions propagated by bosons as are all known interactions. The conclusion is that the non-coupled positive and negative energy worlds are perfectly stable, with positive and negative energy particles minimizing the absolute value of their energies:

6 Actions and Hamiltonians under Time reversal and Parity

6.1 Negative integration volumes?

Starting from the expression of the Hamiltonian density for a positive energy neutral scalar field:

$$T^{00}(x,t) = \left(\frac{\partial \phi(x,t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial \phi(x,t)}{\partial x_i}\right)^2 + m^2\phi^2(x,t)$$

and applying time reversal we get:

$$\left(\frac{\partial \tilde{\phi}(x,-t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial \tilde{\phi}(x,-t)}{\partial x_i}\right)^2 + m^2\tilde{\phi}^2(x,-t)$$

with $T\phi(x,t)T^{-1} \equiv \tilde{\phi}(x,-t)$ From such expression, a naive free Hamiltonian density for the scalar field$\tilde{\phi}(x,t)$ may be proposed:

$$\tilde{T}^{00}(x,t) = \left(\frac{\partial \tilde{\phi}(x,t)}{\partial t}\right)^2 + \sum_{i=1,3}\left(\frac{\partial \tilde{\phi}(x,t)}{\partial x_i}\right)^2 + m^2\tilde{\phi}^2(x,t)$$

It thus happens that $\tilde{T}^{00}(x,t)$ is manifestly positive since it is a sum of squared terms. We of course cannot accommodate negative energy fields with positive Hamiltonian densities so following the procedure used to obtain negative kinetic energy terms for a phantom field, we just assumed in the previous sections a minus sign in front of this expression. But how could we justify this trick if time reversal does not provide us with this desired minus sign? One possible solution appears when we realize that according to general relativity, actually T^{00} is not a spatial energy density but rather$\sqrt{g}T^{00}$ where $g \equiv -Det\, g_{\mu\nu}$. This is also expected to still remain positive because of a rather strange mathematical choice in general relativity: integration volumes such as dt, d^4x, d^3x are not signed and should not flip sign under time reversal or parity transformations. Let us try the more natural opposite way: $t \to -t \Rightarrow dt \to -dt$ and $x \to -x \Rightarrow dx \to -dx$, natural in the sense that this is naively the straightforward mathematical way to proceed and let us audaciously imagine that for instance a negative 3-dimentional volume is nothing else but the image of a 3-dimentional positive volume in a mirror. Then, the direct consequence of working with signed volumes is that the general relativistic integration element $d^4x\sqrt{g}$ is not invariant anymore under coordinate transformations (such as P or T) with negative Jacobian (it is often stated that the absolute value of the Jacobian is imposed by a fundamental theorem of integral calculus[2]. But should not this apply only to change of variables and not general coordinate transformations?). We are then led to choose an invariant integration element under any coordinate transformations: this is $d^4x\left|\frac{\partial\xi}{\partial x}\right|$, where $\left|\frac{\partial\xi}{\partial x}\right|$ stands for the Jacobian of the transformation from the inertial coordinate system ξ^α to x^μ. Because $\left|\frac{\partial\xi}{\partial x}\right|$ is not necessarily positive as is $\sqrt{g}$ in general relativity, it will get reversed under P or T transformations affecting Lorentz indices only so that spatial charge density $\left|\frac{\partial\xi}{\partial x}\right| J^0$, scalar charge $Q = \int \left|\frac{\partial\xi}{\partial x}\right| J^0 d^3x$, spatial energy-momentum densities $\left|\frac{\partial\xi}{\partial x}\right| T^{\mu 0}$ and energy-momentum four-vector $P^\mu = \int \left|\frac{\partial\xi}{\partial x}\right| T^{\mu 0} d^3x$ should transform accordingly. For instance, it is often stated that a unitary time reversal operator is not allowed because it would produce the not acceptable charge reversal. This analysis is no more valid if the Jacobi determinant flips its sign. Indeed, though J^0, as all four-vector time components, becomes negative, the spatial charge density $\left|\frac{\partial\xi}{\partial x}\right| J^0$ and scalar charge $Q = \int \left|\frac{\partial\xi}{\partial x}\right| J^0 d^3x$ remain positive under unitary time reversal. It is also worth checking what is now the effect of unitary space inversion: $P^\mu = \int \left|\frac{\partial\xi}{\partial x}\right| T^{\mu 0} d^3x$ transforms under Parity as $T^{\mu 0}$ times the pseudo-scalar Jacobi determinant $\left|\frac{\partial\xi}{\partial x}\right|$, so that:

$$P^0 \Rightarrow -P^0,\ P^i \Rightarrow P^i$$

$Q = \int \left|\frac{\partial\xi}{\partial x}\right| J^0 d^3x$ also transforms under Parity as J^0 times the pseudo-scalar Jacobi determinant $\left|\frac{\partial\xi}{\partial x}\right|$, so that:

$$Q \Rightarrow -Q$$

So, if unitary Parity has the same effect on various fields, currents and energy densities as in conventional quantum field theory, it now produces a flip in the energy and charge signs but does not affect momentum! Anyway, we see that the signed Jacobi determinant could do the good job for providing us with the desired minus signs. However, working with negative integration volumes amounts to give up the usual definition of the integral which insures that it is positive definite. If we are not willing to give up this definition, another mechanism should be found to provide us with the necessary minus sign. The issue will be reexamined and a more satisfactory solution described in [5].

7 Interactions between positive and negative energy fields ?

Postulate the existence of a new inertial coordinate system $\tilde{\xi}$ such that $\left|\frac{\partial\tilde{\xi}}{\partial x}\right|$ is negative. This can be achieved simply by considering the two time reversal conjugated (with opposite proper times) inertial coordinate systems ξ and $\tilde{\xi}$. We may then define the positive energy quantum $F(x)$ fields (resp negative energy $\tilde{F}(x)$ fields) as the fields entering in the action with positive $\left|\frac{\partial\xi}{\partial x}\right|$ (resp negative $\left|\frac{\partial\tilde{\xi}}{\partial x}\right|$) entering in the integration volume so that the energy $P^0 = \int \left|\frac{\partial\xi}{\partial x}\right| T^{00} d^3x$

(resp $\tilde{P}^0 = \int \left|\frac{\partial\tilde{\xi}}{\partial x}\right| \tilde{T}^{00} d^3x$) is positive (resp negative). The action for positive energy matter and radiation is then as usual:

$$S = \int d^4x \left|\frac{\partial \xi}{\partial x}\right| \left\{ L(\Psi(x), \frac{\partial \xi^\alpha}{\partial x^\mu}(x)) + L(A_\mu(x), \frac{\partial \xi^\alpha}{\partial x^\mu}(x)) + J_\mu(x) A^\mu(x) \right\}$$

Similarly, the action for negative energy matter and radiation is:

$$\tilde{S} = \int d^4x \left|\frac{\partial \tilde{\xi}}{\partial x}\right| \left\{ L(\tilde{\Psi}(x), \frac{\partial \tilde{\xi}^\alpha}{\partial x^\mu}(x)) + L(\tilde{A}_\mu(x), \frac{\partial \tilde{\xi}^\alpha}{\partial x^\mu}(x)) + \tilde{J}_\mu(x) \tilde{A}^\mu(x) \right\}$$

Hence positive energy fields move under the influence of the gravitational field $\frac{\partial \xi^\alpha}{\partial x^\mu}$, while negative energy fields move under the influence of the gravitational field $\frac{\partial \tilde{\xi}^\alpha}{\partial x^\mu}$. Then, the mixed coupling in the form $J_\mu(x)\tilde{A}^\mu(x)$ that we might have naively hoped is not possible just because the integration volume must be $d^4x \left|\frac{\partial \xi}{\partial x}\right|$ for $F(x)$ type fields and $d^4x \left|\frac{\partial \tilde{\xi}}{\partial x}\right|$ for $\tilde{F}(x)$ type fields. Indeed coherence requires that in the action the negative Jacobian be associated with negative energy fields $\tilde{F}(x)$ involving negative energy quanta creation and annihilation operators. This is a good new since it is well known that couplings between positive and negative energy fields lead to an unavoidable stability problem due to the fact that energy conservation keeps open an infinite phase space for the decay of positive energy particles into positive and negative energy particles. A scenario with positive and negative energy fields living in different metrics also provides a good way to account for the undiscovered negative energy states. However the two metrics should not be independent if we want to introduce a connection at least gravitational between positive and negative energy worlds, mandatory to make our divergences gravitational effects actually cancel. In [5] we shall explicit this dependency between the two conjugated metrics and the mechanism that gives rise through the extremum action principle to the negative source terms in the Einstein equation. It will be clear that this mechanism only works properly if, as in general relativity, we keep working with Jacobi determinants absolute values and do not give up the usual definition of integrals.

8 Maximal C, P and baryonic asymmetries

One of the most painful concerns in High Energy Physics is related to our seemingly inability to provide a satisfactory explanation for the maximal Parity violation observed in the weak interactions. The most popular model that may well account, through the seesaw mechanism, for the smallness of neutrino masses is quite disappointing from this point of view since parity violation is just put in by hand, as it is in the standard model, in the form of different spontaneous symmetry breaking scalar patterns in the left and right sectors. The issue is just postponed, and we are still waiting for a convincing explanation for this trick. Actually, one gets soon convinced that the difficulty comes from the fact that Parity violation apparently only exists in the weak interaction.

Much more easy would be the task to search for its origin if this violation was universal. And yet, quite interestingly, it seems possible to extend parity violation to all interactions, just exploiting the fundamental structure of fermion fields and at the same time explain why this is only detectable and apparent in the weak interactions. There exists four basic degrees of freedom, solutions of the Dirac field equations: these are $\psi_L(x), \psi_R(x), \psi_{cL}(x), \psi_{cR}(x)$ but two of them suffice to create and annihilate quanta of both charges and helicities: for instance the usual $\psi(x) = \psi_L(x) + \psi_R(x)$ may be considered as the most general Dirac solution:

$$\psi(x) = \frac{1}{(2\pi)^{3/2}} \int\limits_{p,\sigma} u(p,\sigma) a_c(p,\sigma).e^{i(px)} + v(p,\sigma) a^\dagger(p,\sigma) e^{-i(px)} \; d^3p$$

But another satisfactory base, as far as our concern is just to build kinetic_interaction terms and not mass terms, could be the pure left handed $\psi_L(x) + \psi_{cL}(x)$ field making use of the charge conjugated field.

$$\psi_c(x) = \frac{1}{(2\pi)^{3/2}} \int\limits_{p,\sigma} u(p,\sigma) a(p,\sigma)\, e^{i(px)} + v(p,\sigma) a_c^\dagger(p,\sigma) e^{-i(px)} \; d^3p$$

Indeed, from a special relativistic mass-less Hamiltonian such as

$$H_L^0 = \int d^3x [\psi_L^\dagger(-i\alpha.\nabla)\psi_L] \; + \int d^3x [\psi_{cL}^\dagger(-i\alpha.\nabla)\psi_{cL}]$$

the same normal ordered current and physics as the usual one are derived when requiring various global symmetries to become local (this is checked in the Annex).

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =: \overline{\Psi}\gamma_\mu \Psi(x) :$$

Assume now that the corresponding general Dirac field built out of only right handed components is not redundant with the previous (as is generally believed because except for a Majorana particle, both create and annihilate quanta of all charges and helicities) but lives in the conjugated metric, an assumption which we shall later justify. This then would be from our world point of view a negative energy density field. This manifestly maximal parity violating framework would not allow to detect any parity violating behavior in those interactions involving only charged Dirac particles in their multiplets, because the charge conjugated left handed field $\psi_{cL}(x)$ can successfully mimic the right handed field $\psi_R(x)$. However, in any interaction involving a completely neutral e.g Majorana fermion, $\psi_{cL}(x)$ could not play this role anymore resulting as in the weak interaction in visible maximal parity and charge violation (we claim that though no symmetry forbids it, the one degree of freedom Majorana field for a neutrino cannot be associated simultaneously with the two degrees of freedom of the Dirac charge field, since this amounts to duplicate the Majorana kinetic term and appears as an awkward manipulation, therefore one has to choose which electron/positron charge is associated with the neutral particle (neutrino) in the multiplet, this resulting in maximal charge violation and making the already present parity violation manifest). Even neutral-less fermion multiplets as in the quark sector of the weak interactions could then have inherited this parity and charge violation provided their particles lived together with neutral fermion particles in higher dimensional groups before symmetry breaking occurred producing their separation into distinct multiplets.

Now what about mass terms? For charged fields, coupling with a positive energy right handed field must take place to produce the chirality flipping mass term. But the right handed field is not there. It may be that no bare mass term is explicitly allowed to appear in an action and that a new mechanism should be found to produce interaction generated massive propagators starting from a completely mass-less action. Let us guess that such scenario is not far from the one which is actually realized in nature, because maximal Charge and Parity violation, and the related bayonic asymmetry of the universe has otherwise all of the characteristics of a not solvable issue.

But why should right handed chiral fields be negative energy density fields? Because this is what the pseudo-vector behavior under Parity of the operator four-momentum told us in section VI.1. Remember however that the unitary parity conjugated field creates positive energy point-like quanta and can be viewed as a positive point-like energy field (this is a standard QFT result). This four-vector behavior under parity of the one particle state four-momentum (an object we called a basic four-vector in section III.2) seems to be in contradiction with the pseudo-vector behavior of the four-momentum field operator. Actually the measured energy of the particle is obtained by acting with the energy operator on the one particle state ket. So there is no contradiction because this measurement is of course performed on a non zero three dimensional volume and we have to admit that the measured energy of the particle we get is negative from our world point of view as a result of the particle being living in an enantiomorphic 3-dimensionnal space. In other words, from our world point of view, the parity conjugated field has a negative energy **density,** which we may consider as a positive energy per negative inertial 3-volume, so that it leads to a negative energy when integrated on a general coordinate 3-volume (as if it was a parity scalar, the behavior of this 3-volume under a parity transformation plays no role in our discussion since this is just one of the general coordinate transformations). Then the PT fields are again positive integrated energy (energy measured in a finite volume) fields but oppositely charged (charge measured in a finite volume), i.e describing anti-particles (see VI.1) living in our world metric and interacting with their PT symmetric fields describing particles.

In short, we believe that recognizing the universality of Parity violation, i.e the fact that we are living in a left chiral world, is also an interesting approach to the issue. It then suffices to introduce the right chiral parity conjugated world (its action) to plainly restore Parity invariance of the total action. Eventually it may be, as already Sakharov suggested in 1967 [11] , that we are living in a left chiral positive energy world with its particles and antiparticles while the conjugated world is from our world point of view a right chiral negative energy world with its particles and antiparticles.

9 Synthesis

Let us gather the main information we learned from our investigation of negative energies in Relativistic QFT indicating that the correct theoretical framework for handling them should be found in a modified GR.

- The TheoreticaI Viewpoint

In second quantification, all relativistic field equations admit genuine negative energy field solutions creating and annihilating negative energy quanta. Unitary time reversal links these fields to the positive energy ones. The unitary choice, usual for all other symmetries in physics also allows to avoid the well known paradoxes associated with time reversal. Positive and negative energy fields vacuum divergences we encounter after second quantization are unsurprisingly found to be exactly opposite. The negative energy fields action must be maximised. However there is no way to reach a coherent theory involving negative energies in flat space-time. Indeed, if positive and negative energy scalar fields are time reversal conjugated, their Hamiltonian densities and actions must also be so which we shall find to be only possible in the context of general relativity thanks to the metric transformation under discrete symmetries.

- The Phenomenological Viewpoint

In a mirror negative energy world which fields remain non coupled to our world positive energy fields, stability is insured and the behavior of matter and radiation is as usual. Hence, it's just a matter of convention to define each one as a positive or negative energy world. Only if they could interact, would we expect hopefully promising new phenomenology since many outstanding enigmas, among which are the flat galactic rotation curves, the Pioneer effect, the universe flatness, acceleration and its voids, indicate that repelling gravity might play an important role in physics. On the other hand, negative energy states never manifested themselves up to now, strongly suggesting that a barrier is at work preventing the two worlds to interact except through gravity.

- The Main Issues

A trivial cancellation between vacuum divergences is not acceptable since the Casimir effect shows evidence for vacuum fluctuations. But in our approach, the positive and negative energy worlds will be maximally gravitationally coupled in such a way as to only produce exact cancellations of vacuum energies gravitational effects. Also, a generic catastrophic instability issue arises whenever quantum positive and negative energy fields are allowed to interact. If we restrict the stability issue to our modified gravity we will see that this disastrous scenario is also avoided. At last, allowing both positive and negative energy virtual photons to propagate the electromagnetic interaction simply makes it disappear. The local gravitational interaction will be treated very differently in our modified GR so that this unpleasant feature also be avoided.

- Outlooks

A left-handed kinetic and interaction Lagrangian can satisfactorily describe all known physics except mass terms which anyway remain problematic in modern physics. This strongly supports the idea that the right handed chiral fields might be living in another world (where the 3-volume reversal under parity presumably would make these fields acquire a negative energy density) and may provide an interesting explanation for maximal parity violation observed in the weak interaction.

If the connection between the two worlds is fully reestablished above a given energy threshold, then loop divergences naturally would get cancelled thanks to the positive and negative energy virtual propagators compensation. Such reconnection might take place through a new transformation process allowing particles to jump from one metric to the conjugated one [4] presumably at places where the conjugated metrics meet each other.

10 Conclusion

Of course, negative energy matter remains undiscovered at present and the stability issue strongly suggests that making it interact with normal matter requires new non standard interaction mechanisms. However, considering the seemingly many related theoretical and phenomenological issues and recalling the famous historical examples of equation solutions that were considered unphysical for a long time before they were eventually observed, we believe it is worth trying to understand how negative energy solutions should be handled in GR. We will propose a special treatment for discrete symmetry transformations in GR. A new gravitational picture will be derived in [5] opening rich phenomenological and theoretical perspectives and making us confident that the approach is on the right way.

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12 Annex

For a purely left-handed kinetic lagrangien,

$$L_{kin} = -\bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L - \bar{\Psi}_{Lc} \gamma^\mu \partial_\mu \Psi_{Lc}$$

Gauge invariance yields interaction terms :

$$L_{kin} + L_{int} = -\bar{\Psi}_L(\gamma^\mu[\partial_\mu + ieA_\mu])\Psi_L - \bar{\Psi}_{Lc}(\gamma^\mu[\partial_\mu - ieA_\mu])\Psi_{Lc}$$

from which follows the QED current :

$$[\bar{\Psi}\gamma_\mu(\frac{1-\gamma_5}{2})\Psi(x) - \bar{\Psi}_c\gamma_\mu(\frac{1-\gamma_5}{2})\Psi_c(x)]$$

Useful formula ([6] p219&225)

$$u^+(q,\sigma) = (u^*(q,\sigma))^T = (-\beta C v(q,\sigma))^T = -v(q,\sigma)^T C^T \beta^T \Rightarrow u^+(q,\sigma) = v(q,\sigma)^T C\beta$$

$$v^+(q,\sigma) = (v^*(q,\sigma))^T = (-\beta C u(q,\sigma))^T = -u(q,\sigma)^T C^T \beta^T$$
$$\Rightarrow v^+(q,\sigma) = u(q,\sigma)^T C\beta \ (1)$$

then

$$u^+(q',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2} u(q,\sigma) = v(q',\sigma')^T C\gamma_\mu \frac{1-\gamma_5}{2} u(q,\sigma)$$
$$v^+(q,\sigma)\beta\gamma_\mu \frac{1+\gamma_5}{2} v(q',\sigma') = u(q,\sigma)^T C\gamma_\mu \frac{1+\gamma_5}{2} v(q',\sigma')$$

using

$$\begin{array}{l} (C\gamma_\mu \frac{1-\gamma_5}{2})^T = \frac{1-\gamma_5^T}{2}\gamma_\mu^T C^T = -\frac{1-\gamma_5^T}{2}\gamma_\mu^T C \\ = \frac{1-\gamma_5^T}{2} C\gamma_\mu = C\frac{1-\gamma_5}{2}\gamma_\mu = C\gamma_\mu \frac{1+\gamma_5}{2} \end{array}$$

we obtain the first useful formula

$$u^+(q',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2} u(q,\sigma) = v^+(q,\sigma)\beta\gamma_\mu \frac{1+\gamma_5}{2} v(q',\sigma')$$

From (1) we get

$$\begin{array}{c} v^+(q,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} u(q',\sigma') = u(q,\sigma)^T C\gamma_\mu \frac{1-\gamma_5}{2} u(q',\sigma') \\ = u(q',\sigma')^T C\gamma_\mu \frac{1+\gamma_5}{2} u(q,\sigma) \end{array}$$

but

$$v^+(q',\sigma')\beta = u(q',\sigma')^T C$$

which leads to the second useful formula

$$v^+(q,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2} u(q',\sigma') = v^+(q',\sigma')\beta\,\gamma_\mu \frac{1+\gamma_5}{2} u(q,\sigma)$$

Computation of the left-handed current

$$\overline{\Psi}\gamma_\mu \frac{1-\gamma_5}{2}\Psi(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2}u(p,\sigma).e^{i(-p'x+px)}a^\dagger(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2}v(p,\sigma).e^{i(p'x-px)}a_c(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2}u(p,\sigma).e^{i(p'x+px)}a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1-\gamma_5}{2}v(p,\sigma).e^{i(-p'x-px)}a^\dagger(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$\overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2}v(p',\sigma').e^{i(px-p'x)}a(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2}u(p',\sigma').e^{i(-px+p'x)}a_c^\dagger(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2}v(p',\sigma').e^{i(-px-p'x)}a_c^\dagger(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p,\sigma)\beta\gamma_\mu \frac{1-\gamma_5}{2}u(p',\sigma').e^{i(px+p'x)}a(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$$\overline{\Psi_c}\gamma_\mu \frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2}u(p,\sigma).e^{i(px-p'x)}a(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2}v(p,\sigma).e^{i(-px+p'x)}a_c^\dagger(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2}v(p,\sigma).e^{i(-px-p'x)}a_c^\dagger(p,\sigma)a^\dagger(p',\sigma')d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu \frac{1+\gamma_5}{2}u(p,\sigma).e^{i(px+p'x)}a(p,\sigma)a_c(p',\sigma')d^3pd^3p'$$

$\overline{\Psi_c}\gamma_\mu\frac{1-\gamma_5}{2}\Psi_c(x) =$
$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu\frac{1+\gamma_5}{2}\, u(p,\sigma).e^{i(px-p'x)}a^\dagger(p',\sigma')a(p,\sigma)d^3pd^3p'$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu\frac{1+\gamma_5}{2}\, v(p,\sigma).e^{i(-px+p'x)}a_c(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu\frac{1+\gamma_5}{2}\, v(p,\sigma).e^{i(-px-p'x)}a^\dagger(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu\frac{1+\gamma_5}{2}\, u(p,\sigma).e^{i(px+p'x)}a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} u^*(p,\sigma)\beta\gamma_\mu\frac{1+\gamma_5}{2}\, u(p,\sigma).d^3p$$

$$+\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} v^*(p,\sigma)\beta\gamma_\mu\frac{1+\gamma_5}{2}\, v(p,\sigma).d^3p$$

$$\overline{\Psi}\gamma_\mu\frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi_c}\gamma_\mu\frac{1-\gamma_5}{2}\Psi_c(x) =$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu\, u(p,\sigma).e^{i(px-p'x)}a^\dagger(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu\, v(p,\sigma).e^{i(-px+p'x)}a_c(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} u^*(p',\sigma')\beta\gamma_\mu\, v(p,\sigma).e^{i(-px-p'x)}a^\dagger(p',\sigma')a_c^\dagger(p,\sigma)d^3pd^3p'$$

$$\frac{1}{(2\pi)^3}\int\limits_{p,p',\sigma,\sigma'} v^*(p',\sigma')\beta\gamma_\mu u(p,\sigma).e^{i(px+p'x)}a_c(p',\sigma')a(p,\sigma)d^3pd^3p'$$

$$-\frac{1}{(2\pi)^3}\int\limits_{p,\sigma} v^*(p,\sigma)\beta\gamma_\mu\, v(p,\sigma).d^3p$$

At last:

$$\overline{\Psi}\gamma_\mu\frac{1-\gamma_5}{2}\Psi(x) - \overline{\Psi_c}\gamma_\mu\frac{1-\gamma_5}{2}\Psi_c(x) =: \overline{\Psi}\gamma_\mu\Psi(x) :$$

For a Majorana field, $\Psi_c(x)$ is not there and we are left only with a chiral kinetic term:

$$\overline{\Psi}\gamma_\mu\frac{1-\gamma_5}{2}\Psi(x)$$

We believe that such term cannot be duplicated to be found associated in multiplets with both $\Psi(x)$ and $\Psi_c(x)$ of a Dirac field, so that the above chiral kinetic term will necessarily result in a chiral interaction term in which parity and charge violation explicitly manifest themselves.