The Legacy of Andrei Sakharov

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Abstract

The fact that the standard model of cosmology Lambda-Cold Dark Matter (ΛCDM) cannot account for the presence of immense voids in the very large-scale structure, as well as the early birth of first-generation stars and galaxies, combined with the impossibility of explaining the absence of observations of primordial antimatter, means that we are forced to consider a paradigm shift, which is what a new bimetric model represents. This new model has its origins in the one proposed by Andrei Sakharov in 1967. We retrace this genesis, focusing on the model's physical and mathematical coherence.

1 Introduction

Between 1967 and 1980, A. Sakharov published articles ([1, 2, 3]) presenting a cosmological model with two universes, linked by an initial singularity, the Big Bang. The first is our own universe. The second is described by him as a twin universe. The time arrows of these two universes are antiparallel, and they are enantiomorphic, or mirrored. Through such a model, he proposed a possible answer to the lack of observation of primordial antimatter. For over half a century now, cosmology has been unable to conceal one of its greatest weaknesses. Not only has no answer been found to the fact that one particle of matter in a million escaped complete annihilation, but no explanation has yet been proposed for the absence of observation of the corresponding quantity of primordial antimatter. Sakharov's attention was focused on the violation of CP symmetry. To re-establish a generalized symmetry, he hypothesized that our universe would be endowed with a twin in which these violations would be reversed. Based on the fact that matter arises from the assembly of quarks and antimatter from the assembly of antiquarks, he hypothesized that the former reaction would have been faster than the latter in our sheet of universe, with the opposite situation in its twin. Thus, in our matter universe we would find a small remnant of matter, associated with an equivalent remnant of free-state antiquarks, with the opposite situation in this twin universe, which would therefore contain antimatter with a corresponding remnant of free-state quarks. However exotic this model may seem, we have to agree that it's the only answer we've been given to this loss of half the cosmic content. Thus it's only logical that we should try to examine the aspects of such a model.

2 The significance of this inversion of time

This T-symmetry refers to the inversion of the time coordinate. In 1970 [4], contributing to the development of symplectic geometry and its application to physics, mathematician J.-M. Souriau provided the physical interpretation of this inversion of the time coordinate. The Gramm matrix defining the Minkowski space is :

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1}$$

Its isometry group is the Poincaré group:

$$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}. \tag{2}$$

Where L is the matrix representing the Lorentz group, axiomatically defined by:

$$L^{\mathsf{T}} G L = G, \tag{3}$$

and where C is the quadrivector of space-time translations:

$$C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}. \tag{4}$$

A very detailed commentary on the work can be found in reference [5]. The approach is based on the introduction of the space of motions as a dual of the Lie algebra of the group. This space of motions is thus a vector space of dimension 10, that of the group. Within these dimensions, one is identified with the energy E, of the motion, three others with its impulse, three with the spin (not quantized) and three others with a 3-vector qualified by Souriau as "passage f". Together, they form a torsor:

$$\mu = (M, P) = \begin{pmatrix} M & -P \\ P^{\mathsf{T}} & 0 \end{pmatrix}, \tag{5}$$

with :

$$M = \begin{pmatrix} 0 & -s_{\mathrm{z}} & s_{\mathrm{y}} & f_{\mathrm{x}} \\ s_{\mathrm{z}} & 0 & -s_{\mathrm{x}} & f_{\mathrm{y}} \\ -s_{\mathrm{y}} & s_{\mathrm{x}} & 0 & f_{\mathrm{z}} \\ -f_{\mathrm{x}} & -f_{\mathrm{y}} & -f_{\mathrm{z}} & 0 \end{pmatrix} \qquad P = \begin{pmatrix} E \\ p_{\mathrm{x}}c \\ p_{\mathrm{y}}c \\ p_{\mathrm{z}}c \end{pmatrix}. \tag{6}$$

If we opt for a coordinate system accompanying motion, then the 3-vector f is null. Seven parameters thus define a motion, defining a geodesic trajectory of Minkowski space. The action of the group on its motion space allows us to define all possible motions. This is written (equations (13.107) in chapter 13 of reference [4]):

$$M' = LML^{\mathsf{T}} + CP^{\mathsf{T}}L^{\mathsf{T}} - LP^{\mathsf{T}}C, \tag{7}$$

and

$$P' = LP. \tag{8}$$

It's the second equation that sheds light on the physical significance of this inversion of the time coordinate. The Lorentz group has four related components:

1. $\{L_{\mathrm{n}}\}$ is the neutral component, which contains the neutral element of the group and reverses neither space nor time;

2. $\{L_{\mathrm{s}}\}$ reverses space, but not time. It is synonymous with P-symmetry;

Combining these two related components forms the orthochronous subgroup or restricted Lorentz group:

$$\{L_{\mathrm{o}}\} = \{L_{\mathrm{n}}\} \cup \{L_{\mathrm{s}}\}.$$

3. $\{L_{\mathrm{t}}\}$ reversing time but not space;

4. $\{L_{\mathrm{st}}\}$ reversing both time and space.

By grouping these last two components together, we form the antichronous subset:

$$\{L_{\mathrm{a}}\} = \{L_{\mathrm{t}}\} \cup \{L_{\mathrm{st}}\}.$$

Finally, $\{L\} = \{L_{\mathrm{o}}\} \cup \{L_{\mathrm{a}}\}$ represents the complete Lorentz group.

The Poincaré group inherits this property. Classically, physics limits its use to its orthochronous components. Note that:

$$\{L_{\mathrm{st}}\} = -\{L_{\mathrm{n}}\}, \tag{9}$$

and

$$\{L_{\mathrm{t}}\} = -\{L_{\mathrm{s}}\}. \tag{10}$$

We can then present the complete Poincaré group as:

$$\begin{pmatrix} \lambda L_{\mathrm{o}} & C \\ 0 & 1 \end{pmatrix}, \qquad \text{with } \lambda = \pm 1. \tag{11}$$

$\lambda = +1$ gives the restricted, orthochronous Poincaré group.
$\lambda = -1$ gives the elements of the antichronous, time-reversing subgroup.
We can see that:

$$M' = L_{\mathrm{o}} M L_{\mathrm{o}}^{\mathsf{T}} + \lambda C P^{\mathsf{T}} L_{\mathrm{o}}^{\mathsf{T}} - \lambda L_{\mathrm{o}} P C^{\mathsf{T}}, \tag{12}$$

and

$$P' = \lambda L_{\mathrm{o}} P. \tag{13}$$

This gives the result of reference [4]: time inversion is synonymous with energy inversion. This also leads to mass inversion [4]:

$$m = \mathrm{sign}(E)\sqrt{E^2 - p^2} \tag{14}$$

where the vector P is called the *stress–energy vector associated with* μ. The vector p is called the *linear momentum of* P, and the scalar E is called the *energy of* P.

Here we discover the physical interpretation of the model proposed by A. Sakharov: his second universe contains particles with negative energy and negative mass, when they have them.

3 Introduction of electric charge

This is synonymous with charge conjugation and matter-antimatter duality. The geometrical interpretation of this C-symmetry was given in 1964 by J.-M. Souriau in chapter V of reference [4]. The chapter is entitled “Relativity in 5 dimensions”. The latter is then inscribed in Kaluza space, whose Gramm matrix is

$$\hat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}. \tag{15}$$

This fifth dimension is of the space type. The associated dynamic group is

$$\begin{pmatrix} \Lambda & \Gamma \\ 0 & 1 \end{pmatrix}, \tag{16}$$

where Λ is the 5D analog of the Lorentz group and axiomatically defined by:

$$\Lambda \hat{G}^{\mathsf{T}} \Lambda = \hat{G}, \tag{17}$$

and where ζ, is the fifth dimension:

$$\varGamma = \begin{pmatrix} \varDelta t \\ \varDelta x \\ \varDelta y \\ \varDelta z \\ \varDelta \zeta \end{pmatrix}. \tag{18}$$

Whereas the Lorentz group is of dimension 5, this group is of dimension 10, bringing the number of dimensions of the isometry group of Kaluza space to 15. Adding a fifth dimension has the effect of endowing the moment space with additional dimensions, five in this case. In 2014 [6], the author restricted the extension of the group $\varLambda$ under bounded space to:

$$\begin{pmatrix} 1 & 0 \\ 0 & \lambda L_{\mathrm{o}} \end{pmatrix}. \tag{19}$$

Translating this according to the group's action on 5D space gives:

$$\begin{pmatrix} 1 & 0 & \phi \\ 0 & \lambda L_{\mathrm{o}} & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta + \phi \\ \lambda L_{\mathrm{o}} \xi + C \\ 1 \end{pmatrix}. \tag{20}$$

This limits us to the subgroup of translations in this fifth dimension. The treatment by constructing the action of this new group on the dual of its Lie algebra endows the moment with an additional scalar, which can be identified with the electric charge q with:

$$q' = q, \tag{21}$$

$$M' = L_{\mathrm{o}} M L_{\mathrm{o}}^{\mathsf{T}} + \lambda C P^{\mathsf{T}} L_{\mathrm{o}}^{\mathsf{T}} - \lambda L_{\mathrm{o}} P C^{\mathsf{T}}, \tag{22}$$

and

$$P' = \lambda L_{\mathrm{o}} P. \tag{23}$$

If we are satisfied with this subgroup, we obtain the constancy of this electric charge. But in reference [7], the author decides to rely on the orthochronous isometry subgroup, of dimension 15, with its two connected components. The action of the group reveals, in addition to the scalar q, a quadrivector Q. But this is the equivalent of the passage f, which appeared in the Poincaré group. Like the latter, it is cancelled out when we opt for a coordinate system linked to the particle. It is therefore not a characteristic of motion, a relevant attribute of momentum.

In 4D, the Lorentz group can be interpreted as the "group of hyperbolic rotations in 4D space". As such, it preserves the length of the energy-impulse quadrivector P:

$$P G^{\mathsf{T}} P = E^2 - p^2 c^2. \tag{24}$$

In the same way, we can consider that the group represents "hyperbolic rotations in 5D".
Consider the pentavector:

$$\varPi = \begin{pmatrix} P \\ q \end{pmatrix} = \begin{pmatrix} E \\ p_{\mathrm{x}} c \\ p_{\mathrm{y}} c \\ p_{\mathrm{z}} c \\ q \end{pmatrix}. \tag{25}$$

The Group's action maintains its length:

$$\mathit{\Pi}\hat{G}^{\mathsf{T}}\mathit{\Pi} = E^2 - p^2c^2 - q^2. \tag{26}$$

But in approach [7], these transformations no longer a priori preserve the electric charge q, which then becomes dependent on the chosen coordinate system. In reference [7], taking up the approach initiated in [8], the author opts for a closed fifth dimension, in which the radius of this "universe tube" becomes very small, of the order of Planck's length. He then rediscovers the invariance of electric charge and concludes [7], we quote:

> In this paper, we revisit the Kaluza-Klein theory from the perspective of the classification of elementary particles based on the coadjoint orbit method. The keystone conjecture is to consider the electric charge as an extra momentum on an equal footing with the mass and the linear momentum. We study the momentum map of the corresponding symmetry group $\hat{\mathbb{G}}_1$ which conserves the hyperbolic metric. We show that the electric charge is not an invariant, *i.e.* it depends on the reference frame, which is in contradiction with the experimental observations. In other words, it is not the symmetry group of the Universe today as we know it. To avert this paradox, we scale the fifth coordinate and consider the limit when the cylinder radius ω vanishes. For the corresponding group $\hat{\mathbb{G}}_0$ also of dimension 15, the charge is an invariant then independent of the frame of reference and the observer. On this ground, we propose a cosmological scenario in which the elementary particles of the early Universe are classified from the momenta of the group $\hat{\mathbb{G}}_1$, next the three former dimensions inflate quickly while the fifth one shrinks, leading to the 4D era in which as today the particles are characterized by the momenta of the group $\hat{\mathbb{G}}_0$. By this mechanism, the elementary particles can acquire electric charge as a by-product of the $4+1$ symmetry breaking of the Universe. This work opens the way to the geometric quantization of charged elementary particles.

The expression for this characteristic dimension of this universal tube is given in [8] on page 412 :

$$\frac{\hbar}{e}\sqrt{\frac{\chi}{2\pi}}, \tag{27}$$

χ being the Einstein constant taken equal to [8]:

$$\chi = -\frac{8\pi G}{c^2} = 1.856 \times 10^{-27}\,\mathrm{cm\,g^{-1}}. \tag{28}$$

By introducing numerical values, this characteristic length is 3.782×10^{-32} cm. Dividing by 2π gives us the order of magnitude of Planck's length. In this view, the quantization of electric charge and its constancy are derived from the closure of the extra dimension associated with the decrease in the characteristic dimension associated with it.

This group refers to an extension of Poincaré's group, i.e. to a field-free, curvature-free universe. This construction of a five-dimensional relativity was suggested in 1964 in reference [8] and has been taken up again more recently in [7]. Note that it is in [8], page 413, that the link between charge conjugation and fifth-dimensional inversion is first mentioned.

By generalizing [9], we can envisage an extension of space-time to a space with $4+p$ dimensions, all of which may see their characteristic dimensions reduced, like that of this fifth dimension, each of these collapses leading to the emergence and quantization of new quantum numbers, baryonic, leptonic, unique etc., the electric charge being only the first of these.

4 Introduction of matter-antimatter symmetry

The above technique can be extended and the letter ζ no longer designates one additional dimension, but p additional dimensions, according to:

$$\zeta = \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \end{pmatrix}. \tag{29}$$

The Gramm matrix becomes:

$$\widehat{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & -1 & 0 & 0 & \dots & 0 \\ 0 & 0 & -1 & 0 & \dots & 0 \\ & & & & & \\ 0 & 0 & 0 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & -1 \end{pmatrix}. \tag{30}$$

If we extend the group from the restricted Poincaré group and its orthochronous subgroup, the isometry group of this $4+p$-dimensional space is written as:

$$\begin{pmatrix} \widehat{\Lambda}_o & \widehat{\Gamma} \\ 0 & 1 \end{pmatrix}, \tag{31}$$

with

$$\widehat{\Lambda}_o \widehat{G}^{\mathsf{T}} \widehat{\Lambda}_o = \widehat{G}. \tag{32}$$

The closure and shortening of the characteristic lengths associated with the additional p dimensions leads to quantization and constancy of the resulting quantum numbers. If the basic group is the orthochronous restricted Poincaré group, the number of related components remains limited to two. In passing, we can return to a simplified formulation where the group extension of is limited to p translations along p additional dimensions, ensuring the constancy of the p quantum charges that result:

$$\begin{pmatrix} 1 & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & 1 & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & \phi_p \\ 0 & 0 & \dots & 0 & L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta_1 + \phi_1 \\ \zeta_2 + \phi_2 \\ \dots \\ \zeta_p + \phi_p \\ L_o \xi + C \\ 1 \end{pmatrix}. \tag{33}$$

By introducing a charge symmetry, a C-symmetry, we double this number of related components by writing this group according to:

$$\begin{pmatrix} \mu & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & \mu & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mu & 0 & \phi_p \\ 0 & 0 & \dots & 0 & L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \mu\zeta_1 + \phi_1 \\ \mu\zeta_2 + \phi_2 \\ \dots \\ \mu\zeta_p + \phi_p \\ L_o \xi + C \\ 1 \end{pmatrix}, \qquad \mu = \pm 1. \tag{34}$$

The corresponding group action is

$$q_1' = \mu q_1 \, ; \, q_2' = \mu q_2 \, ; \ldots ; \, q_p' = \mu q_p, \tag{35}$$

$$M' = L_o M L_o^\mathsf{T} + C P^\mathsf{T} L_o^\mathsf{T} - L_o P C^\mathsf{T}, \tag{36}$$

and

$$P' = L_o P. \tag{37}$$

Symmetry $\mu = -1$ reverses all quantum charges.

5 Group associated with A. Sakharov's model: the Janus group

If we want to construct a group that translates the T-symmetry invoked by Sakharov, we'll replace L_o by λL_o with $\lambda = \pm 1$. But, as proposed in [6], we can translate what had already been proposed [1], we quote:

> All phenomena corresponding to $t < 0$ are, in this hypothesis, assumed to be CPT images of phenomena corresponding to $t > 0$.

$$\begin{pmatrix} \lambda\mu & 0 & \ldots & 0 & 0 & \phi_1 \\ 0 & \lambda\mu & \ldots & 0 & 0 & \phi_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \lambda\mu & 0 & \phi_p \\ 0 & 0 & \ldots & 0 & \lambda L_o & C \\ 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \ldots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda\mu\zeta_1 + \phi_1 \\ \lambda\mu\zeta_2 + \phi_2 \\ \ldots \\ \lambda\mu\zeta_p + \phi_p \\ \lambda L_o \xi + C \\ 1 \end{pmatrix}, \qquad \text{with} \quad \begin{cases} \mu = \pm 1, \\ \lambda = \pm 1. \end{cases} \tag{38}$$

The value $\lambda = -1$ is then synonymous with CPT-symmetry.

6 Topology of the Janus model

Let's consider a universe closed in all its dimensions, including space and time (see figure 1 on the following page).

Diametrically opposed, antipodal points can be brought into coincidence. The image is then that of a $\mathbb{P}^2$ projective. The north and south poles, one representing the Big Bang and the other the Big Crunch, come into coincidence. The sphere cannot be paved without the presence of these two singularities. The same applies to any sphere $\mathbb{S}^{2n}$ if n is even, especially if this dimension is 4. This geometry was proposed in [10].

The figure 2 on page 10 shows how this coincidence of antipodal regions generates this T-symmetry. On the $\mathbb{S}^2$ sphere, the direction of time is given by the orientation of the meridian curves. This orientation is shown on the left at the new state of maximum expansion, when space is identified with the sphere's equator. During this folding of the $\mathbb{S}^2$ sphere, described in reference [11] page 65, the vicinity of this equator is configured as the two-folds cover of a Möbius strip with three half-turns (see figure 2 on page 10 on the right).

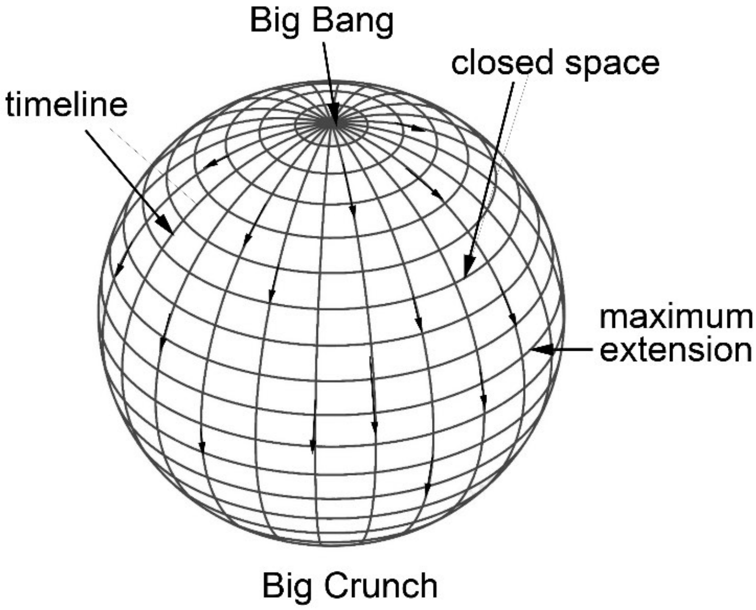

Figure 1: 2D didactic image of a closed universe, a sphere $\mathbb{S}^2$.

In the figure 3 on the next page, we evoke the appearance of T-symmetry by manipulating the vicinity of a meridian line. In addition, we evoke the possible elimination of the Big Bang - Big Crunch double singularity by replacing them with a tubular passage, which then gives this geometry the nature of the two-fold cover of a Klein bottle.

For enantiomorphy and P-symmetry to appear, the operation would have to be performed on a larger sphere. This aspect can be highlighted by considering the conjunction of antipodal regions in the vicinity of a meridian line, which is then configured according to the two-sheet covering of a half-turn Möbius strip. The figure 4 on the following page illustrates this enantiomorphic situation.

By bringing the antipodal points of even-dimensional spheres into coincidence, we locally create a configuration associating two T-symmetrical sheets. By adding further dimensions, the coincidence of the antipodes creates a two-sheet CPT-symmetric coating configuration of a projective space. In the case of the sphere $\mathbb{S}^2$, which corresponds only to a 2D didactic image, the image of the projective $\mathbb{P}^2$ is its immersion in, which corresponds to the surface described in 1903 by the German mathematician Werner Boy [12]. See figure 5 on page 11. In this figure, we show how the coincidence of the antipodal points of the equator of the sphere $\mathbb{S}^2$ gives the two-sheet covering of a Möbius ribbon with three half-turns.

In this section, we show that the P- and T-symmetries invoked by A. Sakharov can be manifested as consequences of a purely topological structure, the covering of a projective $\mathbb{P}^4$.

7 First attempt to introduce negative masses into the model

Using dynamical group theory, we showed that this T-symmetry was synonymous with the introduction of negative masses into the cosmological model. A. Sakharov's primordial antimatter would therefore be endowed with negative mass. This first step is far from anecdotal since, if we neglect it, we admit to losing nothing less than half the universe from the outset. Is it then possible to introduce negative masses into the standard model of general relativity?

A first idea would be to consider that the field comes from two sources, represented by two tensors, the first referring to a positive mass content and the second to a negative mass content:

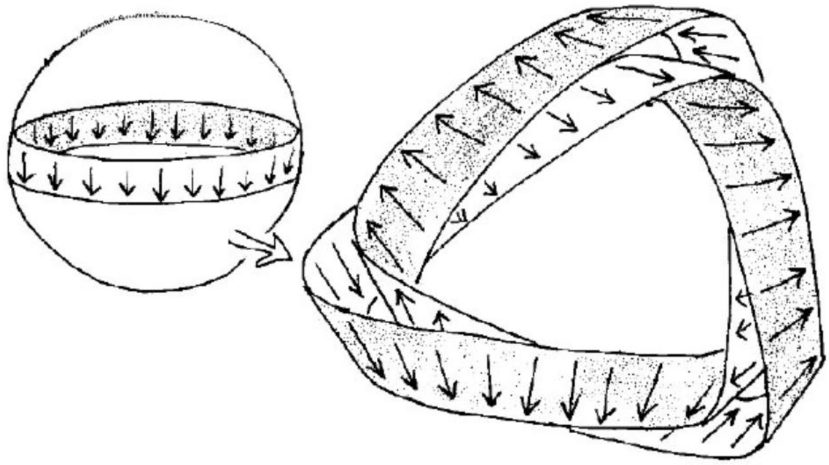

Figure 2: How the coincidence of antipodal regions creates T-symmetry. Drawing extracted from [11], page 65.

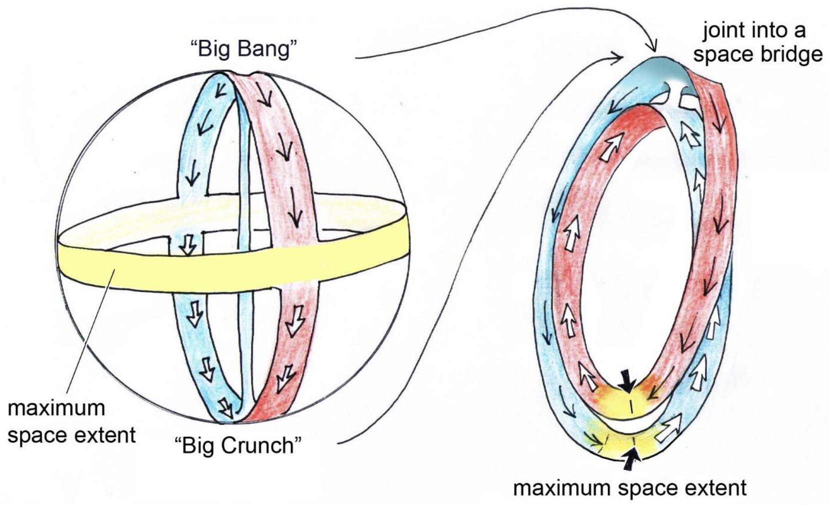

Figure 3: Coincidence of antipodal regions on a sphere $\mathbb{S}^2$, according to the two-folds cover of a half-turn Möbius strip, with the appearance of T-symmetry.

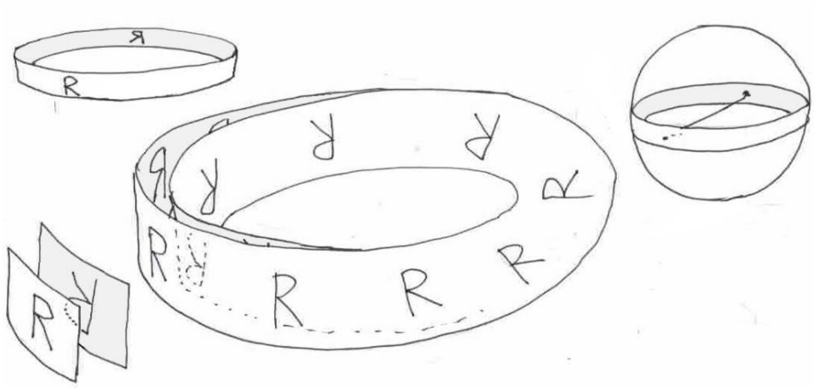

Figure 4: P-symmetry as a consequence of contacting antipodal region neighborhoods on an $\mathbb{S}^2$ sphere.

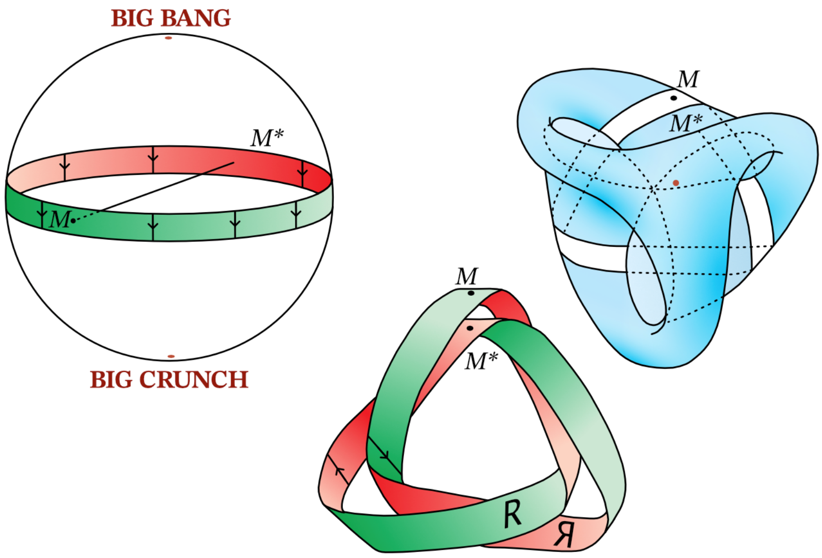

Figure 5: Boy's surface, immersion of the $\mathbb{P}^2$ projective in $\mathbb{R}^3$.

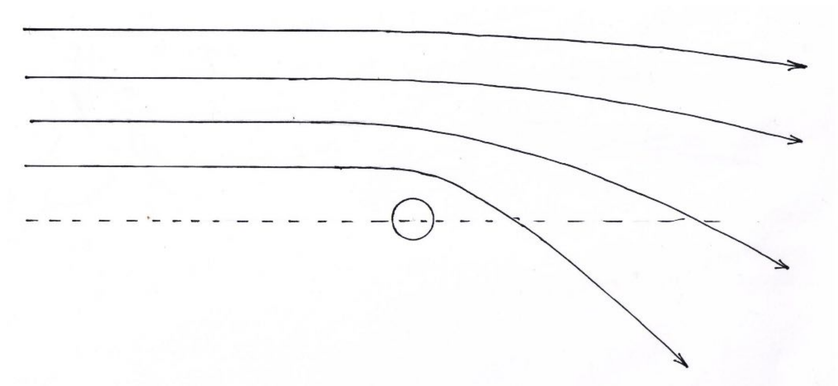

Figure 6: Geodesics created by a positive mass.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi \left[T^{(+)}_{\mu\nu} + T^{(-)}_{\mu\nu} \right]. \tag{39}$$

We can then consider the metric solution corresponding to a region where the field is created, firstly by a positive mass content:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T^{(+)}_{\mu\nu}. \tag{40}$$

Geodesics are given by a solution in the form of an external metric:

$$\mathrm{d}s^2 = \left(1 - \frac{2GM^{(+)}}{c^2 r}\right) c^2\,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 - \dfrac{2GM^{(+)}}{c^2 r}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right). \tag{41}$$

The geodesics evoke an attraction (see figure 6).

In this context, our single field equation provides only a single family of geodesics, which the test particles, with both positive and negative masses, must follow. We deduce that:

- Positive masses attract both positive and negative masses.

  Now consider the field created by a negative mass $M^{(-)}$:

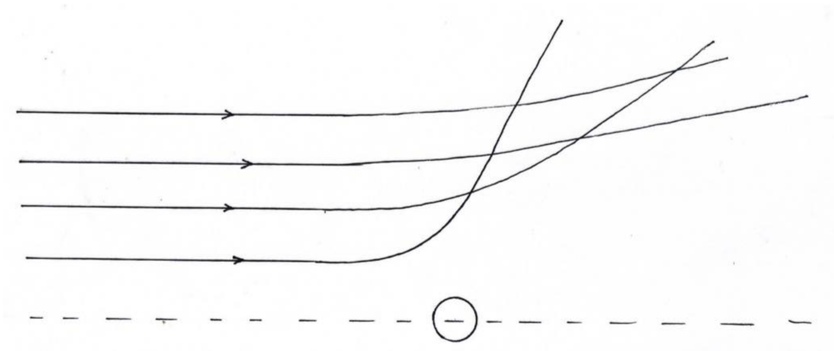

Figure 7: Geodesics created by negative mass.

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \chi T^{(-)}_{\mu\nu}. \tag{42}$$

The solution then corresponds to the metric:

$$\mathrm{d}s^2 = \left(1 + \frac{2G\left|M^{(-)}\right|}{c^2 r}\right) c^2 \,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1 + \dfrac{2G\left|M^{(-)}\right|}{c^2 r}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \,\mathrm{d}\varphi^2\right). \tag{43}$$

The geodesics then represent a repulsion (see figure 7).

- Negative masses repel both positive and negative masses.

  This result was illustrated in 1957 by H. Bondi [13]. If two masses of identical absolute values but opposite signs are brought together, the positive mass flees, pursued by the negative mass. Both then accelerate uniformly, but without any energy input, since the energy of the negative mass is itself negative. This phenomenon is known as runaway. What's more, this scheme violates the action-reaction principle. In 1957, the conclusion was reached that it was physically impossible to include negative masses in the cosmological model. This would only be possible at the price of a profound paradigmatic shift, not by denying the achievements of general relativity, but by considering its extension in a wider geometric context.

  We deduce that negative masses repel both positive masses and negative masses.

8 A paradigm shift to escape the crisis of today's cosmology

In the mid-1970s, the excessive rotation speeds of stars in galaxies had already led specialists to propose the existence of dark matter, ensuring their cohesion. In 2011, the discovery that the cosmic expansion was accelerating was attributed to a new, unknown ingredient known as dark energy. Over the decades, all attempts to assign an identity to these new components ended in failure.

In 2017 [14], Hélène Courtois, Daniel Pomarède, Brent Tully and Yeudi Hoffman produced the first very-large-scale mapping of the universe, in a cube of one and a half billion light-years across, with the Milky Way, our observation point, at the center (see figure 8 on the following page). By subtracting the radial component of the velocity linked to the expansion motion, they indicate the trajectories followed by the masses. A dipolar structure appears. One formation, the Shapley attractor, comprising hundreds of thousands of galaxies, attracts galaxies to itself. But, symmetrically to this formation, 600 million light-years from the Milky Way, there is an immense

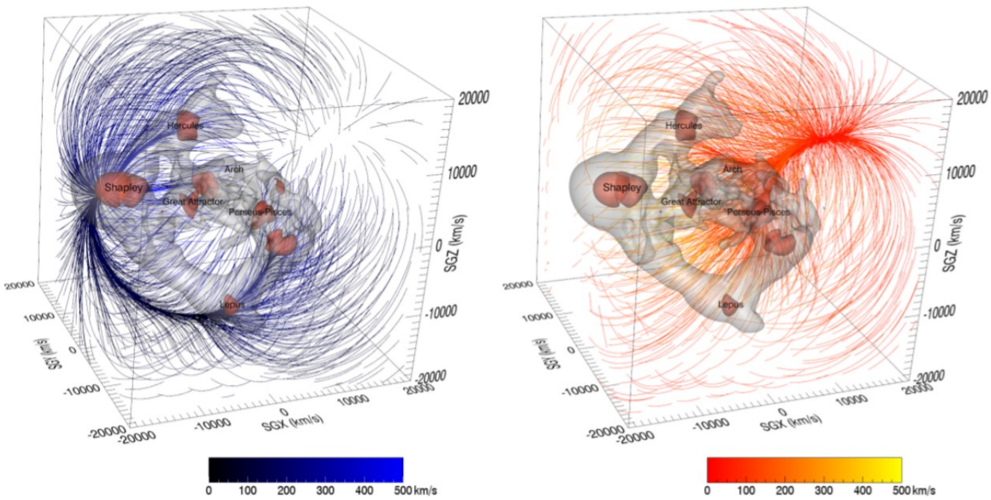

Figure 8: The figure from [14] shows the location of the Dipole Repeller (highlighted by the red circle) within the large-scale structure of the universe. The Dipole Repeller is a hypothesized region of space where galaxies are pushed away from, counteracting the attractive force of the Shapley Supercluster.

void, some one hundred million light-years across, which, on the contrary, repels galaxies, and to which we give the name of dipole repeller. To date, no theory has been able to explain the existence of this vast void. While the idea of a gap in dark matter, positive and attractive, has been evoked, it doesn't hold water, as no mechanism has been found to give rise to it. Since 2017, several other such voids have been detected and located.

The launch of the James Webb Space Telescope has only added to this crisis [15]. The Standard Model ΛCDM proposes a hierarchical mechanism for the birth of stars and galaxies. Gravitational instability appears as soon as matter and radiation are decoupled. The scenarios for the formation of both stars and galaxies in this model make use of the attributes conferred on hypothetical dark matter. But even with these parameters, it's impossible to imagine galaxies forming before a billion years. The Hubble Space Telescope was already able to obtain images in the near infrared. Early images of distant objects appeared to show groups of mini-galaxies. But the James Webb Space Telescope showed that these objects were nothing other than HII regions belonging to barred spiral galaxies, fully formed, hosting old stars, only 500 million years old.

For decades, the Standard Model ΛCDM has relied on its ability to account for CMB fluctuations as gravito-acoustic oscillations, by adapting the numerous parameters relating to dark matter, dark energy and, in particular, the value of the Hubble constant. This desire to match observational data has resulted in a Hubble constant value of $67\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This is significantly lower than the value of $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ deduced from direct observation of standard candles.

All these factors are creating a deep crisis within the specialist community, and some voices are beginning to be heard, suggesting the need to consider a profound paradigm shift. This is what the Janus cosmological model[1] proposes.

Since we are unable to introduce negative masses into the general relativity model, let's consider a profound change of geometric paradigm, already evoked in the previous sections under the aspect of group theory and topology. The motion of positive masses, immersed in the gravitational field, takes place according to geodesics that we consider to be derived from a first metric $g_{\mu\nu}$. We will therefore describe the motion of negative masses using a second set of geodesics, derived from a second metric $\overline{g}_{\mu\nu}$. We thus have a manifold, whose points are marked

[1] see section 9 on the next page, where this model is developped.

by the coordinates $\{x^0, x^1, x^2, x^3\}$, equipped with a pair of metrics $(g_{\mu\nu}, \overline{g}_{\mu\nu})$. We shall neglect the action of electromagnetic fields and consider only the field of gravity. From the metrics and we can construct Ricci tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ and their associated Ricci scalars R and $\overline{R}$.

9 The Janus Cosmological Model

To build this model, we'll start with the action:

$$J = \int_{D4} \left[(R + S + \sigma)\sqrt{-g} + \left(\overline{R} + \overline{S} + \overline{\sigma}\right)\sqrt{-\overline{g}} \right] \mathrm{d}^4x. \tag{44}$$

Terms involving Ricci scalars are treated as classically, in the derivation of the Einstein equation from the Hilbert-Einstein action. It comes to:

$$\begin{aligned} \delta J = &\int_{D4} \left[\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}S\right)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\sigma\right)}{\delta g^{\mu\nu}} \right] \sqrt{-g}\delta g^{\mu\nu} \,\mathrm{d}^4x \\ &+ \int_{D4} \left[\frac{\delta\overline{R}}{\delta\overline{g}^{\mu\nu}} + \frac{\overline{R}}{\sqrt{-\overline{g}}} \frac{\delta\sqrt{-\overline{g}}}{\delta\overline{g}^{\mu\nu}} + \frac{1}{\sqrt{-\overline{g}}} \frac{\delta\left(\sqrt{-\overline{g}}\overline{S}\right)}{\delta\overline{g}^{\mu\nu}} + \frac{1}{\sqrt{-\overline{g}}} \frac{\delta\left(\sqrt{-\overline{g}}\overline{\sigma}\right)}{\delta\overline{g}^{\mu\nu}} \right] \sqrt{-\overline{g}}\delta\overline{g}^{\mu\nu} \,\mathrm{d}^4x. \end{aligned} \tag{45}$$

Posing:

$$\frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}S\right)}{\delta g^{\mu\nu}} = -\chi T_{\mu\nu}, \tag{46}$$

$$\frac{1}{\sqrt{-\overline{g}}} \frac{\delta\left(\sqrt{-\overline{g}}\overline{S}\right)}{\delta\overline{g}^{\mu\nu}} = -\chi\overline{T}_{\mu\nu}, \tag{47}$$

$$\frac{1}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\sigma\right)}{\delta g^{\mu\nu}} = \chi t_{\mu\nu}, \tag{48}$$

and

$$\frac{1}{\sqrt{-\overline{g}}} \frac{\delta\left(\sqrt{-\overline{g}}\overline{\sigma}\right)}{\delta\overline{g}^{\mu\nu}} = \chi\overline{t}_{\mu\nu} \tag{49}$$

Hence the system:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \chi\left[T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}}t_{\mu\nu}\right], \tag{50a}$$

$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\overline{g}_{\mu\nu} = -\chi\left[\overline{T}_{\mu\nu} + \sqrt{\frac{g}{\overline{g}}}\overline{t}_{\mu\nu}\right]. \tag{50b}$$

$T_{\mu\nu}$ is the source tensor of the field acting on positive masses and positive-energy photons. The term $\sqrt{\frac{\overline{g}}{g}}$ is the source of this field attributed to the action of negative masses on these positive masses. The tensor $t_{\mu\nu}$ is therefore referred to the interaction tensor. The terms $\overline{T}_{\mu\nu}$ and $\overline{t}_{\mu\nu}$ are the corresponding source elements of the second equation, $\overline{t}_{\mu\nu}$ being the second corresponding interaction tensor. Let's write this system of coupled equations in mixed notation:

$$R^\nu_\mu - \frac{1}{2} R g^\nu_\mu = \chi \left[T^\nu_\mu + \sqrt{\frac{\overline{g}}{g}} t^\nu_\mu \right], \tag{51a}$$

$$\overline{R}^\nu_\mu - \frac{1}{2} \overline{R} \overline{g}^\nu_\mu = -\chi \left[\overline{T}^\nu_\mu + \sqrt{\frac{g}{\overline{g}}} \overline{t}^\nu_\mu \right]. \tag{51b}$$

General relativity produces only a limited number of exact solutions. We'll follow the same logic.

10 Construction of a time-dependent, homogeneous and isotropic solution

Given the symmetry assumptions, the metrics then have the FLRW form. The variable x^0 is the common chronological coordinate (time marker).

$$g_{\mu\nu} = \mathrm{d}x^{0\,2} - a^2 \left[\frac{\mathrm{d}u^2}{1 - k u^2} + u^2 \,\mathrm{d}\theta^2 + u^2 \sin^2\theta \,\mathrm{d}\varphi^2 \right], \tag{52a}$$

$$\overline{g}_{\mu\nu} = \mathrm{d}x^{0\,2} - \overline{a}^2 \left[\frac{\mathrm{d}u^2}{1 - \overline{k} u^2} + u^2 \,\mathrm{d}\theta^2 + u^2 \sin^2\theta \,\mathrm{d}\varphi^2 \right]. \tag{52b}$$

The determinants of the two metrics are

$$g = -a^6 \sin^2\theta, \qquad \overline{g} = -\overline{a}^6 \sin^2\theta. \tag{53}$$

As shown in reference [6] the treatment of the two equations leads to the compatibility relation:

$$\rho c^2 a^3 + \overline{\rho} \overline{c}^2 \overline{a}^3 = E = \mathrm{cst}. \tag{54}$$

This translates into conservation of energy, extended to both populations. The exact solution, referring to two dust universes, corresponds to:

$$k = \overline{k} = -1 \tag{55}$$

and:

$$a^2 \frac{\mathrm{d}^2 a}{\mathrm{d}x^{0\,2}} = -\frac{4\pi G}{c^2} E, \tag{56a}$$

$$\overline{a}^2 \frac{\mathrm{d}^2 \overline{a}}{\mathrm{d}x^{0\,2}} = +\frac{4\pi G}{\overline{c}^2} E. \tag{56b}$$

A theoretical model is of no interest if it cannot be compared with observational data. The evolution of the positive species will correspond to an acceleration if the energy E of the system is negative. This provides a physical interpretation of the acceleration of the cosmic expansion ([16, 17]), which then follows from the fact that the energy content is predominantly negative. Numerical data have been successfully compared with observational data [18]. The corresponding curve is shown in figure 9 on the following page.

To complete the model, we now need to provide exact stationary solutions. We'll restrict ourselves to SO(3) symmetry.

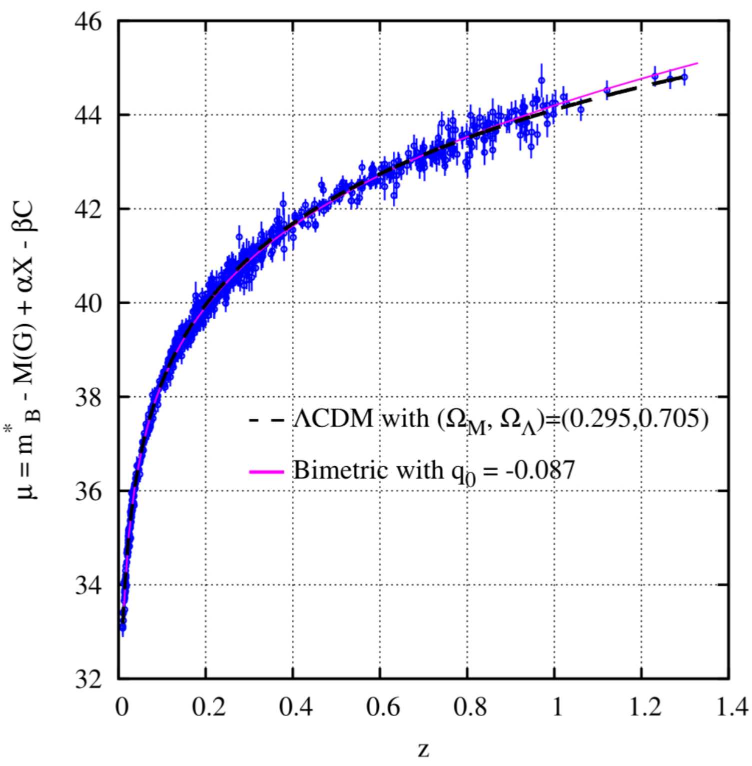

Figure 9: Comparison of observed and theoretical magnitudes as a function of z redshift.

11 Interaction laws and observational consequences

From the metrics we note and we know how to construct covariant derivatives that we note:

$$\partial^\nu \quad \text{and} \quad \overline{\partial}^\nu \tag{57}$$

By construction we have:

$$\partial^\nu\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) = \overline{\partial}^\nu\left(\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\overline{g}_{\mu\nu}\right) = 0. \tag{58}$$

The corresponding covariant derivatives of the two second members must therefore also be zero, which corresponds to the Bianchi conditions, implying:

$$\partial^\nu T_{\mu\nu} = 0, \tag{59}$$

$$\partial^\nu\left[\sqrt{\frac{\overline{g}}{g}}t_{\mu\nu}\right] = 0, \tag{60}$$

$$\overline{\partial}^\nu \overline{T}_{\mu\nu} = 0, \tag{61}$$

and

$$\overline{\partial}^\nu\left[\sqrt{\frac{g}{\overline{g}}}\overline{t}_{\mu\nu}\right] = 0. \tag{62}$$

In stationary conditions, the square roots of the ratios of the determinants behave like constants, reflecting an "apparent mass effect". Conditions (60) and (62) can therefore be replaced by:

$$\partial^{\nu} t_{\mu\nu} = 0, \tag{63}$$

and

$$\overline{\partial}^{\nu} \overline{t}_{\mu\nu} = 0. \tag{64}$$

Let's write the system of equations in mixed notation, replacing the square roots, which have become constant, by the positive constants b^2 and $\overline{b}^2$:

$$R^{\nu}_{\mu} - \frac{1}{2} R g^{\nu}_{\mu} = \chi \left[T^{\nu}_{\mu} + b^2 t^{\nu}_{\mu} \right], \tag{65a}$$

$$\overline{R}^{\nu}_{\mu} - \frac{1}{2} \overline{R} \overline{g}^{\nu}_{\mu} = -\chi \left[\overline{T}^{\nu}_{\mu} + \overline{b}^2 \overline{t}^{\nu}_{\mu} \right]. \tag{65b}$$

Using the Newtonian approximation, in both populations the non-zero tensor terms reduce to:

$$T^0_0 = \rho c^2 > 0 \qquad t^0_0 = \overline{\rho c}^2 < 0 \qquad \overline{T}^0_0 = \overline{\rho c}^2 < 0 \qquad \overline{t}^0_0 = \rho c^2 > 0. \tag{66}$$

In our system of coupled field equations, the presence of a minus sign in front of the second member of the second equation gives the following interaction laws:

- Masses of the same sign attract each other;
- Masses of opposite signs repel each other.

We have thus eliminated the runaway effect.

The first conclusion to be drawn is that where one of the two types of mass is present, the other is absent, as immediately confirmed by simulations [19]. This is the case in the vicinity of the Sun, and under these conditions the first equation is identified with Einstein's 1915 equation. The model is therefore in line with all the classical local observational data of general relativity: Mercury's perihelion advance, deflection of light rays by the Sun. The model therefore does not invalidate that of general relativity, but presents itself as its extension, made essential to integrate the new observational data, which can no longer be managed by introducing the hypothetical components of dark matter and dark energy.

We have seen, in our construction of the unsteady solution, that negative energy dominates. The model is thus profoundly asymmetrical. Negative mass content will now replace the dark matter + dark energy pair. By the way, going back to the original idea, inspired by the work of Andrei Sakharov, this allows us to attribute a well-defined identity to these components. They are invisible, insofar as negative masses emit photons of negative energy that our optical instruments cannot capture. They are therefore simply copies of our own antimatter, assigned a negative mass. We then have a new distribution of contents (see figure 10 on the next page).

At the moment of decoupling, when the gravitational instability can play its role (we must then speak of joint gravitational instabilities), the characteristic Jeans time is shorter for negative masses:

$$\overline{t}_{\mathrm{J}} = \frac{1}{\sqrt{4\pi G |\overline{\rho}|}} \ll t_{\mathrm{J}} = \frac{1}{\sqrt{4\pi G \rho}}. \tag{67}$$

The result will be a regular distribution of negative-mass conglomerates of spheroidal antihydrogen and negative-mass antihelium. These will behave like immense negative-mass protostars. As soon as their temperature causes hydrogen reionization, their contraction will cease. These formations will then radiate in the red and infrared wavelengths. But their cooling time is then large compared to the age of the universe, which means that these objects will no longer evolve.

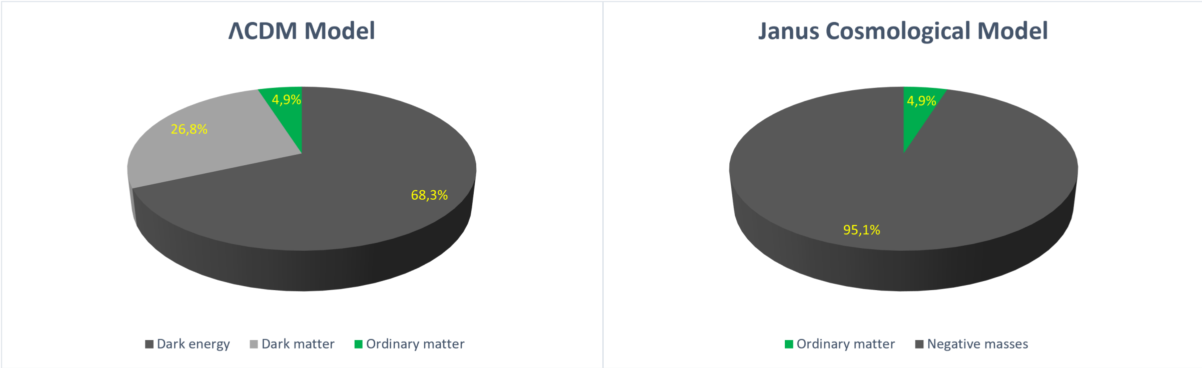

Figure 10: Comparative contents of the ΛCDM and Janus models.

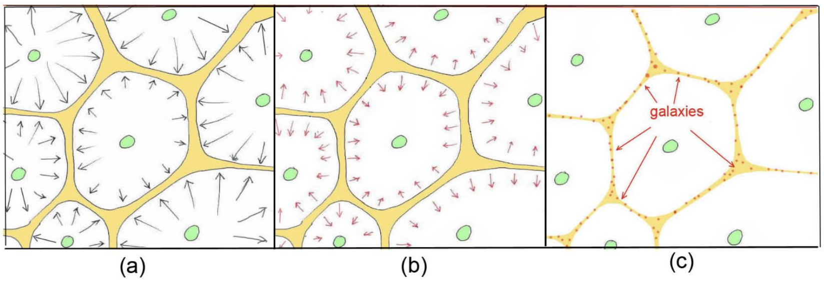

Figure 11: Early rapid star and galaxy formation.

The history of this negative world is totally different from our own. It will not give rise to stars, galaxies or planets. It will contain no atoms heavier than negative-mass antihelium. And there will be no life. And, as we'll see later: these negative formations are deliberately situated within the Newtonian approximation.

But there's another very important point. When these spheroidal conglomerates form, they confine the positive mass to the residual space, giving it a lacunar structure, comparable to joined soap bubbles. The negative mass is thus distributed in the form of thin plates, sandwiched between two negative conglomerates that exert a strong back pressure on it. The positive mass is thus violently compressed and heated. However, due to its plate-like arrangement, it can cool down very quickly through the emission of radiation (see figure 11).

The result is a pattern of first-generation star and galaxy formation totally different from the standard one. This configuration had been the subject of simulations [19] since the first, heuristic, approach to the model, and the fact that objects all form within the first hundred million years was one of its predictions, largely confirmed by JWST data.

The lacunar structure, advocated as early as 1995 [19], predicted the existence of large voids, which the discoveries of the dipole repeller and other similar large voids have also confirmed.

Once this lacunar structure has been formed, matter tends to concentrate along the segments common to three gaps, forming filaments (see figure 12 on the next page). The nodes of this distribution will only develop into galaxy clusters.

12 The problem of the model's mathematical consistency

This is ensured in an isotropic, homogeneous and unsteady situation, the required condition being the generalized conservation of energy expressed by equation (54). We now turn to the case of stationary solutions, limiting ourselves to those that satisfy SO(3) symmetry. Bianchi conditions must then be satisfied, i.e. relations (59), (61), (63) and (64).

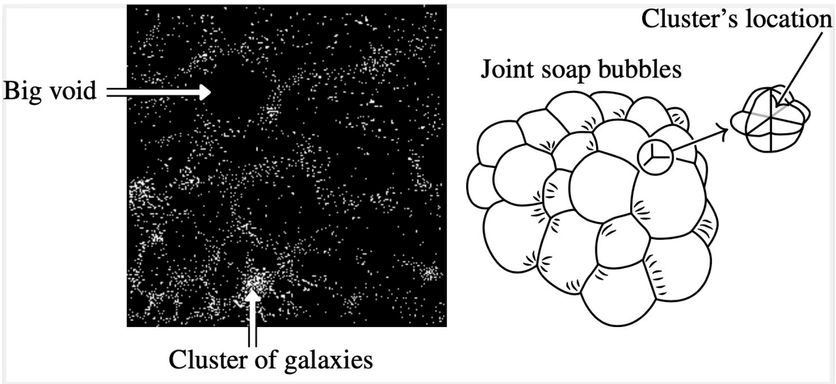

Figure 12: Structure of positive mass in contiguous bubbles.

First, we'll show the existence of asymptotic consistency in Newtonian approximation situations. Let's recall its various aspects:

- Velocities must be negligible compared to the speed of light. This is the case for velocities $\langle v\rangle$ and $\langle\overline{v}\rangle$ of thermal agitation in both media, which are involved in the definition of pressures and in both media. After decoupling:

$$\varepsilon p = \frac{\varepsilon\rho\langle v\rangle}{3} \quad \text{and} \quad \varepsilon\overline{p} = \frac{\varepsilon\overline{\rho}\langle\overline{v}\rangle}{3}. \tag{68}$$

- Curvature effects must be neglected. We translate this by saying that the values of the radial coordinate must be large in front of the length characterizing curvature effects, i.e. the Schwarzschild radius.

12.a Newtonian approximation, field created by a positive mass M

We introduce the Schwarzschild radius, ε being a small parameter:

$$\varepsilon R_{\mathrm{S}} = \varepsilon\frac{2GM}{c^2}. \tag{69}$$

SO(3) symmetry imposes the shapes of the two metrics:

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\,\mathrm{d}x^{0\,2} - \mathrm{e}^{\lambda}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{70a}$$

$$\mathrm{d}\overline{s}^2 = \mathrm{e}^{\overline{\nu}}\,\mathrm{d}x^{0\,2} - \mathrm{e}^{\overline{\lambda}}\,\mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{70b}$$

The construction of a stationary solution then requires to calculate the functions:

$$\nu(r), \quad \lambda(r), \quad \overline{\nu}(r), \quad \text{and} \quad \overline{\lambda}(r). \tag{71}$$

To locate this solution, we need to consider the shapes of the field source tensors:

$$T_{\mu}^{\nu}, \quad t_{\mu}^{\nu}, \quad \overline{T}_{\mu}^{\nu}, \quad \text{and} \quad \overline{t}_{\mu}^{\nu}. \tag{72}$$

Let's start by considering a situation where only positive mass is present. The tensors t_{μ}^{ν} and $\overline{T}_{\mu}^{\nu}$ are then null. The form of the tensor T_{μ}^{ν} is

$$T_{\mu}^{\nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -\varepsilon p & 0 & 0 \\ 0 & 0 & -\varepsilon p & 0 \\ 0 & 0 & 0 & -\varepsilon p \end{pmatrix}. \tag{73}$$

As we're in the Newtonian approximation, ε is very small. With the introduction of the metric (70a) and the tensor (73) in the first field equation we are led to introduce the function $m(r)$ such that:

$$\mathrm{e}^{-\lambda} = 1 - \frac{2m(r)}{r}. \tag{74}$$

The classic calculation then leads to the relationship:

$$m(r) = 2\frac{4\pi G r^2 \rho}{3c^2} \leq \frac{2GM}{c^2} = R_{\mathrm{S}}. \tag{75}$$

We then obtain the Tolman-Oppenheimer-Volkoff equation (TOV). Relation (75) places the small quantity in front of any quantity that will be neglected in the Newtonian approximation:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left(\rho + \varepsilon p/c^2\right)\left(m + 4\pi\varepsilon G p r^3/c^4\right)c^2}{r(r - 2\varepsilon G m/c^2)}. \tag{76}$$

When ε tends to zero (or c tends to infinity) we get:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\rho m c^2}{r^2} = -\frac{G\rho}{r^2}\frac{4\pi r^3 \rho}{3}. \tag{77}$$

The quantity $\frac{4\pi r^3 \rho}{3}$ represents the amount of matter $\mu(r)$ contained inside a sphere of radius r. We know that the force of gravity exerted inside a mass of constant density is equivalent to that exerted by the mass located at the center of the sphere, and that the mass located outside this sphere gives a force of zero. So the quantity $-\frac{G\rho\mu(r)}{r^2}$ is the force of gravity, per unit volume, acting on the matter contained in an elementary volume around a point at distance r from the center. Thus the relation (73), which follows from the Newtonian approximation, expresses that the force of gravity balances the force of pressure. This is the classic Euler relationship.

To ensure the mathematical consistency of the system of two field equations, we therefore need to consider a form of the tensor t_μ^ν that gives back this same Euler relation when the Newtonian approximation is also applied to this solution. This is guaranteed with the form:

$$t_\mu^\nu = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & +\varepsilon p & 0 & 0 \\ 0 & 0 & +\varepsilon p & 0 \\ 0 & 0 & 0 & +\varepsilon p \end{pmatrix}. \tag{78}$$

The calculation leads to the TOV equation equivalent of the second equation:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{\left(\rho - \varepsilon p/c^2\right)\left(m - 4\pi\varepsilon G p r^3/c^4\right)c^2}{r(r + 2\varepsilon G m/c^2)}. \tag{79}$$

This is not identical to equation (76) but, like it, identifies with the Euler equation when ε tending towards zero.

- We deduce that mathematical consistency is asymptotically assured when the conditions satisfy the Newtonian approximation.

It is then possible to construct the two metric solutions describing the geometry inside the sphere of radius r_0 containing matter of constant density ρ.

Posing:

$$\hat{R} = \sqrt{\frac{3c^2}{8\pi G\rho}} \quad \text{and} \quad r_{\mathrm{S}} = \frac{2GM}{c^2} \quad \text{(Schwarzschild's lenght)}.$$

We have used a lower-case letter r_{S} for the Schwarzschild radius to illustrate the fact that in the Newtonian approximation $r_{\mathrm{S}} \ll r_0$. Conversely, for the characteristic radius associated with the interior metric I have used a capital letter $\hat{R}$, because then $\hat{R} \gg r_0$. The first interior metric corresponds to the classical solution:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{1-\frac{r_0^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1-\frac{r^2}{\hat{R}^2}}\right]^2 \mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1-\frac{r^2}{\hat{R}^2}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{80a}$$

The second is

$$\mathrm{d}\overline{s}^2 = \left[\frac{3}{2}\sqrt{1+\frac{r_0^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1+\frac{r^2}{\hat{R}^2}}\right]^2 \mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1+\frac{r^2}{\hat{R}^2}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{80b}$$

These metrics connect with external metric solutions:

$$\mathrm{d}s^2 = \left(1-\frac{r_\mathrm{S}}{r}\right)c^2\,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{r_\mathrm{S}}{r}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{81a}$$

$$\mathrm{d}\overline{s}^2 = \left(1-\frac{r_\mathrm{S}}{r}\right)c^2\,\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{1-\frac{r_\mathrm{S}}{r}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{81b}$$

Let's now consider the case, still in the Newtonian approximation, where the geometry is determined by the presence of negative mass, corresponding to the dipole repeller.

12.b Newtonian approximation, field created by negative mass $\overline{M}$

We'll follow a similar procedure in every respect. The forms of the two metrics will remain (70a) and (70b), which simply reflect the assumption of SO(3) symmetry. The field equations become:

$$R_\mu^\nu - \frac{1}{2}Rg_\mu^\nu = \chi t_\mu^\nu, \tag{82a}$$

$$\overline{R}_\mu^\nu - \frac{1}{2}\overline{R}g_\mu^\nu = \chi \overline{T}_\mu^\nu. \tag{82b}$$

Our task is to shape the tensors $\overline{T}_\mu^\nu$ and t_μ^ν in such a way that the geodesics derived from the inner and outer metrics reflect repulsion, and that the compatibility conditions of the two equations are satisfied when the Newtonian approximation is applied. This leads to the following ad hoc choices:

$$\overline{T}_\mu^\nu = \begin{pmatrix} \overline{\rho c}^2 & 0 & 0 & 0 \\ 0 & +\varepsilon\overline{p} & 0 & 0 \\ 0 & 0 & +\varepsilon\overline{p} & 0 \\ 0 & 0 & 0 & +\varepsilon\overline{p} \end{pmatrix}, \tag{83}$$

and

$$t_\mu^\nu = \begin{pmatrix} \overline{\rho c}^2 & 0 & 0 & 0 \\ 0 & -\varepsilon\overline{p} & 0 & 0 \\ 0 & 0 & -\varepsilon\overline{p} & 0 \\ 0 & 0 & 0 & -\varepsilon\overline{p} \end{pmatrix}. \tag{84}$$

The expressions are similar, but the signs of the pressure terms are reversed. This is due to the desire to find, in Newtonian approximation, the equivalent of Euler's equation in the object of negative mass, meaning that within it, pressure forms balance the force of gravity. For an observer made up of the same type of matter, this negative matter would behave in exactly the same way as our own. The change in sign is due to the presence of a minus sign in the second member of the second field equation.

Under these conditions, the second equation generates its own “TOV” equation:

$$\frac{\mathrm{d}\overline{p}}{\mathrm{d}r} = -\frac{\left(\overline{\rho} + \varepsilon\overline{p}/\overline{c}^2\right)\left(\overline{m} + 4\pi\varepsilon G\overline{p}r^3/\overline{c}^4\right)\overline{c}^2}{r\left(r - 2\varepsilon G\overline{m}/\overline{c}^2\right)}. \tag{85}$$

With the first equation, we obtain:

$$\frac{\mathrm{d}\overline{p}}{\mathrm{d}r} = -\frac{\left(\overline{\rho} - \varepsilon\overline{p}/\overline{c}^2\right)\left(\overline{m} - 4\pi\varepsilon G\overline{p}r^3/\overline{c}^4\right)\overline{c}^2}{r\left(r + 2\varepsilon G\overline{m}/\overline{c}^2\right)}. \tag{86}$$

Once again, by tending ε to zero or, what amounts to the same thing, by tending $\overline{c}$ to infinity, these two equations can be identified with the Euler equation.

The asymptotic consistency of the Newtonian approximation is clearly restored.

Let’s write:

$$\hat{\overline{R}} = \sqrt{\frac{3\overline{c}^2}{8\pi G|\overline{\rho}|}} \quad \text{and} \quad \overline{r}_{\mathrm{S}} = \frac{2G|\overline{M}|}{\overline{c}^2} \quad \text{(Schwarzschild's lenght).} \tag{87}$$

The inner metric of the positive sector becomes:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{1 + \frac{\overline{r}_0^2}{\hat{\overline{R}}^2}} - \frac{1}{2}\sqrt{1 + \frac{r^2}{\hat{\overline{R}}^2}}\right]^2 \mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1 - \frac{r^2}{\hat{\overline{R}}^2}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{88a}$$

and that of negative sector:

$$\mathrm{d}\overline{s}^2 = \left[\frac{3}{2}\sqrt{1 - \frac{\overline{r}_0^2}{\hat{\overline{R}}^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{\hat{\overline{R}}^2}}\right]^2 \mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1 - \frac{r^2}{\hat{\overline{R}}^2}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{88b}$$

These metrics connect with external metric solutions:

$$\mathrm{d}s^2 = \left(1 + \frac{\overline{r}_{\mathrm{S}}}{r}\right)\mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1 + \frac{\overline{r}_{\mathrm{S}}}{r}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2, \tag{89a}$$

$$\mathrm{d}\overline{s}^2 = \left(1 - \frac{\overline{r}_{\mathrm{S}}}{r}\right)\mathrm{d}x^{0\,2} - \frac{\mathrm{d}r^2}{1 - \frac{\overline{r}_{\mathrm{S}}}{r}} - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\varphi^2. \tag{89b}$$

13 Dipole repeller prediction

The Janus model is essentially falsifiable in Popper’s sense. It predicted a large-scale twin structure with large voids. This has been confirmed [14]. It predicted a very early birth of first-generation stars and galaxies. A new prediction this time concerns the magnitude of sources located in the background of the large void. According to the model, the magnitude of the light emitted by these distant sources will be attenuated by negative lensing. This is a novel aspect, since it has been assumed that the two entities, positive and negative, interact only through antigravitation. Photons from these distant sources can then freely pass through the negative-mass conglomerates. This means that both external and internal geodesics must be used. The deflection effect of light rays will be greatest when they graze the surface of the object, with radius. This effect weakens as you move deeper into the object, becoming zero when the photons pass through its center (see figure 13 on the following page).

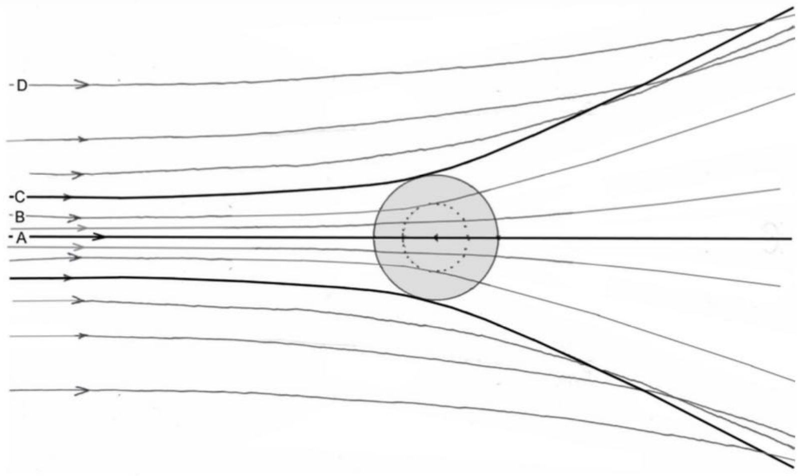

Figure 13: Deflection of photons of positive energy by a negative mass.

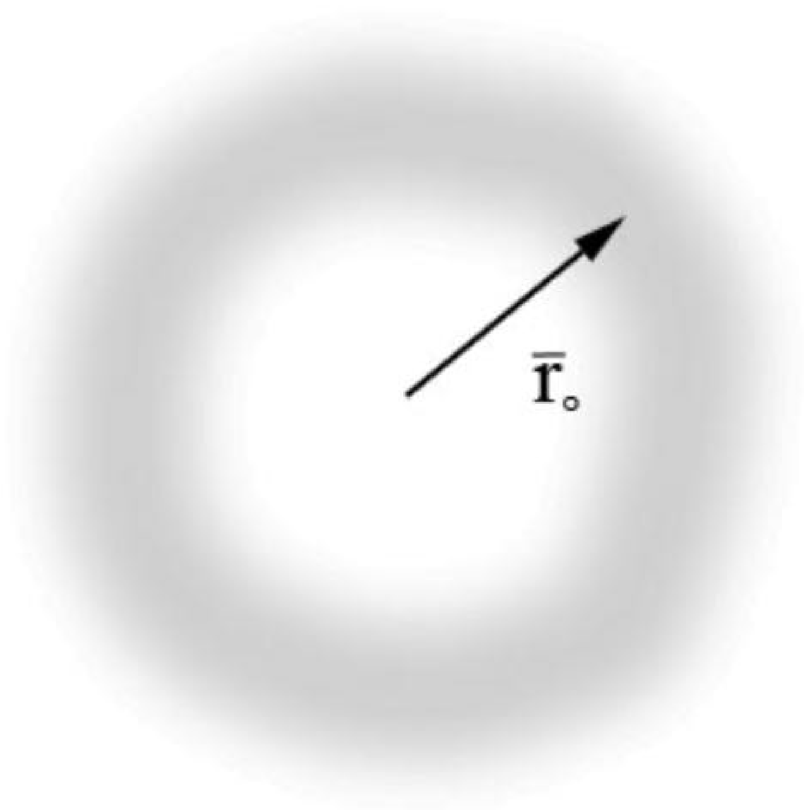

Figure 14: Attenuation of the magnitude of objects in the background of the dipole repeller.

Eventually, we'll be able to map the magnitudes of objects in the background of the dipole repeller. Schematically, their luminosity will be attenuated in a ring-shaped pattern (see figure 14 on the next page).

This measurement will immediately give us the value of the radius $\overline{r}_0$ of this formation.

14 Beyond the Newtonian approximation

These objects are absent in the negative world. In the positive world, objects that deviate from the Newtonian approximation are neutron stars and hypermassive objects located at the center of galaxies, which early images show to be the seat of a strong gravitational redshift effect, darkening their central part. These objects are a priori manageable using the classic pair of outer and inner metrics, taking rotation into account. It should be remembered that we are under no obligation to provide the form of the source tensor of the other sector, in this case an interaction tensor, whose form would be precisely imposed by the Bianchi condition. It's conceivable that one day someone will provide the exact form of this tensor.

But even in the absence of such an object, there is no a priori inconsistency.

15 Conclusion

The genesis of the Janus model spanned several decades. The starting point in 1965 was the attempt by Andrei Sakharov to provide the beginnings of an explanation for the absence of observations of primordial antimatter, which remains a major flaw in the Standard Model ΛCDM, which provides no explanation for such loss of half the universe's content. He therefore suggested a universe structure with two sectors, the second being T-symmetrical to our own. A few years later, in 1970, as an application of symplectic geometry, the mathematician Jean-Marie Souriau showed that this inversion of the time coordinate, this T-symmetry, is synonymous with the inversion of energy and mass. Taking this desire to reconstitute global symmetry a step further, Sakharov envisaged what he called a twin universe as CPT-symmetrical with our own. In this view, the invisible components of the universe boil down to negative-mass antimatter. In 1994, we suggested that this universe structure corresponds to the two folds cover of a projective $\mathbb{P}^4$ by a compact universe with the topology of an $\mathbb{S}^4$ sphere. The two singularities of this spherical universe, the Big Bang and the Big Crunch, then coincide. By substituting a tubular structure, these singularities disappears. This configuration combines two PT-symmetrical sheets. These adjacent sectors are assumed to interact exclusively through gravity. The interaction between positive masses and negative masses in the other sector must therefore be considered. However, the introduction of negative masses is impossible within the framework of general relativity, as it would lead to interaction laws that are incompatible with physics. A bimetric model is therefore envisaged. A system of coupled field equations is then constructed from an action. Its form eliminates the unmanageable runaway effect. The model's interaction laws are such that masses of the same sign attract each other according to Newton's law, while masses of opposite signs repel each other according to anti-Newton's law. As these masses are mutually exclusive, the negative mass can be neglected in the vicinity of the Sun, and the first field equation is then identified with Einstein's equation. In this way, the model is consistent with local relativistic observations, such as the advance of Mercury's perihelion and the deflection of light rays by the Sun. The model is therefore an extension of the general relativity model. An exact, unsteady solution is constructed, revealing a generalized energy conservation law, extended to both sectors. Adaptation of the model to observations, revealing an acceleration of the expansion, then imposes a total dissymmetry between the two entities in presence. The vast majority of negative mass replaces the hypothetical components of dark matter and dark energy. The result is a distribution of 4% visible matter and 96% negative mass, invisible because it emits photons of negative energy that escape our observation instruments. This dissymmetry means that, after decoupling, the negative masses form a regular network of spheroidal conglomerates, while the positive mass, confined to the remaining space, adopts a patchy distribution. The model accounts for the existence of large voids, the dipole repeller having been the first of these. At the center of these large voids are invisible spheroidal conglomerates, behaving like gigantic protostars, with a cooling time greater than the age of the universe. These objects, which emit negative-energy photons corresponding to light in the red and infrared regions [20], do not evolve, nor do they give rise to stars, galaxies or atoms heavier than helium. Life is therefore absent from this negative sector, which is made up of a mixture of negative-mass antihydrogen and antihelium. The model also accounts for the very early birth of first-generation stars and galaxies, as recently demonstrated by the James Webb Space Telescope. We then examine the question of the model's mathematical consistency, i.e. whether the Bianchi conditions are satisfied. We show that these can be asymptotically satisfied under conditions corresponding to the Newtonian approximation. We then settle the question of objects that don't fit into this approximation, essentially located on the positive side, by simply pointing out that we are under no obligation to provide the exact form of the interaction tensor, which is then determined by the zero divergence condition, and that the lack of definition of this tensor in no way invalidates the consistency of a non-linear solution.

References

[1] Andrei D. SAKHAROV. "Violation of CP Invariance, C Asymmetry, and Baryon Asymmetry of the Universe". In: *ZhETF Pisma Redaktsiiu* 5 (Jan. 1967), p. 32.

[2] Andrei D. SAKHAROV. "Baryon asymmetry of the universe." In: *Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki* 76 (Apr. 1979), pp. 1172–1181.

[3] Andrei D. SAKHAROV. "Cosmological models of the universe with reversal of time arrow". In: *Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki* 79 (Sept. 1980), pp. 689–693.

[4] Jean-Marie SOURIAU. *Structure des systèmes dynamiques: maîtrises de mathématiques*. Collection Dunod Université. Dunod, 1970. ISBN: 9782040107529.

[5] Géry de SAXCÉ and Charles-Michel MARLE. *Presentation of Jean-Marie Souriau's book "Structure des systèmes dynamiques"*. June 2023. arXiv: 2306.03106 [math.DS].

[6] Jean-Pierre PETIT and Gilles D'AGOSTINI. "Negative mass hypothesis in cosmology and the nature of dark energy". In: *Astrophysics and Space Science* 354.2 (2014), pp. 611–615. DOI: 10.1007/s10509-014-2106-5.

[7] Géry de SAXCÉ. *Which symmetry group for elementary particles with an electric charge today and in the past?* Mar. 2024. arXiv: 2403.14846 [math-ph].

[8] Jean-Marie SOURIAU. *Géométrie et Relativité*. Paris: Hermann, 1964.

[9] Ryszard KERNER. "Generalization of the Kaluza-Klein theory for an arbitrary non-abelian gauge group". In: *Annales de l'institut Henri Poincaré. Section A, Physique Théorique* 9.2 (1968), pp. 143–152. URL: http://www.numdam.org/item/AIHPA_1968__9_2_143_0/.

[10] Jean-Pierre PETIT. "The missing-mass problem". In: *Nuovo Cimento B Serie* 109.7 (July 1994), pp. 697–709. DOI: 10.1007/BF02722527.

[11] Jean-Pierre PETIT. *Topo The World (The Adventures of Archibald Higgins)*.

[12] Werner BOY. "Über die Curvatura integra und die Topologie geschlossener Flächen". In: *Mathematische Annalen* 57.2 (1903), pp. 151–184. DOI: 10.1007/BF01444342.

[13] Hermann BONDI. "Negative Mass in General Relativity". In: *Reviews of Modern Physics* 29.3 (July 1957), pp. 423–428. DOI: 10.1103/RevModPhys.29.423.

[14] Yehuda Hoffman, Daniel Pomarède, R. Brent Tully, and Hélène M. Courtois. "The dipole repeller". In: *Nature Astronomy* 1.2 (Jan. 2017), p. 0036. doi: 10.1038/s41550-016-0036.

[15] Leonardo Ferreira et al. "Panic! at the Disks: First Rest-frame Optical Observations of Galaxy Structure at $z > 3$ with JWST in the SMACS 0723 Field". In: *The Astrophysical Journal Letters* 938.1 (Oct. 2022), p. L2. doi: 10.3847/2041-8213/ac947c.

[16] Adam G. Riess et al. "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant". In: *The Astronomical Journal* 116.3 (Sept. 1998), pp. 1009–1038. doi: 10.1086/300499.

[17] S. Perlmutter et al. "Measurements of Ω and Λ from 42 High-Redshift Supernovae". In: *The Astrophysical Journal* 517.2 (June 1999), pp. 565–586. doi: 10.1086/307221.

[18] Gilles D'Agostini and Jean-Pierre Petit. "Constraints on Janus Cosmological model from recent observations of supernovae type Ia". In: *Astrophysics and Space Science* 363.7 (2018), p. 139. doi: 10.1007/s10509-018-3365-3.

[19] Jean-Pierre Petit. "Twin universes cosmology". In: *Astrophysics and Space Science* 226.2 (1995), pp. 273–307. doi: 10.1007/BF00627375.

[20] Hasnae Zejli and Hicham Zejli. "Nature of the Dipole Repeller". In: *Journal of Natural Sciences Research* 2 (2023). Discusses the concept of negative mass and the role of the dipole repeller in cosmology, pp. 2–14. url: https://naukaru.ru/en/nauka/article/57604/view.