Challenging the Standard Model

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Abstract : We review new observational data, such as the presence of immense voids in the universe and the early appearance of galaxies, which the Standard Model can no longer account for. We then consider an extension of general relativity, using a system of two coupled field equations to overcome these paradoxes. This model introduces physics-compatible negative mass. It fulfils the functions hitherto assigned to the dark matter-dark energy pair. In addition, taking up A. Sakharov's idea, it explains the non-observation of primordial antimatter, which is assigned a negative mass. In so doing, he gives a precise identity to the invisible components of the universe.

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1 - **Introduction.**

In cosmology and astrophysics, the previous paradigm shift took place in 1915, when Einstein [1] and Hilbert [2] simultaneously published the equation that would become the basis of general relativity. Immediately Karl Schwarzschild presented the two solutions, in the form of an exterior metric [3] and an interior metric [4], which describe the geometry outside and inside a sphere filled with a material of constant density. The two observational confirmations of this new model are immediately mathematically consistent interpretations of the advance of Mercury's perihelion and the deflection of light rays by the masses. The question then arose of how to transmit this new vision of the universe. In English, its propagandist was the American R. Tolman [5]. The Second World War marked a pause in theoretical research in this direction. The emphasis is on technological and warlike applications of the advances made in the pre-war period. But the scientific community was still waiting for confirmation of the model that had emerged from the dynamic version highlighted by A. Friedman and E. Hubble. This became clear in 1967, when the "ash of the Big Bang" was discovered, i.e. the cosmic microwave background at 2.7°K. But two fundamental questions remain: why did one particle in a billion survive after the annihilation of matter and antimatter? And why have we been unable to obtain a single observation of the primordial antimatter content deduced from this? At the time, everything seemed to focus on determining the curvature index, which we linked to a critical density value of 10^{-29} gr/cm^{-3}. This was no mean feat: from the outset, we lost track of half the cosmic content.

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In the early 1970s, measurements of rotation velocities in galaxies became increasingly precise. Vera Rubin published a series of papers, and by the end of the decade the conclusion was clear [6]: the rotational motions of galaxies were far from consistent with Kepler's law. Astrophysicists deduced that a very large quantity of matter produced the gravitational field opposing the centrifugal force linked to rotational motion. They gave it a name: dark matter. All possible models were then considered to clarify its nature, both observationally and with regard to new particles. Measurements of gravitational lensing effects in the vicinity of galaxies and clusters confirm, in the minds of theorists, that this matter does indeed exist. But half a century later, no credible model is available. In this second field, the neutron's sister particle, the neutralino, in a new theory based on supersymmetry, seems to be the most credible candidate. But it refuses to appear, whether in experiments conducted in large particle accelerators, in mines and tunnels, or aboard the International Space Station, in cosmic rays. This failure went hand in hand with the stagnation of observations in particle physics, as the existence of no particle of this new symmetry could be demonstrated. Anyway, cosmologists modified their model, which became known as the CDM (cold dark matter) model. The term “cold” implies that this dark matter is driven by velocities that are small with respect to the speed of light. From 1988 onwards, new satellites made it possible to map the CMB, which has been refined over the years. The CMB now appears homogeneous to the nearest hundred thousandth. This raises a new paradox. Where could such homogeneity emerge from in a non-collisional environment? There are two possible interpretations. Either it is due to a fantastic expansion [7], a new field to which a new particle is associated : the inflaton, either it comes from a mechanism in which all constants vary jointly [8], [9] the secular variation of the speed of light making the cosmological horizon follow very exactly the evolution of the space scale factor of the universe. But thirty-six years later, there are as many inflaton models as there are researchers working on the subject.
Finally, in 2011 ([10],[11],[12]), a Nobel Prize was awarded for a new discovery: the acceleration of the cosmic expansion. This phenomenon is attributed to the presence of the cosmological constant in the field equation, without being able to identify its physical nature. We simply introduce the additional term “dark energy”, also of an unidentified nature. By converting this energy into mass using the mc^2 relation, we obtain a new distribution, where ordinary matter is the only one suitable for observations. The cosmological constant Λ is incorporated into the model, which is now called the Standard Model ΛCDM.

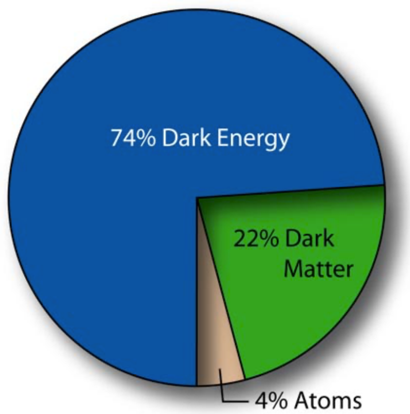

Fig.1 : Matter distribution in ΛCDM model.

2 – **The emergence of a crisis in cosmology and astrophysics**.

The Einstein's model, born in 1915 survive until 2017 with :

- An inflation field
- Dark matter.
- Dark energy.

In 2017, four researchers - Hélène Courtois, Daniel Pomarède, Brent Tully and Yeudi Hoffman - created a map of the universe [13] in a cube one and a half billion light-years across, with the Milky Way at its center. The Doppler effect provides the escape velocities of galaxies, and thus the Hubble velocity field. By subtracting this field, the authors obtain the proper velocities of the objects. The result is a dipolar velocity field. At one end is a formation containing a hundred thousand galaxies, the Shapley attractor. At the other end, six hundred million light-years from the Milky Way, we discover an immense void one hundred million light-years across, the Dipole Repeller, which repels all surrounding matter. Some astrophysicists have suggested that this may reflect the presence of an equivalent void in the general distribution of dark matter. But this doesn't provide the solution, as gravitational instability within this positive-mass dark matter would create conglomerates, not voids.

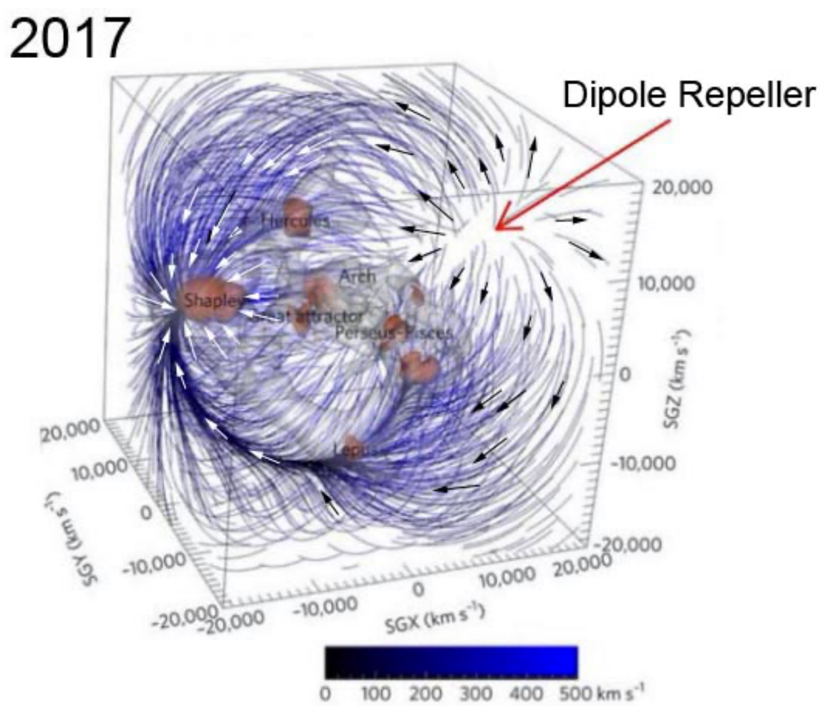

Fig.2 : The dipole repeller

Today, in 2024, observation has revealed the existence of a handful of similar large voids. No theory has been proposed to explain their existence.

A few years later, the first images from the JWST appeared. They caused a panic among the scientific community, as the title of the reference [14] suggests. Successive images show that barred, fully-formed spiral galaxies, harboring stars that are already old, exist at an epoch (prior to the first five hundred million years) where the standard model cannot place them. Within this model framework, it is necessary to assign parameters to dark matter that allow us to model the birth, first of mini-galaxies, and then, through their accretion, of galaxies as we observe them today. What slows down this formation by gravitational instability is the fact that any accretion leads to the heating of matter by compression. This thermal energy, resulting from the conversion of gravitational energy, must be able to be evacuated by radiation. But for an object of radius R, the amount of heat to be evaluated is the cube of this radius, while its radiator, its surface area, is the square of R. So cooling time increases with mass. So it was thought that mini-galaxies would have to form first, as these would merge to produce the more massive galaxies. We therefore expected to find a mass of mini-galaxies, not fully-formed, adult galaxies, for the youngest, high-redshift objects.

By focusing on the analysis of the weak fluctuations observed in the CMB, the specialists had deduced a spectrum of these fluctuations, attributed to a gravito-acoustic mechanism. The modeling of these fluctuations was based on a choice of numerous parameters of different kinds, linked among other things to the invisible components of the universe. Among these choices was the value assigned to Hubble's constant. The curve fit required a value of 67 kms^{-1} Mpc^{-1}. However, the value deduced from direct measurements is different: 70 kms^{-1} Mpc^{-1}.

Since this phenomenon was first demonstrated, it has been known as Hubble tension. A new element of disagreement.

Observation constantly provides new elements that the standard model cannot account for. In 2022, a method based on MgII absoption of the light emitted by quasars in the background revealed an immense arc of galaxies [15] and galaxy clusters at z = 0.8, at a distance of three and a half billion light-years. In 2024, these same researchers discovered an immense ring [16] 1.3 billion light-years in diameter, which they named the Big Ring, located 9.2 billion years away. No explanation for the existence of such formations has yet been found.

When the scientific community is confronted with a problem, it immediately comes up with a new word. Inflation leads to the word inflaton. A very good example can be found in an article published in 2021 by the prestigious English magazine Monthly Notice. In this article, the essential component of the hypermassive object at the center of the Milky Way is named: a compact mass of ... darkinos! This practice is actually quite common. In 1964, when the mathematician proposed extending the cosmological model to a five-dimensional space [18], his work revealed an unknown scalar. It was immediately given a name. It's the dilaton!

Today, we're faced with a wide range of problems.

We pointed out the conflicts between the Standard Model
and observations with the color brown.

Some suggest that the laws of physics are different in these great empty regions. Voices are beginning to be raised suggesting a possible paradigm shift. If we are to opt for such a change, the new model must:

- Provide alternative explanations for all the phenomena for which the standard model provided an explanation. We indicate these points in blue.

- Account for phenomena that defeat the standard model. We'll mark these points with red.

Let's list them:

- The absence of observations of cosmological antimatter
- The nature of the invisible components of the universe
- The reason for the non-observation of these same components
- The explanation for the existence of large cosmic voids.
- The early birth of stars and galaxies

Add: an explanation, other than inflation, for the extreme uniformity of the CMB.

3 – A system of two coupled field equations.

The geometric paradigm of general relativity assimilates the universe to a hyperbolic manifold, endowed with a metric $g_{\mu\nu}$. This universe contains curvatures that are reflected in the existence of a tensor field $R_{\mu\nu}$. Particles with mass then follow the non-zero geodesics derived from this metric $g_{\mu\nu}$. Photons follow zero-length geodesics. The whole is the solution of a field equation:

(1) $$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} = \chi T_{\mu\nu}$$

Its origins lie in an action:

(2) $$A = \int_{D4} \left(R + L_m \right) \sqrt{-g}\, d^4x$$

R is then the Ricci scalar and L_m the matter Lagrangian. By differentiating, $\delta A = 0$, we introduce the matter tensor $T_{\mu\nu}$ through :

(3) $$\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L_m)}{\delta g^{\mu\nu}} = -\chi T_{\mu\nu}$$

In the Janus model, the manifold is equipped with two metrics $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$. Its points are identified by coordinates $\left\{ x^{\circ}, u, \theta, \varphi \right\}$ common to both populations.
The action is:

(4) $$A = \int_{D4} \left[\left(R + L_m + \lambda \right) \sqrt{-g} + \left(\overline{R} + \kappa \overline{L}_m + \kappa \overline{\lambda} \right) \sqrt{-\overline{g}} \right] d^4x$$

With $\kappa = \pm 1$. The différenciation gives :

(5)

$$\delta A = \int_{D4} \left[\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, L_m)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\ \lambda)}{\delta g^{(+)\mu\nu}} \right] \sqrt{-g}\ \delta g^{\mu\nu} d^4x$$
$$+ \int_{D4} \left[\frac{\delta \overline{R}}{\delta \overline{g}^{\mu\nu}} + \frac{\overline{R}}{\sqrt{-\overline{g}}} \frac{\delta \sqrt{-\overline{g}}}{\delta \overline{g}^{\mu\nu}} + \frac{\kappa}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-\overline{g}}\ \overline{L}_m)}{\delta \overline{g}^{\mu\nu}} + \frac{\kappa}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-\overline{g}}\, \overline{\lambda})}{\delta \overline{g}^{\mu\nu}} \right] \sqrt{-\overline{g}}\ \delta \overline{g}^{\mu\nu} d^4x$$

We see that we have two Lagrangians of matter, and therefore a priori two types of matter.
Posing :

(6) $$\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\ L_m)}{\delta g^{\mu\nu}} = -\chi\, T_{\mu\nu}$$

(7) $$\frac{1}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-\overline{g}}\ \overline{L}_m)}{\delta \overline{g}^{\mu\nu}} = -\chi\, \overline{T}_{\mu\nu}$$

(8) $$\frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\ \lambda)}{\delta g^{\mu\nu}} = \chi\, \tau_{\mu\nu}$$

(9) $$\frac{1}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-g}\ \overline{\lambda}\ )}{\delta \overline{g}^{\mu\nu}} = \chi\, \overline{\tau}_{\mu\nu}$$

Whence the system :

(10a) $$R_{\mu\nu} - \frac{1}{2} R\ g_{\mu\nu} = \chi \left[T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}}\ \tau_{\mu\nu} \right]$$

(10b) $$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R}\ \overline{g}_{\mu\nu} = \chi \kappa \left[\overline{T}_{\mu\nu} + \sqrt{\frac{g}{\overline{g}}}\ \overline{\tau}_{\mu\nu} \right]$$

g and $\overline{g}$ are the determinants of the two metrics $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$. The tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ translate the way these two materials m and $\overline{m}$ behave under the effect of the gravitational field acting on them. The second members of both equations represent the sources of these fields.

The gravitational field acting on the particles of matter will be the sum of two source terms:

- $T_{\mu\nu}$ will be the source of the field acting on these particles, emanating from particles of the same nature.

In fact, we know that the source of the field is not the volume density of matter, but the volume density of energy ρc^2. Added to this is a second density, pressure p. In mixed notation, adding the assumption that this medium behaves like a perfect fluid, this tensor $T_\mu^{\ \nu}$ is written as

(11)
$$T_\mu^{\ \nu} = \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

with :

(12)
$$p = \frac{\rho < v^2 >}{3}$$

Where the thermal agitation velocity <v> << c in a Newtonian approximation.

The second term $\left[T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}}\, \tau_{\mu\nu} \right]$ represents the contribution of the masses to the gravitational field acting on the masses $\overline{m}$. The tensor $\tau_{\mu\nu}$ will be referred to as an interaction tensor, and the scalar $\sqrt{\frac{\overline{g}}{g}}$ will reflect an apparent mass effect. It will be assumed that :

(13)
$$\tau_o^{\ o} = \overline{\rho}\, \overline{c}^2$$

Similar situation for terms $\overline{T}_\mu^{\ \nu}$ and $\overline{\tau}_\mu^{\nu}$ with :

(14)
$$\overline{\tau}_o^{\ o} = \rho c^2$$

4 – **Laws of force.**

We will now assume that the masses m are of the ordinary type and that the masses $\overline{m}$ are negative.

Let's consider a region of space where there is a concentration of masses of type m, to the exclusion of masses of type $\overline{m}$. The system becomes :

(15a) $$R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} = \chi \, T_{\mu\nu}$$

(15b) $$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \; \overline{g}_{\mu\nu} = \chi \kappa \sqrt{\frac{g}{\overline{g}}} \; \overline{\tau}_{\mu\nu}$$

Equation (15a) can then be identified with Einstein's equation, without the cosmological constant. As $T_{00} = \rho c^2 > 0$ the conclusion is that positive masses attract positive masses.

The direction of the force translating the action of the masses $m > 0$ on a test-mass $\overline{m} < 0$ will depend on the sign of κ. As $\overline{\tau}^{o}_{o} = \rho c^2 > 0$, if :

- $\kappa = +1 \;\; \rightarrow$ Positive masses attract negative masses.

- $\kappa = -1 \;\; \rightarrow$ Positive masses repel negative masses.

Now consider a region where the field is created by a concentration of negative mass. The system becomes :

(16a) $$R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} = \chi \sqrt{\frac{\overline{g}}{g}} \; \tau_{\mu\nu}$$

(16b) $$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \; \overline{g}_{\mu\nu} = \chi \kappa \, \overline{T}_{\mu\nu}$$

Equation (16a) confirm us to the conclusion that negative masses repel positive ones. Since $\tau^{o}_{o} = \overline{\rho}\,\overline{c}^2 < 0$ the direction of action of one mass $\overline{m} < 0$ on another $\overline{m} < 0$ depends on the sign of κ.

- $\kappa = +1 \;\; \rightarrow$ Negative masses repel negative masses.

- $\kappa = -1 \;\; \rightarrow$ Positive masses attract negative masses.

If we opt for $\kappa = +1$ and put two masses $m > 0$ and $\overline{m} < 0$ and together, the positive mass runs away, pursued by the negative mass. Both masses accelerate uniformly, but without energy input, since the kinetic energy of the negative mass is itself negative. This is a violation of the action-reaction principle. This is the *runaway effect*.

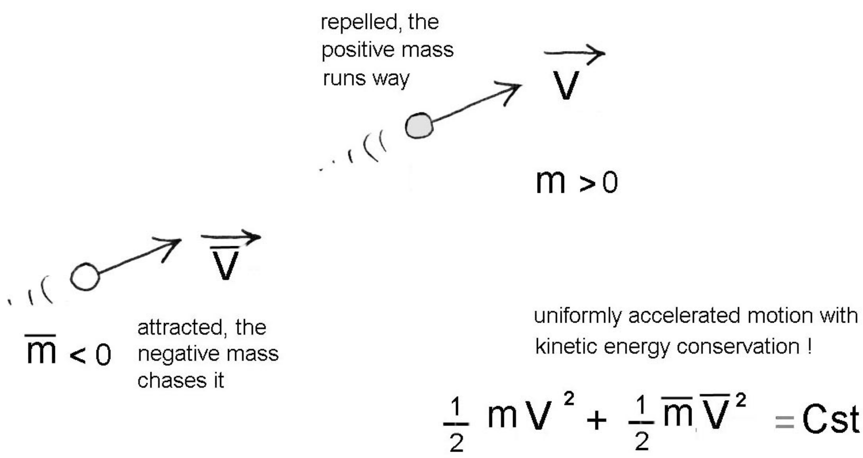

Fig.3 : Runaway effect.

This same paradox was encountered when we tried to introduce negative masses into the general relativity model [15], which had a single metric. But in this new, bimetric model, the choice $\kappa = -1$ allows us to eliminate it, thus obtaining the system of two coupled field equations of the Janus Cosmological Model (JCM).

(17a) $$R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} = \chi \left[T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}}\, \tau_{\mu\nu} \right]$$

(17b) $$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \; \overline{g}_{\mu\nu} = -\chi \left[\overline{T}_{\mu\nu} + \sqrt{\frac{g}{\overline{g}}}\, \overline{\tau}_{\mu\nu} \right]$$

5 – **Bianchi conditions.**

From the metrics $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$ we can create the two covariant derivation operators:

(18) $$\partial^{\nu} \qquad \overline{\partial}^{\nu}$$

We get :

(18a) $$\partial^{\nu} \left(R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} \right) \equiv 0$$

(18b) $$\overline{\partial}^{\nu} \left(\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \; \overline{g}_{\mu\nu} \right) \equiv 0$$

Thus we must have :

(19a) $$\partial^{\nu} T_{\mu\nu} = 0$$

(20b) $$\partial^{\nu}\left(\sqrt{\frac{\overline{g}}{g}}\,\tau_{\mu\nu}\right)=0$$

(21a) $$\overline{\partial}^{\nu}\,\overline{T}_{\mu\nu}=0$$

(21b) $$\overline{\partial}^{\nu}\left(\sqrt{\frac{g}{\overline{g}}}\,\overline{\tau}_{\mu\nu}\right)=0$$

6 – **Time dependent exact homogeneous and isotropic solution.**

In this case, the covariant derivations are reduced to the derivation with respect to the chronological variable x°.

(22) $$\partial^{\nu}=\overline{\partial}^{\nu}=\frac{d}{dx^{\circ}}$$

Our symmetries lead to two FLRW metrics:

(23a) $$g_{\mu\nu}=dx^{\circ 2}-\ a^{2}\left[\frac{du^{2}}{1-ku^{2}}+u^{2}d\theta^{2}+u^{2}\sin^{2}\theta\, d\varphi^{2}\right]$$

(24b) $$\overline{g}_{\mu\nu}=dx^{\circ 2}-\ \overline{a}^{2}\left[\frac{du^{2}}{1-\overline{k}u^{2}}+u^{2}d\theta^{2}+u^{2}\sin^{2}\theta\, d\varphi^{2}\right]$$

The variable u is dimensionless. a and $\overline{a}$ are the space scaling factors of the two sectors. The ratios of the determinants then boil down to the function of the chronological variable x°:

(25) $$\phi=\sqrt{\frac{g}{\overline{g}}}=\frac{a^{3}}{\overline{a}^{3}}$$

By introducing the metrics (23a) and (23b) into the system (17a) and (17b) we obtain an exact solution [19] such that:

(26) $$k=\overline{k}=-1$$

The compatibility condition then appears:

(27) $$a^{3}\rho c^{2}+\overline{a}^{3}\overline{\rho}\,\overline{c}^{2}=Cst$$

This is nothing more than a generalized condition of energy conservation:

(28)
$$\rho c^2 a^3 + \overline{\rho}\,\overline{c}^2 \overline{a}^3 = E = Cst$$

And the exact solutions, for phases dominated by matter are:

(29a)
$$a^2 \frac{d^2 a}{dx^{\circ 2}} = -\frac{4\pi G}{c^2} E$$

(29b)
$$\overline{a}^2 \frac{d^2 \overline{a}}{dx^{\circ 2}} = +\frac{4\pi G}{c^2} E$$

Here we find a way of conforming to an observational datum. All we have to do is assume that the overall energy of the system is predominantly negative. Under these conditions, equation (29a) accounts for the acceleration of the cosmic expansion. This is an alternative interpretation to that given by the model, which attributes the phenomenon to the presence of the cosmological constant. There is, however, an important difference: the standard model leads to an exponential acceleration, whereas in the interpretation provided by the Janus model, the acceleration tends towards zero at infinity. The expansion becomes linear at infinity. This solution has been successfully confronted with observational data [20].

The Janus model is therefore profoundly asymmetrical. This has immediate consequences. At the moment of decoupling, the Jeans time characteristic of negative mass will be shorter:

(30)
$$\overline{t}_J = \frac{1}{\sqrt{4\pi G |\overline{\rho}|}} \ll t_J = \frac{1}{\sqrt{4\pi G \rho}}$$

Under these conditions, this population will give rise to a regular set of spheroidal conglomerates that will confine the positive mass to the remaining space, giving it a lacunar structure.

→ The locations of these conglomerates will be marked by huge voids, the dipole repeller being one of them.

→ When this very large-scale structure is formed, the material will be compressed by the adjacent negative-mass conglomerates and will heat up. But its geometry will allow radiative cooling to take place just as rapidly. Destabilized, the positive mass will immediately give rise to first-generation stars, clusters and galaxies, predating the first hundred million years. This phenomenon is consistent with the early presence of fully-formed galaxies.

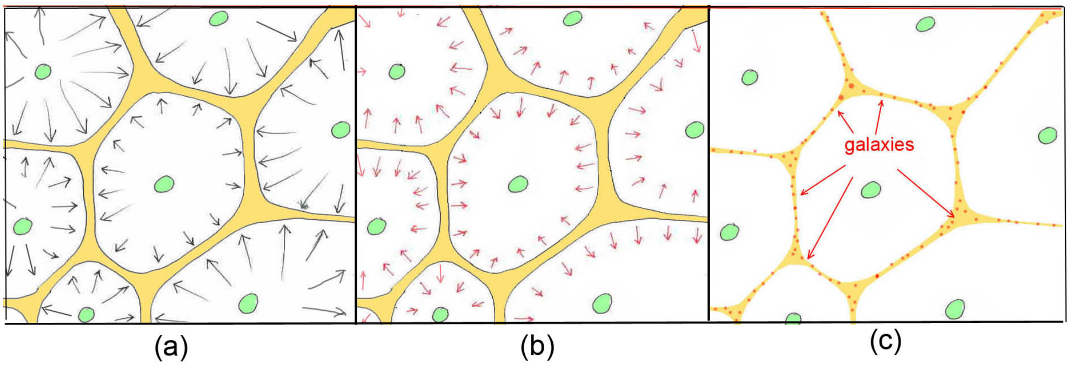

Fig.4 : Rapid star and galaxy formation.

Infiltrating between galaxies, the negative mass will exert a counter-pressure on the galaxies, ensuring their confinement. This explains the flatness of the rotation curves at the periphery, as well as the strong gravitational lensing effects, hitherto attributed to the presence of a dark matter halo.

This negative mass replaces both dark matter and dark energy, giving us the following equivalent distribution:

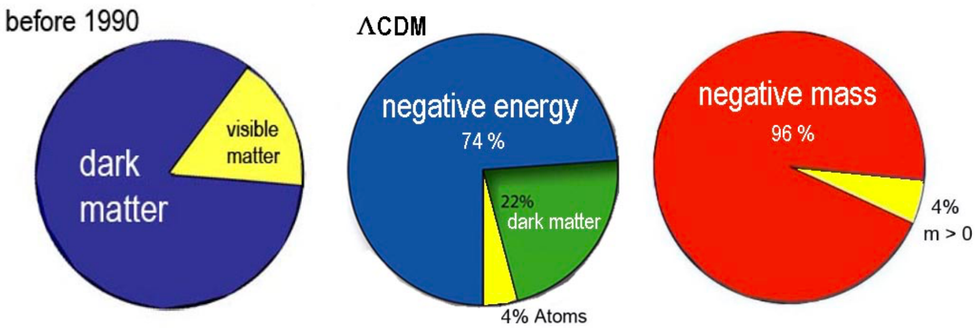

Fig.5 : Comparative mass distribution.

7 – **Stationary solutions in** SO(3) **symmetry and in the Newtonian approximation.**

The force laws we have obtained show a mutual exclusion of masses of opposite signs. This allows us to restrict ourselves to situations where only one of the two mass populations is present.

7a – **When only positive mass is present**.

The system becomes :

(31a)
$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \chi T_{\mu\nu}$$

(31b) $$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\ \overline{g}_{\mu\nu} = -\chi\sqrt{\frac{g}{\overline{g}}}\,\overline{\tau}_{\mu\nu}$$

The coefficient $\sqrt{\frac{g}{\overline{g}}}$ is a simple positive constant, which can be integrated into the interaction tensor $\overline{\tau}_{\mu\nu}$. The two conditions to be satisfied are

(32) $$\partial^{\nu} T_{\mu\nu} = 0$$

(33) $$\overline{\partial}^{\nu}\overline{\tau}_{\mu\nu} = 0$$

Conditions (32) and (33) are obviously satisfied in a vacuum, outside the mass, where $T_{\mu\nu} = \overline{\tau}_{\mu\nu} = 0$. With regard to the geometry inside this mass, we'll start by restricting ourselves to the case where velocities are low compared to the speed of light and curvature remains low, which corresponds to the Newtonian approximation.

(34) $$\varepsilon T_{\mu}^{\ \nu} = \begin{pmatrix} \varepsilon\rho c^2 & 0 & 0 & 0 \\ 0 & -\varepsilon^2 p & 0 & 0 \\ 0 & 0 & -\varepsilon^2 p & 0 \\ 0 & 0 & 0 & -\varepsilon^2 p \end{pmatrix}$$

Posing :

(35) $$ds^2 = e^{(1+\varepsilon\nu)}dx^{\circ 2} - e^{(1+\varepsilon\lambda)}dr^2 - r^2 d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

In other words:

(36) $$g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon\,\gamma_{\mu\nu}$$

where $\eta_{\mu\nu}$ is the Lorentz metric. Series expansion of the equation yields the Poisson equation.

(37) $$\varepsilon\Delta\gamma_{oo} = -\varepsilon\chi\rho$$

and the TOV equation ([21] , [22], Tolman-Oppenheimer-Volkoff).

(38) $$\frac{\varepsilon dp}{dr} = -\frac{(\varepsilon\rho + \varepsilon^2 p/c^2)(\varepsilon m + 4\pi\varepsilon G\,\varepsilon^2 p r^3/c^4)c^2}{r(r - 2\varepsilon^2 G m/c^2)}$$

With :

(39) $$\varepsilon m(r) = \frac{4\pi r^3 \varepsilon\rho}{3}$$

We get :

(40) $$\frac{\varepsilon^2 dp}{dr} = -\frac{\varepsilon^2\rho\, m c^2}{r^2} = -\frac{\varepsilon^2 G\rho}{r^2}\frac{4\pi r^3\rho}{3}$$

In other words, the Euler equation, translating the equilibrium between the force of pressure and the force of gravity.
Equation (33) then determines the form of the interaction tensor $\overline{\tau}_{\mu\nu}$.
We express it as a limited development corresponding to the Newtonian approximation:
Newtonienne :

(41) $$\varepsilon\overline{\tau}_{\mu}^{\nu} = \begin{pmatrix} \varepsilon\rho c^2 & 0 & 0 & 0 \\ 0 & +\varepsilon^2 p & 0 & 0 \\ 0 & 0 & +\varepsilon^2 p & 0 \\ 0 & 0 & 0 & +\varepsilon^2 p \end{pmatrix}$$

We'll give the metric the form:

(42) $$d\overline{s}^2 = e^{(1+\varepsilon\overline{\nu})} dx^{\circ 2} - e^{(1+\varepsilon\overline{\lambda})} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

The calculation then leads to a compatibility equation:

(43) $$\frac{\varepsilon^2 dp}{dr} = -\frac{(\varepsilon\rho - \varepsilon^2 p/c^2)(\varepsilon m - 4\pi\varepsilon^2 G p r^3/c^4)c^2}{r(r + 2\varepsilon G m/c^2)}$$

which is not identical to (38), but is the same as Euler's equation (40) in the Newtonian approximation, i.e. when the terms in ε^2 are neglected. Bianchi's condition is therefore satisfied asymptotically.

→ Note that this Newtonian approximation corresponds to the vast majority of astrophysical phenomena.

The geodesics outside the masses, the only ones that fall within the scope of our physics, i.e. that lead to measurements, are given by :

(44) $$\varepsilon r_s = \frac{2\varepsilon G M}{c^2}$$

(45) $$ds^2 = \left(1 - \frac{\varepsilon r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{\varepsilon r_s}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta \, d\varphi^2$$

We cannot measure the geodesics followed by negative-energy objects. But, in the case of this Newtonian approximation, these would derive from the metric :

(46)
$$d\bar{s}^2 = \left(1+\frac{\varepsilon r_s}{r}\right)c^2 dt^2 - \frac{dr^2}{1+\frac{\varepsilon r_s}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

This confirms the fact that positive mass repels negative mass and negative-energy photons.

This is the case of the solar system. The first equation is then identified with Einstein's, with no cosmological constant. In this way, the model satisfies the classic verifications of general relativity: Mercury's perihelion is advanced, and light rays are deflected by the mass of the Sun.

But there are objects that fall outside this approximation. These are :

- Neutron stars
- Hypermassive objects located at the center of galaxies.

In these cases, a Newtonian solution is not appropriate. The conditions of mathematical compatibility only apply to geometries inside masses. A The metric then has the non-linearized form:

(47)
$$d\bar{s}^2 = e^{\bar{\nu}} dx^{\circ 2} - e^{\bar{\lambda}} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

With :

(48)
$$m(r) = \frac{4\pi r^3 \rho}{3}$$

It gives the TOV equation:

(49)
$$\frac{dp}{dr} = -\frac{(\rho + p/c^2)(m + 4\pi G p r^3/c^4)c^2}{r(r - 2Gm/c^2)}$$

with :

(50)
$$R_s = \frac{2GM}{c^2}$$

The metric giving the geodesics outside this hyperdense positive mass is the classical Schwarzschild outer metric:

(50)
$$ds^2 = \left(1-\frac{R_s}{r}\right)c^2 dt^2 - \frac{dr^2}{1-\frac{R_s}{r}} - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi^2$$

We are then under no obligation to provide the expression of the interaction tensor $\bar{\tau}_{\mu\nu}$. This form, after applying Newtonian approximation, should simply tend to the Euler equation (40). We can thus conclude that the system of field equations (10a) , (10b) can be used to describe the geometry inside and outside a positive mass, even a hyperdense one.

7b – **When only negative mass is present**.

By integrating the constant $\sqrt{\frac{\overline{g}}{g}}$ into the interaction tensor $\tau_{\mu\nu}$, the system of field equations becomes :

(51a)
$$R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} = \chi \tau_{\mu\nu}$$

(51b)
$$\overline{R}_{\mu\nu} - \frac{1}{2} \overline{R} \; \overline{g}_{\mu\nu} = -\chi \overline{T}_{\mu\nu}$$

This corresponds to the geometry of the dipole repeller, which is linked to the presence of a negative-mass spheroidal object. These are the first objects to form, immediately after decoupling. This phenomenon leads to spheroidal objects, which then heat up. The constituent atoms ionize, stopping the contraction. These objects can then be compared to immense protostars, whose cooling time exceeds the age of the universe..

Objects at the center of large voids, of which the dipole repeller is an example, are immense spheroidal masses of atoms which, when brought to the ionization temperature of the medium, radiate in the red and infrared. These structures do not evolve. They constitute the objects of the negative world, which is devoid of stars, galaxies and planets.

→ Life is therefore absent from this negative world.

These structures, on the other hand, fall within the Newtonian approximation. We'll need to construct the metric which alone lends itself to observation. To do this, we need to produce tensors that satisfy Bianchi's conditions for asymptotically zero covariant derivatives. We propose:

(52)
$$\varepsilon \, \overline{T}_{\mu}^{\nu} = \begin{pmatrix} \varepsilon \overline{\rho} \overline{c}^2 & 0 & 0 & 0 \\ 0 & +\varepsilon^2 \, \overline{p} & 0 & 0 \\ 0 & 0 & +\varepsilon^2 \, \overline{p} & 0 \\ 0 & 0 & 0 & +\varepsilon^2 \, \overline{p} \end{pmatrix}$$

(53)
$$\varepsilon \, t_{\mu}^{\nu} = \begin{pmatrix} \varepsilon \overline{\rho} \overline{c}^2 & 0 & 0 & 0 \\ 0 & -\varepsilon^2 \, \overline{p} & 0 & 0 \\ 0 & 0 & -\varepsilon^2 \, \overline{p} & 0 \\ 0 & 0 & 0 & -\varepsilon^2 \, \overline{p} \end{pmatrix}$$

Equation (51b) gives rise to the relationship:

(54) $$\varepsilon^2 \frac{d\overline{p}}{dr} = -\frac{(\varepsilon\overline{\rho} + \varepsilon^2\overline{p}/\overline{c}^2)(\varepsilon\overline{m} + 4\pi\varepsilon^2 G\,\overline{p}r^3/\overline{c}^4)\overline{c}^2}{r(r - 2\varepsilon G\,\overline{m}/\overline{c}^2)}$$

while equation (51a) gives:

(55) $$\frac{d\overline{p}}{dr} = -\frac{(\overline{\rho} - \varepsilon\overline{p}/\overline{c}^2)(\overline{m} - 4\pi\varepsilon G\,\overline{p}r^3/\overline{c}^4)\overline{c}^2}{r(r + 2\varepsilon G\,\overline{m}/\overline{c}^2)}$$

In the Newtonian approximation, these equations become compatible. We must then finalize the calculation of the metric $g_{\mu\nu}$. It comes with:

(56) $$\hat{\overline{R}} = \sqrt{\frac{3\overline{c}^2}{8\pi G\,\varepsilon|\overline{\rho}|}} \qquad \varepsilon\overline{r}_s = \frac{2\varepsilon G|\overline{M}|}{\overline{c}^2}\ (\text{Schwarzschild's lenght})$$

We've used a lower-case letter to designate the Schwarzschild length $\overline{r}$, which, in the Newtonian approximation of weak curvatures, is small in front of r. On the other hand, the characteristic quantity $\hat{\overline{R}}$ associated with the inner metric is larger than r. The outer metric is :

(57) $$ds^2 = \left(1 + \frac{\varepsilon\overline{r}_s}{r}\right)dx^{\circ 2} - \frac{dr^2}{1 + \frac{\varepsilon\overline{r}_s}{r}} - r^2 d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

And the interior metric:

(58) $$ds^2 = \left[\frac{3}{2}\sqrt{1 + \frac{\varepsilon\overline{r}_o^2}{\hat{\overline{R}}^2}} - \frac{1}{2}\sqrt{1 + \frac{\varepsilon r^2}{\hat{\overline{R}}^2}}\right]^2 dx^{\circ 2} - \frac{dr^2}{1 + \frac{\varepsilon r^2}{\hat{\overline{R}}^2}} - r^2 d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

Since we are dealing with low curvatures, both can be extended into a series.:

(59) $$ds^2 = \left(1 + \frac{\varepsilon\overline{r}_s}{r}\right)dx^{\circ 2} - \left(1 - \frac{\varepsilon\overline{r}_s}{r}\right)dr^2 - r^2 d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

(60) $$ds^2 = \left[1 + \frac{3}{2}\frac{\varepsilon\overline{r}_o^2}{\hat{\overline{R}}^2} - \frac{1}{2}\frac{\varepsilon r^2}{\hat{\overline{R}}^2}\right]dx^{\circ 2} - \left(1 + \frac{\varepsilon r^2}{\hat{\overline{R}}^2}\right)dr^2 - r^2 d\theta^2 - r^2\sin^2\theta\, d\varphi^2$$

These two metrics are connected to the surface of the star $r = r_o$ (assimilated to a sphere of constant density $\varepsilon\overline{\rho}$). A prediction can be deduced from these two metrics. Sooner or later, astronomers will create a map showing the magnitudes of light emitted by objects located in the background of this dipole repeller zone.

A weak but measurable negative lensing effect should then be apparent. The drawing below is for illustrative purposes only, and is an exaggeration of the effect.

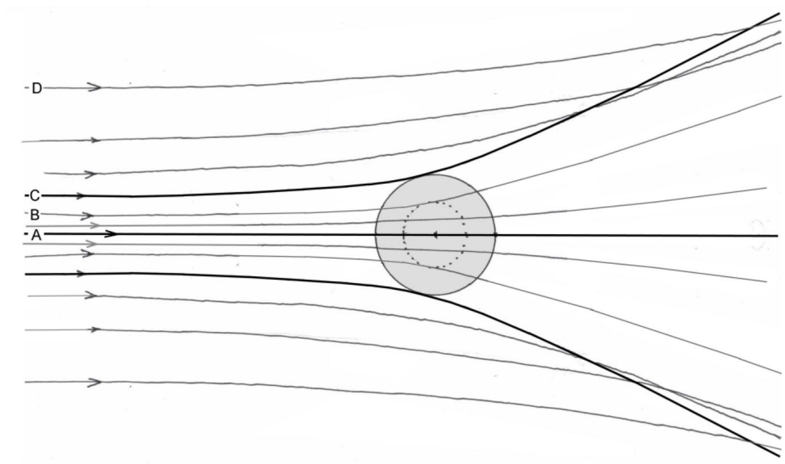

Fig.6 : Negative lensing due to negative mass

As can be seen, the effect is greatest when the photons' trajectory skims the surface of the negative-mass star. It then tends towards zero when this trajectory passes through the center of the object.

→ The effect of magnitude attenuation on the shape of a ring is therefore significant, giving an indication of the diameter of this invisible object.

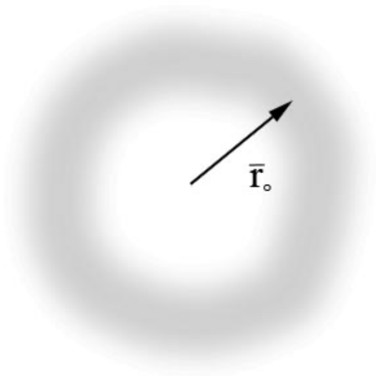

Fig.7 : Prediction of the ring attenuation effect of the magnitude of background objects

8 – **Primordial antimatter**

We know that there is currently no model to account for the absence of observed primordial antimatter. Let's take up the idea proposed by Andrei Sakharov in 1967 ([23], [24] ,[25]). Starting from the violation of CP symmetry, he hypothesized the existence of a second universe, which he described as a Twin, linked to our own by the Big Bang singularity.

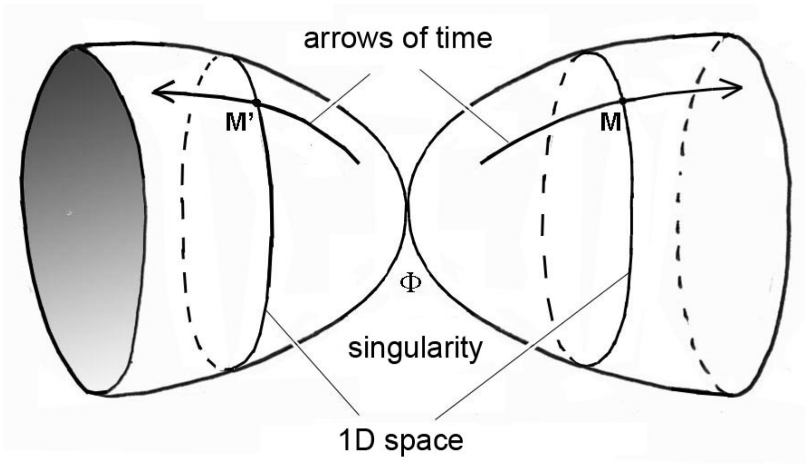

Fig.8 : 2D didactic image of the Sakharov universe.

In this second universe, the violation of CP symmetry would be inverse. Overall, he proposes a CPT twin universe symmetrical with our own. In our own universe, matter is synthesized from quarks, while antimatter is synthesized from antiquarks. Sakharov therefore suggests that in our universe, the synthesis of matter from quarks would have been slightly faster than the synthesis of antimatter from antiquarks. The situation is reversed in the second universe.

The composition of our universe would therefore be :

- Matter
- A corresponding remnant of antiquarks in the free state
- Photons from annihilations.

And in a second universe:

- Antimatter
- A corresponding remnant of quarks in the free state
- Photons from annihilations.

This is the only proposal currently in existence. In the Janus cosmological model [19], we begin by exploiting one of the essential results of [26], chapter XIV, equation (14.66): the inversion of time goes hand in hand with the inversion of mass and energy. In the Janus cosmological model, we begin by identifying the contents of the Sakharov twin universe as negative-mass antimatter, associated with a corresponding remnant of negative-energy quarks and a population of negative-energy photons. This negative-mass antimatter is merely the image of our own, with mass changing sign. We can therefore say:

→ The invisible components of the universe are negative-mass antihydrogen and antihelium. Their invisibility is explained by the fact that they emit photons of negative energy, which cannot be captured by our observational instruments.

→ The use of dynamical group theory [19], applied to the Janus model (Janus group), indicates that there are two types of antimatter.

- An antimatter with positive mass C-symmetrical to our own matter, which we can produce in the laboratory.

- A second type of antimatter, PT-symmetrical to our own matter, found between galaxies and in immense conglomerates of negative mass.

→ As predicted by the Janus model, and confirmed by experience, antimatter created in the laboratory behaves like ordinary matter, in the Earth's gravity field, and "falls down".

9 – **Quantum mechanics and negative energy states**.

→ As already pointed out in [19] , the non-existence of negative energy states is just one of the postulates of quantum field theory. We translate this by assuming a priori that the time-reversal operator T must be antilinear and antiunitary. By freeing itself from this constraint, the Janus model suggests a necessary extension of Quantum Mechanics to these negative energy states, as outlined in ([27], [28]).

→ This "contradicts the CPT theorem". But this is not a theorem, but an assertion based on the assumption that the T operator does not invert energy. The CPT motion symmetrical to that of a matter particle corresponds to the motion of its antimatter, affected by a time inversion..

All attempts to quantify the gravitational field have so far been failures. Although no graviton model exists, this has not prevented legions of researchers from taking it for granted. Thus, in reference [17], the authors report, in addition to a mass spectrum of these gravitons, the existence of a gap between light and heavy gravitons.

We conjecture that the key to quantifying the gravity field lies in integrating negative masses into the model. When we set out to quantize the electromagnetic field, a process that gives rise to the photon as an exchange particle, we take into account the shielding effect associated with the reaction of the vacuum, assimilated to a mixture of matter and antimatter. There's an analogy with kinetic plasma theory. The latter is managed using the Boltzmann equation, which has something in common with quantum mechanics in that it admits the existence of particles, but refrains from locating them individually, insofar as the velocity distribution function f is a probability of presence. The formalism involves integro-differential operators whose eigenvalues are the effective cross-sections of collisions. To calculate these effective cross-sections, the force law is given as a function of the distance between the particles. Thus, a law in $\frac{1}{r^5}$ leads to a constant value, which fits quite well with a "billiard ball" model of electrically neutral particles. Introducing an electric potential in $\frac{1}{r}$, Newtonian, early theorists were in for a nasty surprise: the effective cross-section became infinite.

It was the Dutch physicist P. Debye who provided the key. By acting on its environment, a charged particle caused a shielding effect, the potential becoming $\frac{1}{r}e^{-r/d}$ where d is the Debye distance. Since then, the calculation of the integral giving the effective collision cross-section has been limited to this d value. With regard to the field created by an electrically charged particle, Quantum Mechanics also takes into account a shielding effect linked to the reaction of the vacuum to it. A vacuum then assimilated to a mixture of matter and antimatter. By analogy, we conjecture that managing a vacuum made up of a mixture of masses of both signs would lead to a similar shielding effect, with the characteristic length becoming the Jeans length, and that the infinities encountered in any attempt to quantize the gravity field with a single type of mass would then disappear.

10 – **Topology of the Janus Model.**

In the Janus Cosmological Model (JCM) [19], these two types of matter are made to interact, folding the Sakharov universe in on itself..

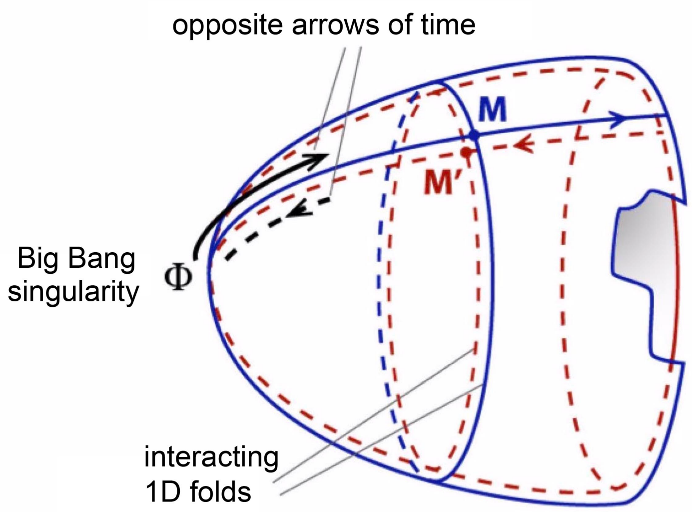

Fig.9: Didactic 2D image of the Janus model.

At this point, the singularity can be replaced by a tube connecting the two universe folds.

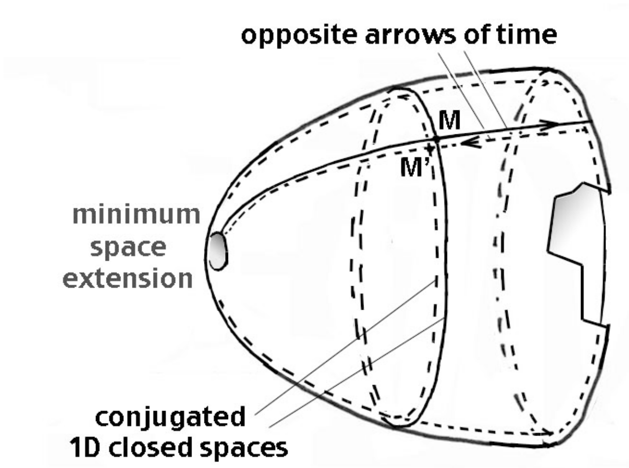

Fig.10 : Same thing without the « initial » singularity.

→ In this way, the Janus model makes the initial cosmological singularity disappear.

But we could do more, i.e. start from a closed space-time, whose didactic 2D image is a sphere S2:

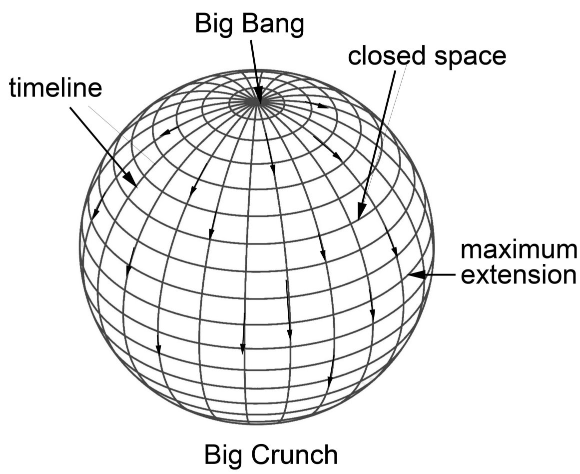

Fig.11 : 2D didactic image of a closed universe

→ In this case, the space-time hypersurface, having the topology of an S4 sphere, is configured according to the two-sheet covering of a P4 projectf, which creates this PT-symmetry between adjacent sheets. The Big Bang and Big Crunch singularities then coincide, and once again, by replacing them with a tube, these singularities disappear and the object becomes the two-sheet covering of a four-dimensional Klein bottle. The universe undergoes a moment of maximum extension, then contracts.

→ This topology does away with the question of a "pre-Big Bang" and "post-Big Crunch" universe. The universe is not "created", it "is".

11 – **Galaxy model**.

Galaxies are collections of masses orbiting both in their own gravitational field and in the field produced by their negative-mass surroundings.

To date, only semi-empirical models of galaxies exist. A mathematically correct model should be based on an exact solution of the Vlasov equation. In this respect, we only have S.Chandrasekhar's exploitation [31] of the solution represented by the spherical Maxwell-Boltzmann distribution function, where Log f is a spherical polynomial as a function of the U, V, W components of the residual velocity, the astrophysical analogy of the thermal agitation velocity. This model has been proposed to describe spheroidal objects such as globular clusters. But one of the difficulties is that the mass of such structures is infinite. An exact elliptical solution was proposed in the 1970s ([29],[30]), also leading to infinite mass. The key assumption concerned the shape of the velocity ellipsoid, hereafter in an axisymmetric system.

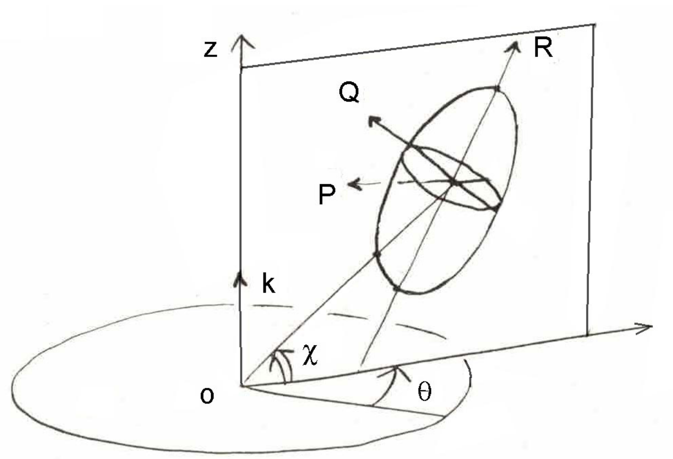

Fig.12 : Extract from reference [30]. Velocity ellipsoid.

The Maxwell Boltzmann solution, with C denoting the residual velocity vector, is:

(61) $$\mathrm{Log}\, f = \varphi(r) - \frac{mC^2}{2kT}$$

In steady state, the elliptical solution is:

(62) $$\mathrm{Log}\, f = \varphi(r) - \frac{mC^2}{2kH} + a\left(< \mathbf{C},\mathbf{r} > \right)^2 + \alpha\left(< \mathbf{C},\mathbf{k}\times\mathbf{r} > \right)^2$$

Constructing the solution then involves determining the functions H , a and α. In the diametral plane, the major axis of the ellipsoid points to the center of the system. At this point, the ellipsoid becomes a velocity sphere. The solution shows the evolution of the ellipsoid's axes as a function of position. We find that the major axis is constant, while the transverse axes, which are equal in the diametral plane, tend towards zero at infinity. Thus, in this plane, at a distance r from the center, we obtain a velocity ellipsoid whose major axis points towards the center of the galaxy. Details of these calculations can be found in [29]. This aspect is consistent with the only measurement available, namely the determination of the velocity ellipsoid for the population of stars in the vicinity of the Sun, whose major axis tends approximately towards the galactic center and whose equal transverse axes have values half that of the galactic center.

This is the beginning of the confirmation of the model.

The density is then obtained by solving Poisson's equation numerically. But with a single population, the galaxy's total mass tends towards infinity. This mass becomes finite if the galaxy is surrounded by a distribution of negative mass, obeying a second Vlasov equation, where the distribution function is assumed to be of the Maxwell Boltzmann type. This work is currently under development.

There is another aspect of galactic dynamics. Specialists in spiral structure are still looking for the mechanism that would allow it to maintain itself. The Janus model shows that perennial spiral structures can be obtained, reflecting the way galaxies continuously transfer energy and angular momentum to their negative-mass surroundings. In collisional environments, transport occurs from close to close, via collisions. This type of dissipative phenomenon cannot arise in a non-collisional system. Instead, it takes place via gravity waves, which manifest themselves both within the galaxy and in its negative-mass environment. Simulations have shown that barred spiral formations can be maintained for up to 40 revolutions [16].

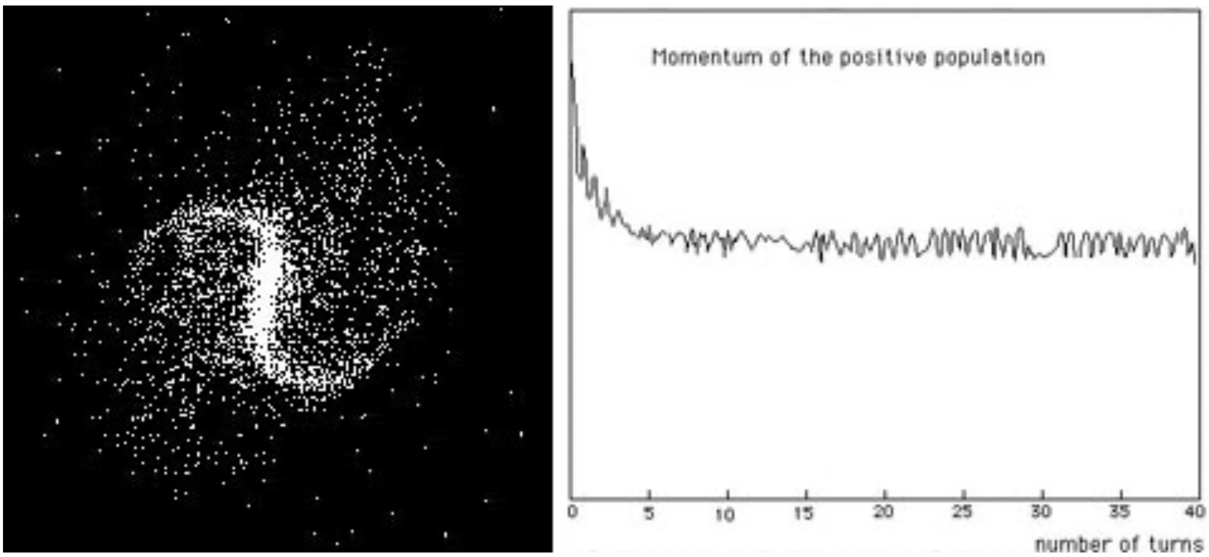

Fig.13 : Loss of angular momentum. Numerical simulation [20]

→ The Janus model is therefore the only one that explains the origin of the galactic spiral structure as the manifestation of a dissipative density-wave phenomenon.

12 – **Conclusion**.

Cosmology and astrophysics are currently facing a major crisis. Over the last five decades, specialists have attempted to integrate new observational data by adding two ad hoc components of unknown nature to the model - dark matter and dark energy - to form a new standard model. In recent years, however, other elements have emerged that the standard model is no longer able to account for. Essentially, the existence of huge voids in the large-scale structure, and the early appearance of galaxies and first-generation stars. Perhaps it's time, not to reject a new creation that has enabled mankind to take a fresh look at the universe, bringing with it so much richness, but to deploy it even more widely, opening the way to even more enlightening discoveries..

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