Janus Cosmological Model Mathematically & Physically Consistent

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We choose not to introduce cosmological constants Λ and $\bar{\Lambda}$. The tensors $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$ represent the contributions to the field from positive and negative masses, respectively. The tensors $\mathcal{T}_{\mu\nu}$ and $\bar{\mathcal{T}}_{\mu\nu}$ will be interaction tensors, representing the action of negative masses on positive masses and the action of positive masses on negative masses, respectively.

In our numerical simulations, we have heuristically opted for the following interaction laws:

- Positive masses attract each other
- Negative masses attract each other
- Masses of opposite signs repel each other.

From a physical perspective, numerical simulations [3] indicate that in regions dominated by positive masses, negative masses are almost nonexistent, and vice versa. Focusing on the Newtonian approximation, in mixed notation, we express the tensors associated with the two types of matter as follows:

$$T_\mu^\nu \approx \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{1.3}$$

$$\bar{T}_\mu^\nu \approx \begin{pmatrix} \bar{\rho} \bar{c}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{1.4}$$

For negative masses to exert mutual attraction, it is necessary to choose a negative gravitational constant, that is, to set $\kappa = -1$. Thus, we write:

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \chi \left[T_\mu^\nu + \mathcal{T}_\mu^\nu \right] \tag{1.5}$$

$$\bar{R}_\mu^\nu - \frac{1}{2} \bar{R} \delta_\mu^\nu = -\chi \left[\bar{T}_\mu^\nu + \bar{\mathcal{T}}_\mu^\nu \right] \tag{1.6}$$

The terms $\mathcal{T}_\mu^\nu$ and $\bar{\mathcal{T}}_\mu^\nu$ represent the interaction tensors. The first, $\mathcal{T}_\mu^\nu$, represents the contribution of negative masses to the field experienced by positive masses, while the second, $\bar{\mathcal{T}}_\mu^\nu$, represents the contribution of positive masses to the field experienced by negative masses. In a region dominated by positive masses, which primarily contribute to the field, the interaction tensor $\mathcal{T}_\mu^\nu$ can be neglected, thus simplifying the first equation.

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \chi T_\mu^\nu \tag{1.7}$$

The equation in question is Einstein's equation, where the cosmological constant is set to zero. This equation is at the origin of local phenomena predicted by general relativity, such as the perihelion precession of Mercury and the deflection of light by the Sun, confirming that the Janus model is in agreement with these tests of general relativity at the local scale. Moreover, in regions dominated by negative masses, the mutual attraction they exert on each other follows from the following equation:

$$\bar{R}_\mu^\nu - \frac{1}{2} \bar{R} \delta_\mu^\nu = -\chi \bar{T}_\mu^\nu \tag{1.8}$$

Since $\bar{\rho} < 0$ and $\bar{p} < 0$, this results in an attraction. This phenomenon also affects photons of negative energy, whose trajectories are altered when they pass near a negative mass, following an attractive trajectory.

The challenge lies in determining the exact form of the interaction tensors $\mathcal{T}_\mu^\nu$ and $\bar{\mathcal{T}}_\mu^\nu$.

2 Construction of a Non-Stationary, Homogeneous, Isotropic Exact Solution [4]

Numerical simulations, relying on the Newtonian approximation and starting from the principle that masses of opposite signs repel each other in a manner proportional to the product of their masses and inversely proportional to the square of the distance separating them, have yielded conclusive results. This leads us to formulate the following hypothesis:

$$\bar{\mathcal{T}}_\mu^\nu \approx \begin{pmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.1}$$

$$\mathcal{T}_\mu^\nu \approx \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.2}$$

If we consider describing the behavior of the system under the assumptions of non-stationarity, isotropy, and homogeneity, the interaction terms can be identified, for the radiative phase where pressures can be neglected, with forms 2.1 and 2.2, where the scalars are then functions of the chronological variable x^0. The metrics then take the following FLRW forms:

$$g_{\mu\nu} = \mathrm{d}x^{0^2} - a^2 \left[\frac{\mathrm{d}u^2}{1 - ku^2} + u^2 \mathrm{d}\theta^2 + u^2 \sin^2\theta \mathrm{d}\phi^2 \right] \tag{2.3}$$

$$\bar{g}_{\mu\nu} = \mathrm{d}x^{0^2} - \bar{a}^2 \left[\frac{\mathrm{d}u^2}{1 - \bar{k}u^2} + u^2 \mathrm{d}\theta^2 + u^2 \sin^2\theta \mathrm{d}\phi^2 \right] \tag{2.4}$$

A search for solutions yields negative curvature indices $k = \bar{k} = -1$.

If we introduce these metrics into the system of equations 2.5 and 2.6, considering 2.1 and 2.2, we obtain a classical system of four equations whose compatibility conditions require having the form 2.7 of the two functions Φ and ϕ of the chronological variable x^0:

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \chi \left[T_\mu^\nu + \Phi \mathcal{T}_\mu^\nu \right] \tag{2.5}$$

$$\bar{R}_\mu^\nu - \frac{1}{2} \bar{R} \delta_\mu^\nu = -\chi \left[\bar{T}_\mu^\nu + \phi \bar{\mathcal{T}}_\mu^\nu \right] \tag{2.6}$$

$$\Phi = \left(\frac{\bar{a}}{a} \right)^3, \quad \phi = \left(\frac{a}{\bar{a}} \right)^3, \quad \phi = \Phi^{-1} \tag{2.7}$$

This then leads to an equation that is none other than that of the conservation of energy and mass. Here, in a similar situation, the compatibility condition is a conservation of energy extended to the two populations:

$$E = \rho c^2 a^3 + \bar{\rho} \bar{c}^2 \bar{a}^3 \tag{2.8}$$

As we will see later, the mathematical consistency of the system of two field equations (Bianchi conditions) translates physically into conservation and balance equations. This leads to obtaining an exact solution in the form of two differential equations:

$$a^2 \frac{\mathrm{d}^2 a}{\mathrm{d}x^{0^2}} = \frac{\chi}{2} E \tag{2.9}$$

$$\bar{a}^2 \frac{\mathrm{d}^2 \bar{a}}{\mathrm{d}x^{0\,2}} = -\frac{\bar{\chi}}{2} E \tag{2.10}$$

To be in agreement with observational data, which suggest that in this phase where the radiation pressure is negligible, the acceleration of the expansion of the positive species is positive, the total energy of the system must be negative [5]. With a negative curvature index, both curves tend towards asymptotes. This situation seems more satisfactory to the physicist than the one emerging from Einstein's equation where this acceleration of expansion is then attributed to the cosmological constant, reflecting an unknown phenomenon. As this energy density does not evolve over time, this results in exponential growth. We note that the coefficients ϕ and Φ from 2.7 can respectively be identified with the factor $\sqrt{\frac{g}{\bar{g}}}$ and its inverse.

3 Constraints Associated with Stationary Solutions

In the system of coupled field equations 1.1 and 1.2, the terms on the left-hand side involve the Ricci tensors $R_{\mu\nu}$ and $\bar{R}_{\mu\nu}$ and the corresponding Ricci scalars R and $\bar{R}$. These terms are calculated from the two metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$.

Using these two metrics, we then calculate the form of two operators known as *covariant derivatives* ∇_μ and $\bar{\nabla}_\mu$. It turns out that, due to their form, the two left-hand sides of both equations identically satisfy the following relation:

$$\nabla_\mu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = 0 \tag{3.1}$$

$$\bar{\nabla}_\mu \left(\bar{R}_{\mu\nu} - \frac{1}{2} \bar{R} \bar{g}_{\mu\nu} \right) = 0 \tag{3.2}$$

The two tensors, $T_{\mu\nu}$ and $\bar{T}_{\mu\nu}$, also satisfy the following condition:

$$\nabla_\mu T_{\mu\nu} = 0 \tag{3.3}$$

$$\bar{\nabla}_\mu \bar{T}_{\mu\nu} = 0 \tag{3.4}$$

We should also have:

$$\nabla_\mu \mathcal{T}_{\mu\nu} = 0 \tag{3.5}$$

$$\bar{\nabla}_\mu \bar{\mathcal{T}}_{\mu\nu} = 0 \tag{3.6}$$

At this stage, it can be said that it is precisely these two conditions that determine the form of these interaction tensors. As for observational data, what is the constraint we must adhere to?

Numerical simulations have shown that the two populations mutually exclude each other. If we limit ourselves to these two extreme situations, the system of coupled field equations 1.5 and 1.6 simplifies, when positive mass dominates, to:

$$R_\mu^\nu - \frac{1}{2} R \delta_\mu^\nu = \chi T_\mu^\nu \tag{3.7}$$

$$\bar{R}_\mu^\nu - \frac{1}{2} \bar{R} \delta_\mu^\nu = -\chi \bar{\mathcal{T}}_\mu^\nu \tag{3.8}$$

The first equation 3.7 reduces to Einstein's equation. This solution can be constructed, both in vacuum and within masses, in which case the mathematical condition 3.3 translates into the equation established in 1916 by Karl Schwarzschild and in 1939 by Tolman, Oppenheimer, and Volkoff, known as the Tolman-Oppenheimer-Volkoff equation [2] [6] [7]. This solution can then extend to the description of the geometry inside a neutron star.

There is no need to explicitly detail the form of the interaction tensor, present on the right-hand side of equation 3.8, since the physical effect that ensues, the deflection of negative energy photons by a positive mass, eludes observation.

When, on the contrary, negative mass dominates, the system becomes:

$$R^{\nu}_{\mu} - \frac{1}{2} R \delta^{\nu}_{\mu} = \chi \mathcal{T}^{\nu}_{\mu} \tag{3.9}$$

$$\bar{R}^{\nu}_{\mu} - \frac{1}{2} \bar{R} \delta^{\nu}_{\mu} = -\chi \bar{T}^{\nu}_{\mu} \tag{3.10}$$

For equation 3.10, condition 3.4 reflects the balance between pressure force and gravity within a body filled with negative mass matter. This yields a condition identical to the classic Tolman-Oppenheimer-Volkoff equation. The challenge then is to give a form to the interaction tensor $\mathcal{T}^{\nu}_{\mu}$ in the first equation 3.9 that satisfies condition 3.5.

4 Interpretation of the Dipole Repeller through the Janus Cosmological Model

When negative masses are predominant as near the Dipole Repeller ([1]), we are faced with potential observational data that will emerge sooner or later and will require a response.

Numerical simulations have shown that as soon as gravitational instability can play its role in both populations, it is negative mass that dominates, given that negative mass density is predominant. In this gravitational instability, we have a characteristic time, given by Jeans, which is:

$$t_j = \frac{1}{\sqrt{4\pi G \rho}} \tag{4.1}$$

$$\bar{t}_j = \frac{1}{\sqrt{4\pi G |\bar{\rho}|}} \tag{4.2}$$

Since $|\bar{\rho}| \gg \rho$, the accretion time of negative masses is shorter. It is therefore within this second population that a semi-regular population of spheroidal conglomerates will establish itself. Observationally, this will create immense voids in the positive world, hundreds of millions of light-years in diameter. In 2017, observation confirmed this prediction [1]. This is the Dipole Repeller, located 600 million light-years from our galaxy (Figure 1).

In the seven years following, other large voids of this kind have been identified. These then behave like immense proto-stars, composed of negative-mass anti-hydrogen and anti-helium. This concentration of matter occurs until the ionization of hydrogen, which then stops this movement. Nevertheless, the geometry inside this spheroid falls within the framework of the Newtonian approximation, that is, when the particle agitation velocities are much smaller than the local speed of light[2] and when curvature effects are negligible[3]. This context also leads to the fact that pressure terms can be neglected.

We then need to propose a form of the source tensors such that the condition of zero covariant derivative is asymptotically satisfied in the limit of weak fields[4].

Hence, if we consider the impact of the presence of negative masses on the geometry of spacetime structured by the metric tensor of the first field equation 3.9 associated with the population of positive masses,

[2] $4\pi r^3 p \ll mc^2$ and $\frac{2Gm}{c^2 r} \ll 1$

[3] The inequality $r \gg 2m$ indicates that we are sufficiently far from the gravitational source for the effects of general relativity to be negligible. Indeed, at large distances (r), the term $\frac{2GM}{c^2}$ becomes very small.

[4] All the details of the following calculations can be found in Hicham Zejli's book on [8].

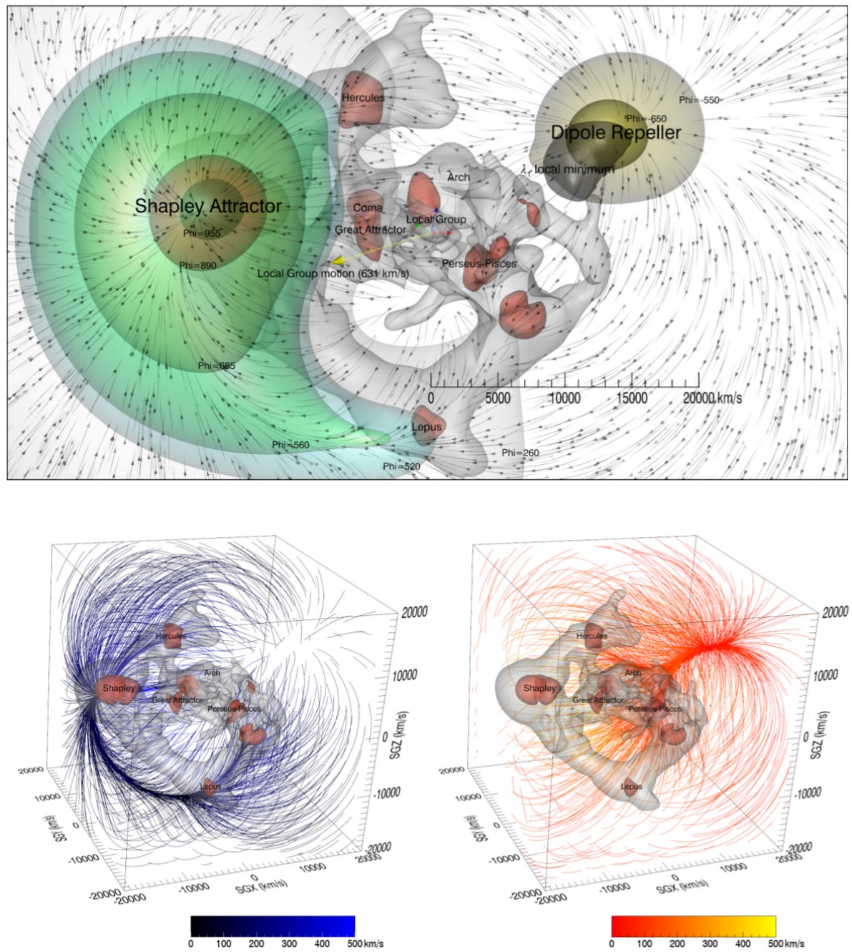

Figure 1: Dipole Repeller from [1]. The figure shows the location of the Dipole Repeller (highlighted by the red circle) within the large-scale structure of the universe. The Dipole Repeller is a hypothesized region of space where galaxies are pushed away from, counteracting the attractive force of the Shapley Supercluster.

we can define the corresponding interaction tensor 4.3 as follows:

$$\mathcal{T}_{\mu}^{\nu} = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & -\bar{p} & 0 & 0 \\ 0 & 0 & -\bar{p} & 0 \\ 0 & 0 & 0 & -\bar{p} \end{pmatrix} \tag{4.3}$$

Thus, the impact of the pressure gradient of negative masses on the geodesics followed by ordinary matter and positive-energy photons according to the field equation 3.9 translates into the following Tolman–Oppenheimer–Volkoff equation:

$$\frac{\bar{p}'}{\bar{c}^2} = -\frac{m - \frac{4\pi G \bar{p} r^3}{\bar{c}^4}}{r(r+2m)} \left(\bar{\rho} - \frac{\bar{p}}{\bar{c}^2}\right) \tag{4.4}$$

Then, it's possible to build the Schwarzschild interior metric solution in this manner:

$$\mathrm{d}s^2 = \left[\frac{3}{2}\sqrt{\left(1 + \frac{{r_n}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1 + \frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}x^{0^2} - \frac{\mathrm{d}r^2}{1 + \frac{r^2}{\hat{r}^2}} - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{4.5}$$

Where r_n is the star radius, and the characteristic radius $\hat{r}$ is defined depending on the star's density ρ as follows:

$$\hat{r} = \sqrt{\frac{3c^2}{8\pi G\rho}} \tag{4.6}$$

And the Schwarzschild Radius is given by :

$$R_s = 2\frac{GM}{c^2} \tag{4.7}$$

This metric can be connected to the Schwarzschild exterior metric:

$$\mathrm{d}s^2 = \left(1 + \frac{2GM}{c^2 r}\right) c^2 \mathrm{d}{x^0}^2 - \frac{\mathrm{d}r^2}{1 + \frac{2GM}{c^2 r}} - r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{4.8}$$

We can deduce that a particle of ordinary matter will undergo a repulsive gravitational field due to the effect of a distribution of negative masses.

Then, when the source of the gravitational field of the second field equation 3.10 is created by a negative mass, we can freely define the following energy-momentum tensor as follows:

$$\bar{T}_\mu^\nu = \begin{pmatrix} \bar{\rho}\bar{c}^2 & 0 & 0 & 0 \\ 0 & \bar{p} & 0 & 0 \\ 0 & 0 & \bar{p} & 0 \\ 0 & 0 & 0 & \bar{p} \end{pmatrix} \tag{4.9}$$

We can therefore deduce the following Tolman–Oppenheimer–Volkoff equation:

$$\frac{\bar{p}'}{\bar{c}^2} = -\frac{\bar{m} + \frac{4\pi G\bar{p}r^3}{\bar{c}^4}}{r(r-2\bar{m})}\left(\bar{\rho} + \frac{\bar{p}}{\bar{c}^2}\right) \tag{4.10}$$

Hence, the interior Schwarzschild metric can be constructed as follows:

$$\bar{\mathrm{d}s}^2 = \left[\frac{3}{2}\sqrt{\left(1 - \frac{\bar{r_n}^2}{\hat{r}^2}\right)} - \frac{1}{2}\sqrt{\left(1 - \frac{r^2}{\hat{r}^2}\right)}\right]^2 \mathrm{d}{x^0}^2 - \frac{\mathrm{d}r^2}{1 - \frac{r^2}{\hat{r}^2}} - r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{4.11}$$

This metric matches the exterior Schwarzschild metric:

$$\bar{\mathrm{d}s}^2 = \left(1 - \frac{2G\bar{M}}{\bar{c}^2 r}\right) \bar{c}^2 \mathrm{d}{x^0}^2 - \frac{\mathrm{d}r^2}{1 - \frac{2G\bar{M}}{\bar{c}^2 r}} - r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right) \tag{4.12}$$

We can deduce that a particle of negative mass will undergo an attractive gravitational field due to the effect of a distribution of negative masses.

Both solutions 4.4 and 4.10 reduces to the Euler equation approximately equal to $-\frac{G\bar{M}(r)\bar{\rho}(r)}{r^2}$ in the Newtonian limit, reflecting hydrostatic equilibrium[5].

The form of these two source tensors satisfies the Bianchi conditions. This would obviously not be the case if the negative mass were to fall outside of this framework. For that, there would need to exist neutron stars of negative mass. However, the characteristic time of evolution of conglomerates of negative mass, their *"cooling time"*, exceeds the age of the universe. These spheroidal conglomerates cannot evolve, so the content of this negative spacetime will be limited to a mixture of negative mass anti-hydrogen and anti-helium. Since nucleosynthesis cannot occur, there can be no anti-galaxies or anti-stars, regardless of their

[5]Where the pressure at the center of this negative mass spheroid is balanced by the negative gravitational force depending on density and mass

mass. Consequently, there cannot exist anti-neutron stars.

Moreover, in the case where this negative spacetime would generate hyperdense stars through an as yet unknown mechanism, it would then be necessary to reconsider the form of these tensors. However, the current configuration satisfies all currently available and potentially available observational data.

Photons of positive energy emitted by sources located behind the Dipole Repulsor will experience a significant decrease in their magnitude due to the negative gravitational lensing effect. These photons then freely traverse this vast void. The effect will be maximal when the photons brush past this spheroidal conglomerate, where the entirety of the mass must be taken into account. However, it will be negligible when these photons pass through the central neighborhood (Figure 2).

Thus, we predict that when a map is established by the JWST telescope, the invisible mass will manifest its presence by a brightness attenuation, not over the entire disk, but in a ring.

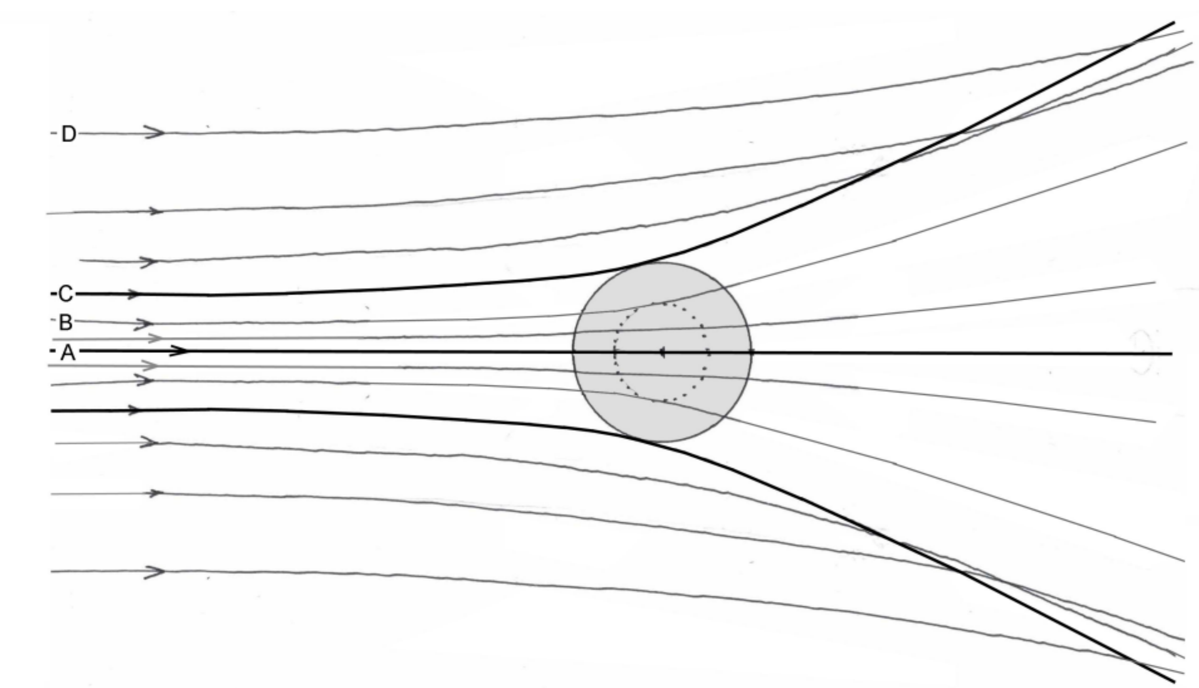

Figure 2: Deviation of positive-energy photons by a negative mass. The trajectories, when the curvature remains moderate, are very close to hyperbolas. The angle of deviation reaches a maximum (C) when the geodesic is tangent to the limit of the mass. It then decreases steadily to zero at very large distances (D). The angle of deviation is null, due to symmetry, when the geodesic passes through the center of the mass (A).

5 Lagrangian Derivation of an Action Yielding the Coupled Field Equations of the Janus Model

Let us now consider the interaction between two entities: ordinary matter with positive mass interacting with negative mass through gravitational effects. This model involving negative mass takes into account the influence of both dark matter and dark energy.

We can describe this system of two entities with respective metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. Let R and $\bar{R}$ be the corresponding Ricci scalars. We then consider the following two-layer action[6]:

$$A = \int_{\mathscr{E}} \left(\frac{1}{2\chi} R + S + \mathcal{S} \right) \sqrt{|g|}\, \mathrm{d}^4x + \int_{\mathscr{E}} \left(\frac{\kappa}{2\bar{\chi}} \bar{R} + \bar{S} + \bar{\mathcal{S}} \right) \sqrt{|\bar{g}|}\, \mathrm{d}^4x \tag{5.1}$$

The terms S and $\bar{S}$ will give the source terms related to the populations of the two entities, while the terms $\mathcal{S}$ and $\bar{\mathcal{S}}$ will generate the interaction tensors. χ and $\bar{\chi}$ are the Einstein gravitational constants for each entity. g and $\bar{g}$ are the determinants of the metrics $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$. For $\kappa = \pm 1$, we apply the principle of least action. The Lagrangian derivation of this action gives us:

$$\begin{aligned} 0 &= \delta A \\ &= \int_{\mathscr{E}} \delta \left(\frac{1}{2\chi} R + S + \mathcal{S} \right) \sqrt{|g|}\, \mathrm{d}^4x \\ &\quad + \int_{\mathscr{E}} \delta \left(\frac{\kappa}{2\bar{\chi}} \bar{R} + \bar{S} + \bar{\mathcal{S}} \right) \sqrt{|\bar{g}|}\, \mathrm{d}^4x \\ &= \int_{\mathscr{E}} \delta \left[\frac{1}{2\chi} \left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{|g|}} \frac{\delta \sqrt{|g|}}{\delta g^{\mu\nu}} \right) \right. \\ &\quad \left. + \frac{1}{\sqrt{|g|}} \frac{\delta(\sqrt{|g|} S)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{|g|}} \frac{\delta(\sqrt{|g|} \mathcal{S})}{\delta g^{\mu\nu}} \right] \delta g^{\mu\nu} \sqrt{|g|}\, \mathrm{d}^4x \\ &\quad + \int_{\mathscr{E}} \delta \left[\frac{\kappa}{2\bar{\chi}} \left(\frac{\delta \bar{R}}{\delta \bar{g}^{\mu\nu}} + \frac{\bar{R}}{\sqrt{|\bar{g}|}} \frac{\delta \sqrt{|\bar{g}|}}{\delta \bar{g}^{\mu\nu}} \right) \right. \\ &\quad \left. + \frac{1}{\sqrt{|\bar{g}|}} \frac{\delta(\sqrt{|\bar{g}|} \bar{S})}{\delta \bar{g}^{\mu\nu}} + \frac{1}{\sqrt{|\bar{g}|}} \frac{\delta(\sqrt{|\bar{g}|} \bar{\mathcal{S}})}{\delta \bar{g}^{\mu\nu}} \right] \delta \bar{g}^{\mu\nu} \sqrt{|\bar{g}|}\, \mathrm{d}^4x \end{aligned} \tag{5.2}$$

For any variation $\delta g^{\mu\nu}$ and $\delta \bar{g}^{\mu\nu}$, we locally obtain:

$$\frac{1}{2\chi} \left(\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{|g|}} \frac{\delta \sqrt{|g|}}{\delta g^{\mu\nu}} \right) + \frac{1}{\sqrt{|g|}} \frac{\delta(\sqrt{|g|} S)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{|g|}} \frac{\delta(\sqrt{|g|} \mathcal{S})}{\delta g^{\mu\nu}} = 0 \tag{5.3}$$

$$\frac{\kappa}{2\bar{\chi}} \left(\frac{\delta \bar{R}}{\delta \bar{g}^{\mu\nu}} + \frac{\bar{R}}{\sqrt{|\bar{g}|}} \frac{\delta \sqrt{|\bar{g}|}}{\delta \bar{g}^{\mu\nu}} \right) + \frac{1}{\sqrt{|\bar{g}|}} \frac{\delta(\sqrt{|\bar{g}|} \bar{S})}{\delta \bar{g}^{\mu\nu}} + \frac{1}{\sqrt{|\bar{g}|}} \frac{\delta(\sqrt{|\bar{g}|} \bar{\mathcal{S}})}{\delta \bar{g}^{\mu\nu}} = 0 \tag{5.4}$$

[6]Integration over $\mathscr{E}$ using the element d^4x is a method for computing the total action in the bimetric spacetime, reflecting the four-dimensional nature of this bimetric universe. This implies considering the entire spacetime as the domain of integration, integrating the contributions from each point to the action. The term d^4x represents an infinitesimal element of hypervolume of this bimetric spacetime, used to "measure" each segment during integration. Thus, it is a multiple volume integral performed over the four dimensions of spacetime, accumulating contributions to the total action from each four-dimensional volume segment corresponding to each metric.

Let us then introduce the following tensors:

$$T_{\mu\nu} = -\frac{2}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}S)}{\delta g^{\mu\nu}} = -2\frac{\delta S}{\delta g^{\mu\nu}} + g_{\mu\nu}S \tag{5.5}$$

$$\bar{T}_{\mu\nu} = -\frac{2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\bar{S})}{\delta \bar{g}^{\mu\nu}} = -2\frac{\delta \bar{S}}{\delta \bar{g}^{\mu\nu}} + \bar{g}_{\mu\nu}\bar{S} \tag{5.6}$$

$$\mathcal{T}_{\mu\nu} = -\frac{2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} \tag{5.7}$$

$$\bar{\mathcal{T}}_{\mu\nu} = -\frac{2}{\sqrt{|g|}}\frac{\delta(\sqrt{|\bar{g}|}\bar{\mathcal{S}})}{\delta \bar{g}^{\mu\nu}} \tag{5.8}$$

We obtain then from equations 5.7 and 5.8:

$$\sqrt{\frac{|\bar{g}|}{|g|}}\mathcal{T}_{\mu\nu} = \sqrt{\frac{|\bar{g}|}{|g|}}\frac{-2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} = \frac{-2}{\sqrt{|g|}}\frac{\delta(\sqrt{|g|}\mathcal{S})}{\delta g^{\mu\nu}} = -2\frac{\delta \mathcal{S}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{S} \tag{5.9}$$

$$\sqrt{\frac{|g|}{|\bar{g}|}}\bar{\mathcal{T}}_{\mu\nu} = \sqrt{\frac{|g|}{|\bar{g}|}}\frac{-2}{\sqrt{|g|}}\frac{\delta(\sqrt{|\bar{g}|}\bar{\mathcal{S}})}{\delta \bar{g}^{\mu\nu}} = \frac{-2}{\sqrt{|\bar{g}|}}\frac{\delta(\sqrt{|\bar{g}|}\bar{\mathcal{S}})}{\delta \bar{g}^{\mu\nu}} = -2\frac{\delta \bar{\mathcal{S}}}{\delta \bar{g}^{\mu\nu}} + \bar{g}_{\mu\nu}\bar{\mathcal{S}} \tag{5.10}$$

Introduced into equations 5.3 and 5.4, we can thus deduce the coupled field equations describing the system of the two entities:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \chi\left(T_{\mu\nu} + \sqrt{\frac{|\bar{g}|}{|g|}}\mathcal{T}_{\mu\nu}\right) \tag{5.11}$$

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \kappa\bar{\chi}\left(\bar{T}_{\mu\nu} + \sqrt{\frac{|g|}{|\bar{g}|}}\bar{\mathcal{T}}_{\mu\nu}\right) \tag{5.12}$$

Where $\mathcal{T}_{\mu\nu}$ and $\bar{\mathcal{T}}_{\mu\nu}$ are the interaction tensors of the system of the two entities corresponding to the *"induced geometry"*, meaning how each matter distribution on one layer of the universe contributes to the geometry of the other[7].

[7]Interaction between populations of positive and negative masses.

6 Conclusion

This model sheds light on several fundamental aspects of the universe:

- It describes the temporal evolution of the universe in its material phase, based on the assumption that the involved mediums are isotropic and homogeneous. In this context, positive masses are accelerated. With a negative curvature index, the model anticipates a decrease of this acceleration towards zero, eventually leading to a linear expansion of the universe. This conclusion contrasts with predictions stemming from the use of a cosmological constant, which suggest an asymptotic expansion.

- It accounts for stationary states corresponding to an SO(3) symmetry, thus illustrating the model's ability to reproduce configurations classically associated with Einstein's equation. For an SO(2) symmetry, the model, like Einstein's, only describes the geometry of empty space, where mathematical coherence conditions are inherently respected.

- The analysis of the exact instationary solution provided by the model reveals results in accordance with observations, underlining its adequacy with real phenomena observed in the universe [5].

- It predicts a patchy structure for positive mass, forming around spaces left by conglomerates of negative mass in a spheroidal shape, arranged in a regular manner. This prediction received initial observational confirmation with the discovery of the Dipole Repeller in 2017 [1]. Subsequently, the detection of numerous other cosmic voids, each extending over a hundred million light-years in diameter, further strengthens this theory.

- The model also predicts a decrease in the apparent magnitude of objects located behind the Dipole Repeller in an annular region, a phenomenon applicable in the long run to all similar formations. This attenuation is due to the effect of these massive negative structures on the distribution of light in the universe.

In summary, our model offers a comprehensive understanding of phenomena classically associated with Einstein's equation, while providing predictions verifiable by observation.

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