Geometry Drives the Today's Necessary Paradigmatic Change

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Abstract:

It is recalled that the construction of special relativity and then general relativity was based on changes in the geometric paradigm. Starting from the difficulties encountered by the ΛCDM model, we propose this change of geometric paradigm represented by the Janus cosmological model, specifying its topological bases, its structure as a two-sheet covering of a projective P_4 and its associated dynamical group structure. We show he such geometry of spacetime goes with PT-symmetry. This is completed by the construction of its system of coupled field equations from an action

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1 - **Introduction**.

Recent observations show that the current cosmological model, the ΛCDM model, which is considered to be standard, is confronted with a growing number of contradictions. These include the discovery of the lacunar structure of the universe, the early birth of galaxies and first-generation stars revealed by the JWST telescope, the lack of detection of primeval dark matter and the impossibility of giving an identity to dark energy. As a result, a growing number of scientists agree that it is now legitimate to envisage a paradigm shift, i.e. an attempt to go beyond the framework imposed by General Relativity. This last model is a perfect illustration of the geometric nature of successive paradigm shifts. Prior to 1905, space and time were considered as independent variables. A space that was also assumed to be Euclidean, i.e. defined by a Euclidean metric, operating in three dimensions of space $\{ x_1 , x_2 , x_3 \}$

(1) $$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

The corresponding Gramm matrix is the unit matrix:

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(2) $$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The isometry group being Euclid's group is:

(2) $$\begin{pmatrix} a & C \\ 0 & 1 \end{pmatrix}$$

Where a belongs to the orthogonal group and where C is the translation vector :

(3) $$C = \begin{pmatrix} \Delta x^1 \\ \Delta x^2 \\ \Delta x^3 \end{pmatrix}$$

The advent of Special Relativity can be summed up by the integration of time and space into a four-dimensional space-time. $\{ x_o , x_1 , x_2 , x_3 \}$, defined by its Lorentzian metric:

(2) $$ds^2 = (dx^o)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2$$

The corresponding Gramm matrix is:

(3) $$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The isometry group becomes the Poincaré group, represented by the matrices:

(4) $$\begin{pmatrix} L & C \\ 0 & 1 \end{pmatrix}$$

Where C is the vector of space-time translations:

(5) $$C = \begin{pmatrix} \Delta x^o \\ \Delta x^1 \\ \Delta x^2 \\ \Delta x^3 \end{pmatrix}$$

And where L represents the Lorentz group, defined by :

(6) $$L^t G L = G$$

The transition to General Relativity means that space-time is given a curvature. The metric is no longer the Lorentzian metric (2) but a more general metric:

(7) $$ds^2 = \sum_{\mu\nu} g_{\mu\nu}\, dx^{\mu}\, dx^{\nu}$$

$g_{\mu\nu}$ being the metric tensor.

This General Relativity is then equipped with a field equation, introduced in 1915 [1] from the Hilbert-Einstein action:

(8) $$J = \int_{D4} (\, R - 2\Lambda - \chi\, L \,)\, \sqrt{-g}\; d^4x$$

Where R is the Ricci scalar, L the Lagrangian of matter, g the determinant of the metric, χ the Einstein constant and Λ the cosmological constant . Its derivation leads to the field equation:

(9) $$R_{\mu\nu} + \Lambda g_{\mu\nu}\ \frac{1}{2} R\, g_{\mu\nu} = \chi\, T_{\mu\nu}$$

Space-time then has the structure of a Lorentzian manifold M_4.

In stationary situations, theorists relied on the metric solutions introduced in 1916 by K. Scharzchild ([2],[3]). However, from the late 1970s onwards, in order to account for the flat velocity profiles at the periphery of galaxies and the high velocities of galaxies in clusters, theorists began to consider the existence of positive-mass dark matter of an unknown nature that escaped observation.

For unsteady situations, until 2011 they used solutions to the cosmological constant-free equation, based on an assumption of homogeneity and isotropy. But observations in 2011 showed that the cosmic expansion was accelerating ([4], [5], [6]). The cosmological constant was then reintroduced into the equation.

There is one final point: the absence of observations of primordial antimatter. The photons that make up the background radiation at 2.7°K are supposed to result from the annihilation of primordial matter-antimatter pairs. Normally, this annihilation should have been complete and it remains a mystery why any matter particle survived. Nor do we understand why its antimatter equivalent has not been detected. This is no mean feat, in the sense that we are losing half the universe from the outset.

2 – **When André Sakharov was the first to envisage a change in geometry**.

At the stage we have just described, the universe is an orientable M_4 manifold, including a singular point, the Big Bang. There is no limit to cosmic expansion. Cosmic acceleration gives space-time a negative curvature index.

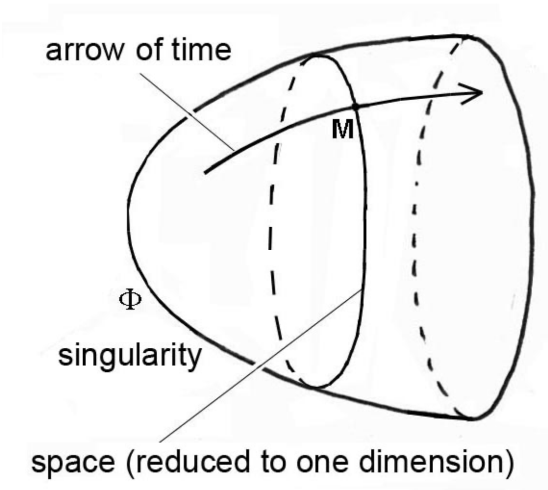


Fig. 1 : The standard universe model

But the universe then presents a matter-antimatter dissymmetry. The existence of the latter is proven: the reactions in which the C-symmetry appears take place in different timescales. In 1967, Sakharov ([7],[8],[9]) therefore imagined reconstituting global symmetry by endowing the knowable universe with a twin universe, CPT-symmetric with our own.

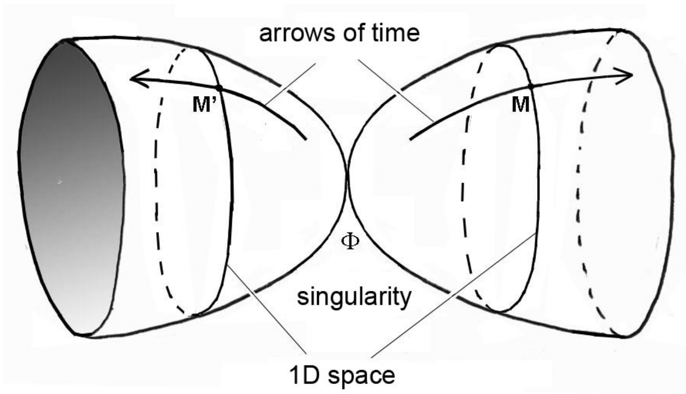


Fig. 2 : The twin worlds of A. Sakharov.

Considering that matter arises from the union of quarks and antimatter from antiquarks, as well as the dissymmetry already observed, he proposes that the synthesis of matter in our universe was faster than the synthesis of antimatter. After annihilation, photons from the annihilations, a small quantity of matter and an equivalent remnant of antiquarks would remain in our side of the universe. The situation being reversed in the twin universe. This theory is currently the only explanation for the proven absence of cosmological antimatter. Note that the Big Bang singularity can be replaced by a tubular passage.:

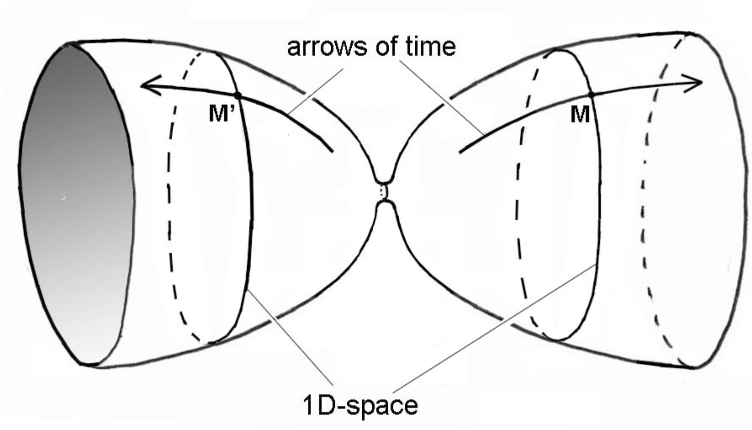


Fig.3 : The Sakharov model, without singularity.

3 – **The Janus model: how to fold this structure in on itself** [10].

Before considering this, it is necessary to assume that the observable universe has the topology of a sphere S2. Below is the didactic image in 2D.

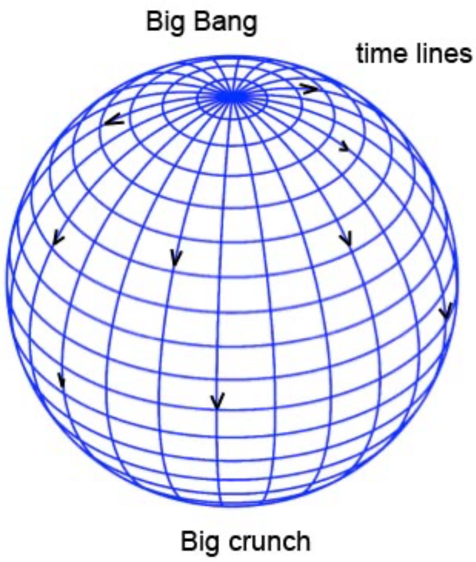


Fig.4 : Didactic image of spherical space-time.

In this didactic image, space, closed in on itself, is represented by a simple circle which, starting from a first Big Bang singularity, experiences a state of maximum expansion, at the equator, then contracts as it converges towards a second singularity, known as the Big Crunch, these two points being antipodal.:

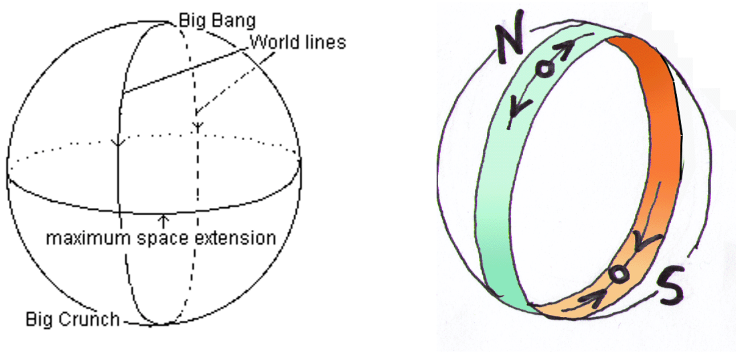


Fig. 5 : The Big Bang and the Big Crunch, antipodal.

The idea of having points in space-time interact with points located at space-time antipodes was considered in 1994 [10]. The universe can then be considered as the two-folds cover of an inorientable projective P_4. To illustrate this idea, let us show how a 2D spacetime can then be configured as a two-folds cover of a P_2 projective, which can be represented by the surface imagined in 1902 by the mathematician Werner Boy [11].

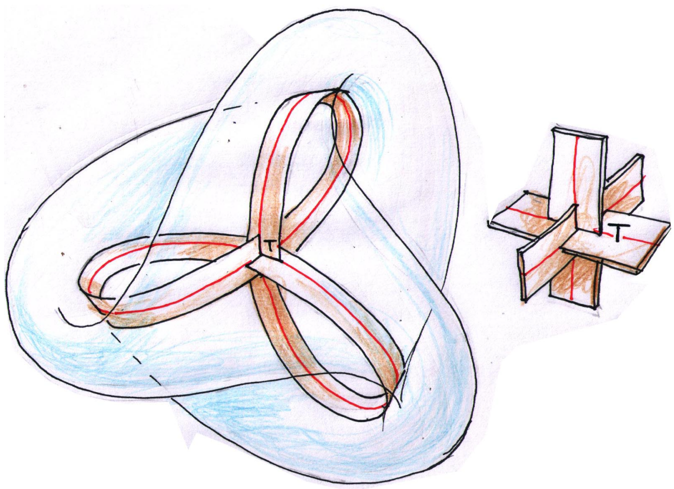


Fig. 6 : Boy's surface, the vicinity of its equator and the triple point.

This triple point arises only from the way in which the projective space P2 is represented as an immersion in R3. Similarly, the projective does not have a helical self-intersection curve.

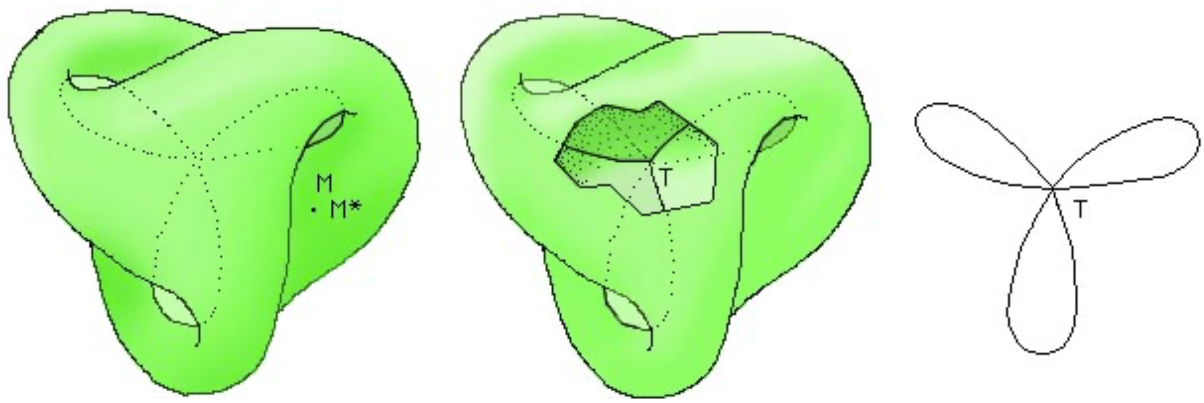


Fig.7 : Boy's surface, triple point and self-intersection curve.

This only results from the mode of representation in the form of an immersion in R3. Just as the self-intersection curve of the Klein bottle is only the result of this mode of representation.

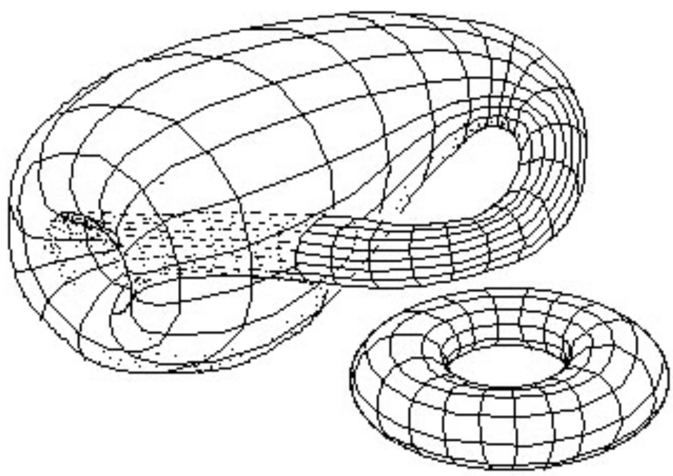
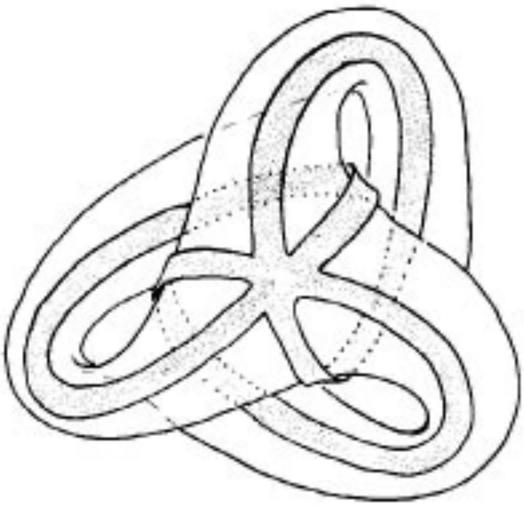


Fig. 8 : Klein bottle and T_2 torus.

The T_2 torus is itself the result of the two-folds covering of this Klein bottle K_2. We can endow the Boy surface with meridians, converging at its single pole. The Euler Poincaré characteristic of the Boy surface is unity. If we decide to tessellate it, this tessellation will therefore include a single tessellation singularity of order unity [12]. Since the characteristic of a sphere of even dimension is 2, this is also the characteristic of the 2D sphere, which therefore has two tessellation singularities of order unity, its poles. During the coating process, these two antipodal poles will coincide. Let's look at what happens to the meridians of the sphere, which are the neighbors of its time lines. These will be configured as a two-sheet covering of the meridians of Boy's surface. Here are three meridians of the surface.



Fg.9 : Méridiens de la surface de Boy.

And its equator:

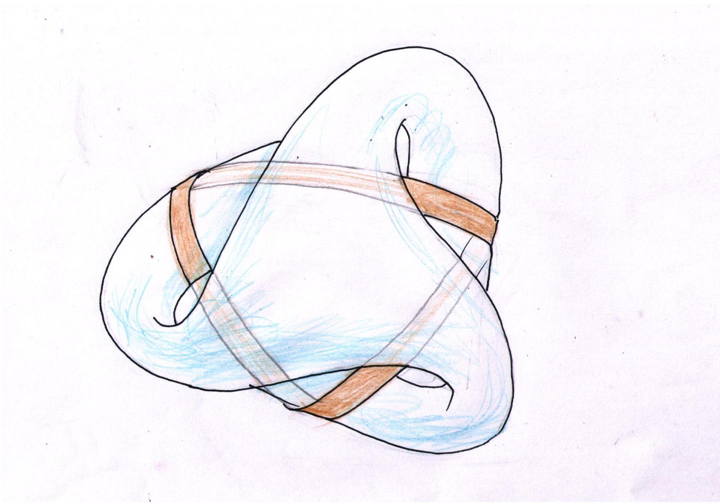


Fig.10 : The equator of Boy's surface: a Möbius strip with 3 half-turns.

Next, the equator and one meridian:

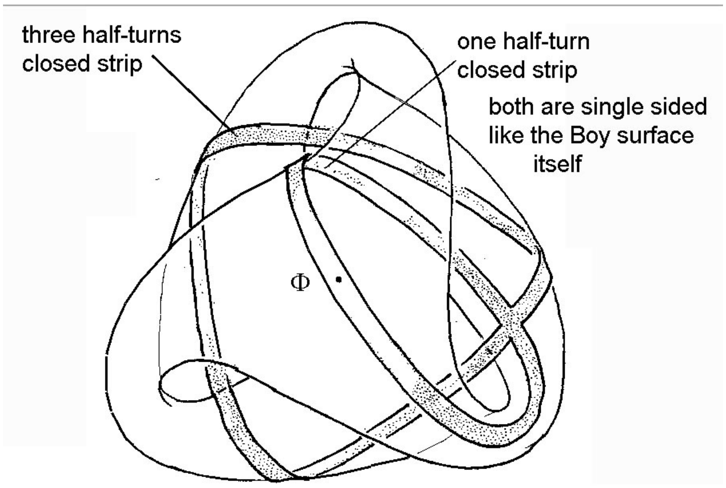


Fig. 11 : Boy's surface, equator and a meridian.

The coating operation will invert the arrow of time, as can be seen in the vicinity of a meridian time line, which is then configured as a two-sheet coating of a half-turn Möbius strip.:

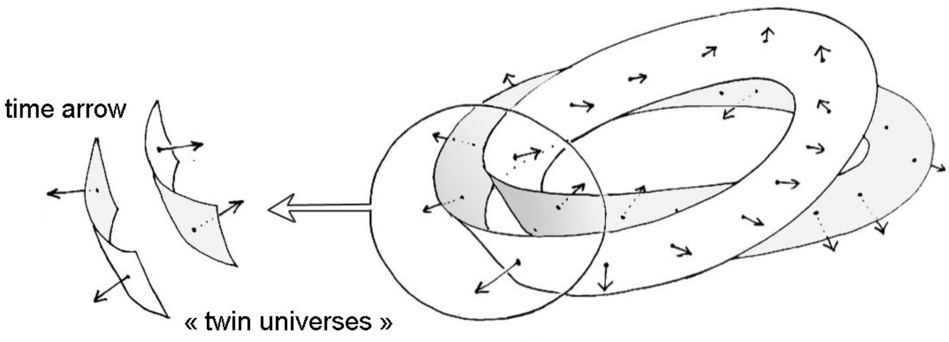


Fig.12 : Reversing time on a meridian.

This can also be seen by bringing antipodal points on the equator of a sphere S2 into coincidence.

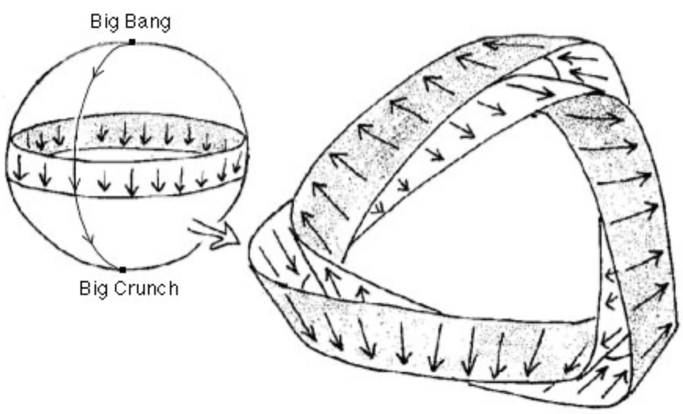
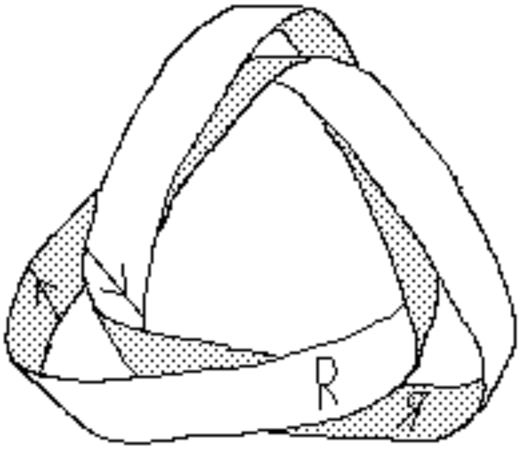


Fig.13 : Time reversal on the equator.

This coating operation also induces an enantiomorphy of adjacent surface portions. Here is an example of the folding of the equator of the sphere S2 according to the two-sheet coating of a Möbius strip with three half-turns.



14 – The mirror symmetry induced by the two-sheet covering of a projective P_2.

In relation to the Sakharov model, we obtain the following:

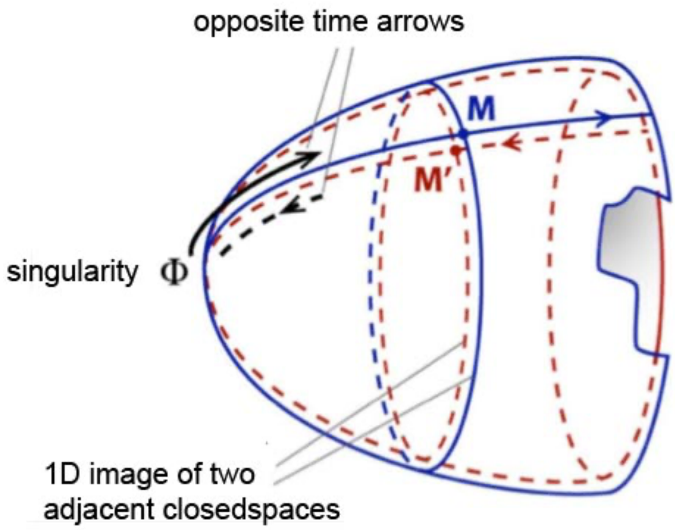


Fig.15 : Didactic image of the Janus model.

Note in passing that we can then make the singularity at the common pole disappear by replacing it with a small tube.

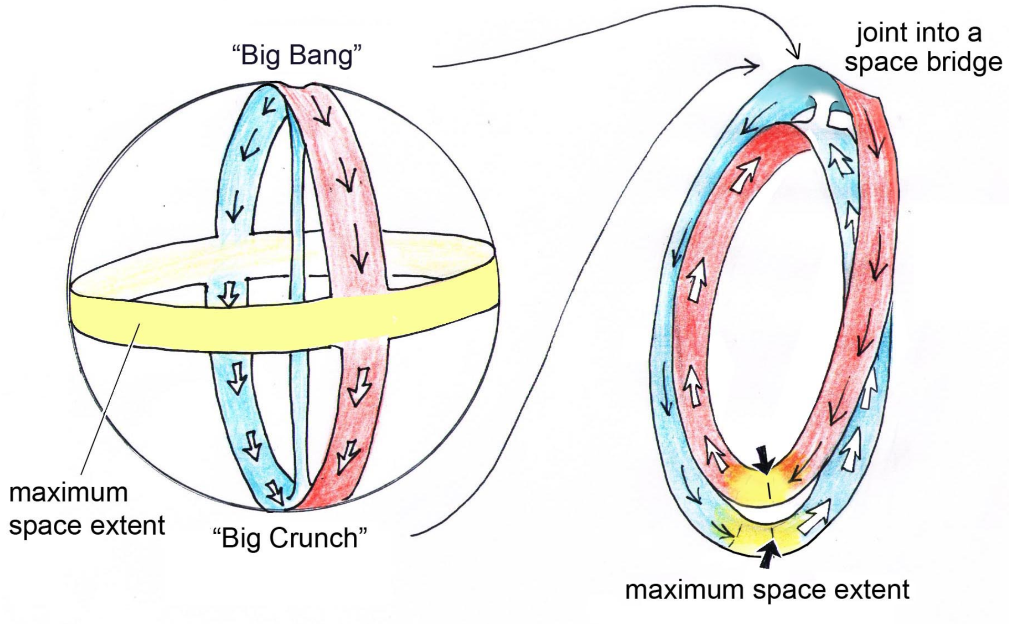


Fig.16 : The polar singularity disappears.

In other words:

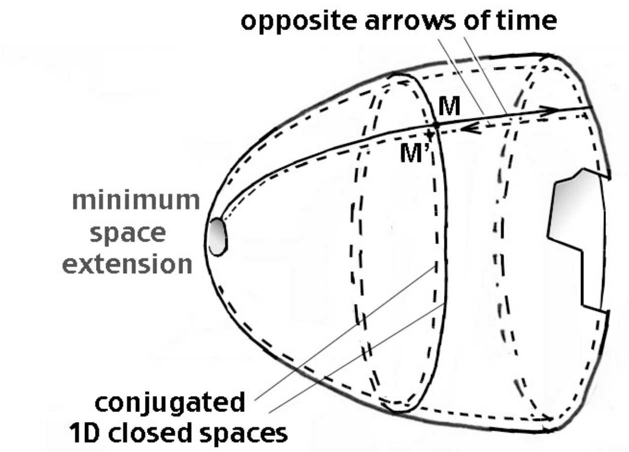


Fig.17 : Cosmological model with no Big Bang singularity.

The object, whose Euler-Poincaré characteristic is then zero, corresponds to the configuration given to a toric space-time in a two-sheet covering of a Klein bottle K_2.

All these operations can be extended to a spacetime with the topology of an orientable sphere S4, which is then configured as a two-sheet covering of a projective P_4. We know that P_N projectives are inorientable when N is even, and orientable when N is odd.

These geometric considerations lead us to consider the interaction between two adjacent regions of PT-symmetric space-time

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4 – **Interpretation of T-symmetry according to dynamic group theory.**

The first Janus group, incorporating this PT-symmetry [13], is the dynamic group [14].:

(10) $$\begin{pmatrix} \lambda L_o & C \\ 0 & 1 \end{pmatrix} \qquad \lambda = \pm 1 \text{ t}$$

L_o is then the subgroup of the complete Lorentz group, or restricted Lorentz group, limited to its two orthochronous components. The value $\lambda = -1$ then corresponds to this PT-symmetry. The Lorentz group has four connect components.

Let's call L_n the neutral component, which contains the neutral element of the group.

L_s is the component that inverts space but not time.

L_t is the component that inverts time but not space.

L_{st} is the component that inverts both space and time.

The first two form the orthochronous subgroup :

(11) $$L_o = L_n \cup L_s$$

The next two are the two antichronous components::

(12) $$L_a = L_t \cup L_{st}$$

It is clear that:

(13) $$L_a = - L_o$$

This configuration would therefore cause two adjacent regions to interact, with the action of adjacent masses being equivalent to that of negative masses

3 – **Extension incorporating** CPT-**symmetry.**

The fact of endowing masses with an electric charge has been considered as the fact that they evolve in a five-dimensional Kaluza space ([15], [16]). If we consider this fifth dimension as a simple fibre, this additional symmetry, in the form of a simple translation along this fifth dimension, will lead to the constancy of a scalar, the electric charge. The associated group structure with its action on a five-dimensional space is then [13]:

(14) $$\begin{pmatrix} 1 & 0 & \phi \\ 0 & L & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} \qquad \textit{with} \qquad \xi = \begin{pmatrix} x^o \\ x^1 \\ x^2 \\ x^2 \end{pmatrix}$$

The Janus group [13] translates the CPT-symmetry:

$$(15)\qquad \begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{avec} \qquad \begin{matrix} \lambda = \pm 1 \\ \mu = \pm 1 \end{matrix}$$

It then has eight connex components. The choice (λ = 1 ; μ = - 1) inverts the fifth dimension, which is the geometric translation of C-symmetry [17].

The choice (λ = - 1 ; μ = 1) translates CPT-symmetry.

Note: if we want this configuration to reflect a configuration in the covering of a projective P_N, N must be even, otherwise the projective is orientable and we lose the symmetries on time and space. The extension can therefore be carried out with an even number of additional dimensions, using the dynamic group:

$$(16)\qquad \begin{pmatrix} \lambda\mu & 0 & \dots & 0 & 0 & \phi_1 \\ 0 & \lambda\mu & \dots & 0 & 0 & \phi_2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda\mu & 0 & \phi_p \\ 0 & 0 & \dots & 0 & \lambda L_o & C \\ 0 & 0 & \dots & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \dots \\ \zeta_p \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda\mu\zeta_1 + \phi_1 \\ \lambda\mu\zeta_2 + \phi_2 \\ \dots \\ \lambda\mu\zeta_p + \phi_p \\ \lambda L_o \xi + C \\ 1 \end{pmatrix} \qquad \mu = \pm 1$$

Each additional dimension is associated with an invariant scalar. The first is the electric charge q. The next are the baryonic, leptonic, muonic charges, etc .

If this representation of charges is correct, then their number should be even, which would fit in with the 10-dimensional extension of string theory.

5- **Integration of negative masses in the cosmological model.**

If these masses of opposite signs interact, then according to what laws? If we base this action on Einstein's equation, we come up against an unmanageable paradox. If the source of the gravitational field is a positive mass, the metric derived from this hypothesis gives geodesics that suggest attraction. :

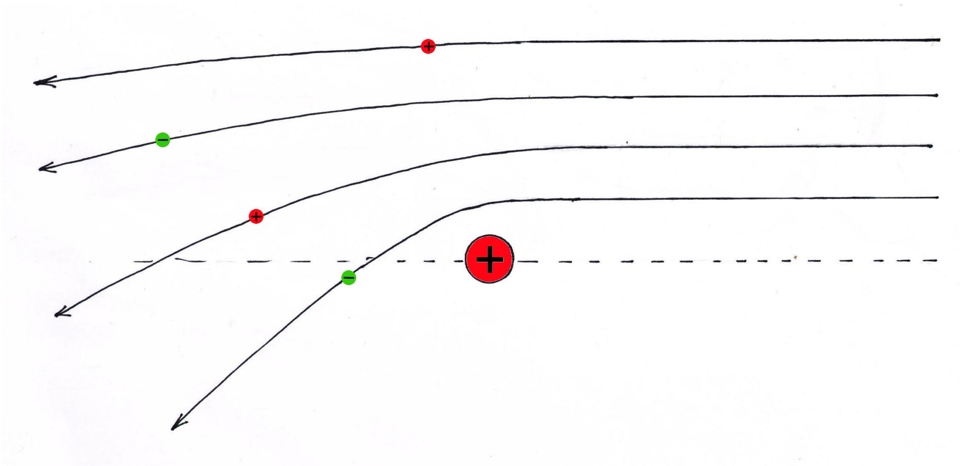

Fig.18 : Geodesics associated with a field created by a positive mass.

Positive masses are represented in red and negative masses in green. Since there is only one family of geodesic solutions, we can deduce that positive masses attract their counterparts just as well as negative masses. Now let's look at the field created by a negative m:

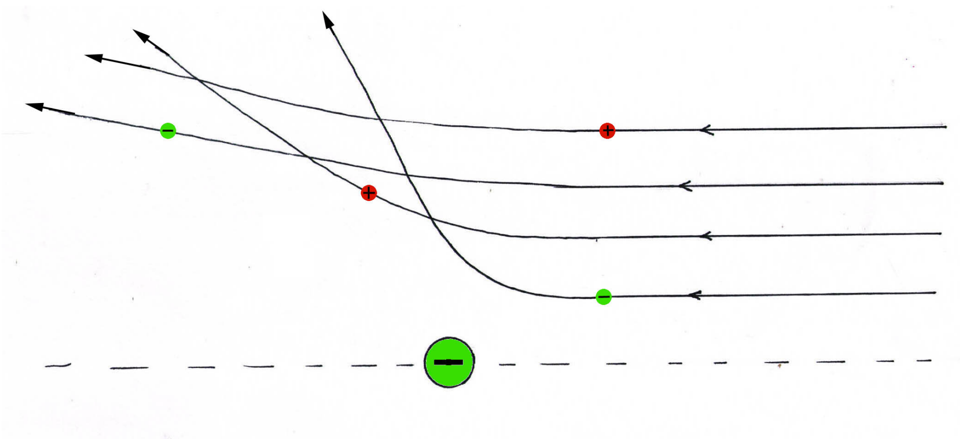

Fig 19 : Geodesics associated with a field created by a negative mass.

Conclusion: negative masses repel their counterparts just as well as positive masses. All this was studied and demonstrated in 1954 by Herman Bondi ([18], [19]). Let's place two masses of opposite signs in front of each other:

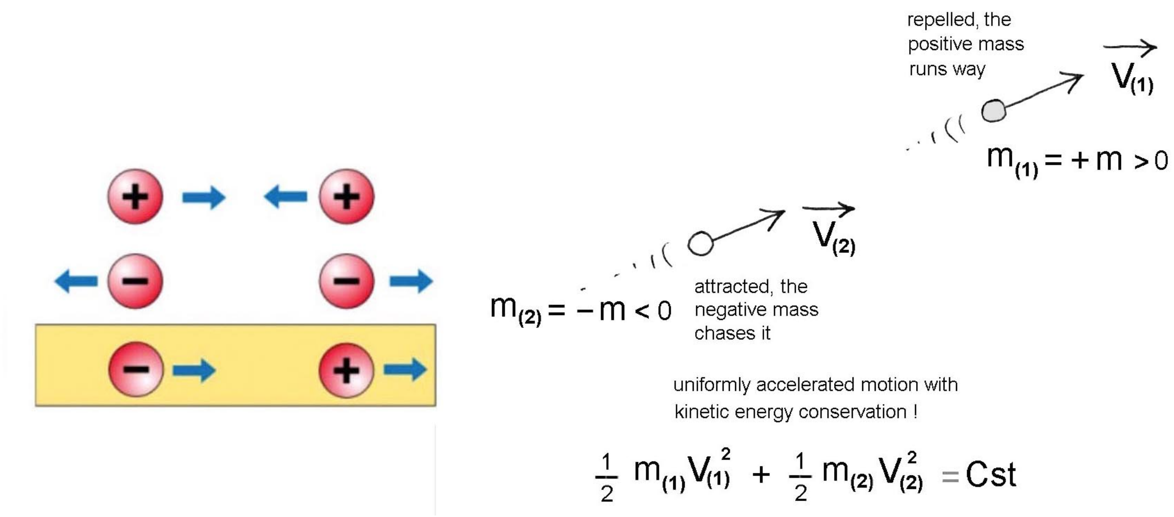

Fig. 20 : Runaway effect.

Positive mass flees, pursued by negative mass. The action-reaction principle is violated. The acceleration of the couple takes place at constant total energy, since that of the negative mass is itself negative. At the time, the conclusion was that it was impossible to include negative masses in a cosmological model. It is indeed impossible with a single metric. But it becomes possible if we assign two different metrics to adjacent folds, $g_{\mu\nu}$ and $\overline{g}_{\mu\nu}$. These two metric fields will then generate their own field of Ricci tensors $R_{\mu\nu}$ and $\overline{R}_{\mu\nu}$ as well as the corresponding Ricci scalars R and $\overline{R}$. We will therefore have two matter fields $T_{\mu\nu}$ and $\overline{T}_{\mu\nu}$. To these we will add two interaction tensors $t_{\mu\nu}$ and $\overline{t}_{\mu\nu}$.

It will then be possible to construct a system of two coupled field equations [23] through the action

The action is:

(17)
$$J = \int_{D4} \left[\left(R + S + \sigma \right) \sqrt{-g} + \left(\overline{R} + \overline{S} + \overline{\sigma} \right) \sqrt{-\overline{g}} \right] d^4x$$

The terms involving Ricci scalars are treated in the traditional way, in the derivation of the Einstein equation from the Hilbert-Einstein action. It follows that:

(18)
$$\delta J = \int_{D4} \left[\frac{\delta R}{\delta g^{\mu\nu}} + \frac{R}{\sqrt{-g}} \frac{\delta \sqrt{-g}}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\, S\,)}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\ \sigma\ )}{\delta g^{(+)\mu\nu}} \right] \sqrt{-g}\ \delta g^{\mu\nu} d^4x$$
$$+ \int_{D4} \left[\frac{\delta \overline{R}}{\delta \overline{g}^{\mu\nu}} + \frac{\overline{R}}{\sqrt{-\overline{g}}} \frac{\delta \sqrt{-\overline{g}}}{\delta \overline{g}^{\mu\nu}} + \frac{1}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-\overline{g}}\, \overline{S})}{\delta \overline{g}^{\mu\nu}} + \frac{1}{\sqrt{-\overline{g}}} \frac{\delta(\sqrt{-\overline{g}}\, \overline{\sigma}\ )}{\delta \overline{g}^{\mu\nu}} \right] \sqrt{-\overline{g}}\ \delta \overline{g}^{\mu\nu} d^4x$$

Posing:

(19)
$$\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,S)}{\delta g^{\mu\nu}} = -\chi\, T_{\mu\nu}$$

(20)
$$\frac{1}{\sqrt{-\overline{g}}}\frac{\delta(\sqrt{-\overline{g}}\;S)}{\delta \overline{g}^{\mu\nu}} = -\chi\, \overline{T}_{\mu\nu}$$

(21)
$$\frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\;\sigma)}{\delta g^{\mu\nu}} = \chi\, t_{\mu\nu}$$

(22)
$$\frac{1}{\sqrt{-\overline{g}}}\frac{\delta(\sqrt{-g}\;\overline{\sigma}\,)}{\delta \overline{g}^{\mu\nu}} = \chi\, \overline{t}_{\mu\nu}$$

We get the system of coupled field equations :

(23a)
$$R_{\mu\nu} - \frac{1}{2}R\; g_{\mu\nu} = \chi\left[\; T_{\mu\nu} + \sqrt{\frac{\overline{g}}{g}}\; t_{\mu\nu}\right]$$

(23b)
$$\overline{R}_{\mu\nu} - \frac{1}{2}\overline{R}\; \overline{g}_{\mu\nu} = -\chi\left[\; \overline{T}_{\mu\nu} + \sqrt{\frac{g}{\overline{g}}}\; \overline{t}_{\mu\nu}\right]$$

If we focus on the solutions in Newtonian approximation, we immediately obtain the direction of the forces:

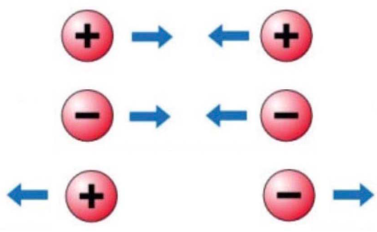

Fig.21 : Direction of forces in the Janus model.

This satisfies the action-reaction principle. This is due to the minus sign in the second member of the second equation. This system has been developed in the form of an unsteady solution, showing that the acceleration of the expansion is due to the asymmetry of the model [22], with a pre-eminence of the negative masses [13], which then drives the construction of the structure on a very large scale [20]. The study of geodesics in a stationary situation then reveals the confinement of positive masses by their environment of negative masses [21].

The establishment of this asymmetry, resulting from an instability of the system, which was initially symmetrical, will be the subject of a subsequent article.

Conclusion :

We recall that the transitions, first to special relativity and then to general relativity, are based on paradigmatic changes of a purely geometric nature. After discussing the growing difficulties encountered by the Standard Model, we move towards a topological solution. After evoking Andréi Sakharov's attempt, in which his twin universes did not interact, we fold this model back on itself by opting for a two-sheet covering of a projective P_4. We then show that this operation leads to a PT-symmetry. Extending this model to two CPT-symmetric entities, we consider the addition of extra dimensions, extending the approach initiated by Kaluza. The group structure is constructed. We then show that this extension implies the addition of a necessarily even number of extra dimensions. We show that a bimetric description is required and, on the basis of an action, we produce a system of coupled equations that satisfies the action-reaction principle, thus escaping the unmanageable runaway phenomenon..

References :

[1] A. Einstein : Die Feldgleichungen des Gravitation (The Field Equation of Gravitation). Sitzungsber. XLIV , 8 , 778 and XLVI s. 799 (25 nov 1915)

[2] K. Schwarzschild : Über das Gravitationsfeld Messenpunktes nach der Einsteinschen Theorie. Sit. Deut. Akad. Wiss. 1916

[3] K. Schwarzschild : Über das Gravitionalsfeld einer Kugel Aus incompressibler Flüssigkeit nach der Einsteinschen Theorie. Sitzung der phys. Math. Klasse v.23 märz 1916

[4] Perlmutter, S., et al. 1999, ApJ, 517, 565

[5] Riess A. G. 2000, PASP, 112, 1284

[6] B.Schmidt Observational evidence from supernovae for an accelerating universe and cosmological constant. Astron. J. 116, pp 1009-1038 (1998).

[7] A.D.Sakharov, ZhETF Pis'ma 5 : 32 ; JETP Lett. 5 : 24 (1967)

[8] A.D.Sakharov , ZhETF Pis'ma 76 : 1172 (1979) ; JETP 49 : 594 (1979)

[9] A.D.Sakharov , (1980). Cosmological Model of the Universe with a Time Vector Inversion. ZhETF (Tr. JETP 52, 349-351) (79): 689–693
[10] J.P.Petit : The missing mass problem. Il Nuovo Cimento B Vol. 109 July 1994, pp. 697-710

[11] W.Boy :Über die curvatura integra und die Topologie geschlossener Flächen. 1902

[12] J.P.Petit : Topo the World. http :www://savoir-sans-frontieres.com/JPP/telechargeablesEnglish/topo_the_world_eng.htm

[13] J.P.Petit, G.D'Agostini : Negative Mass hypothesis in cosmology and the nature of dark energy. Astrophysics And Space Sccience,. *A* 29, 145-182 (2014)

[14] J.M. Souriau : Structure of Dynamical Systems. Birkhaüser Ed. Berlin 1997 and Structure des Systèmes Dynamiques, Dunod Ed. 1970.

[15] T.Kaluza. Sitzungsber Preuss Akad. Wiss. Berlin Math. Phys. K1, 966

[16] O.Klein. Z ; Phys. 37, 895

[17] J.M.Souriau : Géométrie et Relativité (in french). Ed. Hermann 1954

[18] Bondi H. : Negative mass in General Relativity. Rev. of Mod. Phys., Vol 29, N°3, july1957

[19] Bonnor W. B. : Negative mass and general relativity. General Relativity and Gravitation Vol. 21, N°11, 1989

[20] J.P.PETIT, Twin Universe Cosmology, Astrophys. and Sp. Science, 226, 273-307, 1995

[21] J.P.Petit, G.D'Agostini : Cosmological Bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe. Modern Physics Letters
A, Vol.29 ; N° 34, 2014 ; Nov 10th

[22] G. DAgostini and J.P.Petit : Constraints on Janus Cosmological model from recent observations of supernovae type Ia, Astrophysics and Space Science, (2018),

[23] J.P.Petit, G. D'Agostini, N.Debergh : Physical and mathematical consistency of the Janus Cosmological Model (JCM). Progress in Physics 2019 Vol.15 issue 1