20 juin 2019.

En 2018 j'ai envoyé au comité du 10° colloque international Friedmann sur la gravitation et la cosmologie l'abstract d'une proposition de communication orale. Celle-ci a été acceptée le 23 mars 2019. Voici le message :

March 23

Dear Dr. J.-P. Petit,

We are glad to inform you that your talk is included in the Program of the 10th Alexander Friedmann International Seminar on Gravitation and Cosmology and 4th Symposium on the Casimir Effect.

Please find attached the official invitation to take part in the 10th Friedmann Seminar and 4th Casimir Symposium. This invitation may also help you to get funding for your trip from your National soarces and for other bureaucratic purposes.

Thank you for your attention. We are looking forward to see you at St.Petersburg.
Yours sincerely On behalf of the Organizing Committee

Dr. Yuri Pavlov, Scientific Secretary

J'étais spécialement intéressé à aller là-bas parce que Yuri Pavlov avait lui-même publié plusieurs papiers disant que lorsque de la matière franchissait la sphère horizon d'un trou noir, sa masse s'inversait. Je lui ai donc envoyé le texte complet de la communication que je comptais présenter là-bas. On le trouvera à la fin de ce document.

En parallèle j'ai commencé à nouer des contacts avec des chercheurs russes, dont Serguei Krasnikov, dont je savais qu'il avait publié plusieurs papiers sur les voyages interstellaires. Nous avons eu plusieurs échanges et nous projetions de nous retrouver là-bas.

C'est alors que j'ai reçu un mail de Yuri Pavlov, le 10 juin :

June 10

Dear Dr. Petit,
I am sorry to recall that you did not respond to my letter (to your e-mails address specified in your Registration form Jean-Pierre.petit@manaty.net) of June 3rd until June 6, as was requested.

Because of this, your talk was not included in the Friedmann Seminar Program as an oral talk. Taking into account that the Program is already overfilled, at the present time the Organizing Committee could include your talk in the Program only as a poster.

Could you, please inform me urgently whether you will participate in the 10th Friedmann Seminar with a poster presentation.

Thank you for your understanding.

Yours sincerely Yu. V. Pavlov, The Scientific Secretary of the Organizing Committee

---

Problème : je n'ai jamais reçu ce message daté du 3 juin. J'ai vérifié, l'adresse jean-pierre.petit@manaty.net fonctionne très bien. Peut-être est-ce parti dans les spams.

Ma décision a été vite prise. Je suis extrêmement fatigué. Aller là-bas pour un simple poster ne paraissait pas raisonnable. Je demande donc aux gens qui m'ont fait un don pour financer cette mission de m'envoyer leurs noms, adresses et montants de leurs dons. Le GESTO les remboursera immédiatement.

Krasnikov, mis au courant, m'a aussitôt écrit :

Dear Jean-Pierre

I consider the appraoch of the organizing comittee inexcusable. Please accept my apologies for it.

A trick of the anti-spam system, I think.

Join the club! 25 years ago I attended this conference and got equally (or even more) boorish reception. Since then I boycott the conference.

Best regards, Serguei

Il y a plus grave. Je n'étais pas le bienvenu au colloque donné à Paris 7 en l'honneur de Jean-Marie Souriau. Découvrant l'événement mi avril, j'ai proposé une communication. Réponse immédiate de Jean-Jacques Szczerciniarz, un des trois organisteurs : « désolé, le programme est bouclé depuis longtemps ». Bon, dis-je, alors je vous envoie un poster. Réponse « le programme poster était une erreur, cela a été supprimé ».

Mais ils se disent « lui interdire totalement de participer à ce colloque semble difficile ». Ils inscrivent alors mon nom dans la liste des participants.

Sur ce je mets en ligne cette vidéo (30.000 vues à ce jour) :

https://www.youtube.com/watch?v=QTV-OFdzfOk&feature=youtu.be

Quelques heures plus tard mon nom est d'abord rayé (...) puis supprimé de la liste. Aucune réponse à mes demandes d'explications.

Les organisateurs précisent « seul ceux seront admis à participer les candidats faisant état d'affiliations académiques ».

Quand Szczerciniarz, philosophe des sciences, finit son exposé, mon collègue Sébastien Michea, régulièrement inscrit et présent dans la salle demande pourquoi mon nom a été supprimé.

Szczerciniarz lui répond (des gens présents ont témoigné par écrit) :

- Nous avions commencé par accepter que Jean-Pierre Petit puisse assister aux exposés, normalement, en posant des questions. Mais les choses ont commencé à déraper. Il a demandé un temps de parole (...).

(effectivement, j'avais demandé à Szczerciniarz de m'octroyer 5 minutes sur son propre temps de parole pour que je puisse dire aux présents que je tiendrais à la sortie des photocopies de la communication que je voulais présenter. Thème de cette communication : l'extension de la mécanique symplectique de Souriau aux complexes, un sujet qui aurait cent fois mérité d'être inclus dans ce programme).

Szczerciniarz a lors dit que cette communication avait été examinée par des experts de la géométrie symplectique, et que ceux-ci avaient jugé que ce que j'avais soumis était sans valeur, parce qu'entaché d'erreurs de calcul. Patrick Iglesias, mathématicien et second co-organisateur a renchéri, en disant que je n'avais pas ma place au sein d'un colloque réservé à des spécialistes que, selon lui, je n'étais pas. Szczerciniarz continué en disant que je gagnerais à soumettre mes travaux à des journaux et à admettre le verdict des experts. Michea a objecté que j'avais publié mes travaux dans ces mêmes revues, en citant mon dernier papier, publié quelques mois plus tôt dans Progress in Physics. Szczerciniarz a répliqué de nouveau en disant « que cette revue n'était pas reconnue par la communauté scientifique ».

J'ai donc envoyé ce calcul (d'action coadjointe du groupe d'isométrie de l'espace de Minjkowski complexifié sur son moment) a différents spécialistes de géométrie symplectique, dont deux avaient d'ailleurs été présents au colloque. J'ai reçu hier les réponses de deux d'entre eux, qui ne trouvent aucune erreur ou incongruité dans ce calcul.

En parallèle j'ai écrit à Szczerciniarz en lui demandant le nom de ces experts qui auraient, selon lui, trouvé des erreurs dans ce calcul.

Je n'ai pas eu de réponse et je pense que je n'en recevrai pas.

Même silence chez un Patrick Iglesiais, à qui j'ai renvoyé ce calcul en lui demandant s'il trouvait une erreur.

Je pense que Szczerciniarz, qui n'est pas mathématicien mais philosophe des sciences, s'est tourné vers Iglesias (ou Lachièze-Rey, dont on a déjà apprécié les jugements abrupts et sans fondement[1]). Pour ces deux-là, il est inutile de lire. Mes travaux … contiennent forcément des erreurs.

Je donnerai à cette affaire toutes les suites nécessaires, car ce n'est rien d'autre que de la diffamation.

Au passage je présenterai ce calcul (matriciel) dans une prochaine vidéo et les « niveau maths sup » pourront le suivre aisément. Mais les pires sourds sont ceux qui ne veulent pas comprendre.

---

[1] https://www.youtube.com/watch?v=Vl541wUXsSs&feature=youtu.be, 44.000 vues

Mon exclusion de ce colloque, donné en souvenir de Jean-Marie Souriau a été un geste d'une totale malhonnêteté. Les gens qui se comportent de cette façon se déshonorent.

Un autre collègue Russe, Vadim Babenko, m'a écrit :

Hello Jean-Pierre,

Yes, I did read your 2018 article on geometrical interpretation of the matter-antimatter duality (extending the works of Souriau) - and I do think it's very interesting and promising approach. The reason I specifically mentioned the 2019 article in my previous message is because, as I see it, it addresses the critique of earlier Janus model by Damour - if I understood it correctly (I found Damour's article only in French and had to use Google translate).

Best wishes, Vadim

Son attention a été attirée, voulant se faire une opinion sur mon modèle Janus, par la critique acérée mise en ligne sur sa page de l'IHES par Thibaud Damour :

http://www.ihes.fr/~damour/publications/JanusJanvier2019-1.pdf

Pour lire ce texte, composé en français, Babenko dit avoir utilisé le traducteur Google. Dans les faits ce document me discrédite au plan international. Damour n'a pas donné suite à ma demande (en lettre simple) d'insertion de mon légitime « droit de réponse scientifique », en l'occurrence du texte de ma publication intitulée « The physical and mathematical consistency of the Janus Cosmologocal model[2] ».

Je lui ai demandé « une heure en tête à tête, sans témoin ni enregistrement, devant un tableau noir ».

Pas de réponse.

Sachant que ce papier a été composé avec l'aide de la mathématicienne Nathalie Deruelle, j'ai par deux fois demandé à celle-ci de me recevoir.

Pas de réponse.

There is something rotten in the kingdom of science

---

[2] http://www.ptep-online.com Vol. 14 ussue 4

Mass inversion through the Schwarzschild sphere

J.P.Petit[3] G. d'Agostini[4] N.Debergh[5]

Manaty Research Group

Abstract :

We interpret the Schwarschild metric in a context of a different topology, through a new coordinate system, which goes with a space bridge linking two Minkowski spaces. This links to our Janus Cosmological Model. After this scenario matter swallowed by black holes is transformed into negative mass, which no longer interact with ordinary matter and can be dispersed in space.

Keywords : Schwarzschild metric mass inversion, throat sphere, topology, singularity cancellation, neutron star

---

Introduction

Let's go back to the original paper published by Karl Schwarzschild in january 1916 [1] . We quote :

> 3- If one calls t the time and x , y , z the rectangular coordinates, the most general line element that satisfies the conditions 1-3 is clearly the following :
>
> $$ds^2 = Fdt^2 - G(dx^2 + dy^2 + dz^2) - H(xdx + ydy + zdz)^2$$
>
> where F , G , are functions of $r = \sqrt{x^2 + y^2 + y^2}$

Clearly t , x , y , z are basically *real* quantities, so that r is also a *real* (an *positive*) quantity.

Few pages further Schwarzschild writes :

> Introduce the auxiliary quantity :
>
> $$R = (r^3 + \alpha^3)^{1/3}$$

---

[3] Jean-Pierre Petit : jean-pierre.petit@manaty.net
[4] Gilles d'Agostini: gilles.dagostini@manaty.net
[5] nathalie.debergh@manaty.net

where α is nothing but the Schwarzschild's length.

Finally he expresses his solution, based on that auxiliary quantity R in his equation (14), we quote again :

$$ds^2 = \left(1-\frac{\alpha}{R}\right)dt^2 - \frac{dR^2}{1-\frac{\alpha}{R}} - R^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\ ,\ R = (r^3+\alpha^3)^{1/3} \quad (14)$$

As the goal of Karl Schwarzschild was to fit the approximate result published by A.Einstein few months ago [2] he did note express his solution in terms of (t, r, θ, φ). Lets do it. We get :

(1)

$$ds^2 = \left(1- \frac{\alpha}{1-(r^3+\alpha^3)^{1/3}}\right)dt^2 - \frac{r^2\, dr^2}{(r^3+\alpha^3)^{2/3}\left(1-\frac{\alpha}{1-(r^3+\alpha^3)^{1/3}}\right)} - (r^3+\alpha^3)^{2/3}(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

On december 1916 D.Hilbert presented his second version of his « Foundation of Physics » [3].

Let's keep in mind that in 1916 physics reduced to gravitational and electromagnetic interaction. The others, strong and weak interaction, were unknown. In 1915, in a first communication D. Hilbert proposed nothing but a definite synthesis of all the knowledge in physics. The General Relatity was a part of the work. As Schwarzschild presented an exact solution of the equations recently introduced by A. Einstein he had to fullfill his first communication, and integrated this new element into a second communication on december 1916 the 23 [3].

In their papers A.Einstein, K.Schwarzschild, H.Weyl, and many others refer to signature (+ - - -). In his communication D.Hilbert insists on the importance of such « new physics » in which long distance forces are converted into pure geometry :

> The geometrical question mentioned above amounts to the investigation, whether and under what conditions the four-dimensional Euclidean pseudo-geometry
>
> $$g_{11} = 1, \quad g_{22} = 1, \quad g_{33} = 1, \quad g_{44} = -1 \qquad g_{\mu\nu} = 0 \quad (\mu \neq \nu) \qquad (35)$$
>
> is a solution, or even the only regular solution, of the basic physical equations.

At this epoch all scientists, A.Einstein, K. Schwarzschild, H.Weyl and many others opted for signature $(+---)$, going with $ds^2 \geq 0$. At the contrary D.Hilbert obviously chooses $(-+++)$. We could wonder why. We find the answer few pages later :

According to Schwarzschild the most general metric conforming to these assumptions is represented in polar coordinates, where

$$w_1 = r\cos\vartheta$$
$$w_2 = r\sin\vartheta\cos\varphi$$
$$w_3 = r\sin\vartheta\sin\varphi$$
$$w_4 = l,$$

by the expression

$$F(r)dr^2 + G(r)(d\vartheta^2 + \sin^2\vartheta d\varphi^2) + H(r)dl^2 \quad (42)$$

where $F(r)$, $G(r)$, $H(r)$ are still arbitrary functions of r. If we put

$$r^* = \sqrt{G(r)},$$

then we are equally justified in interpreting r^*, ϑ, φ as spatial polar coordinates. If we introduce r^* in (42) instead of r and then eliminate the sign $*$, the result is the expression

$$M(r)dr^2 + r^2 d\vartheta^2 + r^2\sin^2\vartheta d\varphi^2 + W(r)dl^2, \quad (43)$$

Hilbert identifies abruptly his intermediate coordinate r* to a radial polar coordinate r. Further, letting l = it Hilbert says he refinds the Schwardschild's line element. This shows that, in his mind, time is fiber, added to a three dimensional space, which is an imaginary dimension, and gives the signature an hyperbolic nature. He does not care about the change of the signs $(+---) \to (-+++)$.

His footnote :

7 To transform the locations $r = \alpha$ to the origin, as Schwarzschild does, is not to be recommended in my opinion; Schwarzschild's transformation is moreover not the simplest that achieves this goal.

shows he has not understood the topologic implications of the identification of the Schwarzschild's intermediate quantity $R = (r^3 + \alpha^3)^{1/3}$ to a radial coordinate. In effect, if we keep in real world :

(2) $$r = \sqrt{x^2 + y^2 + z^2} = \left(R^3 - \alpha^3\right)^{1/3} \geq 0$$

implies R >0.

If one admits to study « what is inside the object », when $r < \alpha$ (Schwarzschild's radius 2m or R_s whatever is the name we choose), then $ds^2 < 0$. Classically one admits that « inside the object » r becomes timeke, while t is spacelike.

Metric and topology

Let us start by a simpler problem. Consider the 2D metric

(3) $$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2 d\varphi^2$$

We assume that out variables r and φ are real quantities. We just ask the question « what happens when $r < R_s$? ».

From (3) we see that this line element is unchanged for any translation along the φ coordinate. It evokes an axisymmetry around a z-axis. We can think about peculiar geodesics of this 2D-surface which are meridian lines. Along such curves we have $d\varphi = 0$ and $d\Sigma^2 = dr^2 + dz^2$, or

(4) $$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} = dr^2 + dz^2$$

This turns into a diffential équation whose solution is :

(5) $$z = \pm 2 R_s \sqrt{1 - \frac{r}{R_s}} \qquad \text{or} \qquad z = R_s + \frac{z^2}{R_s}$$

The meridian lines are lying parabolas and the surface is a 2D diabolo :

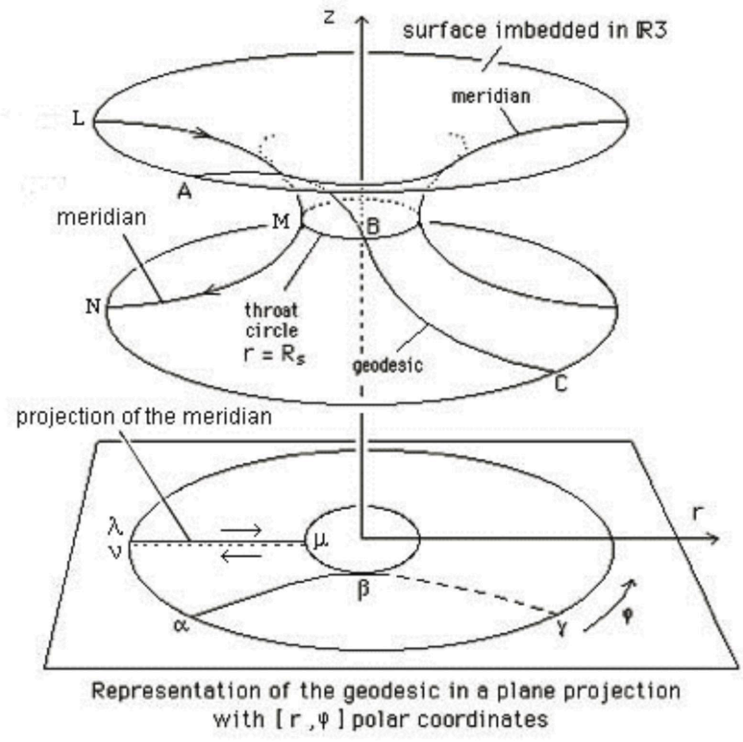

Fig.1 : 2D diabolo

We know that the metric can be expressed in any coordinate system. The system (r, φ) is a source of problem. In effect when r < R_S the elementary distance becomes imaginary, just because we are ou of the object, if we consider this is a real object, described by real coordinates, givong a real elementary length ds .

Can we imagine another coordinate system that would cancel the problem ? The answer is yes, through the coordinate change :

(6) $$\boxed{r = R_s\,(1 + \mathrm{Log}\,\rho\,)}$$

Whence $dr = R_s \mathrm{th}\,\rho\, d\rho$. We get :

(7) $$d\Sigma^2 = \frac{1 + \mathrm{Log}\,\rho}{\mathrm{Log}\,\rho}\; R_s^{\,2}\,\mathrm{th}^2\rho\, d\rho^2 + R_s^{\,2}(1 + \mathrm{Log}\rho)^2 d\varphi^2$$

When $\rho \to \pm 0$ the term $g_{\rho\rho}$ of the metric takes the form $\frac{0}{0}$. Let us perform an expansion into a series around the value $\rho = 0$. We find :

(8) $$\mathrm{ch}\rho = \frac{e^{\rho} + e^{-\rho}}{2} \qquad \frac{\mathrm{ch}\rho}{\rho \to 0} \simeq 1 + \frac{\rho^2}{2} \qquad \frac{\mathrm{Log\,ch}\rho}{\rho \to 0} \simeq \frac{\rho^2}{2} \qquad \frac{\mathrm{th}\rho}{\rho \to 0} \simeq \rho \qquad g_{\rho\rho} \to 2$$

The signature of this metric is (+ +). It remains unchanged whatever are the value of the coordinates .

In this region the determinant of the metric :

(9) $$\det(g_{\mu\nu}) = R_s^{\,4}\,\frac{(1 + \mathrm{Log\,ch}\rho)^3}{\mathrm{Log\,ch}\rho}\,\mathrm{th}^2\rho \;\to\; 2R_s^{\,4}$$

It never vanishes so that if we define an orientation on this surface it holds over all of it. The value of the new coordinate corresponds to figure 2.

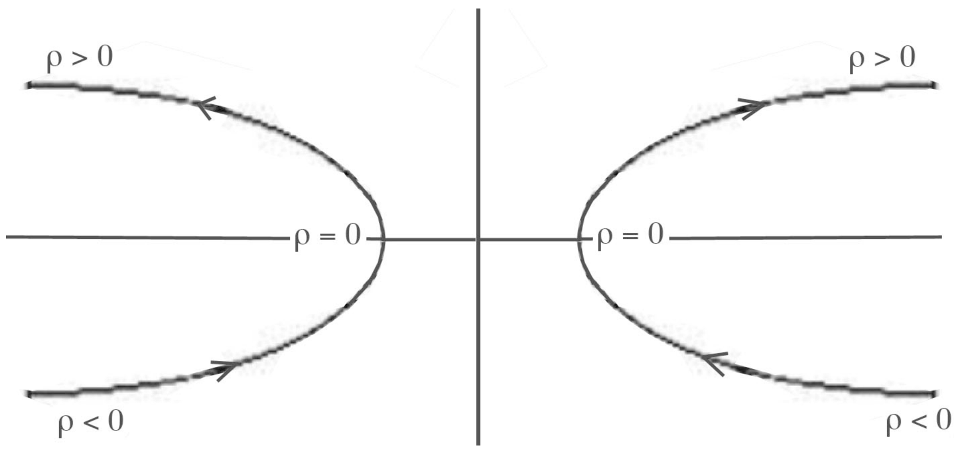

Fig.2 : Meridian of the 2D diabolo

We may compute esasily the geodesics and we get the figure 3.

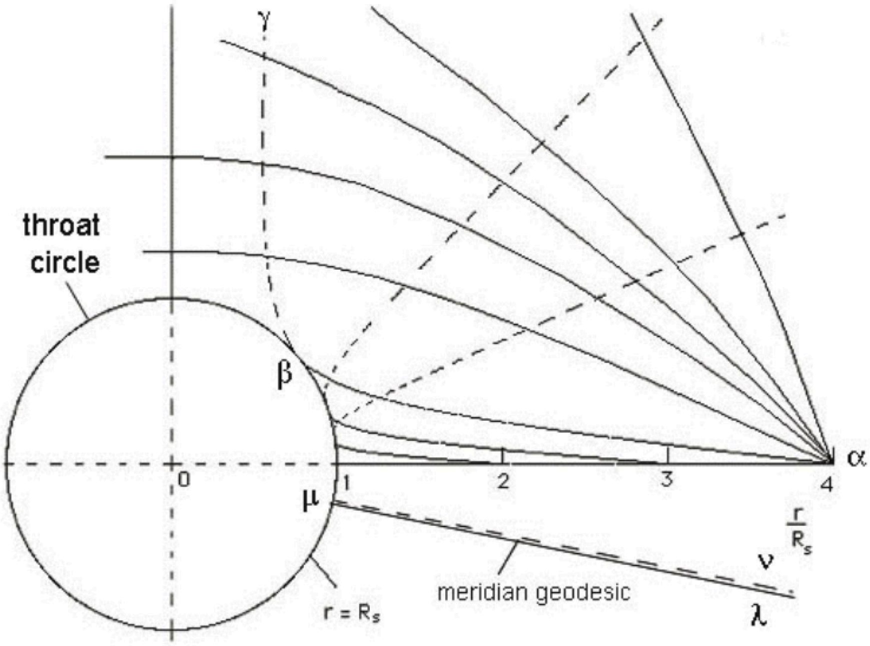

Fig.3 : Geodesics of the 2D diabiolo

Let's take another example. Consider the following 2D metric, with r , R , and $r_o > 0$:

(10) $$d\Sigma^2 = \frac{dr^2}{-r^2 + 2rR + r_o^2 - R^2} + r^2 d\varphi^2 = \frac{dr^2}{D(r)} + r^2 d\varphi^2$$

The polynom D(r) corresponds to the figure 4.

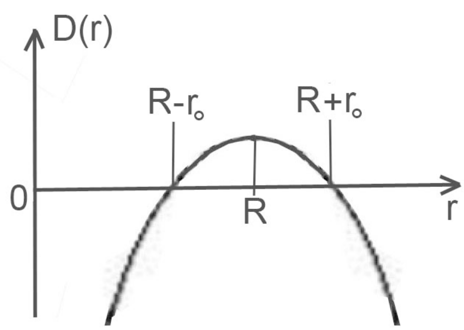

Fig.4 : Polynom D(r)

The signature is (+ +) for $R + r_0 < r < R + r_0$. Outside it is altered. We guesse this comes from a wrong choice of coordinates and we try :

(11) $$r = R + r_o \cos\theta$$

and we get immediatly :

(12) $$d\Sigma^2 = r_o^2 d\theta^2 + (R + r_o \cos\theta)^2 d\varphi^2$$

The signature is unchanged for any values of the new coordinates. This is the metric of a torus.

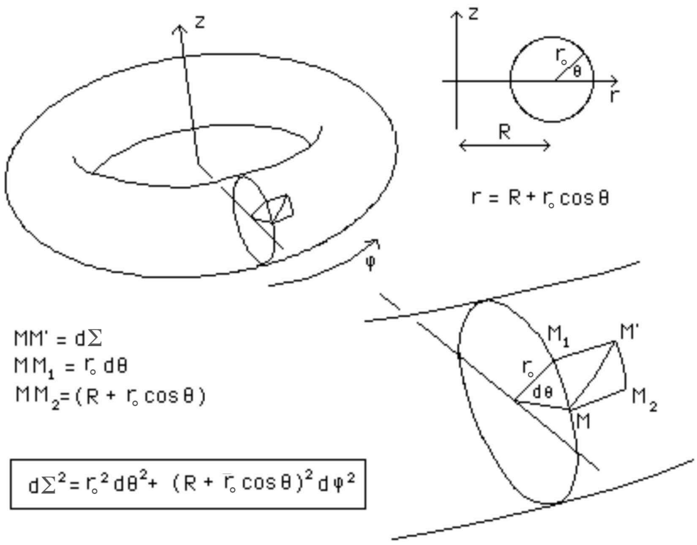

Fig.5 : Metric of the torus

As we can see, if we impose $ds^2 > 0$ and the constancy of the Riemanian signature in such metrics this brings a constainst on the coordinate system which reveals the underlying topology of the object.

Shifting to an orbifold structure.

If we identify the two adjacent points of the 3D diabolo it is like crushing the surface on a plane. The corresponding representation corresponds to the form (3) with r > Rs. This corresponds to an orbifold structure.

If we consider three points on a fold, we can use it to define an orientation. On the figure 6 we see that if a copy of this set glides ont the surface, crosses throat circle and comes to its initial position we get a couple of objects with inverse orientation. The are enantiomorphic.

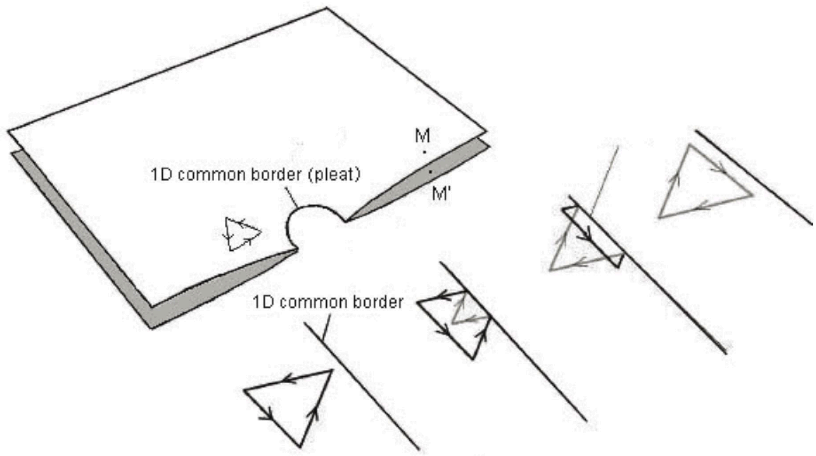

Fig.6 : The 2D diabolo transformed into a 2D orbifold

Extension to a 3D metric

Now, consider the metric :

(13) $$d\Sigma^2 = \frac{dr^2}{1 - \frac{R_s}{r}} + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

If r > Rs the signature remains (+ + +) . Here again we can use de change of coordinates (6) and we get :

(14) $$d\Sigma^2 = \frac{1 + \mathrm{Log}\,\rho}{\mathrm{Log}\,\rho} R_s^{\,2} \, \mathrm{th}^2 \rho \, d\rho^2 + R_s^{\,2} (1 + \mathrm{Log}\rho)^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

This four dimensional hypersurface is everywhere regular. The signature is conserved for any values of the coordinates.

When $\rho \to \pm\infty$ the metric tends to

(15) $$d\Sigma^2 = R_s^{\,2} d\rho^2 + R_s^{\,2} \rho^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

Which is the metric of a 3D euclidean space, in polar coordinates.

This 4D hypersurface is a space bridge linking two 3D euclidean spaces, through a throat sphere.

The determinant is :

(16)

$$\det(g_{\mu\nu}) = R_s^{\,6} \frac{(1+\mathrm{Log\,ch}\rho)^5}{\mathrm{Log\,ch}\rho}\, \mathrm{th}^2\rho \sin^2\theta$$

It is never zero, so that we can give this 4D surface a given orientation. The orientation of space can be figured with a tetraedron. See figure 7.

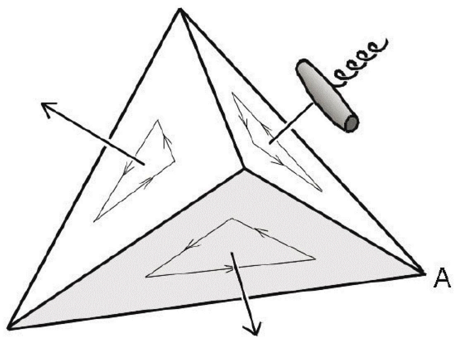

Fig.7 : Oriented tetraedron

If we shape this into a 3D orbifold this links adjacent regions of the two 3D folds and we get a 3D orbifold structure. We can think about a « 3D crushing » into an Euclidean 3D space and figure some geodesics which lie in planes. The geodesics look like parabolas. Consider an observer located in one of the two 3D folds. If he looks at any geodesic this last will seem to disappear after reaching the throat sphere.

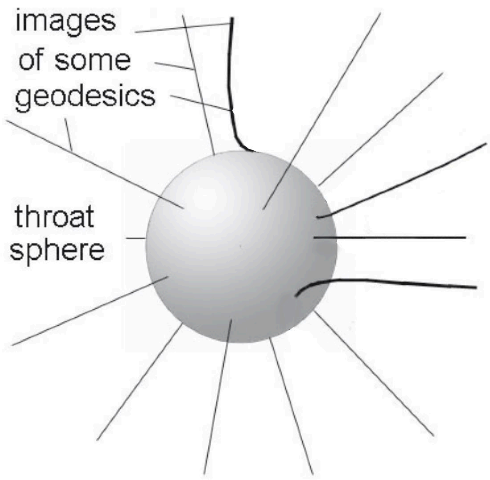

Fig.8 : How our observer see the geodesics.

In the figure 9 we have figured this second part of the geodesics with dotted lines.

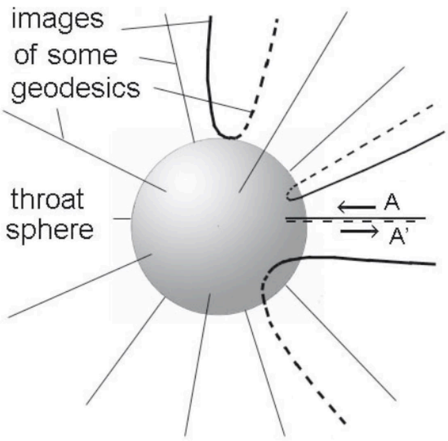

Fig.9 : Completed geodesic lines

If our observer could see the complete geodesic pattern the radial ones would seem to « rebound » on the throat sphere. Similarly on figure 6 we had a apparent « pleat », which comes from our representation schema. I confess it is quite difficult to « see », but when a tetraedron crosses the throat sphere the segments linking two summits remain straight. They donc experience any break, which com from our 3D representation schema. But we will use this last to illustrate the apparent eversion of the tetraedron when its copy crosses the throat sphere and reach its initial position :

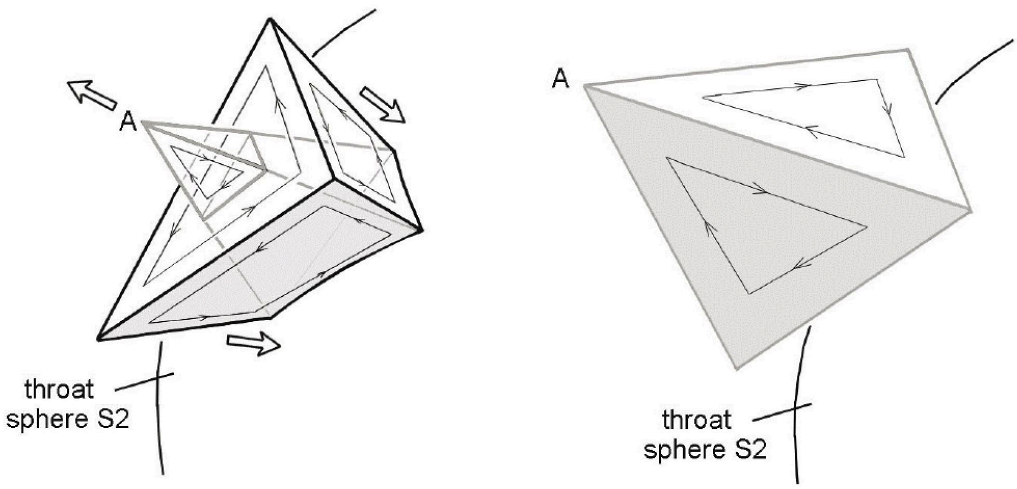

Fig.10 : Eversion of the tetraedron

Back to 2D structure we may imagine that two kinds of objects live in, figured as oriented enantiomorophic triangles (with opposite orientations). We call it « right » and « left » triangles. This last are figured with grey color.

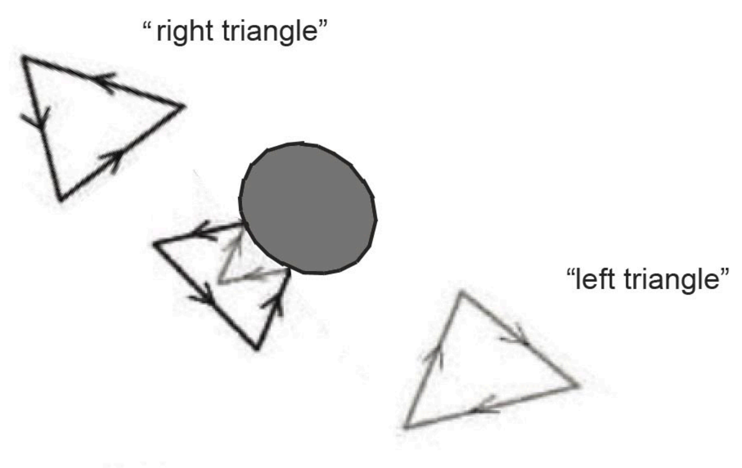

Fig. 11 : Enantiomorphic triangles.

When it reaches and crosses the throat circle a right triangle is converted into a left triangle, and vice-versa. The is nothing « inside » the circle. It looks like a hole, but it is not. It is a space bridge. The black disk does not exist, and just comes from our representation schema.

We get a similar schema in 3D :

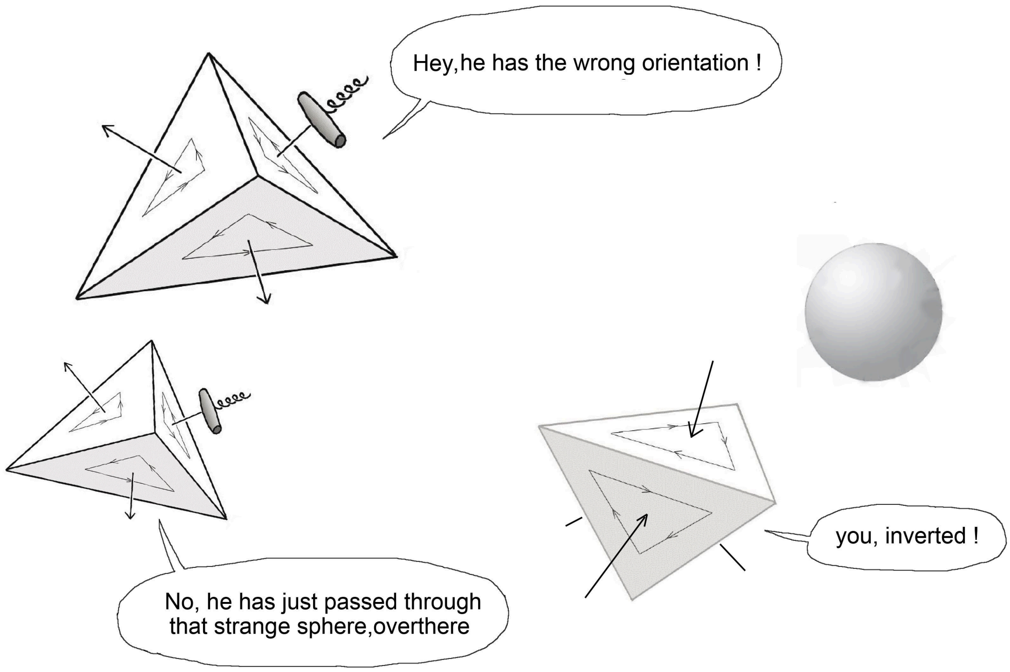

Fig.12 : Enantiomorphic tetraedrons.

Here ainsi, there is nothing « inside » the sphere, which is a throat sphere.

Extension to a 4D Schwarzschild metric

(17)

$$ds^2 = \left(1 - \frac{R_s}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{R_s}{r}} - r^2(\, d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

Apply the coordinate change (6). We get :

(18) $$ds^2 = \frac{\text{Log}\,\rho}{1 + \text{Log}\,\rho}dt^2 - \frac{1 + \text{Log}\,\rho}{\text{Log}\,\rho}\, R_s^{\,2}\, \text{th}^2\rho\, d\rho^2 - R_s^{\,2}(\,1 + \text{Log}\rho\,)^2(\, d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

With this new coordinate system we have cancelled the signature modification. The proper time s remains real for any values of the coordinates.

When $\rho \to \pm\infty$ this metric tends to

(19) $$ds^2 = dt^2 - R_s^{\,2} d\rho^2 - R_s^{\,2}\rho^{\,2}(\, d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

which the Lorentz metric of a flat Minkowski space, so that the geometric structure is a space bridge linking two Minkowski spaces.

The determinant is

(20) $$\det(g_{\mu\nu}) = -\,\mathrm{th}^2\rho\, R_s^{\,6}(1+\mathrm{Log}\,\rho)^4 \sin^2\theta$$

It vanishes on the throat sphere ; which means that the orientation of the space-time is inverted. The two folds are PT-symmetrical.

- The arrows of time are opposite
- The two folds are enantiomorphic

Lets' call $t^{(+)}$ and $t^{(-)}$ the two time-coordinates, with

In general the two folds refer to distinct coordinate sets $\left\{ t^{(+)}, \rho^{(+)}, \theta^{(+)}, \varphi^{(+)} \right\}$ and . $\left\{ t^{(-)}, \rho^{(-)}, \theta^{(-)}, \varphi^{(-)} \right\}$. Let's express the shift to an orbifold structure through :

(21)

$$t^{(-)} = \tau = -\,t^{(+)} \qquad \rho^{(+)} = \rho = -\,\rho^{(-)} \qquad \theta^{(+)} = \theta = \theta^{(-)} \qquad \varphi^{(+)} = \varphi = \varphi^{(-)}$$

We can write the two metrics of the two 4D folds :

Positive fold :

(22)

$$ds^2 = \frac{\mathrm{Log}\,\rho^{(+)}}{1+\mathrm{Log}\,\rho^{(+)}} dt^{(+)2} - \frac{1+\mathrm{Log}\,\rho^{(+)}}{\mathrm{Log}\,\rho^{(+)}} R_s^{\,2}\, \mathrm{th}^2\rho^{(+)}\, d\rho^{(+)2} - R_s^{\,2}(1+\mathrm{Log}\,\rho^{(+)})^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

Negative fold :

(23)

$$ds^2 = \frac{\mathrm{Log}\left|\rho^{(-)}\right|}{1+\mathrm{Log}\left|\rho^{(-)}\right|} dt^{(-)2} - \frac{1+\mathrm{Log}\left|\rho^{(-)}\right|}{\mathrm{Log}\left|\rho^{(-)}\right|} R_s^{\,2}\, \mathrm{th}^2\rho^{(-)}\, d\rho^{(-)2} - R_s^{\,2}(1+\mathrm{Log}\left|\rho^{(-)}\right|)^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

From a physical point of view, the inversion of the time-marker dos not imply the inversion of the proper time.

Positive matter emits positive energy photons. Negative matter emist negative energy photons, so that negative masses are invisible to positive mass observers, and vice versa .

If we figure the geodesics of a test particle of matter that reaches the Schwarzschild sphere it seems to bounce on it. By the way, an observer made of positive mass cannot observe directly the second part of the trajectory, because it is inscribed on the second three-dimensional fold. For such an observer the mass swallowed by the Schwarzschild sphere seems to disappear completely.

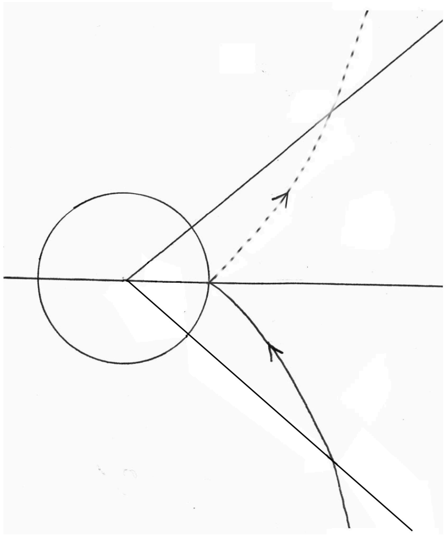

FiG. 13 : Paths of a positive test particle reaching the Schwarzschild sphere
Dotted line : this part is not visible for a positive mass observer

This bouncing phenomenon is an artifact due to the representation of such a space brige as the two folds cover of a 3D-space-time. Let's calculate, in the $\{\tau, \rho, \theta, \varphi\}$ representation the evolution of the pitch angle

$$\mathrm{tg}\,\beta = \frac{\rho\, d\varphi}{d\rho}$$

at the vicinity of the point $\rho = 0$. In a $\{\tau, r, \theta, \varphi\}$ with c = 1 we have :

(24) $$d\varphi = \pm \frac{1}{r^2} \frac{dr}{\sqrt{\frac{l^2 - 1}{h^2} + \frac{R_s}{h^2 r} - \frac{1}{r^2} + \frac{R_s}{r^3}}}$$

where l and h are the classical parameters of a quasi-Keplerian relativistic trajectory. On the Schwarzschild sphere we get :

$$R_s \left(\frac{d\varphi}{dr}\right)_{r=R_s} = \frac{h}{R_s l}$$

In the $\{\tau, \rho, \theta, \varphi\}$ representation, at the vicinity of $\rho = 0$ the pitch angle β is :

(25) $$\left(\mathrm{tg}\,\beta\right)_{\rho\to 0} = \frac{\rho\, d\varphi}{d\rho} = \frac{r\, d\varphi}{dr}\frac{\rho}{r}\frac{dr}{d\rho} \simeq \frac{h}{R_s^2 l}\rho^2 \to 0$$

In the vicinity of we can write the metrics :

Positive fold :

(26) $$ds^2 \simeq \frac{\rho^2}{2} dt^{(+)2} - 2R_s^2\, d\rho^2 - R_s^2 (\, d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

Negative fold :

(27) $$ds^2 \simeq \frac{\rho^2}{2} dt^{(-)2} - 2R_s^2 d\rho^2 - R_s^2 (\, d\theta^2 + \sin^2\theta\, d\varphi^2\,)$$

The Lagrange equations, in the vicinity of $\rho = 0$ give $\ddot{\rho} = \ddot{\varphi} = 0$. The continuity of the geodesics is ensured.

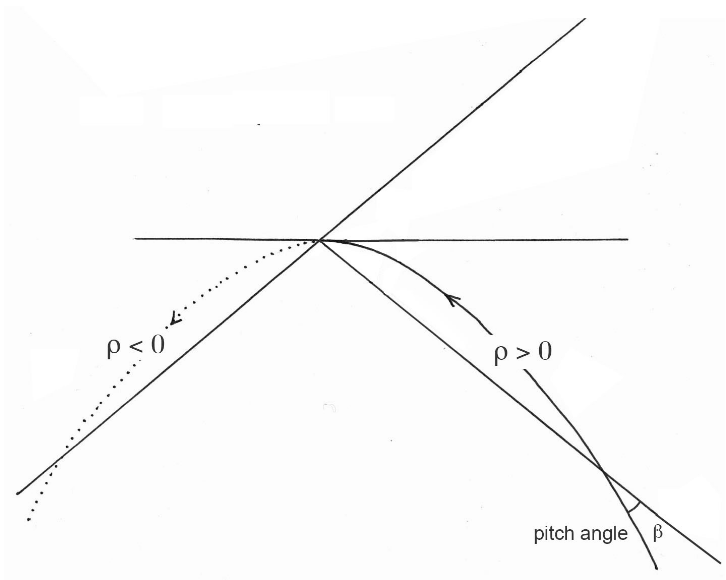

Fig.14 : Polar representation in a $\{\rho, \varphi\}$ frame.
The continuity of the geodesic is ensured

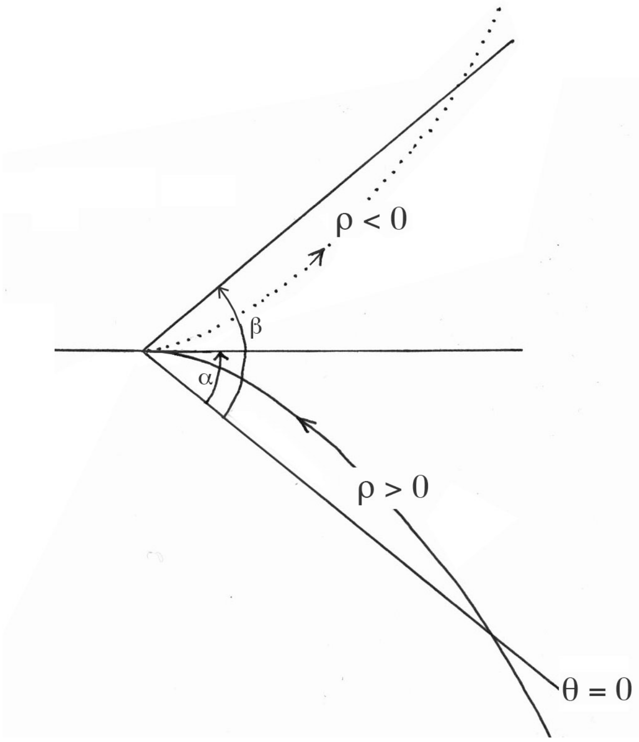

Fig.15 : With $\rho^{(-)} = -\rho^{(+)}$ (enantiomorphy)

Keep in mind that the proper time s is the only physical quantity. $t^{(+)}$ and $t^{(-)}$ are just time-markers. Their inversion, through the throat sphere kust just shows that the energy and the masse are reversed.

Time and mass-inversions

In classical theory of quantum field the energy-inversion is avoided by a ad hoc choice of the T-inversion operator, which is arbitraly defined as anti-linear and anti-unitary. We quote reference [4] page 76 :

> But for any state Ψ of energy $E > 0$, there would be another state $P^{-1}\Psi$ of energy $-E$. *There are no states of negative energy* (energy less than that of the vacuum) , so we are *forced* to *choose* the other alternative : P is *linear* and *unitary* and commutes than anticommutes with *H*. On the other hand, setting $\rho = 0$ in Eq.(2.6.6) yields $T\, i\, H\, T^{-1} = -\, i\, H$. If we supposed that T is linear and unitary we could simply cancel the is, and find
> $T\, H\, T^{-1} = -\, H$, with the again *disastrous conclusion* that for any state Ψ of energy E there is another state $T^{-1}\Psi$ of energy $-\, E$. *To avoid this, we are forced to conclude* that T is antilinear and antiunitary".

Page 104 Weinberg quotes :

> No example are known of particles that furnish unconventional representations of inversions, so these possibilities will not be prushued further here.

That was true until the discovery of the acceleration of the expansion of the universe, in 2011 ([5] , [6] , [7]), which implies a begative dark energy content . Recently N.Deberg showed [8] that the Dirac equation, if equipped with a unitary and unilinear time-inversion operator, ruled negative energy states.

The link between time-inversion and energy and mass inversion can be deduced from the theory of dynamic groups [9], « Inversions of space and time », equation (14.67), if we consider the full Poincaré dynamic group. We quote :

$$(14.67)\qquad \begin{cases} \underline{I_s} : \mathbf{l}\to\mathbf{l}\ ;\quad \mathbf{g}\to-\mathbf{g}\ ;\quad \mathbf{p}\to-\mathbf{p}\ ;\quad E\to E \\ \underline{I_t} : \mathbf{l}\to\mathbf{l}\ ;\quad \mathbf{g}\to-\mathbf{g}\ ;\quad \mathbf{p}\to\mathbf{p}\ ;\quad E\to-E \end{cases}$$

The second mine refers to tiùe-inversion, that gives E-inversion.

A didactic image of time and space inversion processes.

Consider a two-dimension space (x^1,x^2), figured as a flat, equipped with a third dimension, corresponding to a chronological marker x°, the set forming Gaussian coordinates $\left\{ x^\circ, x^1, x^2 \right\}$. See figure 16.

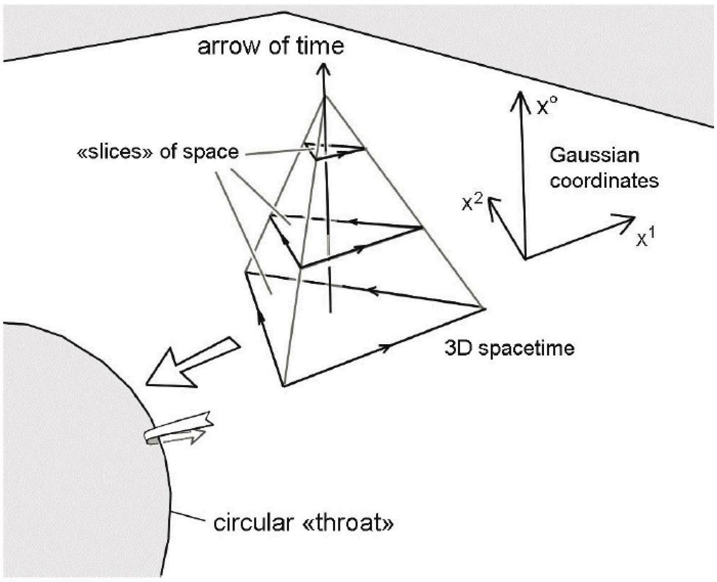

Fig.16 : Building a 3D spacetime

The schema evokes the one of Figure 6, i.e. a 2D space shapped into an orbifold structure with a circular frontier. On figure 17 the eversion of the x° coordinate is figured.

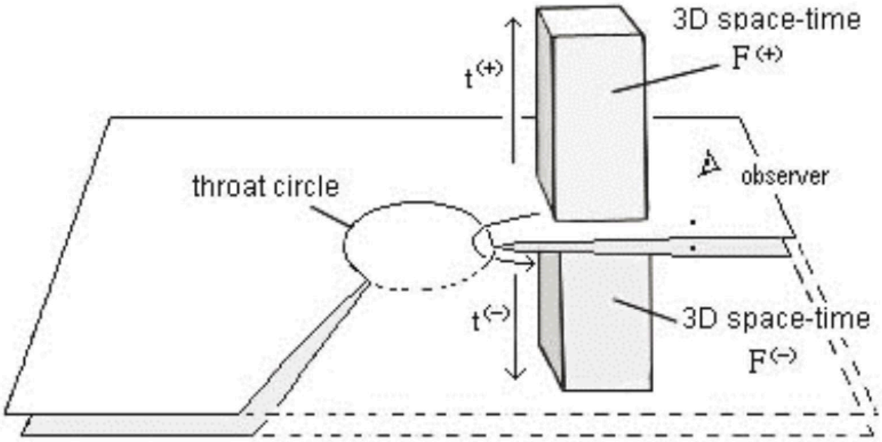

Fig.17 : Didactic image of the inversion of the time-coordinate through a throat circle

Link to the Janus Cosmological Model

In the Janus Cosmological Model we consider a M_4 manifold equipped with two metrics $g^{(+)}_{\mu\nu}$ and $g^{(-)}_{\mu\nu}$. If we add a space like fifth dimension we can write, using the space dimension ρ the corresponding metrics as :

(28)

$$ds^2 = \frac{\mathrm{Log}\,\rho^{(+)}}{1+\mathrm{Log}\,\rho^{(+)}} dt^{(+)2} - \frac{1+\mathrm{Log}\,\rho^{(+)}}{\mathrm{Log}\,\rho^{(+)}} R_s^{\,2} \mathrm{th}^2 \rho^{(+)} d\rho^{(+)2} - R_s^{\,2}(1+\mathrm{Log}\rho^{(+)})^2(d\theta^2 + \sin^2\theta d\varphi^2) - d\zeta^{(+)2}$$

(29)

$$ds^2 = \frac{\mathrm{Log}\left|\rho^{(-)}\right|}{1+\mathrm{Log}\left|\rho^{(-)}\right|} dt^{(-)2} - \frac{1+\mathrm{Log}\left|\rho^{(-)}\right|}{\mathrm{Log}\left|\rho^{(-)}\right|} R_s^{\,2} \mathrm{th}^2 \rho^{(-)} d\rho^{(-)2} - R_s^{\,2}(1+\mathrm{Log}\left|\rho^{(-)}\right|)^2(d\theta^2 + \sin^2\theta d\varphi^2) - d\zeta^{(-)2}$$

The determinant of the metrics is :

(30)

$$\det(g_{\mu\nu}) = \mathrm{th}^2\rho\, R_s^{\,6}(1+\mathrm{Log}\,\rho)^4 \sin^2\theta$$

Here again it vanishes on the throat sphere $\rho = 0$.

The Janus group is :

(31)

$$\begin{pmatrix} \mu\lambda & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} \qquad \text{with } \lambda = \pm 1 \qquad \mu = \pm 1$$

L_o is the restricted Lorentz group (orthochron sus-group). ζ is a fifth dimension.

$$\xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \qquad C = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

$L_a = - L_o$ in the antichron set. So that λL_o is the complete Lorentz group. The Lorentz groupe is defined by :

(32)

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad {}^tL\,G\,L = G$$

Following J.M.Souriau we consider the Lie algebra of the group:

(32)

$$\delta g = \begin{pmatrix} 0 & 0 & \delta\phi \\ 0 & \lambda\delta L_o & \delta C \\ 0 & 0 & 0 \end{pmatrix}$$

and we form the coadjoint action of the group on its Lie algebra :

(33) $$\delta g' = g^{-1} \delta g \, g$$

We add a simple translation ϕ in the fifth dimension. From Noether's theorem this add a component to the momentum which is a scalar q, identified to the electric charge. The detail of the calculation are given in te annex. Detailed calculation is given in the annex and the result is :

(34) $$q' = \lambda \mu \, q$$

(35) $$M' = L_o M \, {}^tL_o + \lambda C \, {}^tP \, {}^tL_o - \lambda \, L_o P \, {}^tC$$

(36) $$P' = \lambda L_o P$$

with

(37)

$$P = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

If we shift from the Schwarzschild's metric to the metrics (22), (23) to the 5D metrics (28), (29) and if the transfer from a fold to the other is ruled by the Janus' group (31) then the transfer from the positive side to the negative one goes with

$$\lambda = -1 \quad \text{and} \quad \mu = -1 \quad \rightarrow \quad E \rightarrow -E \quad \text{and} \qquad m \rightarrow -m \quad q \rightarrow q$$

According to that schema, when a mass is swallowed by a black hole its is inverted, not its charge. From (31) this goes with a CPT-symmetry : it is transformed into negative energy and negative mass matter.

A negative mass emits negative energy photons. So that such ~~antimatter~~ matter is invisible for an observer made of positive mass. It no longer interacts with positive mass matter and interacts only through gravitational force. As shown in [10] and [11] this negative mass material is repelled by ordinary matter and is just scattered away.

As presented in [10] the content of the two sides fits A.Sakharov's idea. If it is assumed that the rate of production of baryons fros quarks and antibaryons frome antiquarks are different in the two sides, the positive side and the negative one, then in ours side mutual annihilations produces positive energye photons, plus a small amount of positive mass matter and the equivalent (ratio 3/1) of remnant positive energy antiquarks. Symmetrical situation in te negative side. So that negative mass matter injected into that negative sector tends to a annihilate with its content, i.e. negative mass antimatter.

The fate of a neutron star

In february 1916 [16] just before he died Karl Schwarzschild published a second paper describing the geometry inside a sphere filled by constant density material. This work was later rewrited and published in english. Schwarzschild was the first to show that when the mass of such sphere reaches a critical value the pressure tends to infinite at its center. We quote in german first :

Im Kugelmittelpunkt ($\chi = 0$) werden Lichtgeschwindigkeit und Druck unendlich, sobald $\cos \chi_a = 1/3$, die Fallgeschwindigkeit gleich $\sqrt{8/9}$ der (natürlich gemessenen) Lichtgeschwindigkeit geworden ist. Es

434 Sitzung der phys.-math. Klasse v. 23. März 1916. — Mitt. v. 24. Februar

ist damit eine Grenze der Konzentration gegeben, über die hinaus eine Kugel inkompressibler Flüssigkeit nicht existieren kann. Wollte man

and in english (translated by S. Antocci in 1999) :

At the center of the sphere ($\chi = 0$) velocity of light and pressure become infinite when $\cos\chi_a = 1/3$, and the fall velocity becomes $\sqrt{8/9}$ of the (naturally measured) velocity of light. Hence there is a limit to the concentration, above which a sphere of incompressible fluid can not exist. If one would apply our equations to values $\cos\chi_a < 1/3$, one would get discontinuities already outside the center of the sphere.

A pressure is a density of energy. When the neutron star gathers matter, due to the emission of solar wind from a companion star, when its mass reaches criticity we may conjecture that such energy concentration could modify the local topology and open a space bridge at its center, making possible to evacuate such excess of mass, introducing a stabilization process, keeping the mass of the star close to a critical value (around 3 solar masses).

Within the neutron star the internal metric is, in the positive fold :

(38)

$$ds^2 = \left[\frac{3}{2}\sqrt{1 - \frac{r_o^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1 - \frac{r^2}{\hat{R}^2}} \right]^2 c^2\, dt^2 - \frac{dr^2}{1 - \frac{r^2}{\hat{R}^2}}$$

$$- r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2) \quad for\ r < r_o \quad with \quad \hat{R}^2 = \frac{3\, c^2}{8\pi G \rho}$$

In the negative fold :

(39)

$$ds^2 = \left[\frac{3}{2}\sqrt{1+\frac{R_s^2}{\hat{R}^2}} - \frac{1}{2}\sqrt{1+\frac{R^2}{\hat{R}^2}} \right] dt^2 - \frac{dR^2}{1+\frac{R^2}{\hat{R}^2}} - R^2(d\theta^2 + \sin^2\theta \, d\varphi^2)$$

Conclusion :

We have built an alternative interpretation of the Schwarschild metric in a context of a different topology, through a new coordinate system, which goes with a space bridge linking two Minkowski spaces. This links to our Janus Cosmological Model. After this scenario matter swallowed by black holes is transformed into negative mass, which no longer interact with ordinary matter and can be dispersed in space.

References :

[1] K. Schwarzschild : Über das Gravitationsfeld Messenpunktes nach der Einsteinschen Theorie. Sit. Deut. Akad. Wiss. 1916 .
http://januscosmologicalmodel.com/pdf/1916-Schwarzschild-exterior-de.pdf

English translation by S.Antoci and A.Loinger
http://januscosmologicalmodel.com/pdf/1916-Schwarzschild-exterior-en.pdf

[2] A.Einstein : Sitzubgsberichte der Köninglich Preussischen Akademie der Wissenschaften zu Berlin, Phys.-Math Klasse 1915
http://januscosmologicalmodel.com/pdf/1915-Einstein-25-nov-de.pdf
English translation :
http://januscosmologicalmodel.com/pdf/1915-Einstein-25-nov-en.pdf

[3] Die Grundlangen der Physik. (Zwerte Mitteilung) in Nachtchriten von Köniniglichen Gesellschaft zu Göttingen. Math-Phys. Klasse 1917 p.53-76. Presented in the Session of 23 december 1916.
http://januscosmologicalmodel.com/pdf/1916-Hilbert-de.pdf
English translation : The Foundations of Physics (second communication) at
http://januscosmologicalmodel.com/pdf/1916-Hilbert-en.pdf

[4] S.Weinberg : The Quantum Theory of Fields. Vol.1 Foundations. Cambridge University Press 2005

[5] Riess, A. G., et al. 1998, ApJ, 116, 1009

[6] Perlmutter, S., et al. 1999, ApJ, 517, 565

[7] Riess A. G. 2000, PASP, 112, 1284

[8] N.Debergh, J.P.Petit and G.D'Agostini : Evidence of negative energies and masses in the Dirac equation through a unitary time-reversal operator. , J. Phys. Comm. **2** (2018) 115012

[9] J.M. Souriau : Structure of Dynamical Systems. Birkhaüser Ed. Berlin 1997 english translation of J.M.Souriau , Structure des Systèmes Dynamiques, Dunod Ed. 1970.

[10] J.P.Petit, G.D'Agostini : Negative Mass hypothesis in cosmology and the nature of dark energy. Astrophysics And Space Sccience,. *A* **29**, 145-182 (2014)

[11] J.P.Petit, G.D'Agostini : Cosmological Bimetric model with interacting positive and negative masses and two different speeds of light, in agreement with the observed acceleration of the Universe. Modern Physics Letters
A, Vol.29 ; N° 34, 2014 ; Nov 10th DOI :10.1142/So21773231450182X

[12] J.P.Petit & G.D'Agostini : Lagrangian derivation of the two coupled field equations in the Janus Cosmological Model. Astrophysics and Space Science 2015, 357 :67
Doi : 10.1007/s10509-015-2250-6

[13] J.P.Petit & G.D'Agostini : Cancellation of the singularity of the Schwarzschild solution with natural mass inversion process. Mod. Phys. Lett. A vol. 30 n°9
2015Doi:10.1142/S0217732315500510

[14] J.P.Petit : A Symplectic Cosmological Model. Progress in Physics, Vol. 14, jan. 2018
http://www.ptep-online.com/2018/PP-52-09.PDF

[15] J.P.Petit : Janus Cosmological Model and the fluctuations of CMB.
Progress in Physics Vol.14 Issue4(october)
http://www.ptep-online.com/2018/PP-55-11.PDF

[16] K.Schwarzschild : Über das Gravitationsfeld einer Kugel aus inkompressibler Flüssegkeit nach der Einsteinschen Theorie. Sittzungder phys.-math. Klasse v.23 März 1916-Mitt.v.24 Februar
http://januscosmologicalmodel.com/pdf/1916-Schwarzschild-interior-de.pdf

English translation (1999) by S. Antoci : On the gravitational field of a sphere of incompressible fluid according to Einstein's theory. 24 Fev. 1916
arXiv :phys/9912033v1/(phys.hist-ph)
http://januscosmologicalmodel.com/pdf/1916-Schwarzschild-interior-en.pdf

[17] A.A Grib and Yu. Pavlov Particles with negative energies in black holes.
Teor. Mat. Fiz. 190 , n°2, 312-324 (2017) arXiv : 1601.02592 (gr-qc)
https://arxiv.org/pdf/1008.3657.pdf

Annex :

The element of the Lie algebra is :
(40)

$$\begin{pmatrix} 0 & 0 & \varpi \\ 0 & \Lambda & \lambda \\ 0 & 0 & 0 \end{pmatrix}$$

The derivation is performed around the neutral element, so that ${}^t\Lambda = -\Lambda$
ω being an antisymmetrical matrix we can write this element :
(41)

$$\begin{pmatrix} 0 & 0 & \varpi \\ 0 & G\omega & \lambda \\ 0 & 0 & 0 \end{pmatrix}$$

To simplify the calculation let us use $L = \lambda L_o$. Then :
(42)

$$\begin{pmatrix} 0 & 0 & \varpi' \\ 0 & G\omega' & \gamma' \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \varpi \\ 0 & L^{-1}G\omega L & L^{-1}G\omega C + L^{-1}\gamma \\ 0 & 0 & 0 \end{pmatrix}$$

But $L^{-1} = G\,{}^tLG$, which gives $G\omega' = G\,{}^tLGG\omega L$. But $GG = I$. To sum up :

(43) $$\varpi' = \varpi$$

(44) $$\omega' = {}^tL\omega L$$

(45) $$\gamma' = G\,{}^tL\omega C + G\,{}^tLG\gamma$$

Let's figure the antisymmetric matrix μ as :
(46)

$$\mu = \begin{pmatrix} M & -P \\ {}^tP & 0 \end{pmatrix} \quad \text{with} \quad {}^tM = -M \quad \text{and} \quad P \in E_4$$

Write the duality relationship as :

(47) $$\frac{1}{2}T_r(M.\omega) + {}^tP.G\gamma = \frac{1}{2}T_r(M'.\omega') + {}^tP'.G\gamma'$$

(48) $$\frac{1}{2}T_r(M.\omega)+{}^tP.G\gamma=\frac{1}{2}T_r(M'.{}^tL\omega L)+{}^tP'.L\omega C+{}^tP\,{}^tLG\gamma$$

The identification on γ terms gives

(49) $${}^tP={}^tP'\,{}^tL \qquad \rightarrow \qquad P=LP'=\lambda L_o P$$

In the trace we can achieve a circular permutation

(50) $$T_r(M'\,{}^tL\omega L)=T_r({}^tLM'\,{}^tL\omega)$$

Identify the ω terms

(51) $$\frac{1}{2}T_r(M.\omega)=\frac{1}{2}T_r(LM'.{}^tL\omega)+{}^tP'\,{}^tL\omega C$$

The second term of the second member is equal to the product of a line-matrix by a column-matrix. This being equal to the reversed product. Hereafter, schematically, the product of a line-matrix by a column-matrix :

(52) $${}^tP'\,{}^tL\omega C=T_r({}^tL\omega C\,{}^tP')$$

In the trace one can achieve a circular permutation :

(53) $${}^tP'\,{}^tL\omega C=T_r(C\,{}^tP'\,{}^tL\omega)$$

Whence

(54) $$\frac{1}{2}T_r(M.\omega)=\frac{1}{2}T_r(LM'.{}^tL\omega)+T_r(C\,{}^tP'\,{}^tL\omega)$$

Keep in mind that ω is an antisymmetric matrix. We know that a matrixs trace is equal to the product of another matrix by a symmetrical matrix. Any matrix can ve symmetrized or antisymmetrized. In addition the trace of the product of a matrix by an ansymmetric is zero. Whence :

(55) $$T_r(A\omega)=T_r\left[\text{antisym}(A)\times\omega\right]$$

We can apply that to the matrix $C\,{}^tP\,{}^tL$ why we take the trace of its product by an antisymmetric matrix ω

(56) $$C\,{}^tP'\,{}^tL = \mathrm{sym}(C\,{}^tP\,{}^tL) + \mathrm{antisym}(C\,{}^tP'\,{}^tL)$$

But :

(57) $$T_r\left[\ \mathrm{sym}(C\,{}^tP'\,{}^tL)\times\omega\ \right] = 0$$

So that :

(58) $$T_r\left[\ C\,{}^tP\,{}^tL\,\omega\ \right] = T_r\left[\ \mathrm{antisym}(C\,{}^tP\,{}^tL)\times\omega\ \right]$$

(59) $$\mathrm{antisym}(C\,{}^tP'\,{}^tL) = \frac{1}{2}\left[\ C\,{}^tP'\,{}^tL + {}^t(C\,{}^tP'\,{}^tL)\ \right]$$

(60) $${}^t(C\,{}^tP'\,{}^tL) = {}^t({}^tP'\,{}^tL)\,{}^tC = L'\,{}^tC$$

(61) $$\mathrm{antisym}(C\,{}^tP'\,{}^tL) = \frac{1}{2}(C{}^tP'\,{}^tL - L\,P'\,{}^tC)$$

(62) $$T_r(C\,{}^tP'\,{}^tL) = \frac{1}{2}T_r(C{}^tP'\,{}^tL - L\,P'\,{}^tC)$$

Finally :

(63) $$M = L\,M'\,{}^tL + C{}^tP'\,{}^tL - L\,P'\,{}^tC$$

(64) $$P = L\,P'$$

With $L = \lambda L_o$ we can write :

(65) $$M' = L_o\,M\,{}^tL_o + \lambda C{}^tP\,{}^tL_o - \lambda L_o\,P\,{}^tC$$

(66) $$P' = \lambda L_o\,P$$

With (46), using the Poincaré group :

(67)
$$a = \begin{pmatrix} \lambda L_o & C \\ 0 & 1 \end{pmatrix}$$

we can write :

(68) $$\mu' = a\,\mu\,{}^{t}a$$

To sum up, considering the Janus group :
(69)
$$\begin{pmatrix} \mu\lambda & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{with } \lambda = \pm 1 \qquad \mu = \pm 1$$

such extension to a five dimensional space just adds the relationship :
(70)
$$q' = \lambda \mu q$$

The physical interpretation is the following :

Considering the movement of a positive mass particle, with electric charge q.

- The elements $(\lambda = 1, \mu = 1)$ give its the possible orthochronic movements. Notice that the set $\{L_o\}$ includes P-symmetry process.

- The elements $(\lambda = 1, \mu = -1)$ reverse both the fifth dimension ζ and the electric charge q. But they don't reverse time, mass and energy. Such movement correspond to classical antimatter. We shall call it « Dirac antimatter ».

- The elements $(\lambda = -1, \mu = -1)$ do not reverse the fifth dimension ζ and the electric charge q. But time, energy and mass are inversed. The correspond to the negative mass particles of the negative sector. They correspond to a CPT symmetry.

- The elements $(\lambda = -1, \mu = 1)$ reverse both the fifth dimension ζ and the electric charge q.In addition they achieve a PT symmetry. It correponds to negative mass antimatter. We shall call it « Feynman antimatter ».